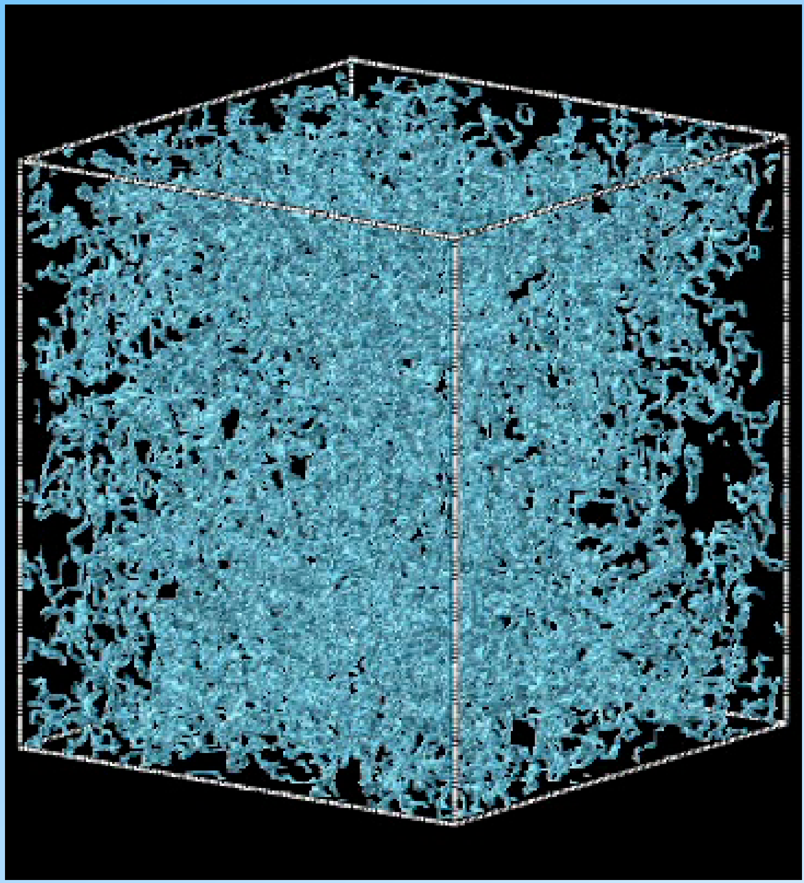
Quantum Hydrodynamics and Turbulence



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Review article

- Progress in Low Temperature Physics Vol.16, eds. W. P. Halperin and M. Tsubota, Elsevier, 2009
- M.T., K. Kasamatsu, arXiv:1202.1863
- M.T., K. Kasamatsu, M. Kobayashi, arXiv:1004.5458

What is “quantum”？

Element of something

What is “quantum mechanics”？

Mechanics with element

Energy, momentum and angular momentum *etc*. are quantized.

The element is determined by the Planck’s constant h.

What is “quantum turbulence”？

Turbulence with some “element”

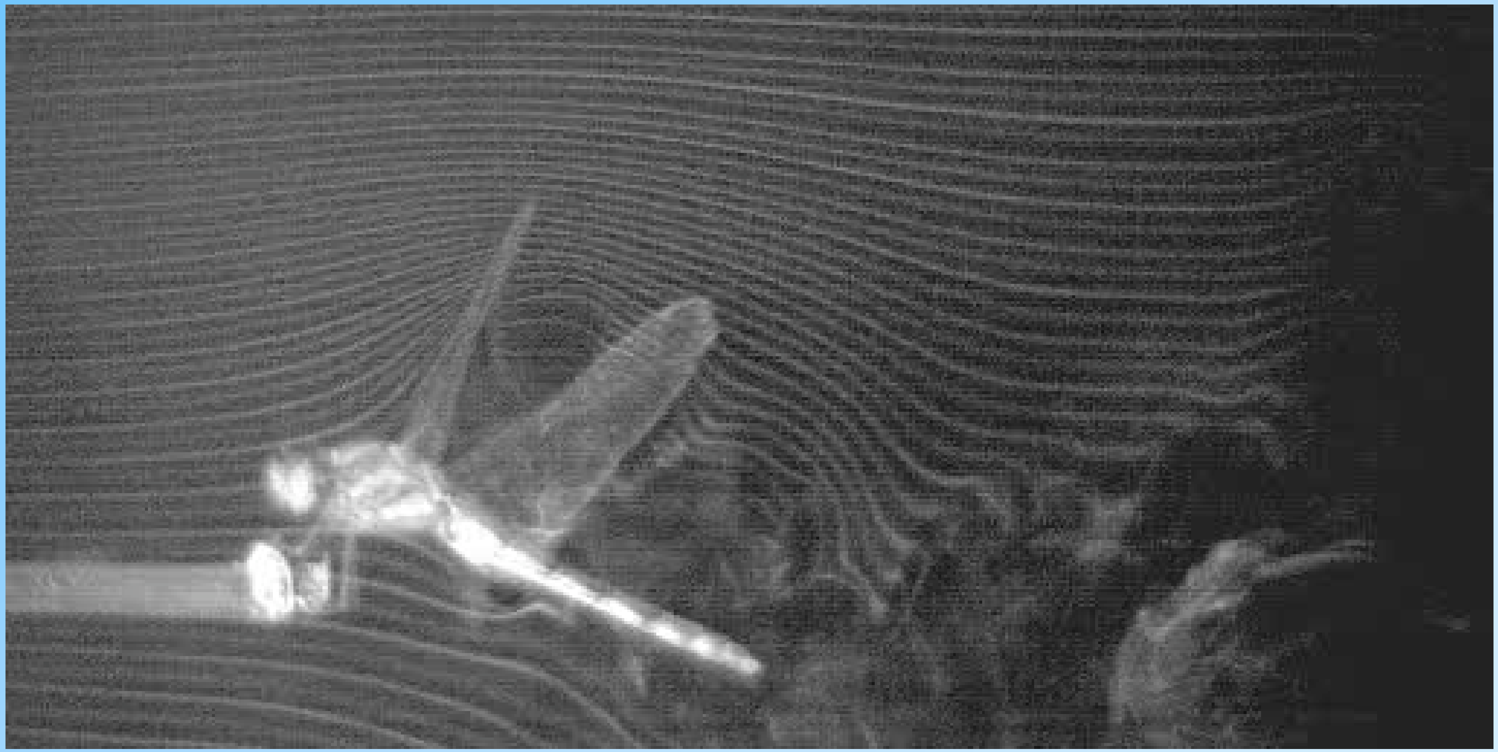
Leonardo Da Vinci
(1452-1519)

Da Vinci observed turbulent flow and found that turbulence consists of many vortices with different scales.

Turbulence is not a simple disordered state but having some structures with vortices.

Certainly turbulence looks to have many vortices.

Turbulence behind a dragonfly

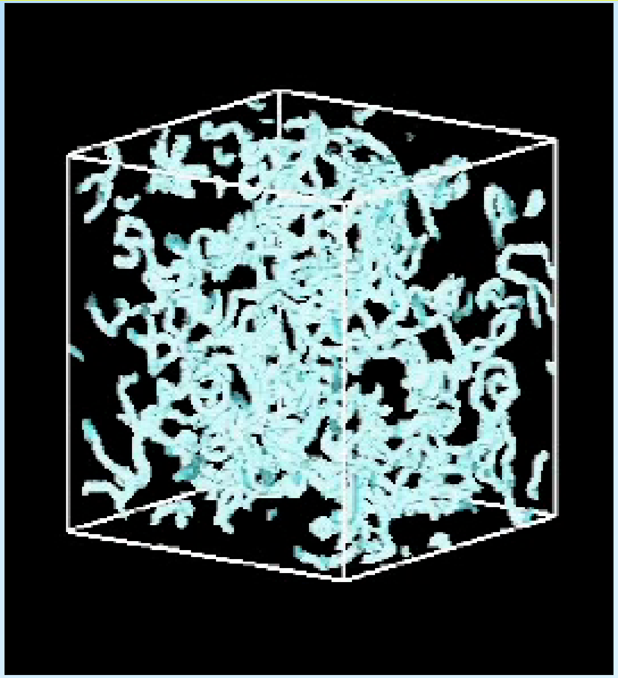


http://www.nagare.or.jp/mm/2004/gallery/iida/dragonfly.html

It is not so straightforward to confirm the Da Vinci message in classical turbulence.

Key concept

The Da Vinci message "*turbulence consists of vortices*" is actually realized in quantum turbulence (QT) comprised of quantized vortices.



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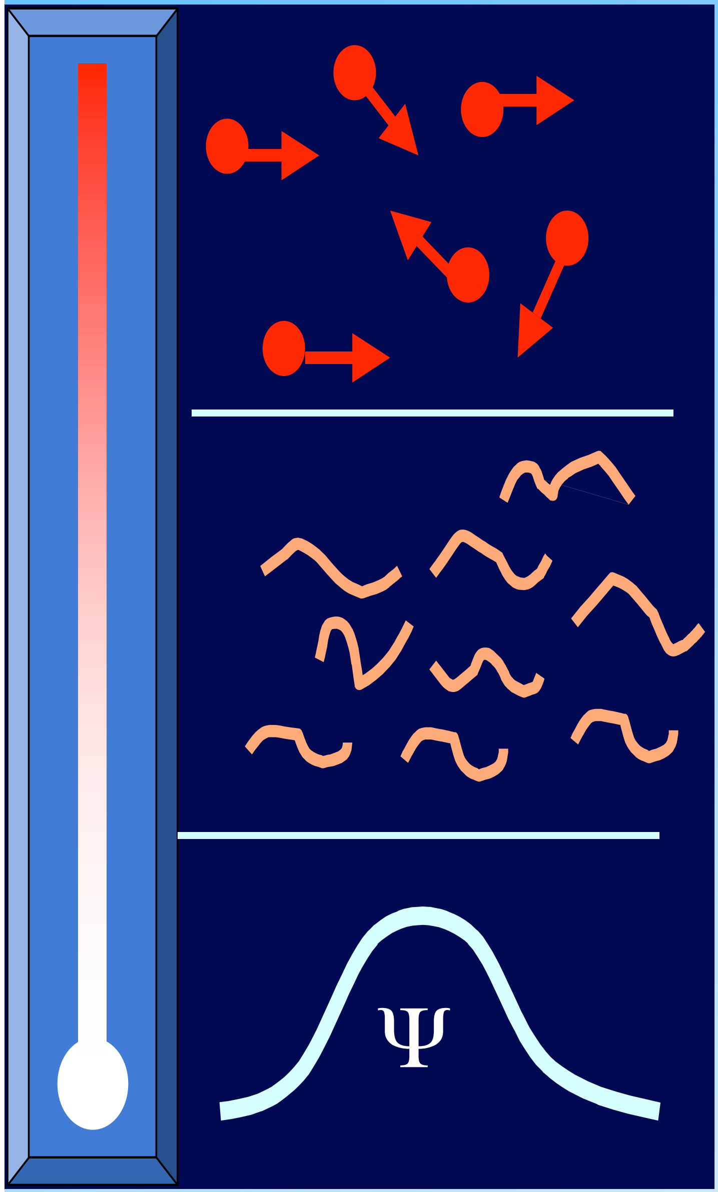
0. Introduction



Quantum mechanics
～ Duality of matter and wave ～

Each atom behaves as a particle at high temperatures.

Thermal de Broglie wave length
～ Distance between particles

Each atom behaves like a wave at low temperatures.

Bose-Einstein condensation (BEC)

Each atom occupies the same single particle ground state. The matter waves become coherent, making a macroscopic wave function Ψ .

Hydrodynamics of the system is described by the macroscopic wave function.

$$\Psi(\mathbf{r},t) = \sqrt{n_0(\mathbf{r},t)}\exp\left[i\theta(\mathbf{r},t)\right]$$

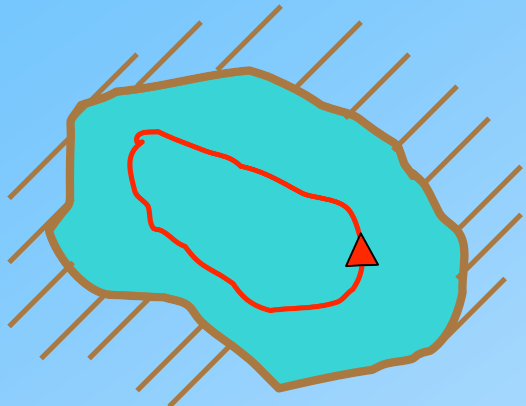
Condensate density $n_0(\mathbf{r},t)$

Superfluid velocity field $\mathbf{v}_s(\mathbf{r},t) \equiv \frac{\hbar}{m}\nabla\theta(\mathbf{r},t)$

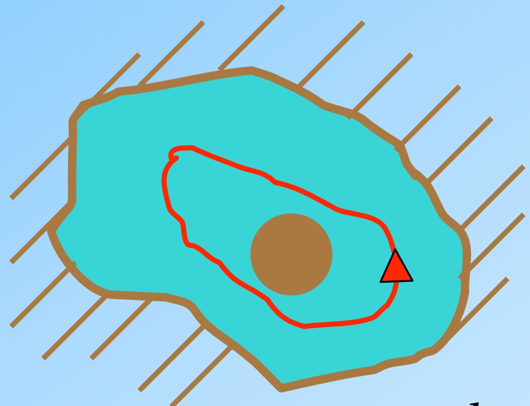
Quantization of circulation

Superfow $\mathbf{v}_s = \dfrac{\hbar}{m}\nabla\theta$

Single-connected region



Multi-connected region



Quantized circulation

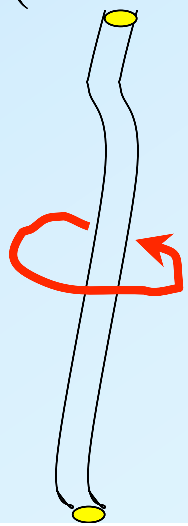
$\kappa = \dfrac{h}{m}$

$\Gamma = \oint_C \mathbf{v}_s \cdot d\mathbf{l} = 0,\ \ \mathrm{rot}\,\mathbf{v}_s = 0$

$\Gamma = \oint_C \mathbf{v}_s \cdot d\mathbf{l} = \dfrac{\hbar}{m}\oint_C \nabla\theta \cdot d\mathbf{l} = \dfrac{h}{m}n$ (n: integer)

A vortex with quantized circulation and vacant core

Quantized vortex



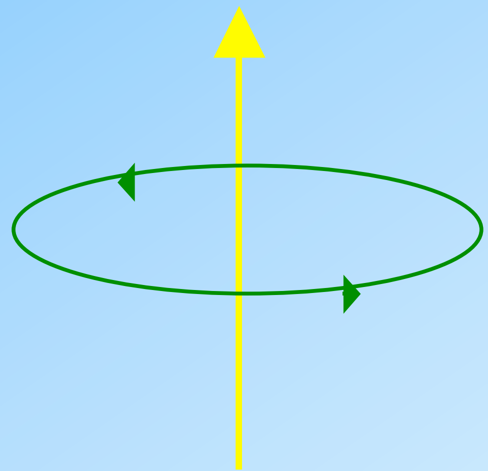
A quantized vortex is a vortex of superflow in a BEC. Any rotational motion in superfluid is sustained by quantized vortices.

(i) The circulation is quantized.

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n \qquad (n = 0, 1, 2, \cdots)$$

$$\kappa = h / m$$

A vortex with $n \geqq 2$ is unstable.



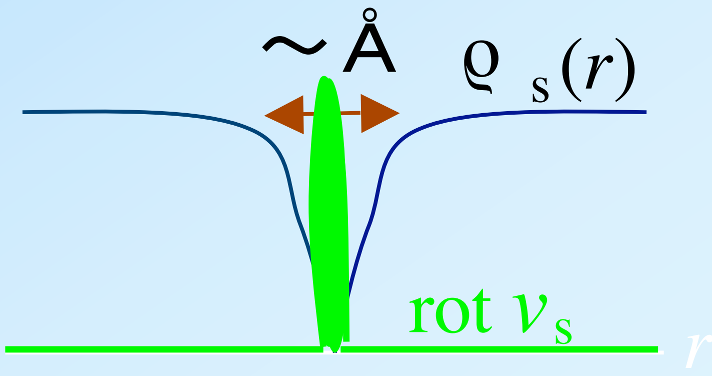
⇒ **Every vortex has the same circulation.**

(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

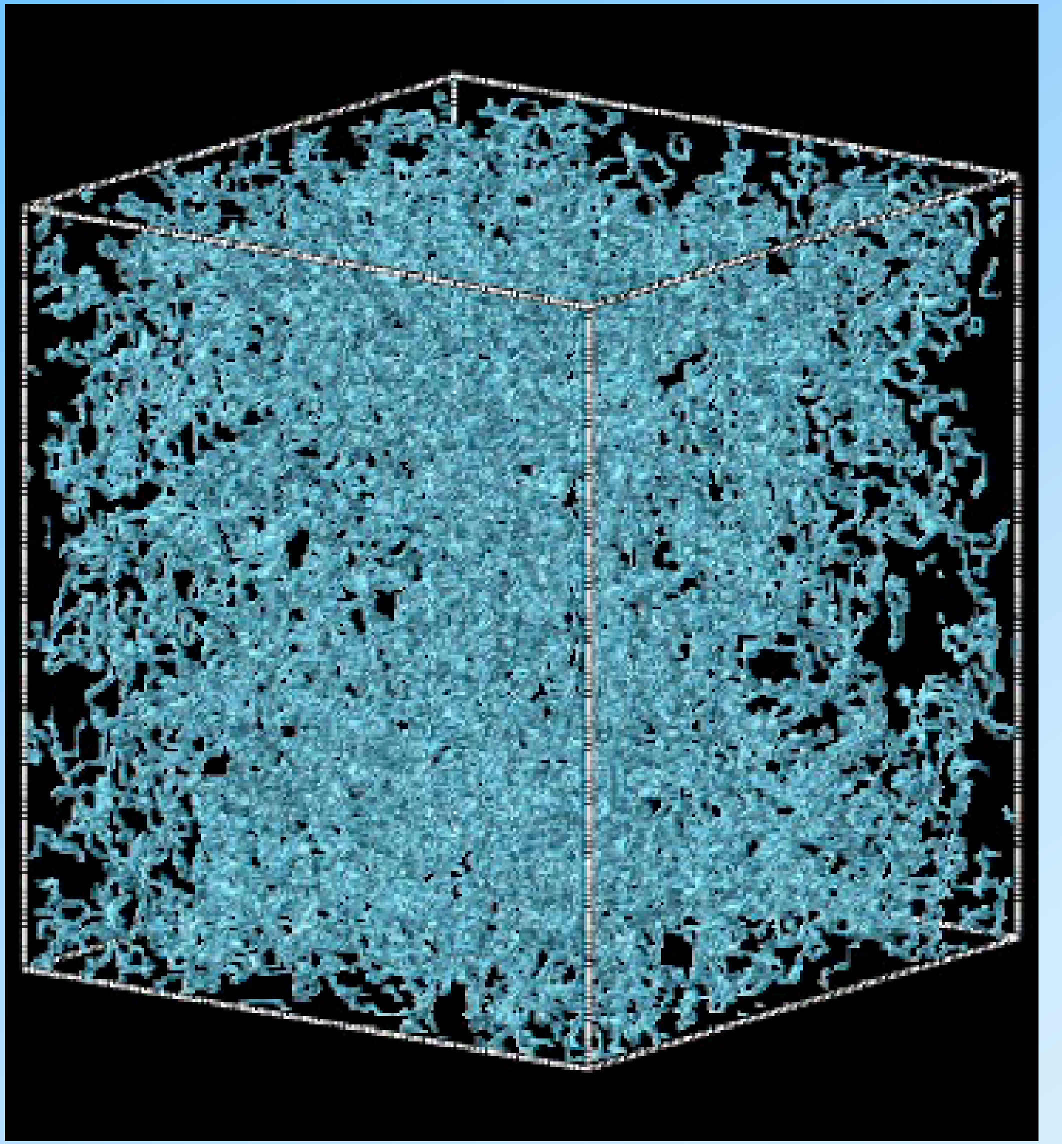
⇒ **The vortex is stable.**

(iii) The core size is very small.

