Infinite dimensional symmetries in the AdS/CFT correspondence

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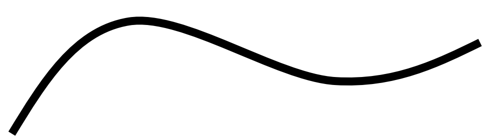
Introduction

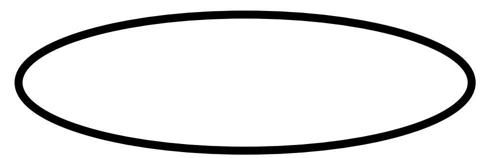
In string theory, fundamental objects are
not point particles (0-dim.) but strings (1-dim.).

Two kinds of strings are contained, open strings and closed strings.

Fundamental objects in string theory



open string



closed string

A single string can represent various particles as oscillating modes.

open string → gauge boson, closed string → graviton

A remarkable feature

> Gauge and gravitational theories are naturally unified
> in string theory.

Dualities between gauge and gravitational theories are realized.

EX

- matrix models [Banks, Fischler, Shenker, Susskind, 1996; Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996]
- matrix string [Dijkgraaf, Verlinde, Verlinde, 1997]
- gauge/gravity duality [Maldacena, 1997; etc.]

The subject in this talk

AdS/CFT correspondence [Maldacena,1997]

String on AdS space ⟷ Conformal Field Theory

AdS/CFT can be applied to investigate strongly coupled dynamics in quantum field theories.

EX Hadron spectrum in QCD, shear viscosity in fluid dynamics

[Sakai-Sugimoto, 2004; Kovtun- Son-Starinets, 2004]

Question

What is the structure behind AdS/CFT to work?

The integrable structure is considered to ensure the duality.

The current understanding

A long range spin chain

[Beisert, Staudacher, 2005; …]

IIB string theory on $AdS_5 \times S^5$

[Bena, Polchinski, Roiban, 2003; …]

D=4, N=4, super Yang-Mills

[Minahan, Zarembo, 2002; …]

Motivation

Integrable deformations of AdS/CFT

Universality classes of classical integrable models

[Belavin, Drinfel'd, 1982]

An integrable model is characterized by an infinite dim. symmetry.

1) Rational class	Yangian algebra
degenerate limit ↑	↓ one-parameter deformation
2) Trigonometric class	quantum affine algebra
degenerate limit ↑	↓ one-parameter deformation
3) Elliptic class	elliptic algebra

Note

AdS/CFT belongs to the rational class.

Our work

A deformation to the trigonometric model.

3D AdS is considered here for simplicity:

Deformation: AdS_3 $\Rightarrow$ warped AdS_3

String theory on AdS_3

Isometry of AdS_3 : $SL(2,R)_L \times SL(2,R)_R$

Classical integrable structure of string theory on AdS_3:

Global symmetries	$SL(2,R)_L$	$SL(2,R)_R$
Hidden symmetries	Yangian	Yangian
Class	rational	rational

cf. classical integrable structure of sigma model on S^3.

[Luscher, Pohlmeyer, 1978; Drinfel'd, 1985; Bernard, 1991; Mackay, 1992; etc.]

String theory on warped AdS_3

Deformation to the trigonometric class

Isometry of AdS_3 : $SL(2,R)_L \times SL(2,R)_R$

one-parameter deformation