⇒ **The order of the coherence length.**



Turbulence comprised of quantized vortices ——> Quantum turbulence (QT)

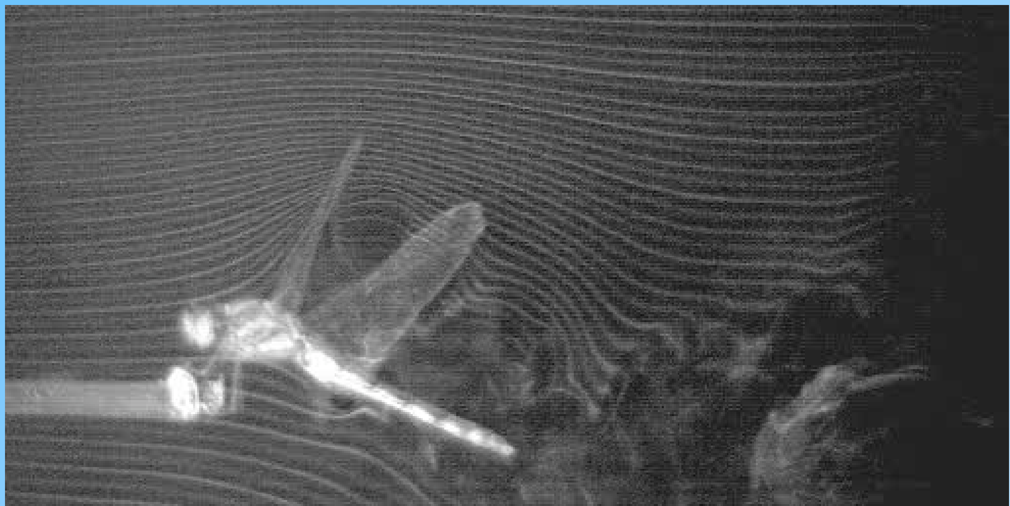


Each filament represents the core of quantized vortices.

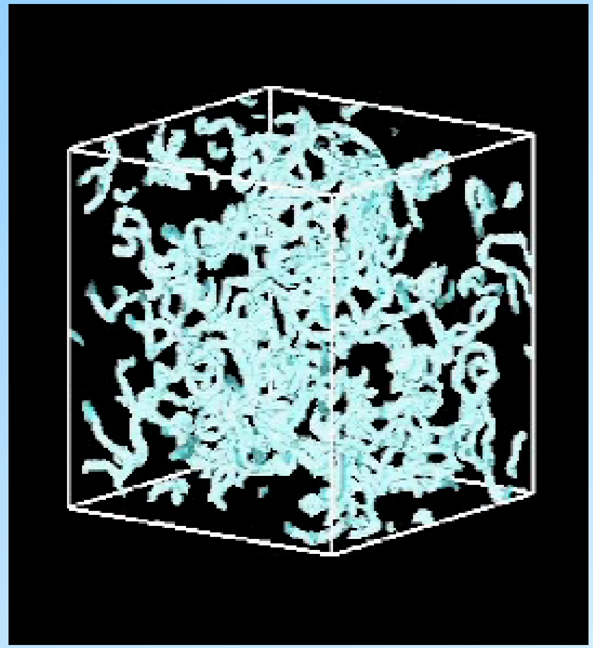
Simulation of the Gross-Pitaevskii model

Classical Turbulence (CT) vs. Quantum Turbulence (QT)

Classical turbulence



Quantum turbulence



Motion of vortex cores

QT can be much simpler than CT, because each element of turbulence is well-defined.

・The quantized vortices are stable topological defects.
・Every vortex has the same circulation.
・Circulation is conserved.

Models available for simulation of QT

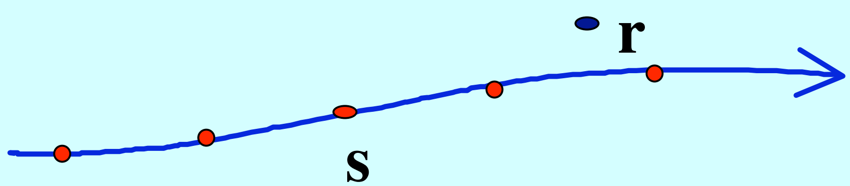
Gross-Pitaevskii (GP) model for the macroscopic wave function

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

$$i\hbar \frac{\partial \Psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 \right] \Psi(\mathbf{r},t)$$

Vortex filament model **Biot-Savart law**

$$\mathbf{v}_s(\mathbf{r}) = \frac{\kappa}{4\pi} \int \frac{(\mathbf{s}-\mathbf{r}) \times d\mathbf{s}}{|\mathbf{s}-\mathbf{r}|^3}$$



A vortex makes the superflow of the Biot-Savart law, and moves with this local flow.

- Brief Research History of QT -

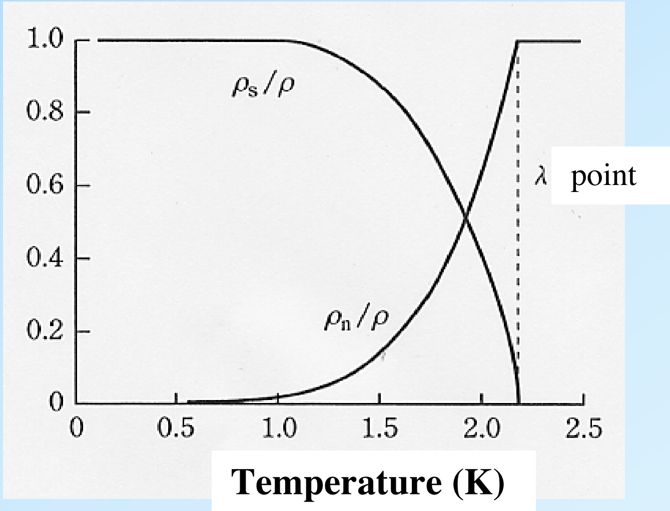
Liquid ^{4}He enters the superfluid state below 2.17 K (λ point) with Bose-Einstein condensation.

Its hydrodynamics are well described by the two-fluid model:

The two-fluid model (Tisza, Landau)

The system is a mixture of inviscid superfluid and viscous normal fluid.

$$\rho = \rho_s + \rho_n \qquad \mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$



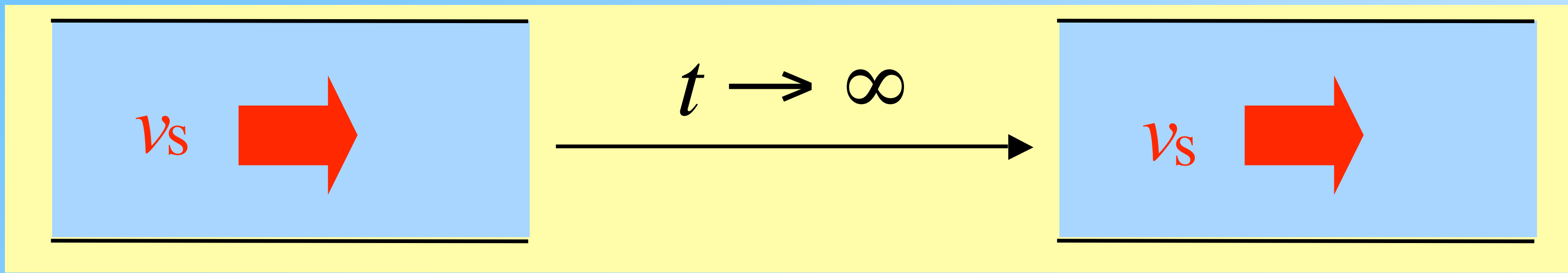
	Density	Velocity	Viscosity	Entropy
Superfluid	$\rho_s(T)$	$\mathbf{v}_s(\mathbf{r})$	0	0
Normal fluid	$\rho_n(T)$	$\mathbf{v}_n(\mathbf{r})$	$\eta_n(T)$	$s_n(T)$

The two-fluid model can explain various experimentally observed phenomena of superfluidity (e.g., the thermomechanical effect, film flow, etc.)

However, …

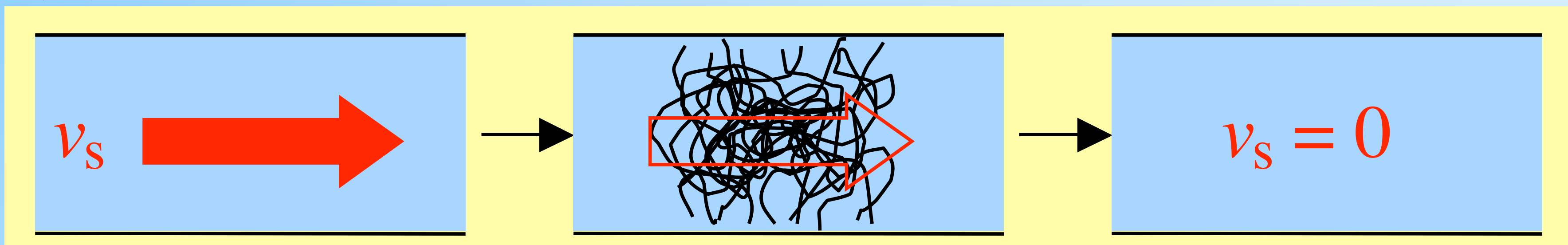
Superfluidity breaks down in fast flow

(i) $v_s < v_c$ (some critical velocity)



The two fluids do not interact so that the superfluid can flow forever without decaying.

(ii) $v_s > v_c$



A tangle of quantized vortices develops. The two fluids interact through mutual friction generated by tangling, and the superflow decays.

1955: **R. P. Feynman** proposed that “superfluid turbulence” consists of a tangle of quantized vortices.

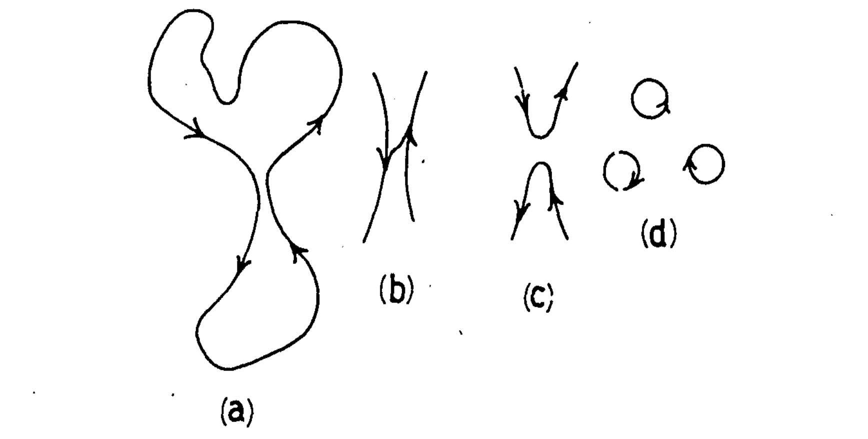
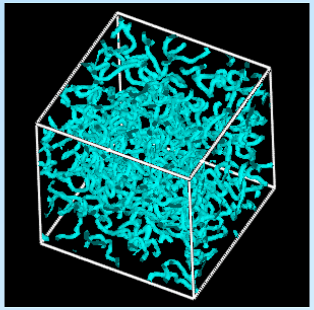


Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

Progress in Low Temperature Physics Vol. I (1955), p.17

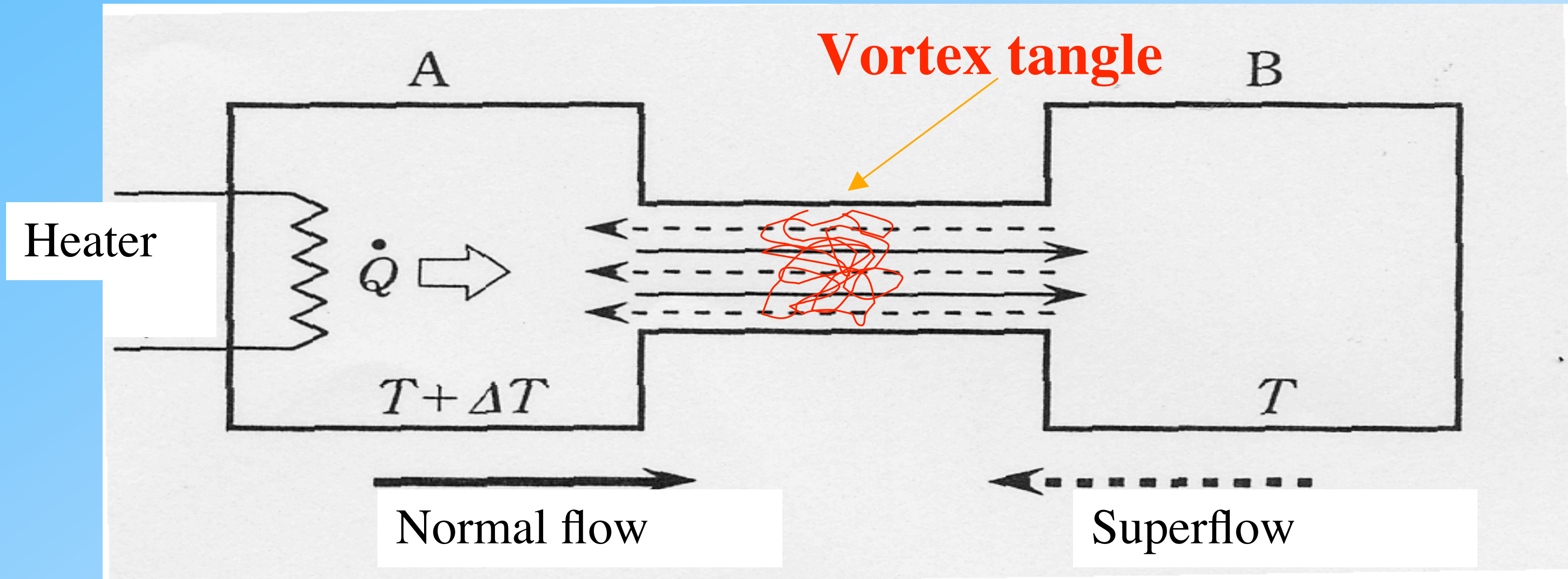
Such a large vortex should break up into smaller vortices like the cascade process in classical turbulence.



1955 – 1957: **W. F. Vinen** observed “superfluid turbulence”.

Mutual friction between the vortex tangle and the normal fluid causes dissipation of the flow.

Many experimental studies were conducted chiefly on thermal counterflow of superfluid ^{4}He.

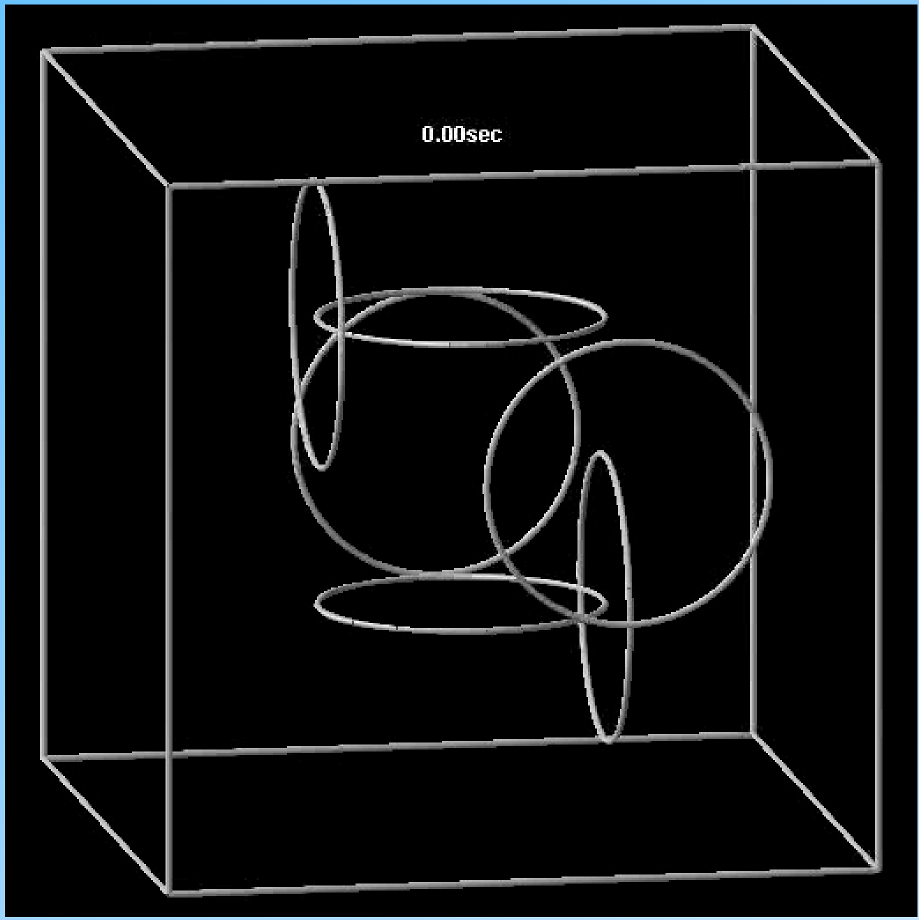


1980s **K. W. Schwarz** Phys. Rev. B38, 2398 (1988)

Performed a direct numerical simulation of the three-dimensional dynamics of quantized vortices and succeeded in quantitatively explaining the observed temperature difference ΔT .

Development of a vortex tangle in a thermal counterflow

Vortex filament model



K. W. Schwarz, Phys. Rev. B38, 2398 (1988).

Schwarz obtained numerically the statistically steady state of a vortex tangle, which is sustained by the competition between the applied flow and the mutual friction.

H. Adachi, S. Fujiyama, MT, Phys. Rev. B81, 104511(2010)(**Editors suggestion**)

We made more correct simulation by taking the full account of the vortex interaction.

V_s ↑ ↓ V_n

Counterflow turbulence has been successfully explained.

Most studies of superfluid tur
thermal counterflow.

⇨ **No analo**

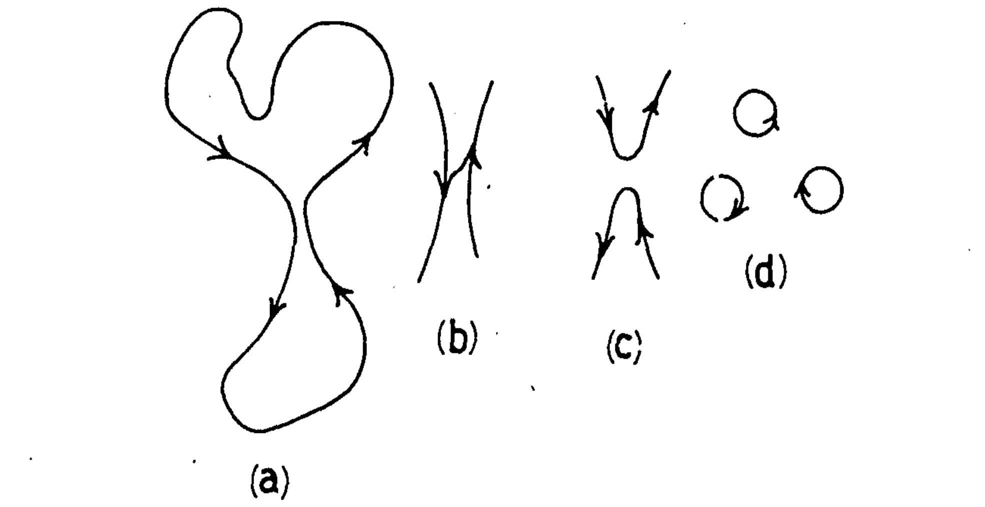


Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

When Feynman drew the above figure, he was thinking of a cascade process in classical turbulence.

What is the relation between superfluid turbulence and classical turbulence ?

New era of quantum turbulence has come!

1. Superfluid helium

   Classical analogue has been considered since 1998.

   ～Energy spectrum of QT, Visualization of QT… ～

2. Atomic Bose-Einstein condensates (BECs)

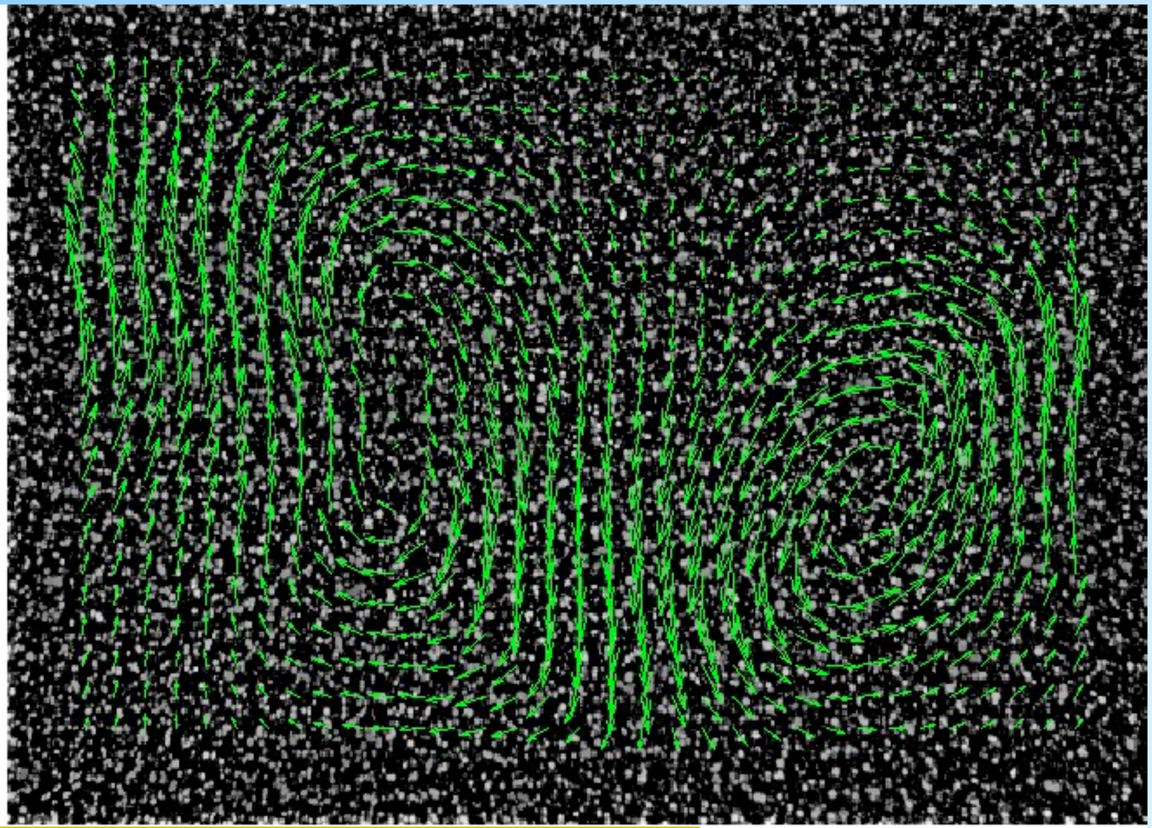
   BEC was realized in 1995.

2. Visualization of QT in superfluid ^{4}He: Coupled dynamics of quantized vortices and particles

Recently PIV(Particle Image Velocimetry) is applied to superfluid helium.

By seeding a fluid with fine tracer particles, we follow their motion by optical technique.

Visualization of the flow!

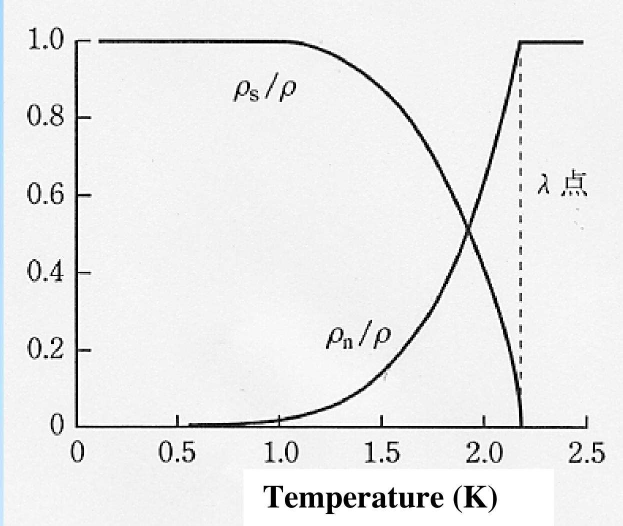


Basic assumption:
The particles should follow the velocity field.

However, it is not so clear whether PIV is available in superfluid helium.

Two-fluid model

	Density	Velocity	Viscosity	Entropy
Superfluid	$\rho_s(T)$	$\mathbf{v}_s(\mathbf{r})$	0	0
Normal fluid	$\rho_n(T)$	$\mathbf{v}_n(\mathbf{r})$	$\eta_n(T)$	$s_n(T)$



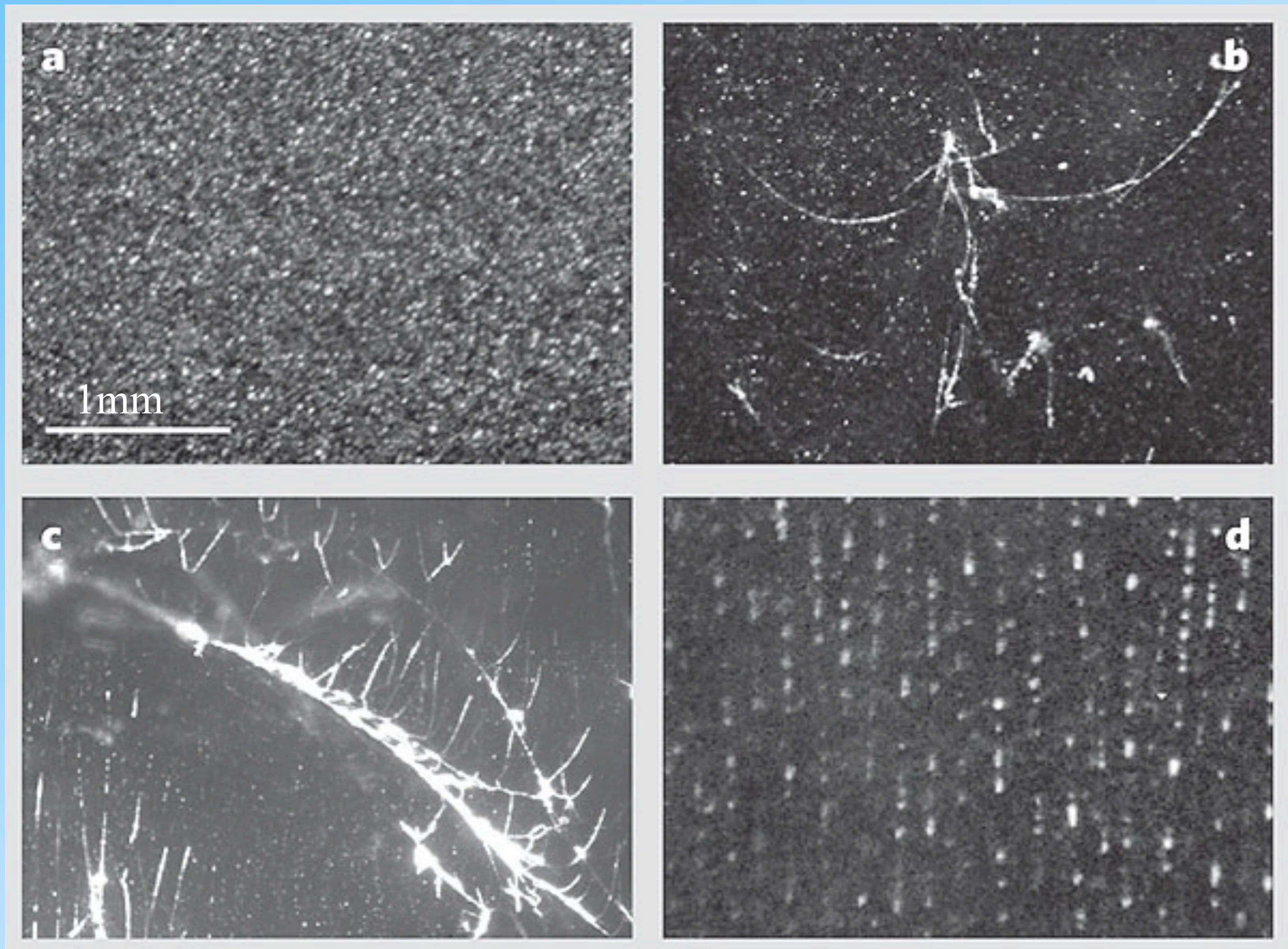
What do the tracer particles follow?

Superflow *No*　Normal flow *Yes*　Quantized vortices *?*

Visualization of quantized vortices by micron-sized solid hydrogen particles

G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, Nature 441, 588(2006)

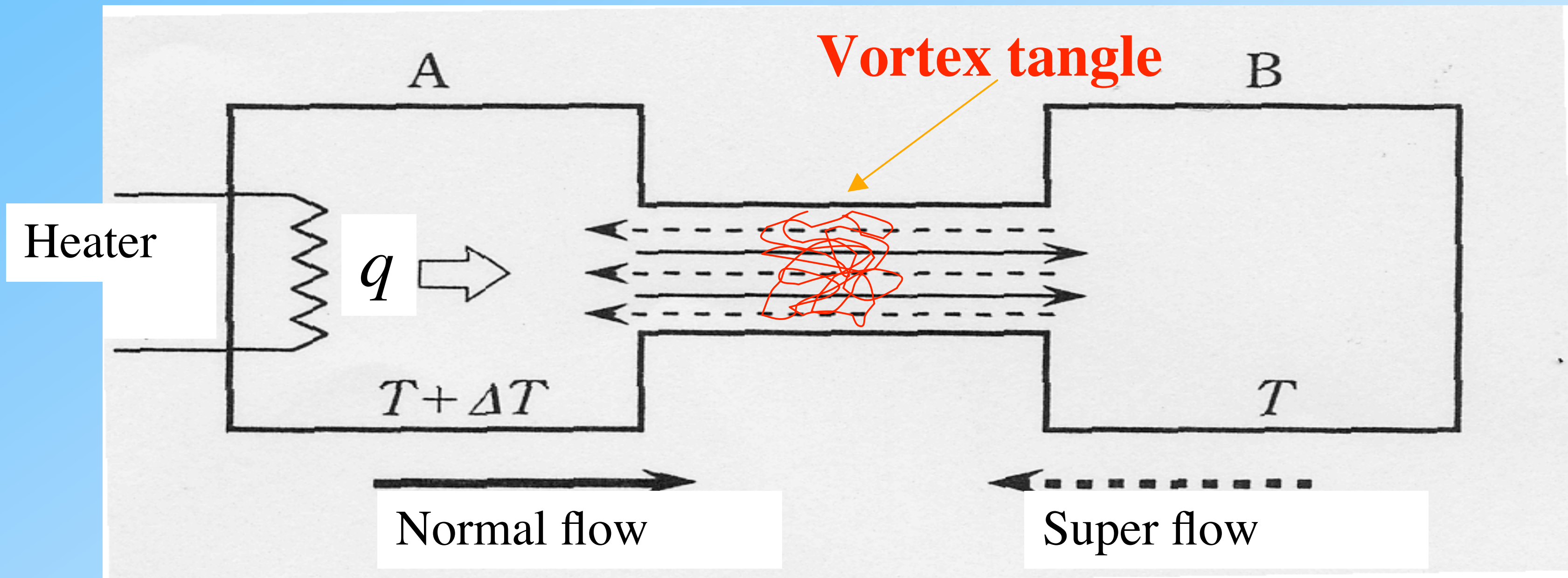
$T > T_\lambda$



$T < T_\lambda$

M. S. Paoletti, R. B. Fiorito, K. R. Sreenivasan, D. P. Lathrop
J. Phys. Soc. Jpn. 77, 111007(2008)

They succeeded in the visualization of thermal counterflow.



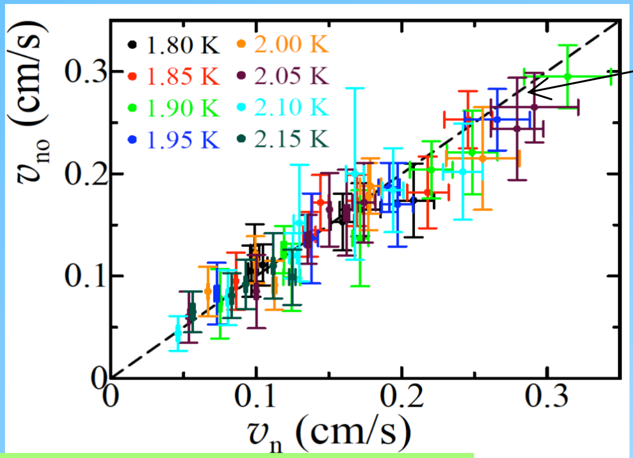
$$v_n = \frac{q}{\rho S T}$$

$$v_s = -\frac{\rho_n}{\rho_s} v_n$$

Observation of the velocity by the solid hydrogen particles in counterflow

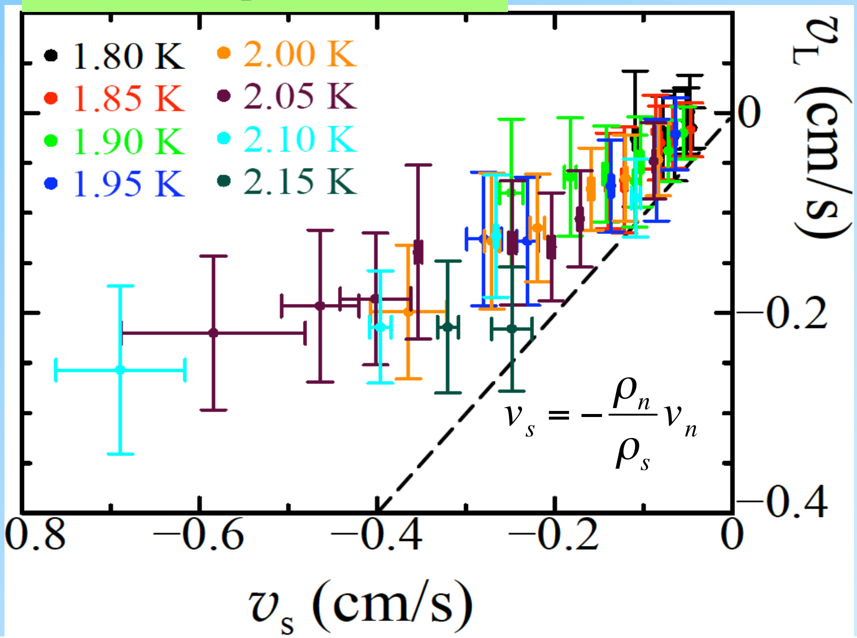
Paoletti,Fiorito,Sreenivasan,and Lathrop, J.Phys. Soc. Jpn. 77,111007(2008)

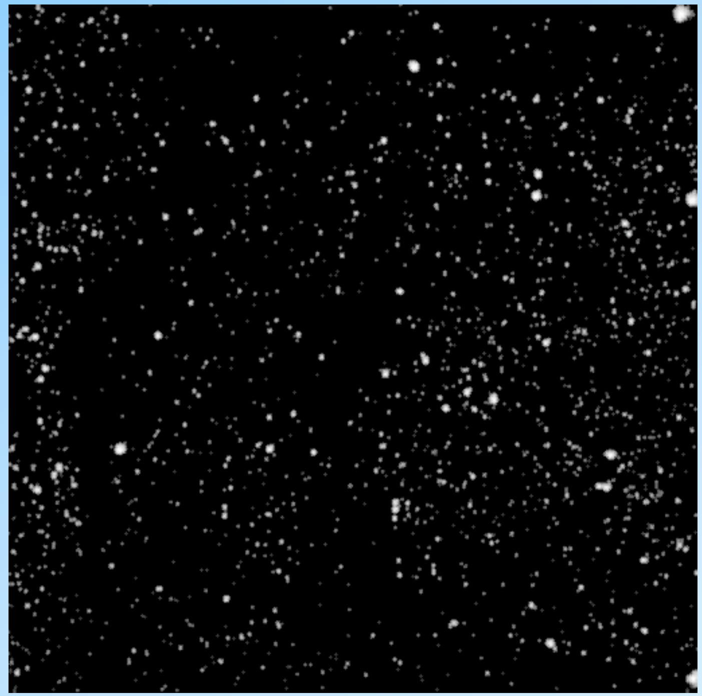
Upward particles

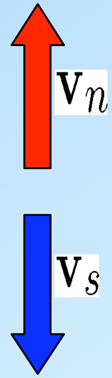


$$v_n = \frac{q}{\rho S T}$$

Downward particles





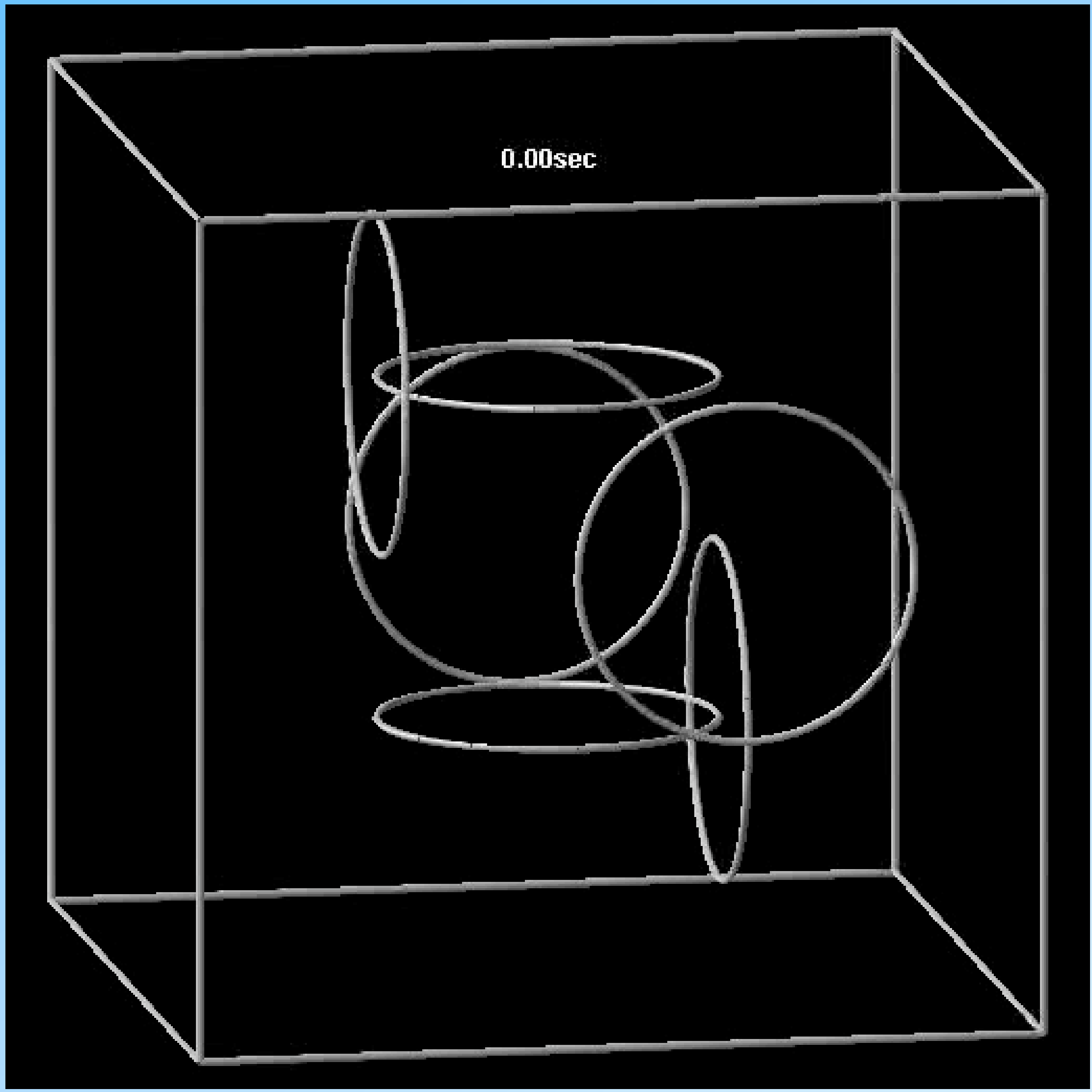


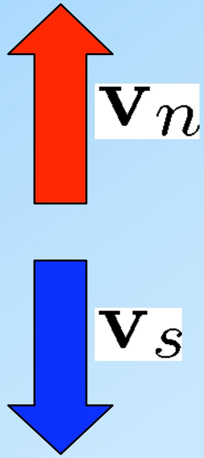
Upward particles just follow v_n.

Downward particles fluctuate, probably affected by quantized vortices. The velocity is smaller than v_s.

We succeeded for the first time in making a statistical steady QT in counterflow.

H. Adachi, S. Fujiyama, MT, PRB81, 104501(2010) (Editors' Suggestion)





BOX (0.1cm)^3

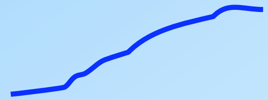
T = 1.6 K

Vns = 0.367cm/s

Periodic boundary conditions for all three directions

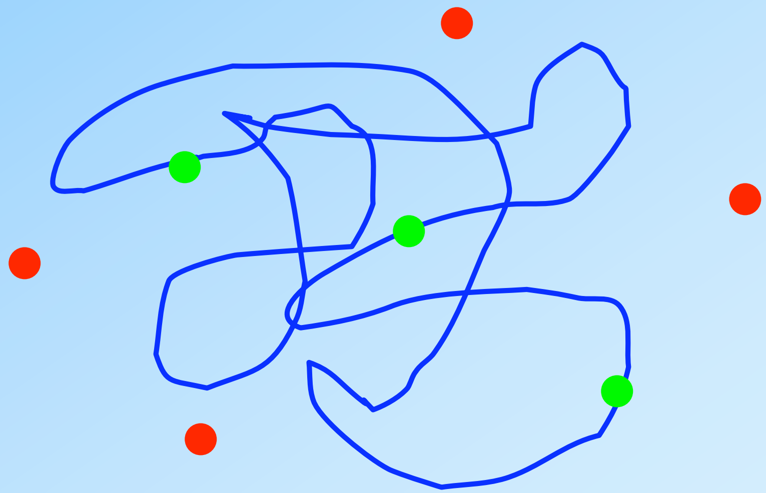
Coupled dynamics of vortices and particles

Players of the game

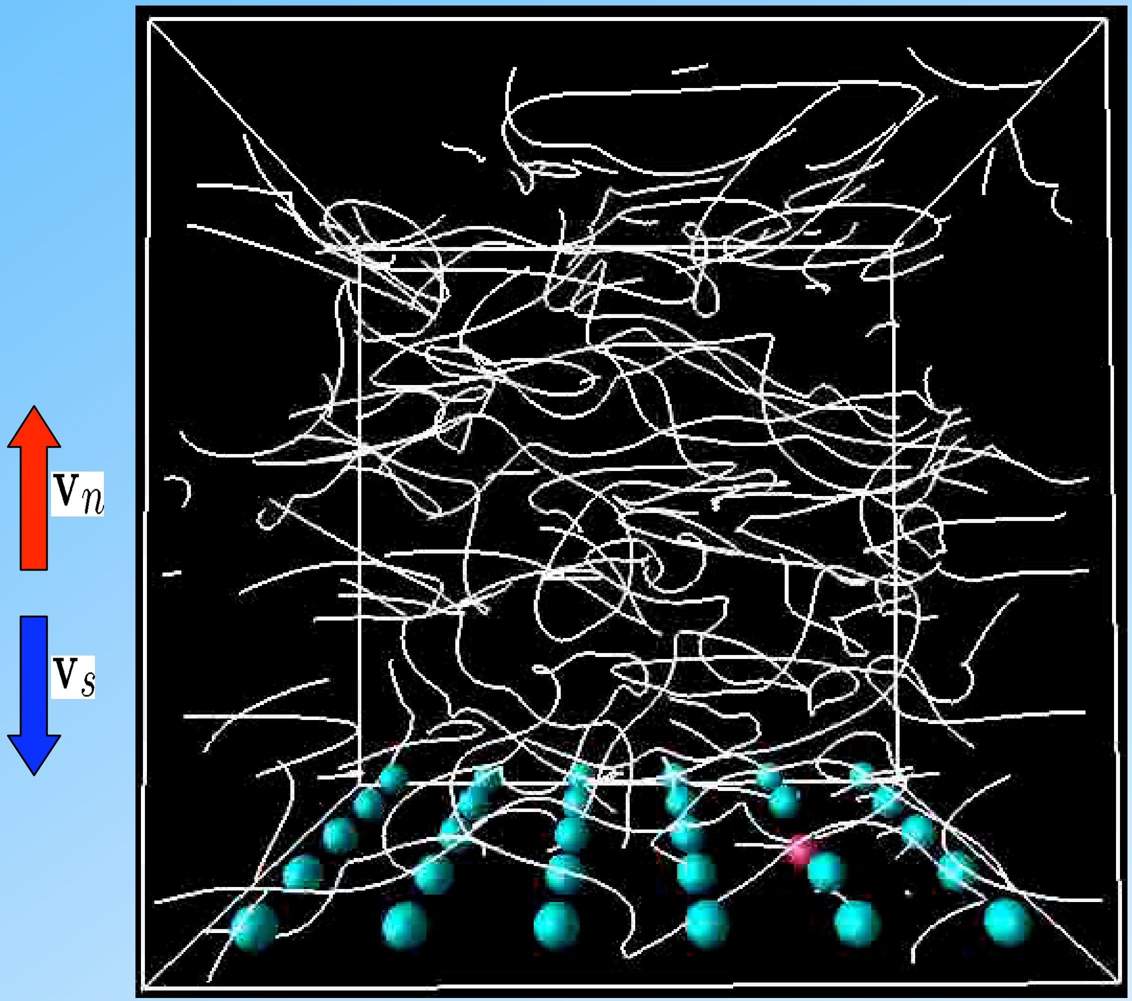
Quantized vortices by the vortex filament model 

Free particles

Particles trapped by vortices



Typical results of the coupled dynamics in thermal counterflow



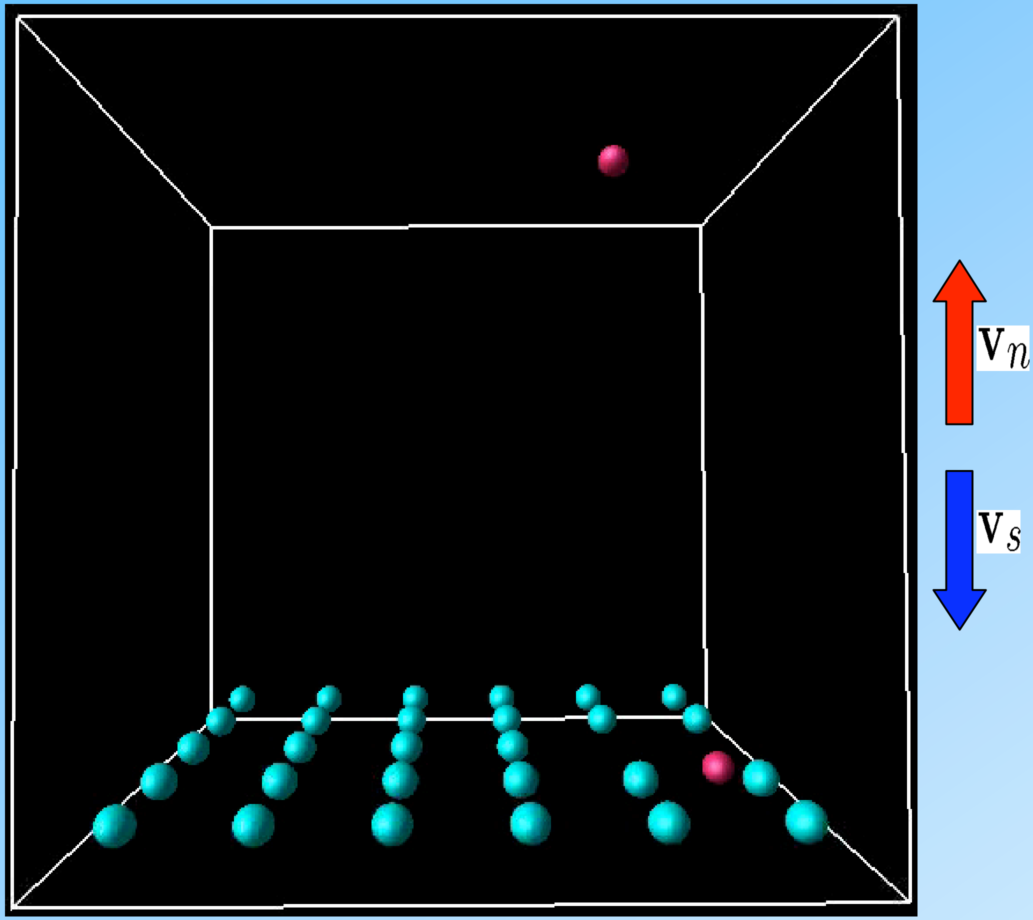
T=1.9K

v_n=0.3cm/s, v_{ns}=0.52cm/s

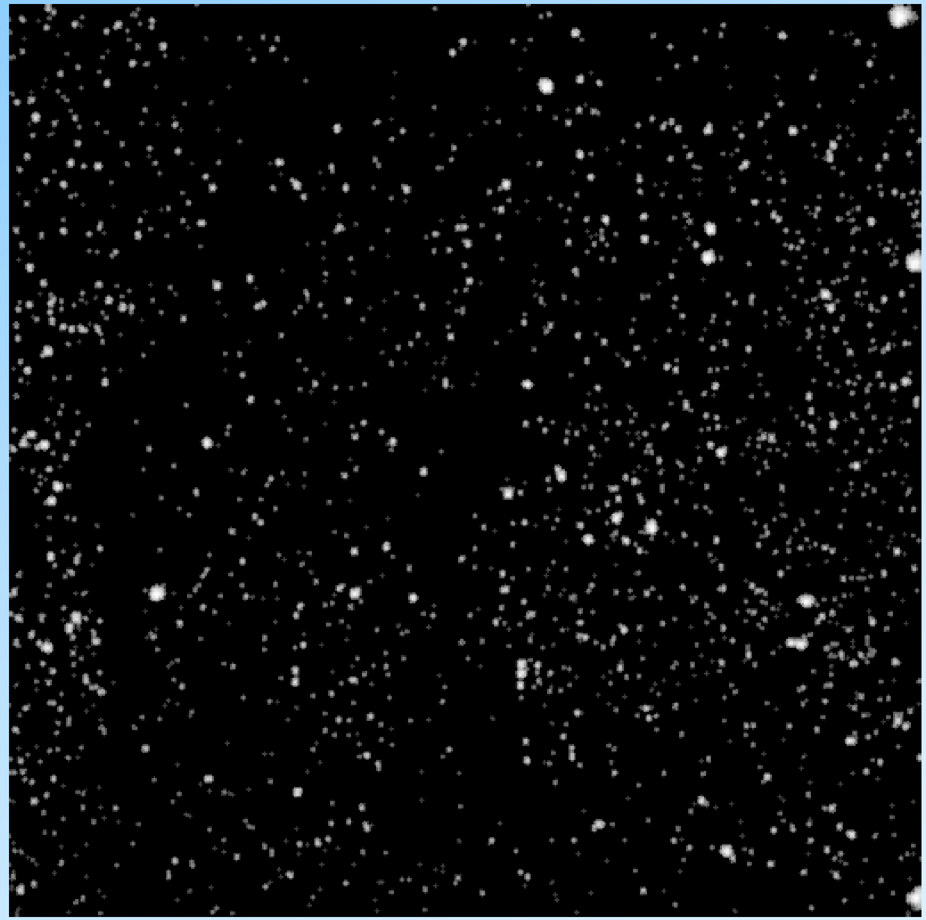
Box size: (1.0 mm)^3

Free particles

Trapped particles



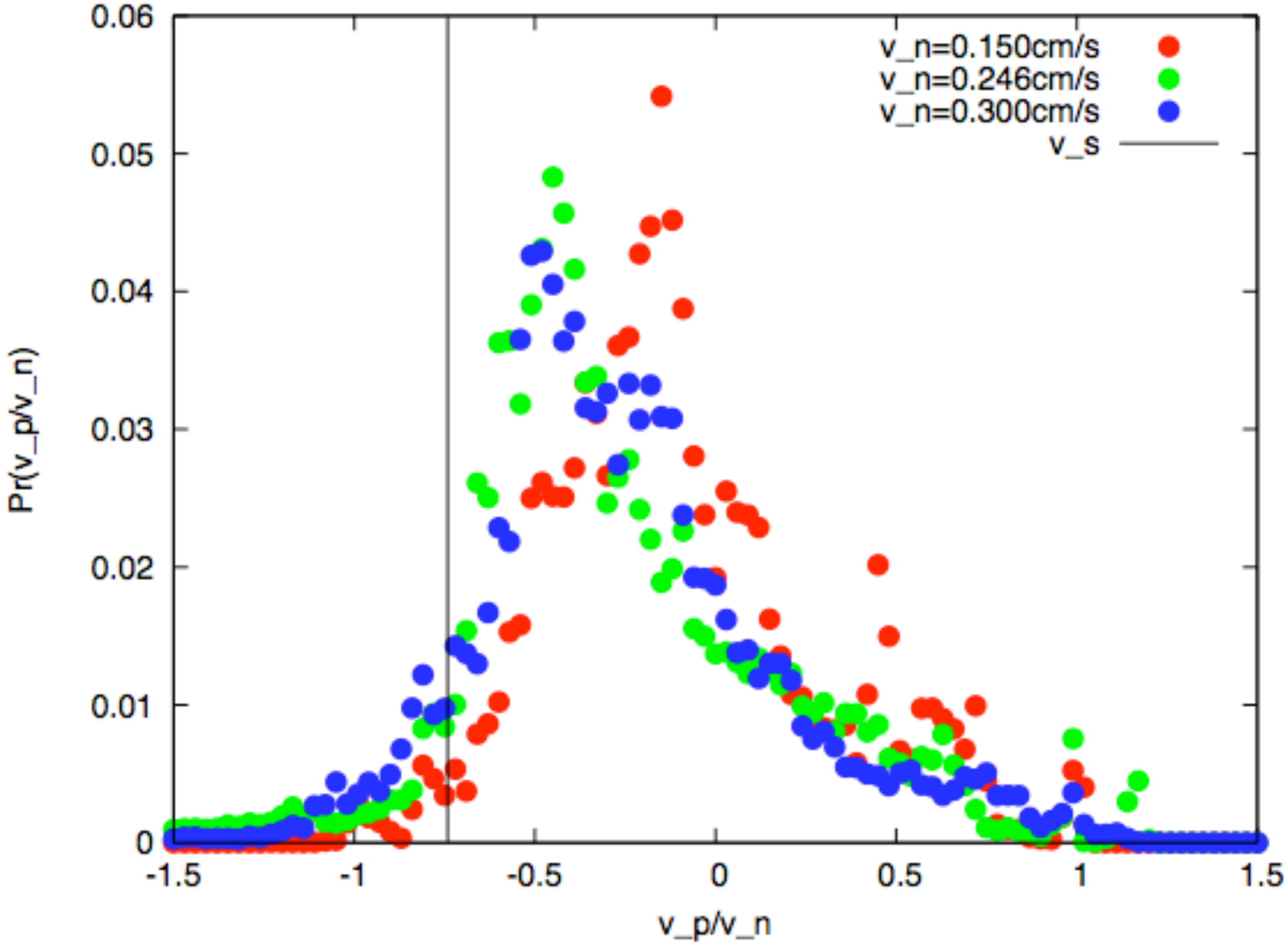
The movie erasing vortices

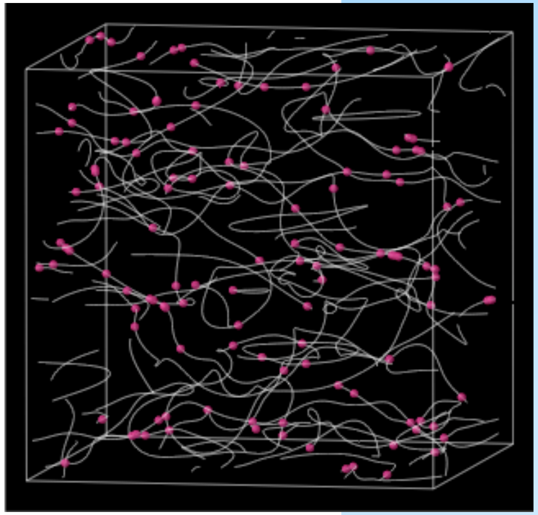


Observation by Paoletti *et al*.

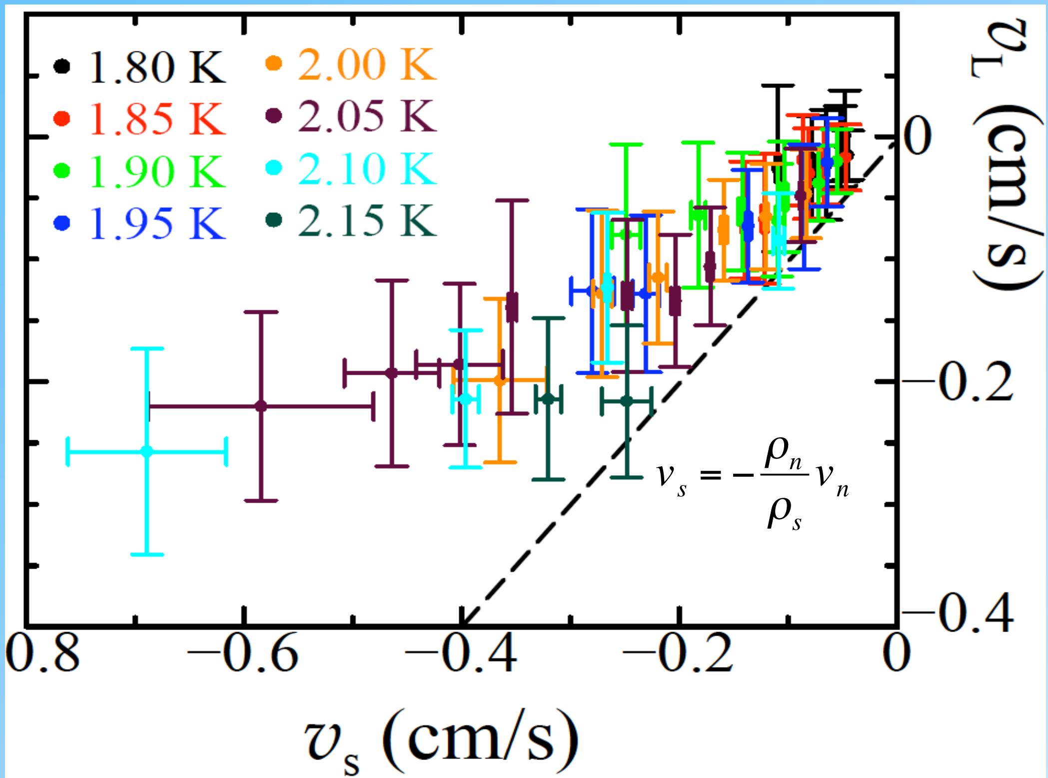
PDF(Probability Density Function) of the velocity of trapped particles

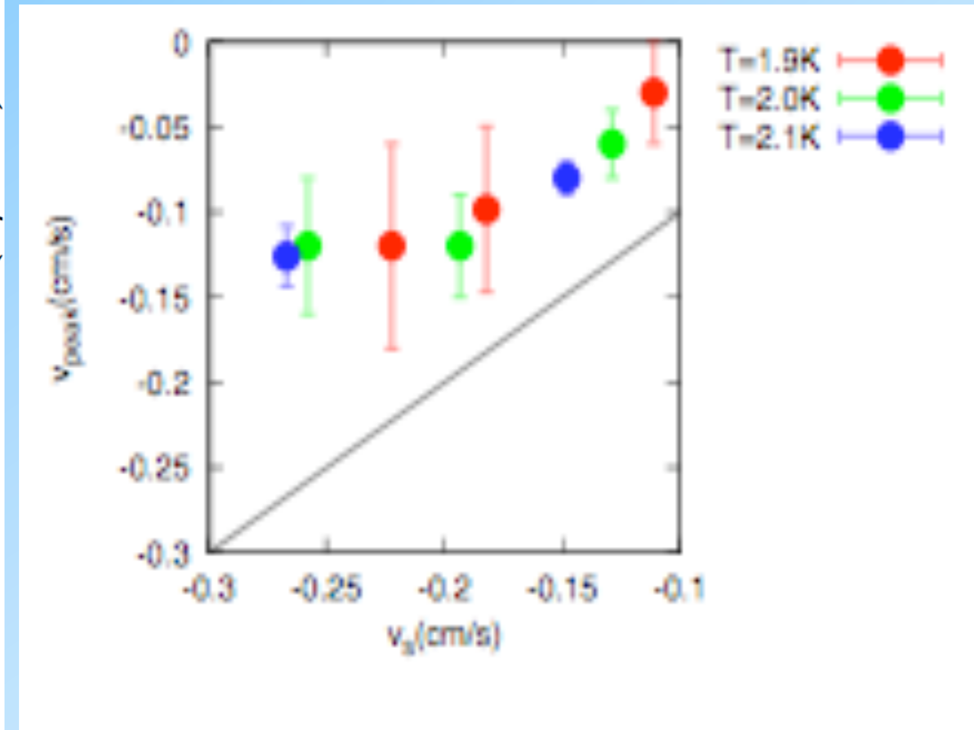
1.9K $v_s = -\frac{\rho_n}{\rho_s} v_n$



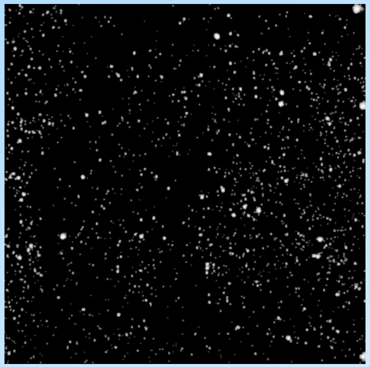


Comparison between observations and simulation

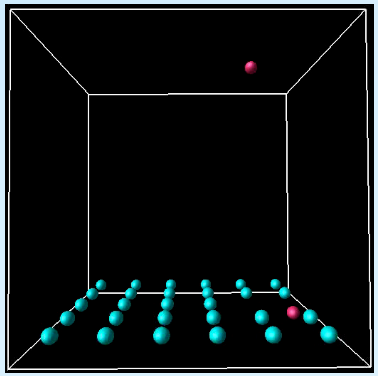


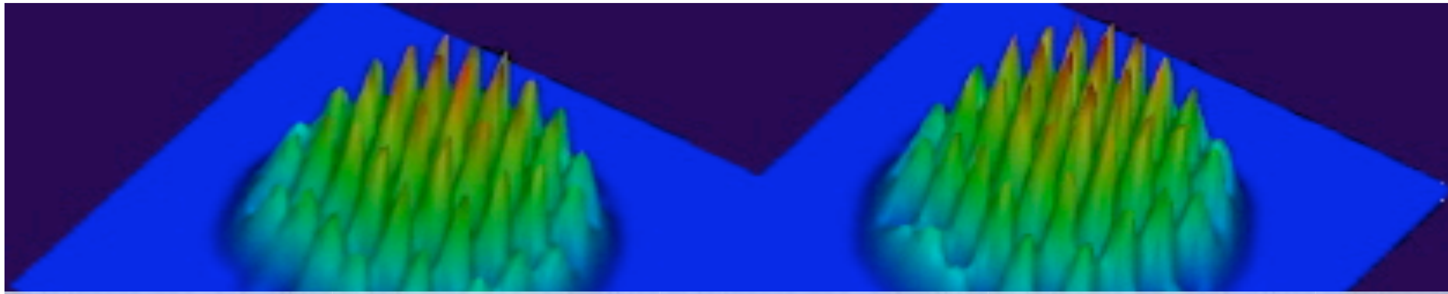


Exp.



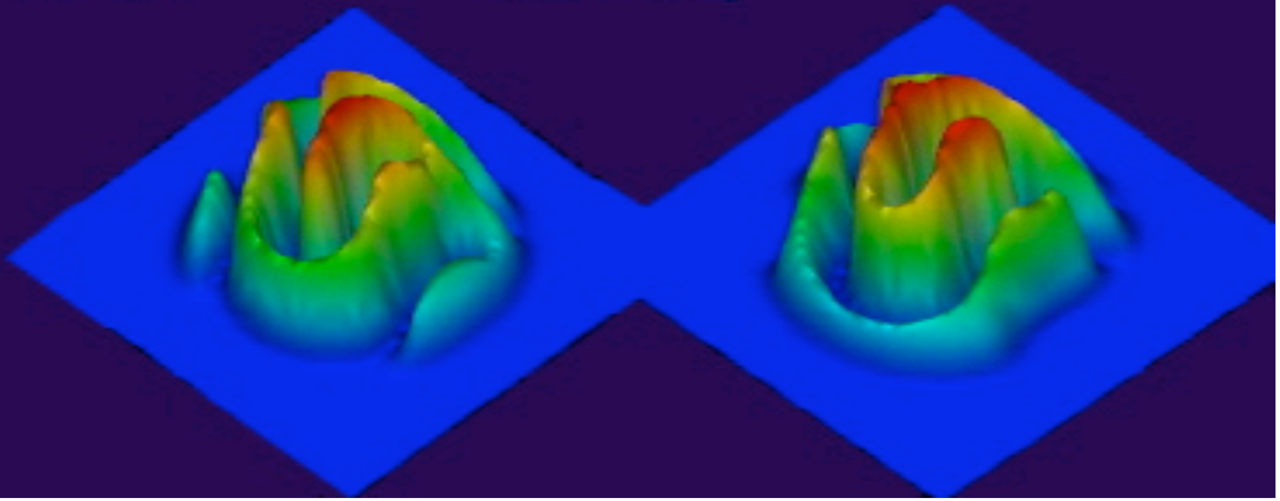
Simulation





3. Quantized vortices in two-component BECs

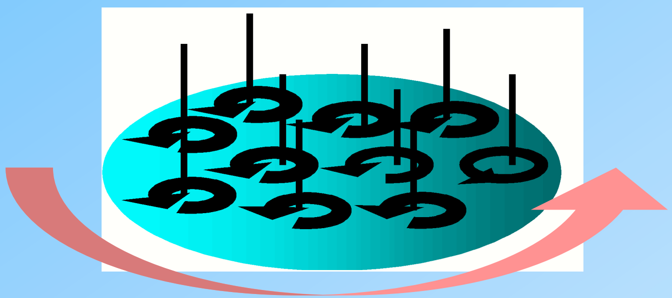
Review article*: *K.Kasamatsu, MT, M.Ueda, Int. J. Mod. Phys. 11, 1835(2005)



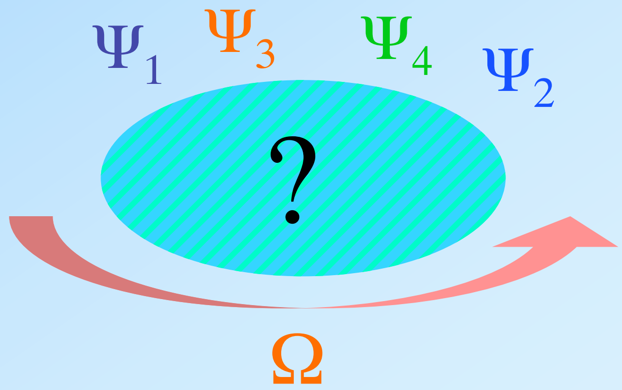
Vortices and hydrodynamics in multi-component BECs

Depending on the symmetry, multi-component order parameters can yield various kinds of topological defects.

superfluid ^{3}He, superconductivity with non-s-wave symmetry (Sr_2RuO_4, UPt_3), bilayer quantum Hall systen, nonlinear optics, nuclear physics, cosmology(Neutron star), …

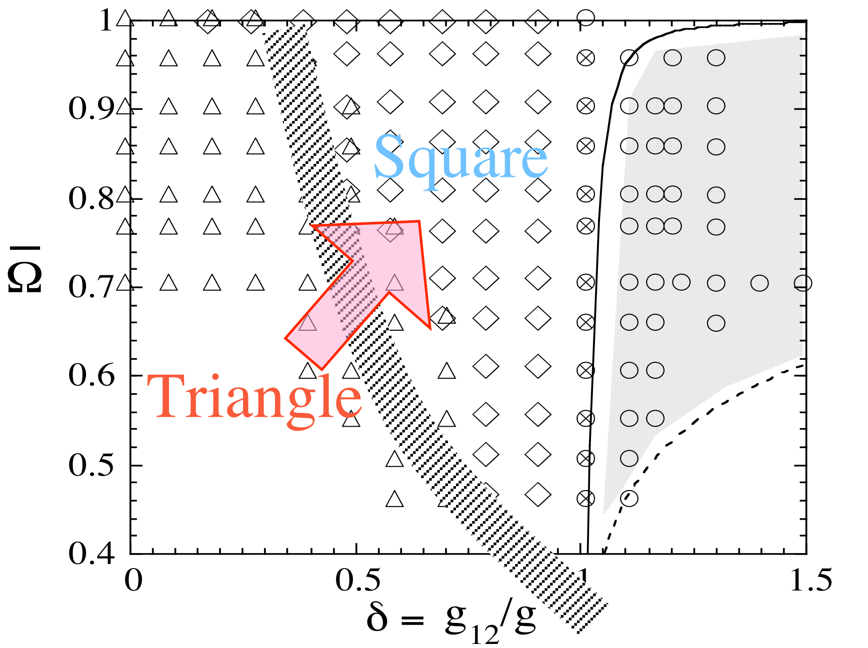


Ω Triangular (Abrikosv)lattice

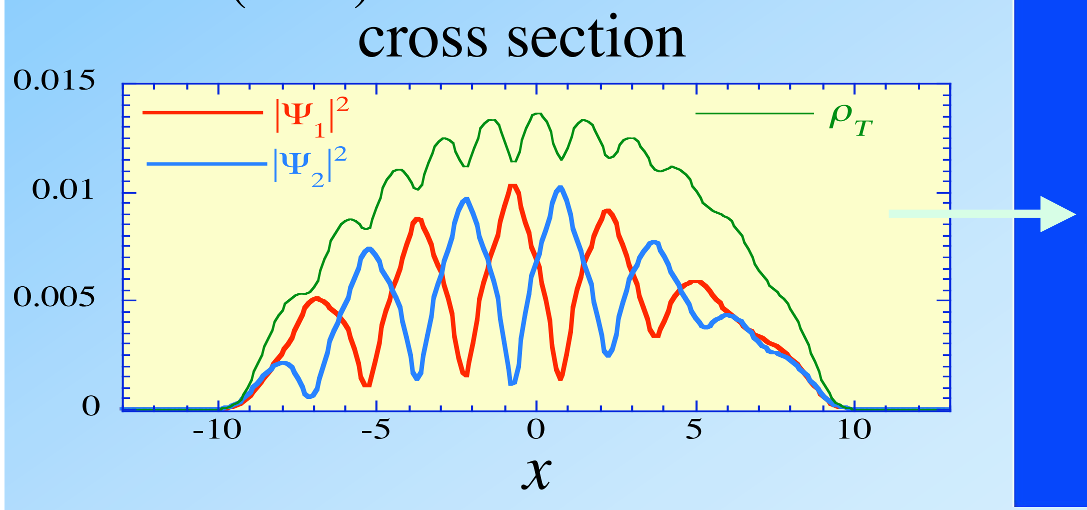


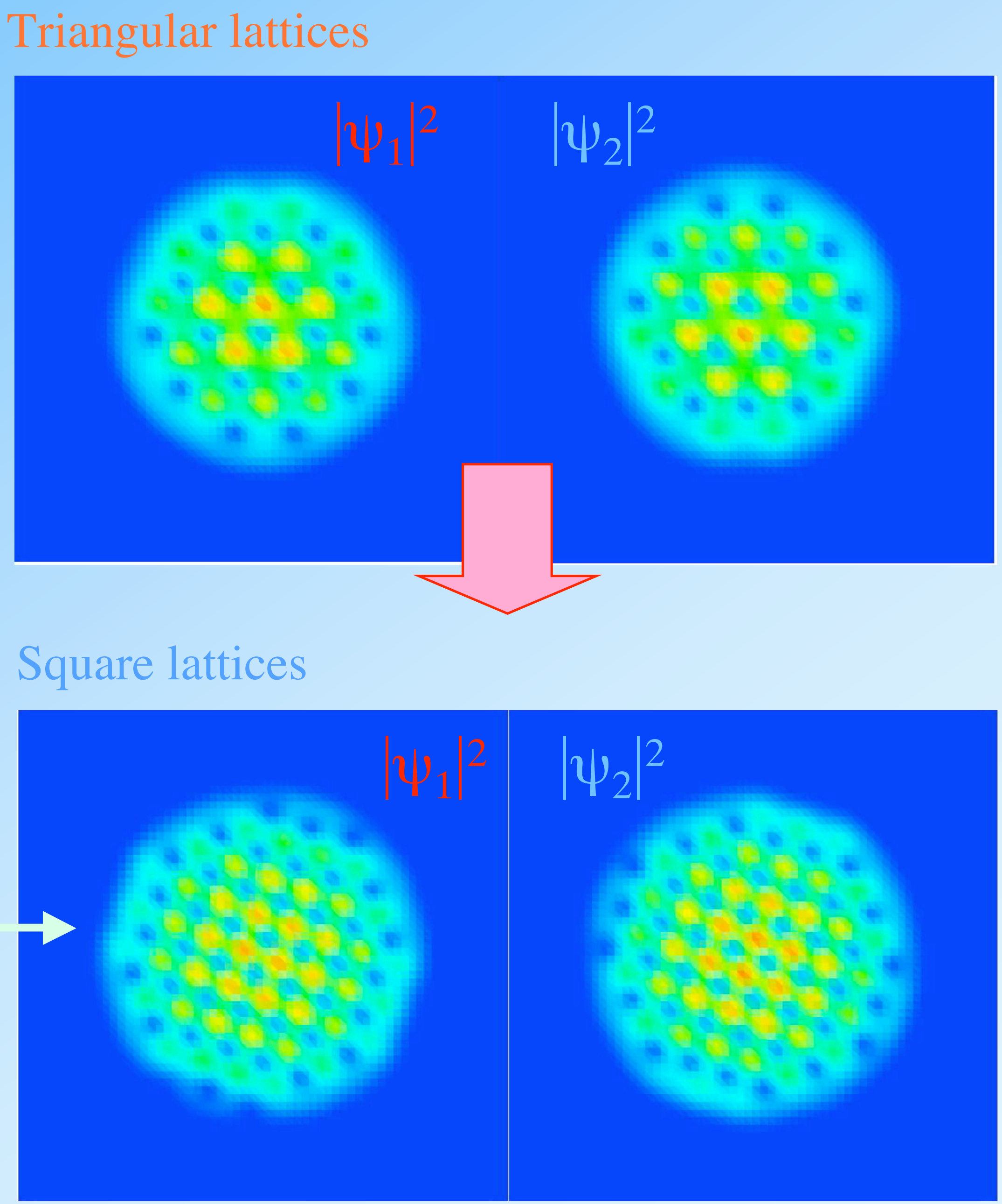
Topological defects in two-component BECs

Vortex lattices in rotating two-component BECs



K. Kasamatsu, MT, M. Ueda, PRL91, 150406 (2003)





Hydrodynamic instability in two-component BECs

Quantum Kelvin-Helmholtz instability (KHI)

H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, MT, PRB81, 094517 (2010)

Crossover between KHI and counterflow instability

N. Suzuki, H. Takeuchi, K. Kasamatsu, MT, H. Saito, PRA81, 063604 (2010)

Counterflow instability and QT
H. Takeuchi, S. Ishino, MT, PRL105, 205301(2010);
S. Ishino, MT, H. Takeuchi, PRA83, 063602(2011)

Rayleigh-Taylor instability

K. Sasaki, N, Suzuki, D. Akamatsu, H. Saito, PRA80, 042704 (2009)

Counterflow of two-component BECs: two-component quantum turbulence

H. Takeuchi, S. Ishino, MT, Phys. Rev. Lett.105, 205301(2010);

S. Ishino, MT, H. Takeuchi, Phys. Rev. A83, 063602(2011)

Motivation

BEC 1

BEC1 can flow without dissipation.

BEC 2

BEC2 can flow without dissipation too.

BEC 1

BEC 2

What happens? Two BECs destroy the superfluidity of each other.

Two-component GP model

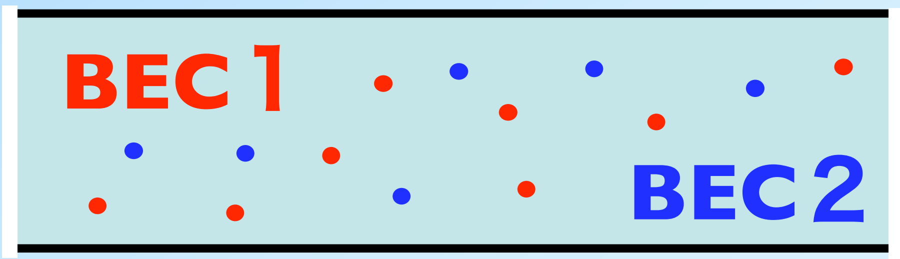
$$i\hbar\frac{\partial}{\partial t}\Psi_1 = \frac{\mathbf{p}_1^2}{2m_1}\Psi_1 + g_{11}|\Psi_1|^2\Psi_1 + g_{12}|\Psi_2|^2\Psi_1$$

$$i\hbar\frac{\partial}{\partial t}\Psi_2 = \frac{\mathbf{p}_2^2}{2m_2}\Psi_2 + g_{22}|\Psi_2|^2\Psi_2 + g_{12}|\Psi_1|^2\Psi_2$$

g_{11}, g_{22} : intracomponent interaction

g_{12} : intercomponent interaction

$g_{11}g_{22} > g_{12}^2$ $\Rightarrow$ The mixture is stable.



Two-component GP model

$$i\hbar\frac{\partial}{\partial t}\Psi_1 = \frac{\mathbf{p}_1^2}{2m_1}\Psi_1 + g_{11}|\Psi_1|^2\Psi_1 + g_{12}|\Psi_2|^2\Psi_1$$

$$i\hbar\frac{\partial}{\partial t}\Psi_2 = \frac{\mathbf{p}_2^2}{2m_2}\Psi_2 + g_{22}|\Psi_2|^2\Psi_2 + g_{12}|\Psi_1|^2\Psi_2$$

g_{11}, g_{22} : intracomponent interaction

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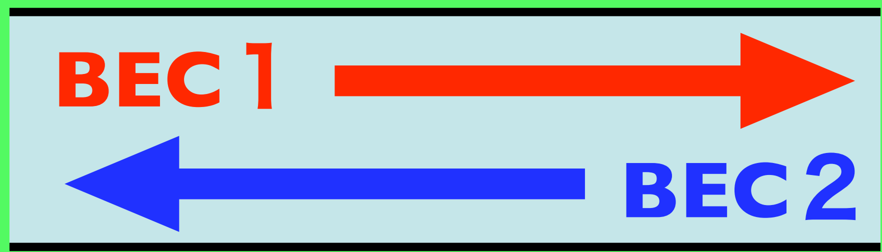
$g_{11}g_{22} > g_{12}^2$ ➡ The mixture is stable.



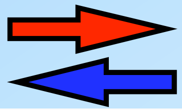
The large relative velocity should make it unstable.

V. I. Yukalov and E. P. Yukalova, Laser Phys. Lett. **1**, 50 (2004)

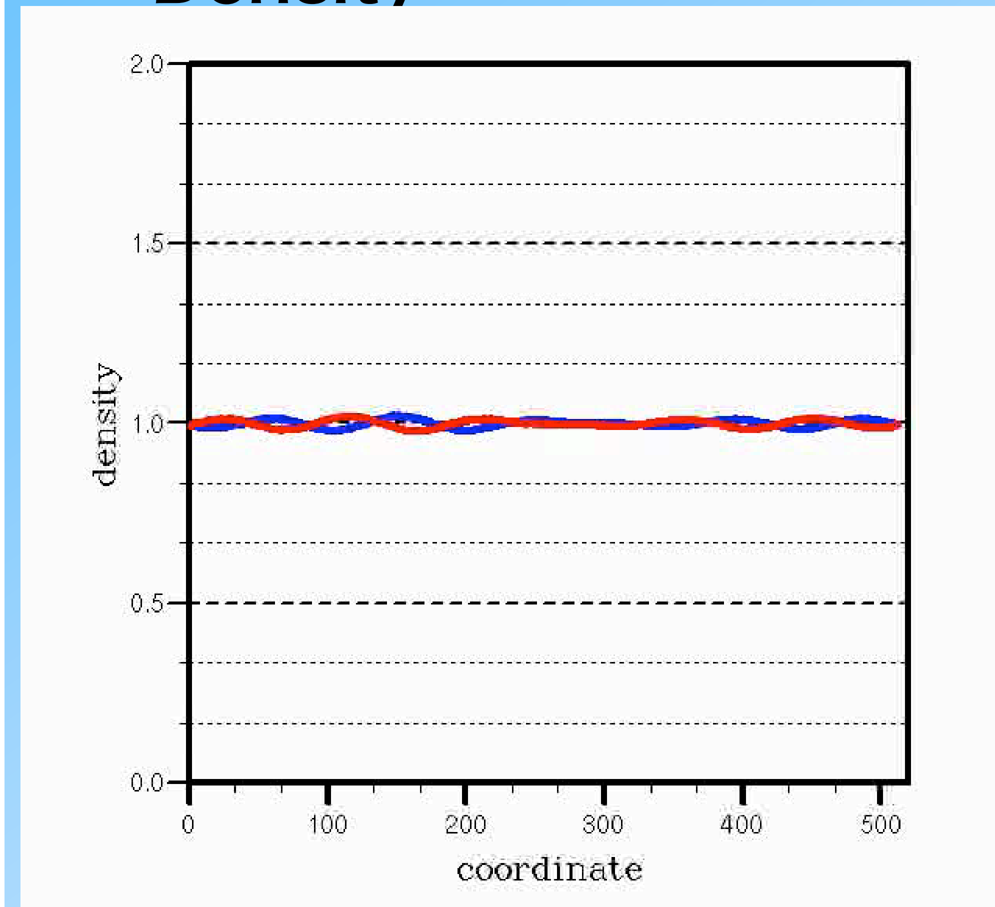
Results (One-dimension)

$n_1 = n_2 = 1.0$, $\mathbf{v}_1 = -\mathbf{v}_2$, $\gamma = 0.5$

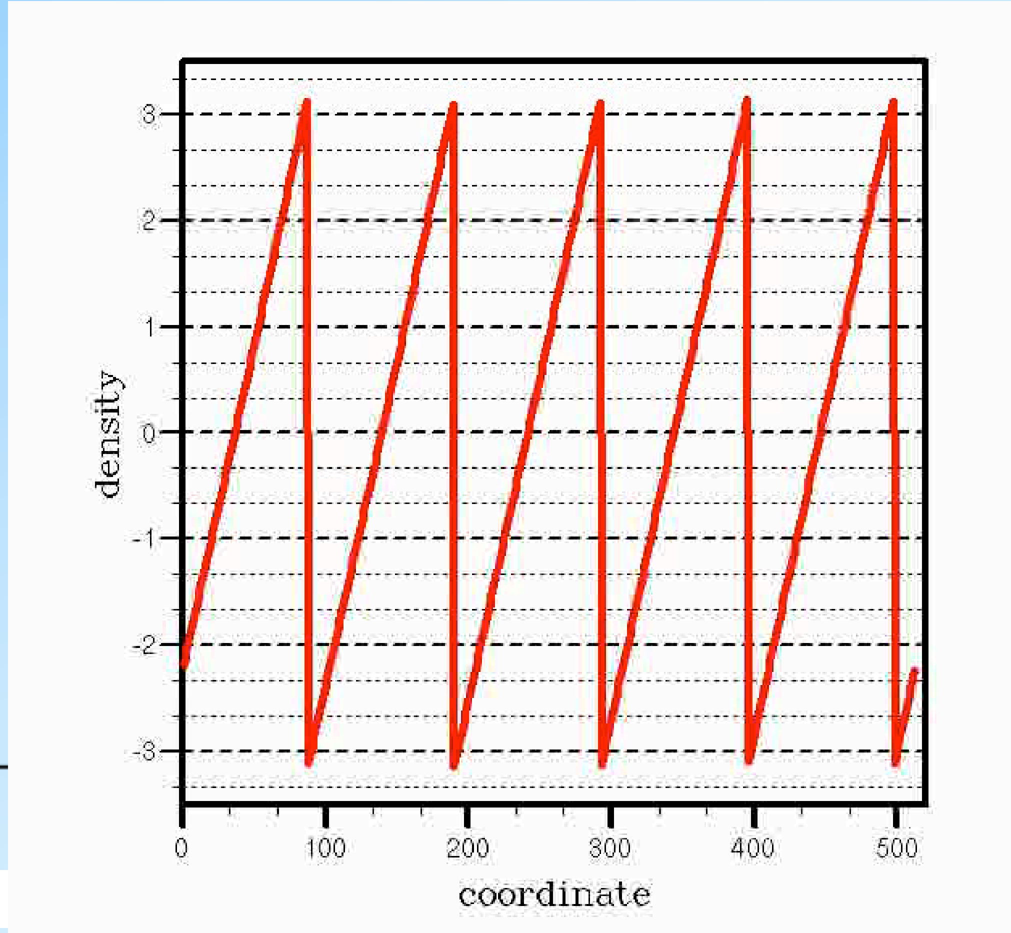
Direction of flow



Density



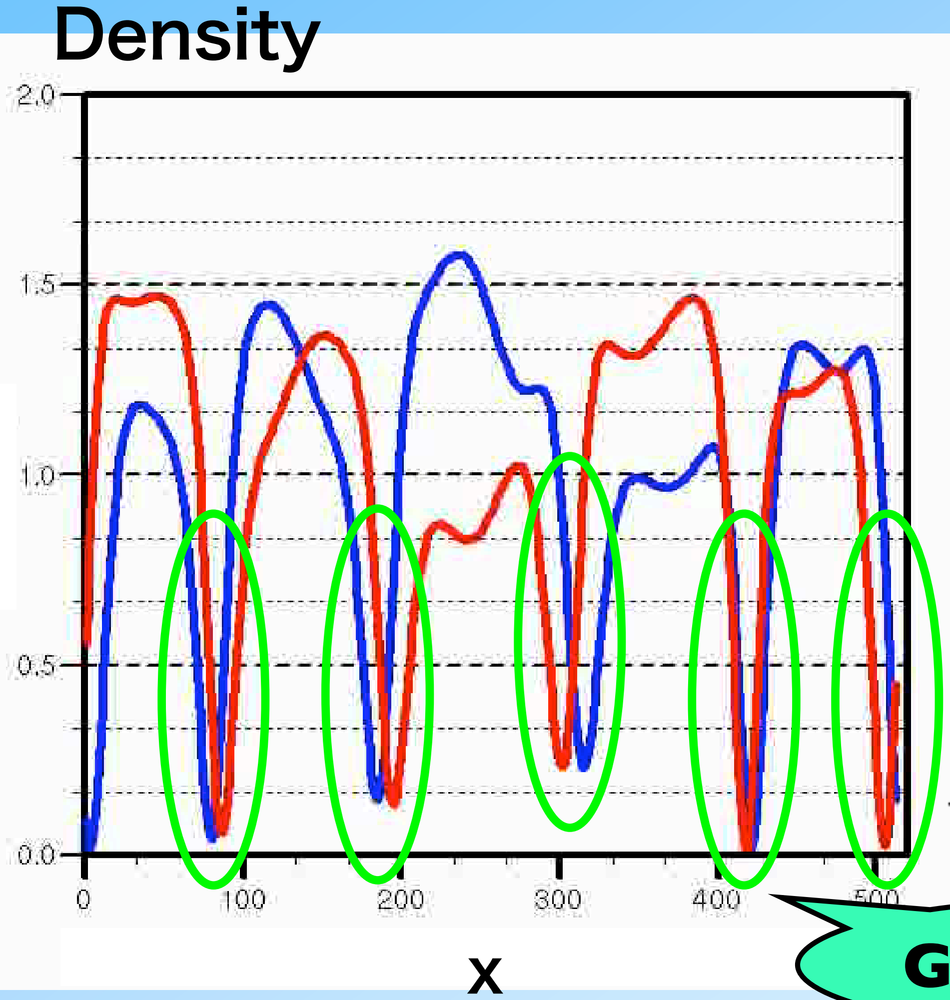
Phase

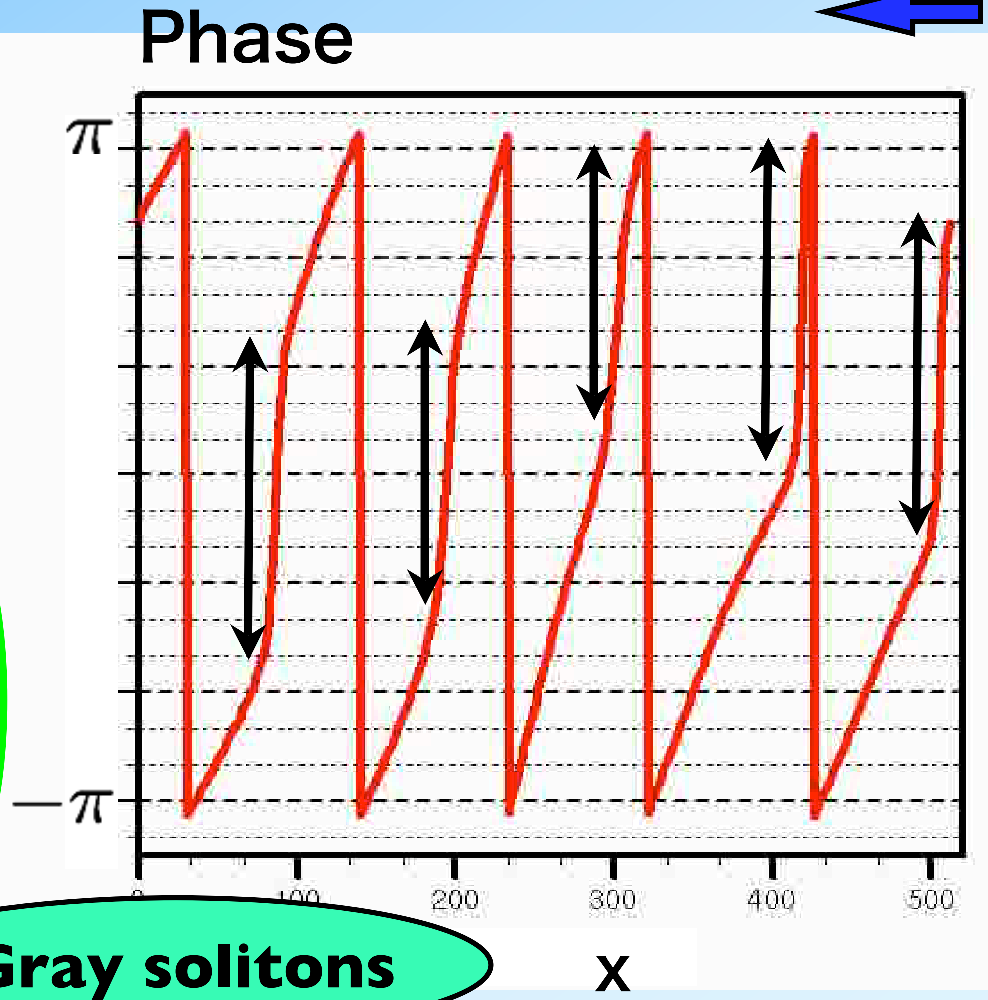


Results (One-dimension)

$n_1 = n_2 = 1.0$, $\mathbf{v}_1 = -\mathbf{v}_2$, $\gamma = 0.5$

Flow direction





Results (two-dimension)

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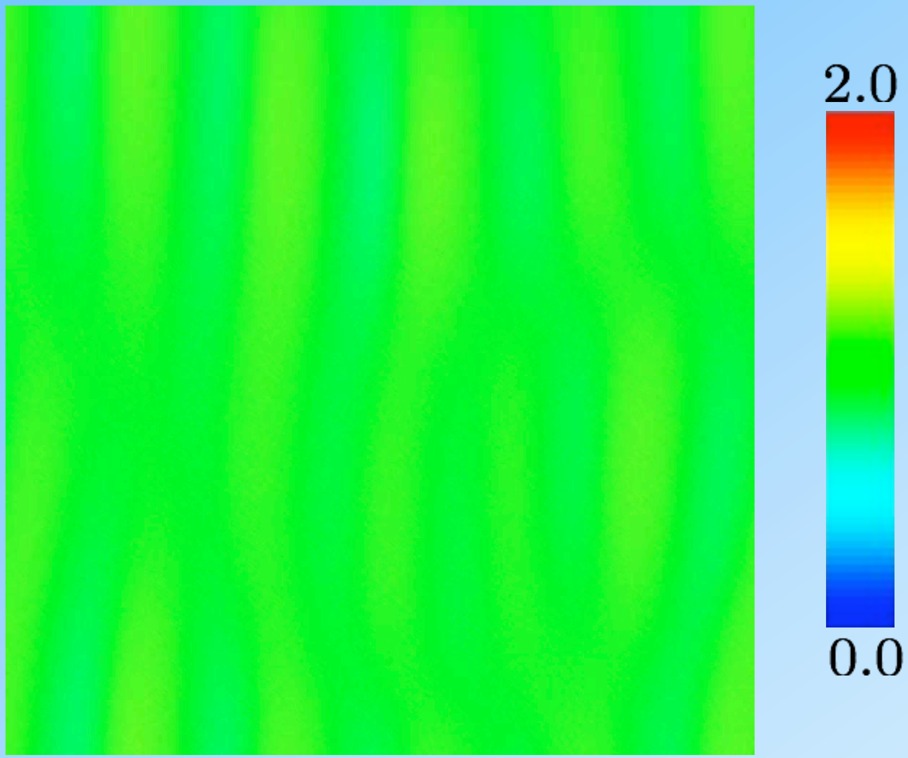
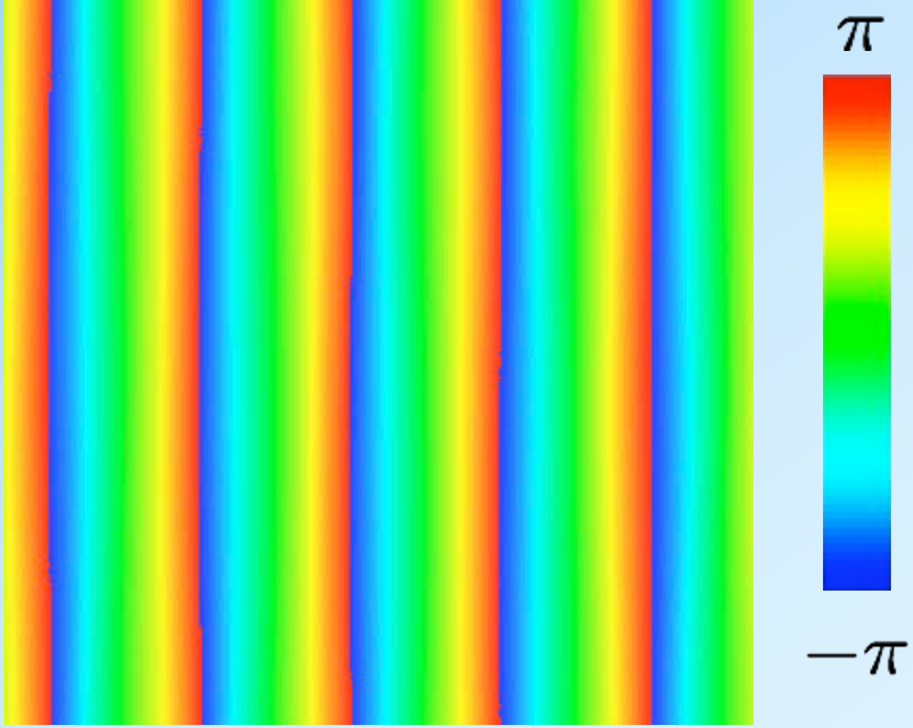
Flow direction

Density



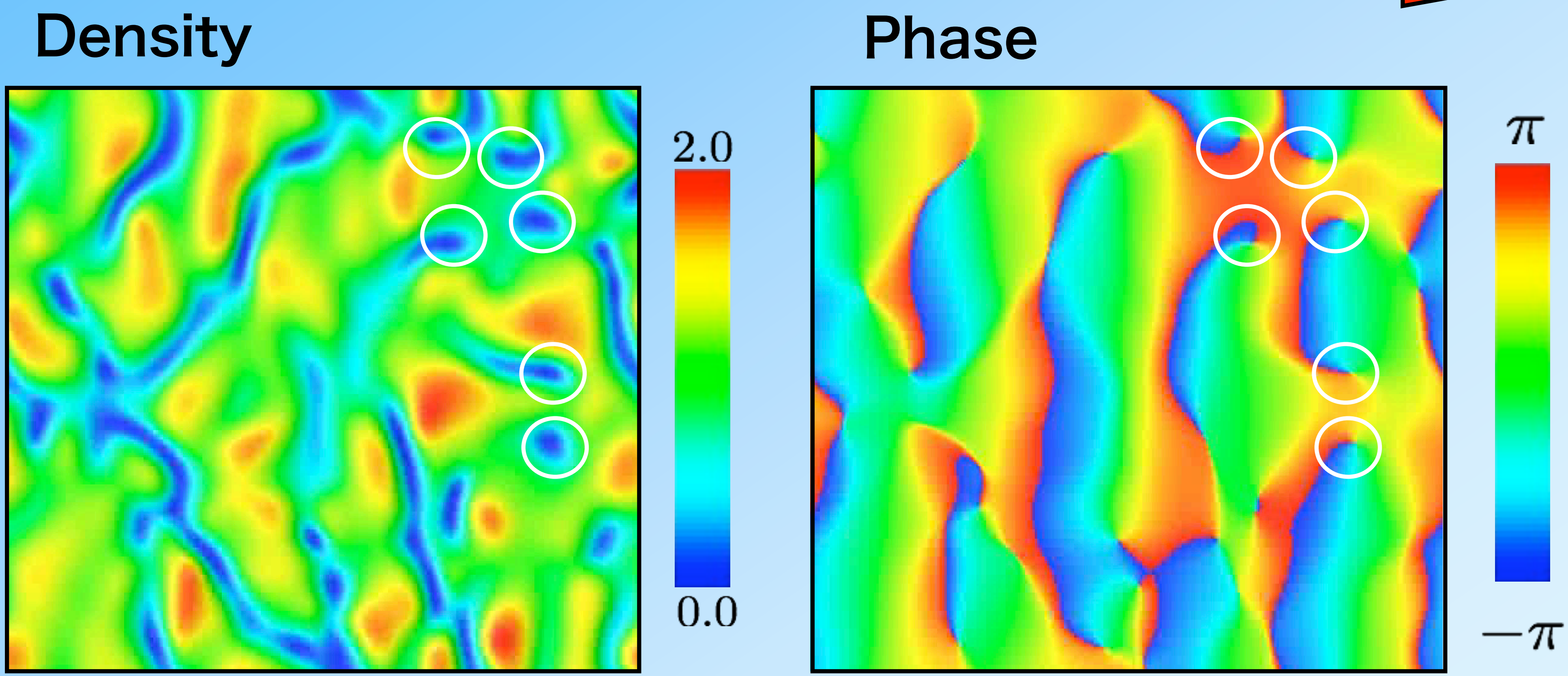
Phase



Results (two-dimension)

$n_1 = n_2 = 1.0$, $\mathbf{v}_1 = -\mathbf{v}_2$, $\gamma = 0.5$

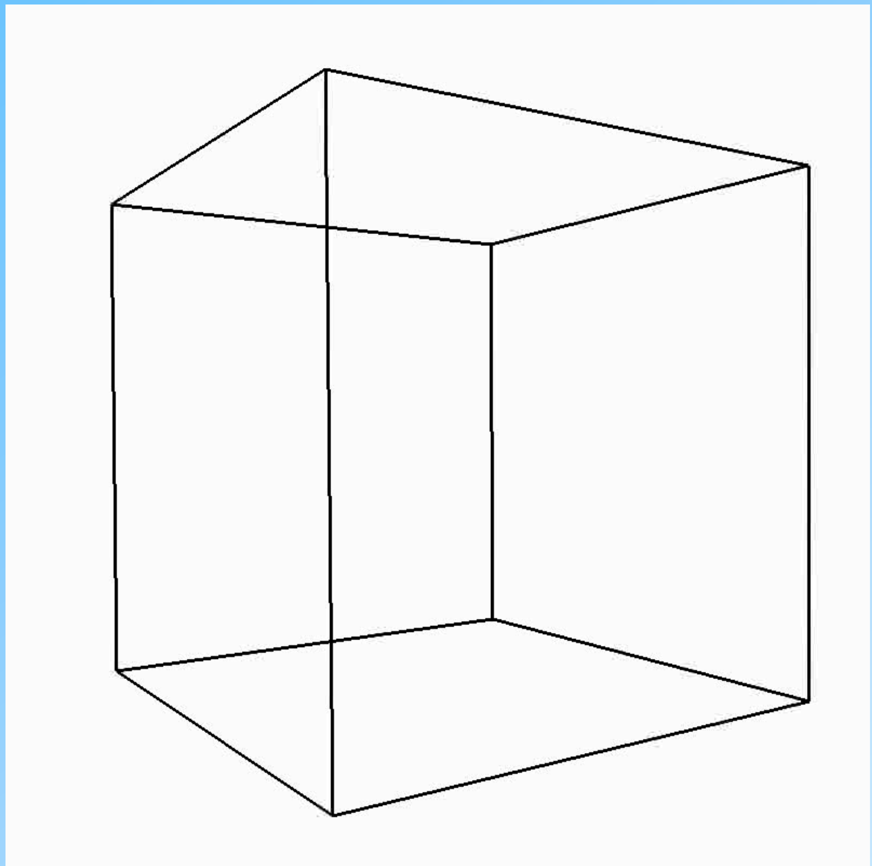
Flow direction

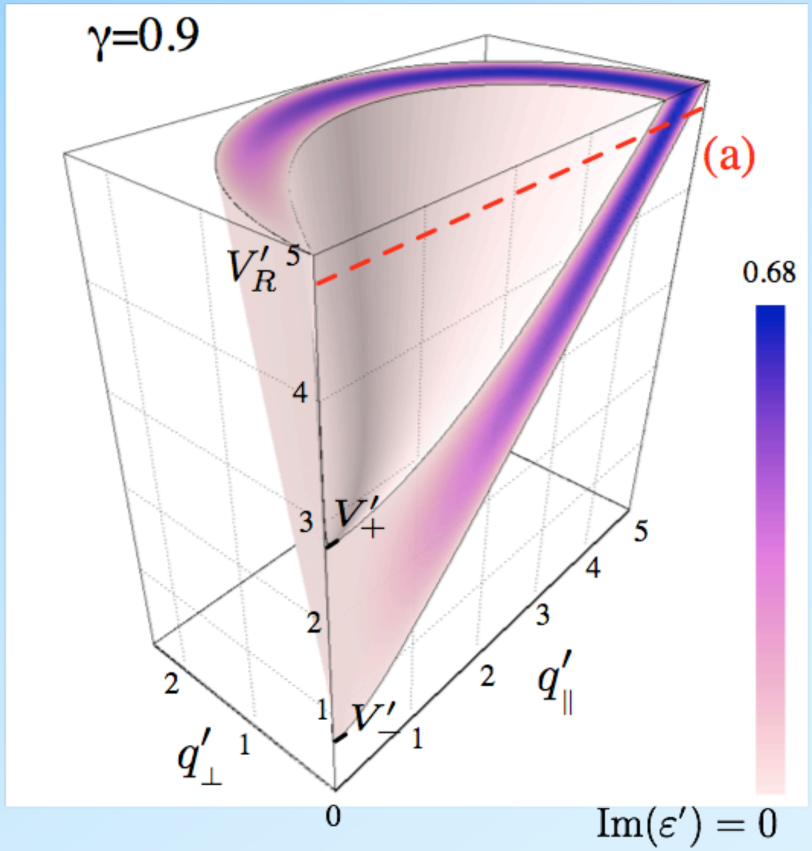


The solitons decay to vortex pairs through snake instability.

3D 2-component QT

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V_R' = 4.71, \;\; \gamma = 0.9$$



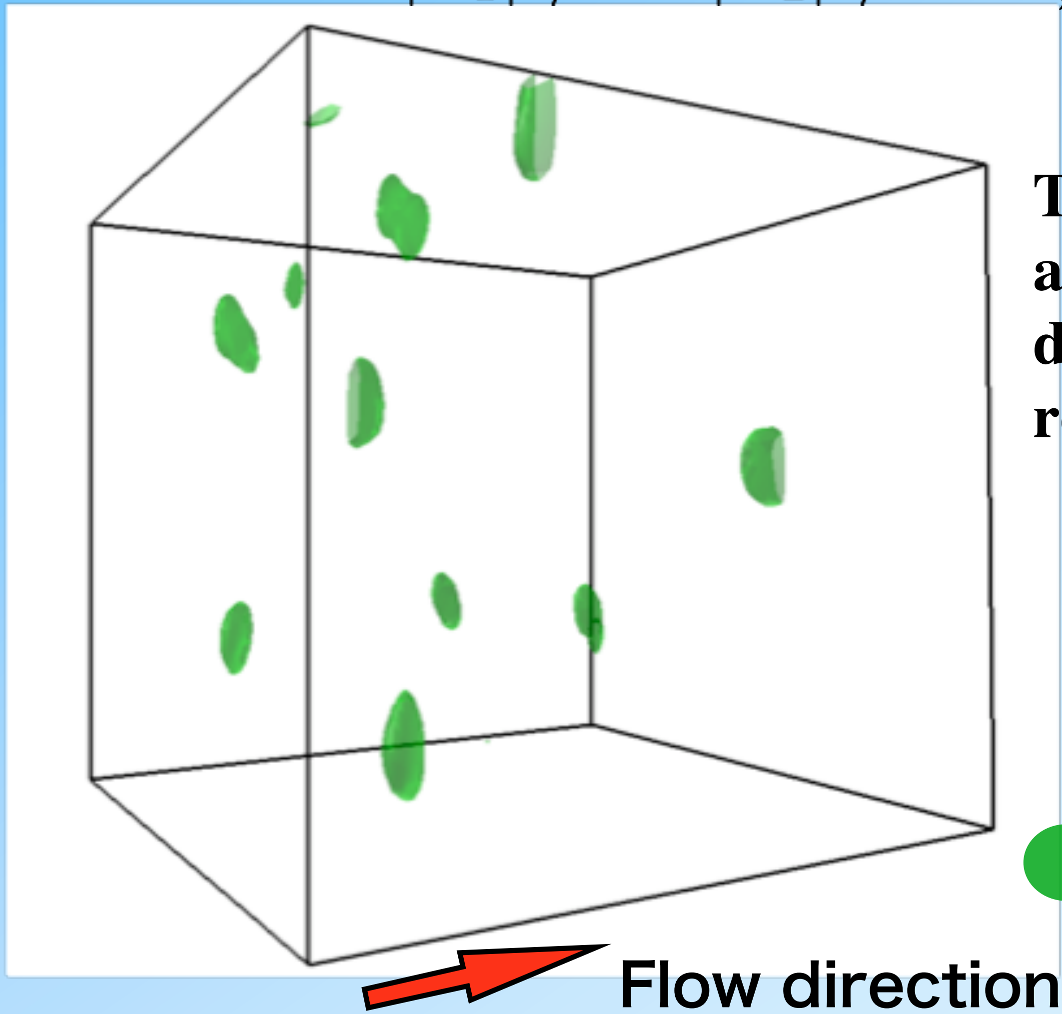


Flow direction

Solitons → Vortex loops → QT

Scenario to turbulence (1)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V_R' = 4.71, \;\; \gamma = 0.9$$



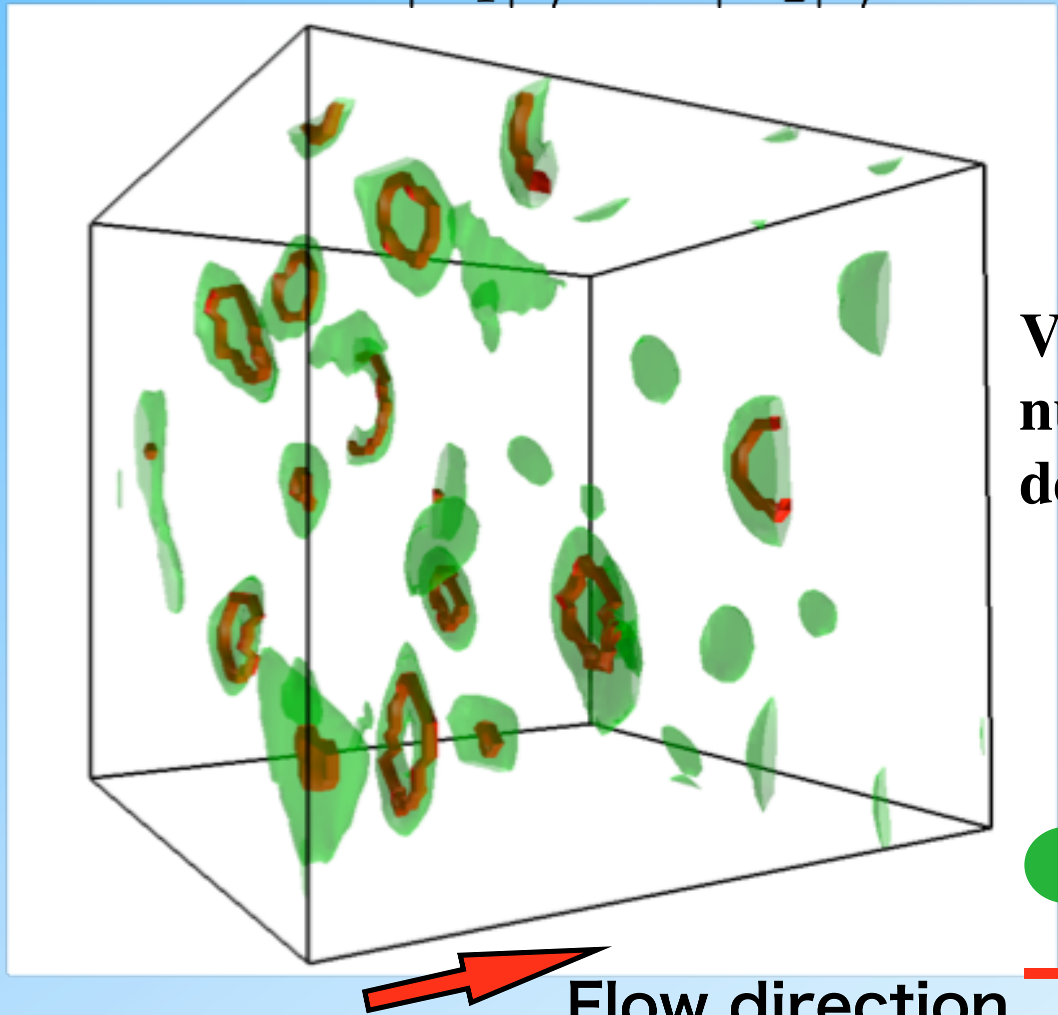
The unstable mode is amplified to lead to the disk-shaped low density regions.

Isosurface of $|\Psi_1|^2/n = 0.1$

Flow direction

Scenario to turbulence (2)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \ \ V'_R = 4.71, \ \ \gamma = 0.9$$



Flow direction

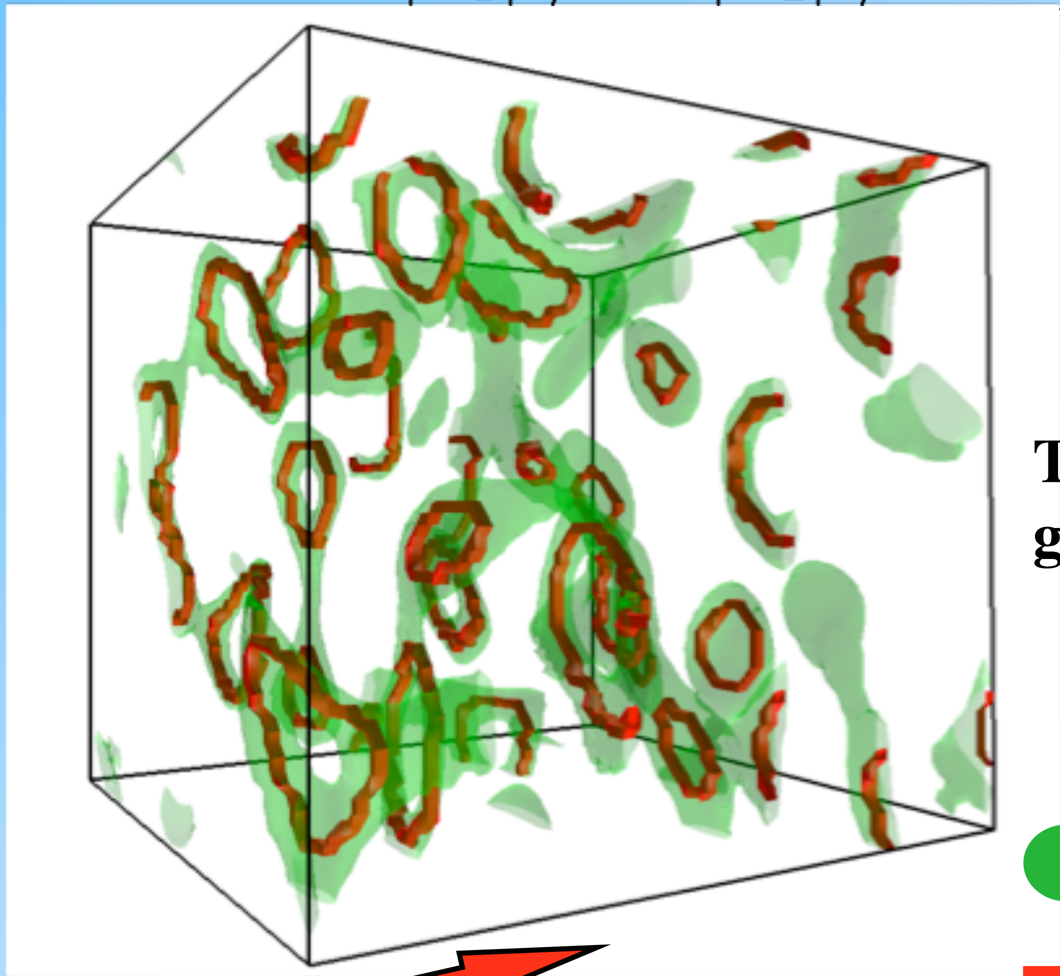
Vortex rings are nucleated inside the low density regions.

Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (3)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V_R' = 4.71, \;\; \gamma = 0.9$$



Flow direction

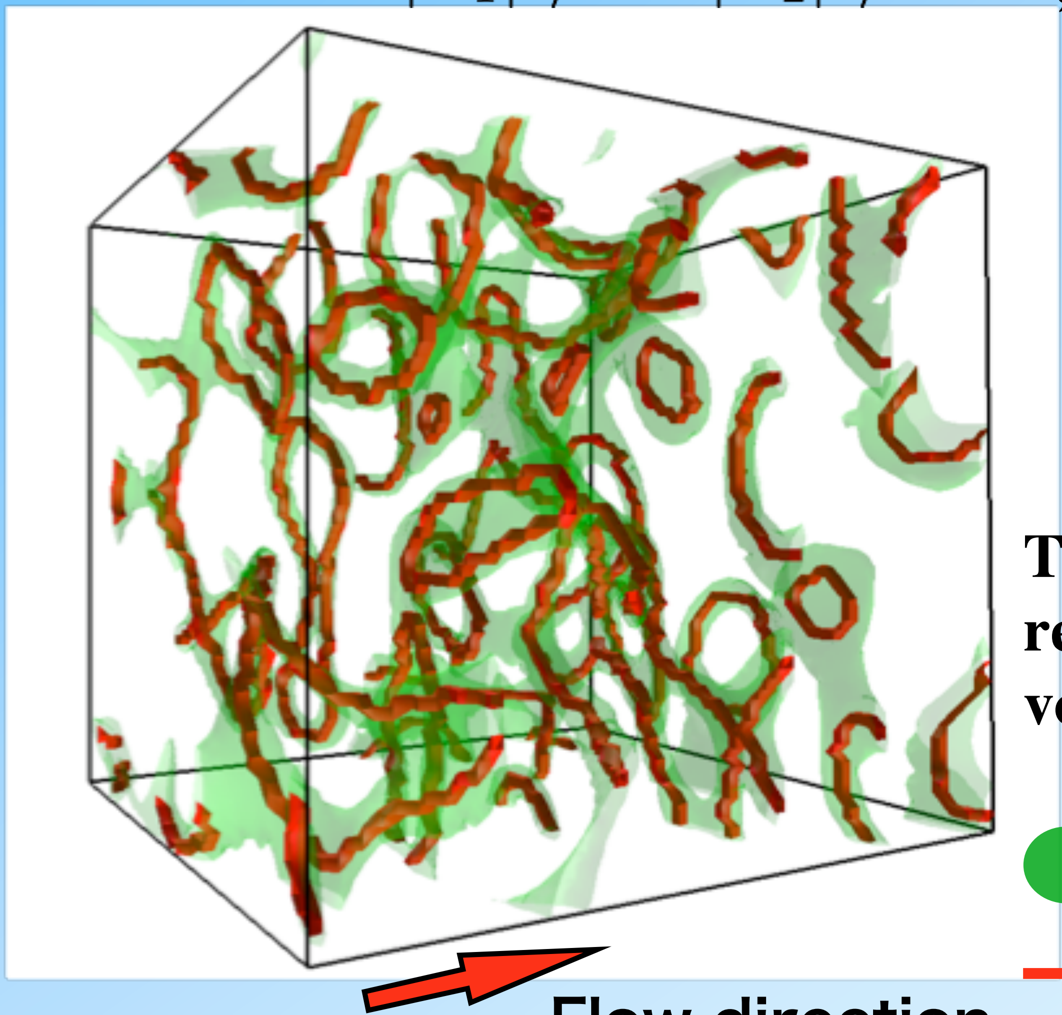
The vortices expand and grow.

Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (4)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V_R' = 4.71, \;\; \gamma = 0.9$$



Flow direction

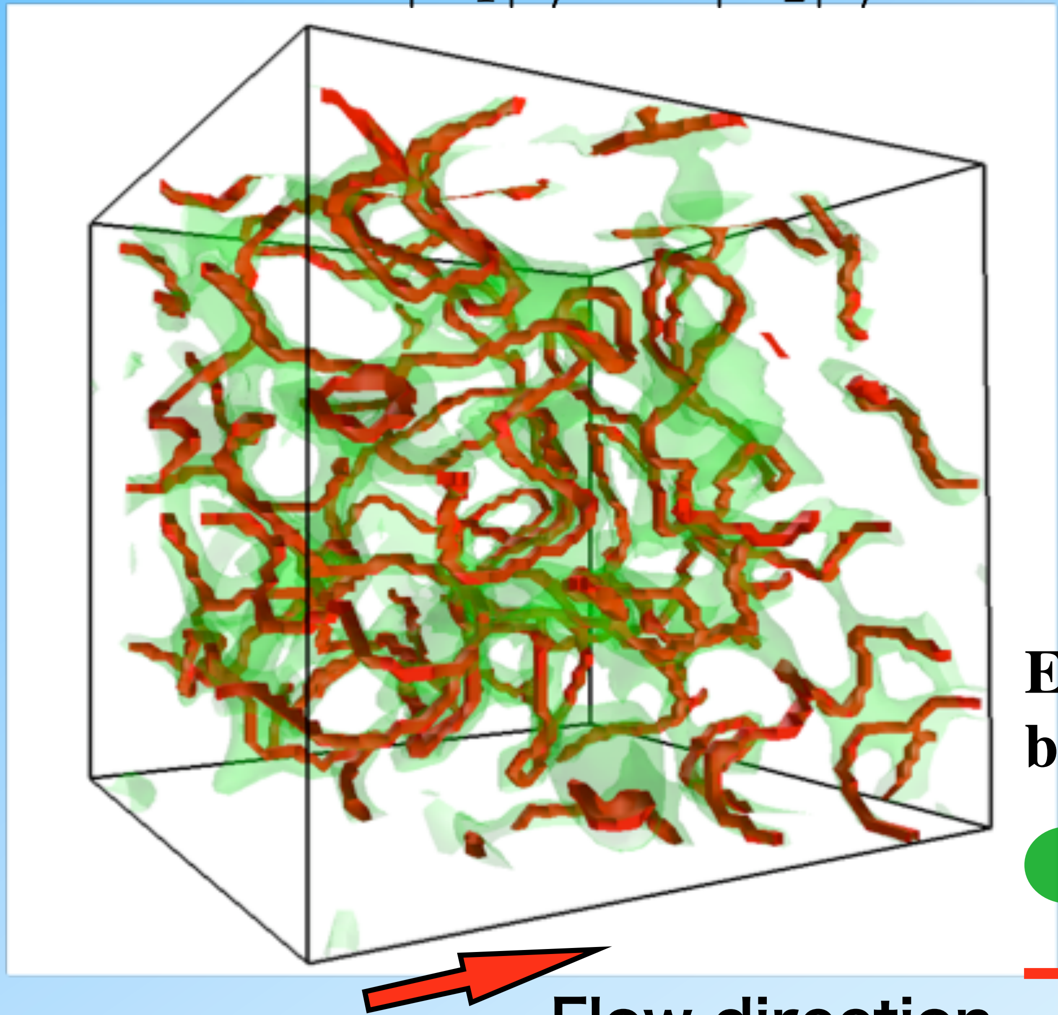
The vortices expand to reconnect with other vortices.

Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (5)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \ \ V_R' = 4.71, \ \ \gamma = 0.9$$

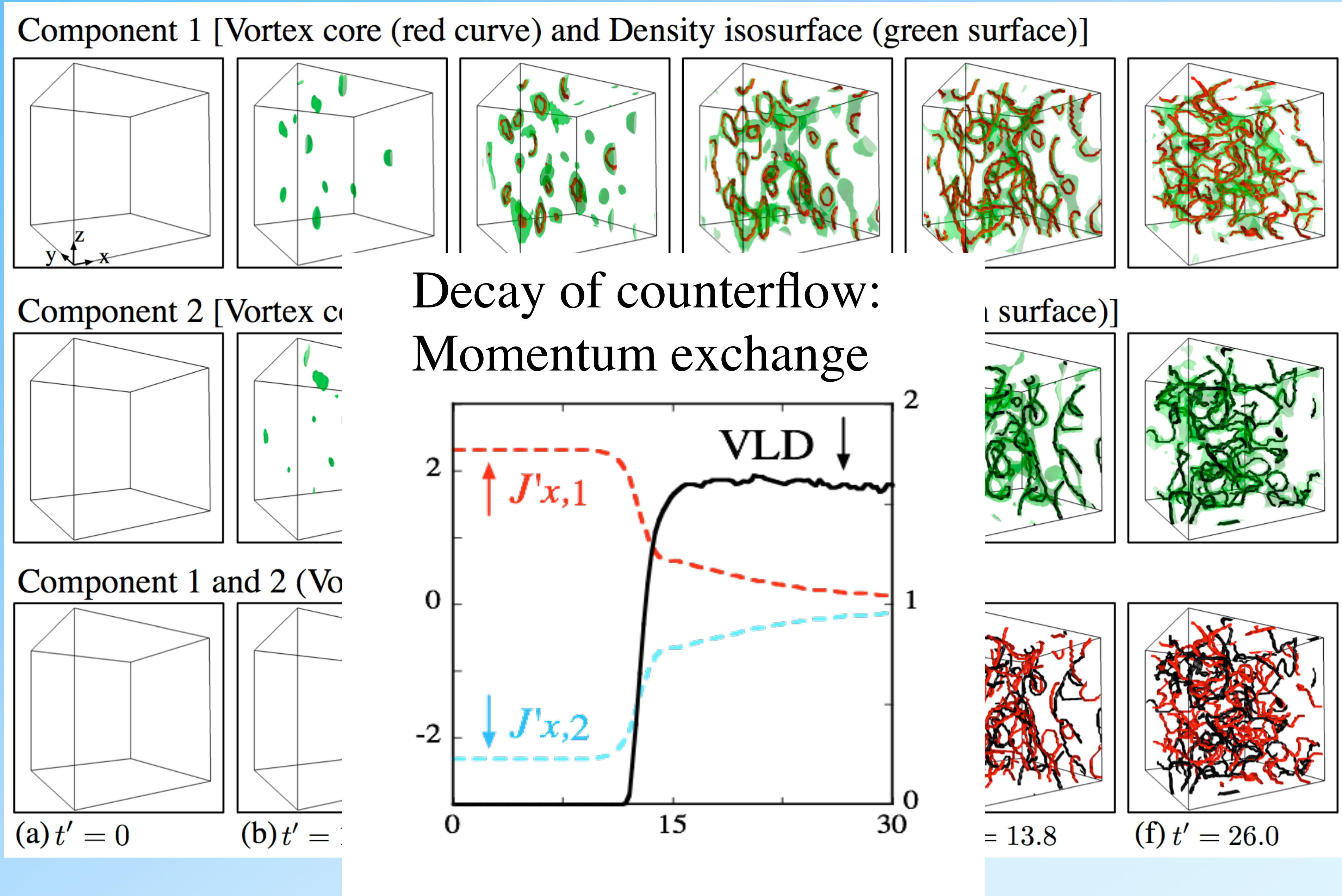


Eventually the vortices become tangled.

Isosurface of $|\Psi_1|^2/n = 0.1$

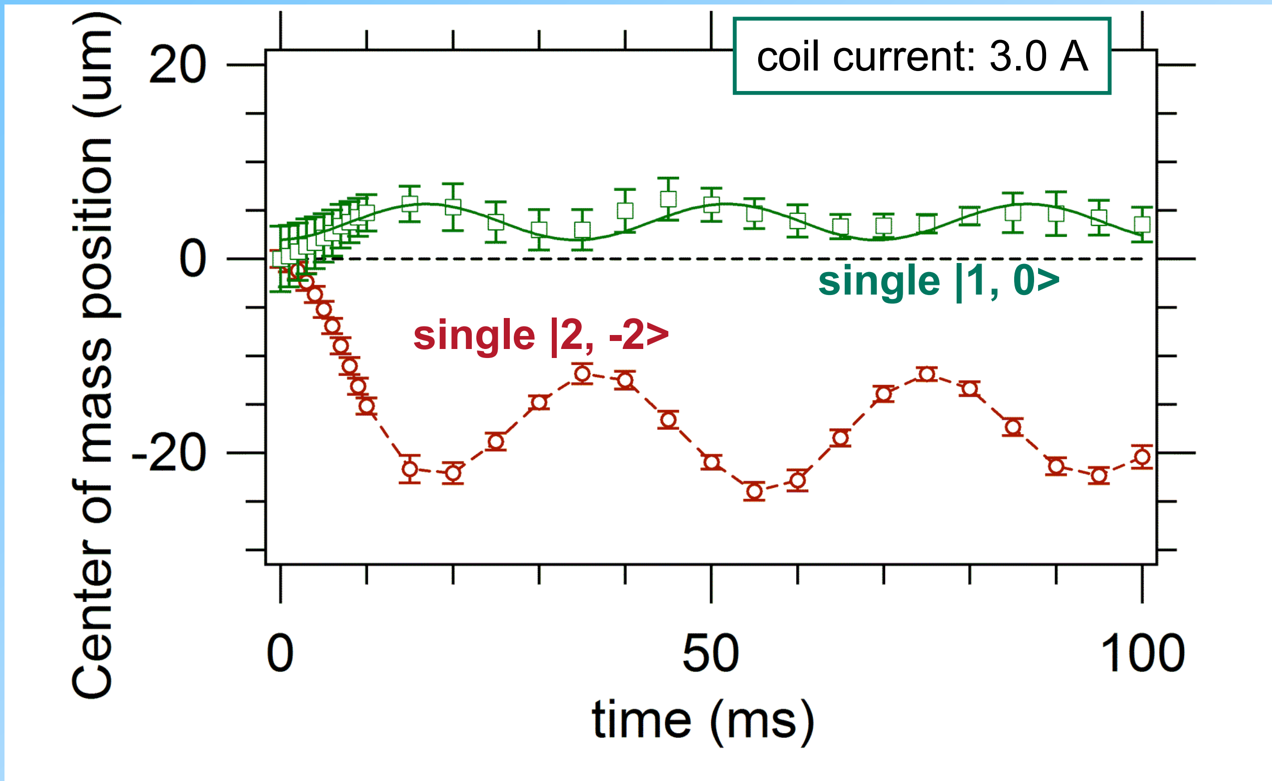
Vortex core of component 1

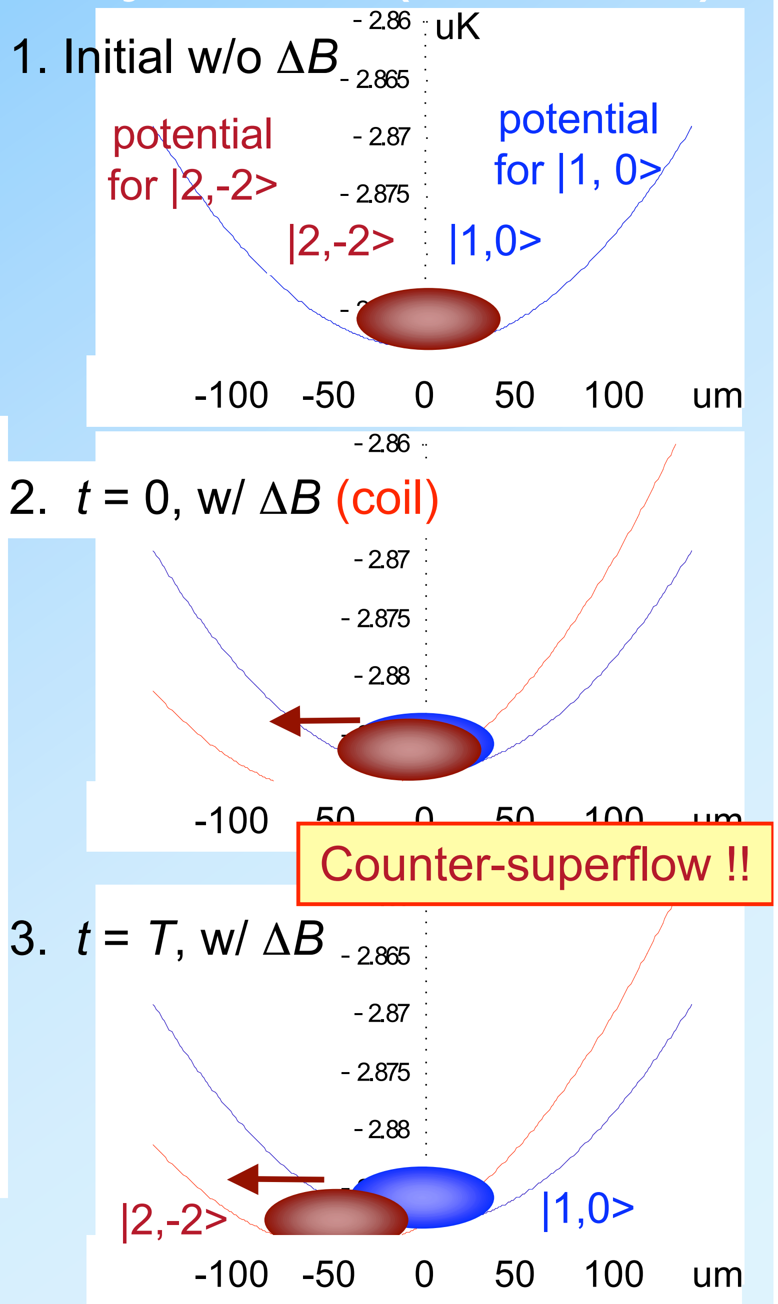
Scenario to binary quantum turbulence



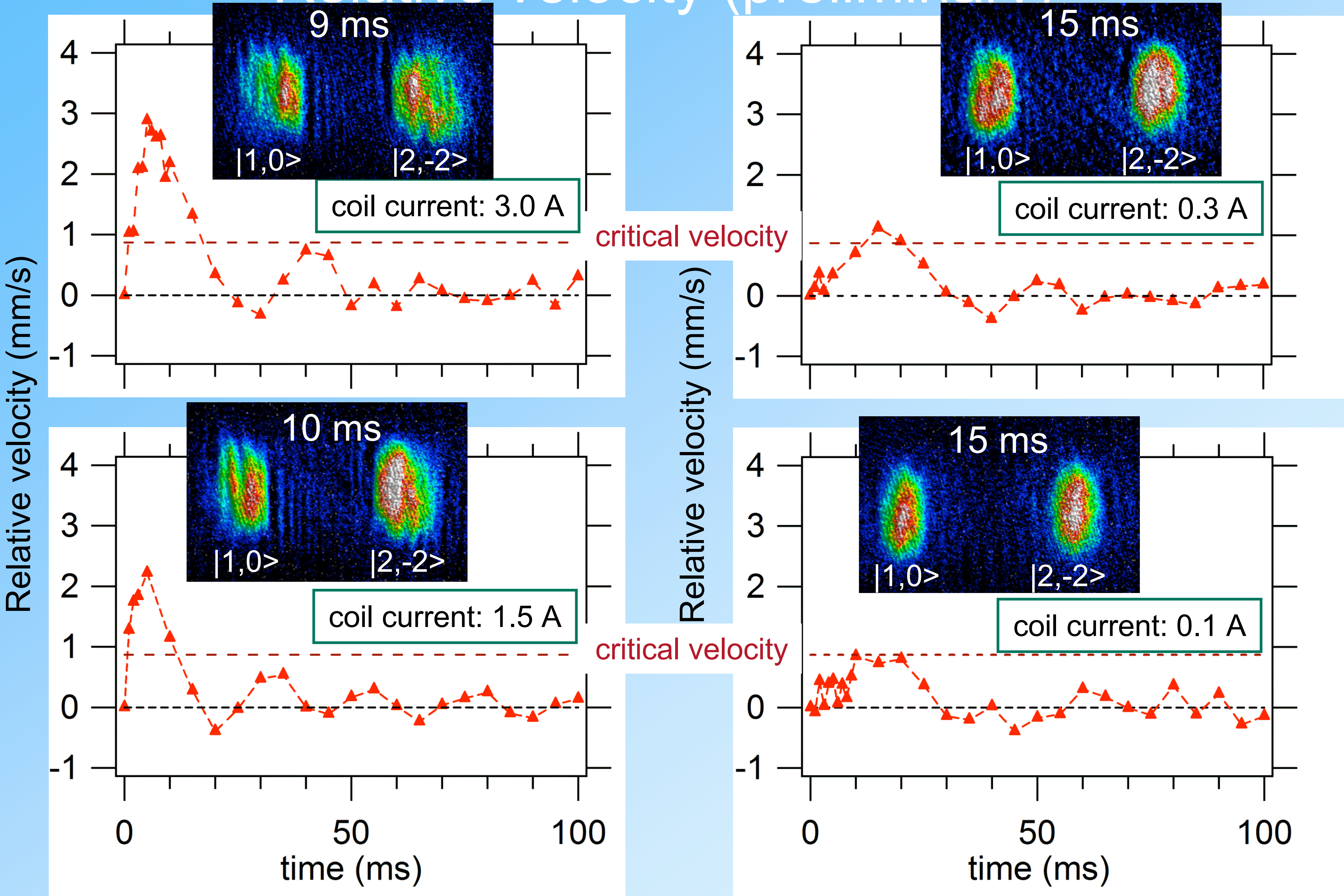
Tojo et al. (Gakushu-in) Counter flow in binary BEC (B=20G)

^{87}Rb	high-field seeker		m_F	low-field seeker	
F=2	**-2**	-1	0	+1	+2
F=1		+1	**0**	-1	





Relative velocity (preliminary)



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   Two-component QT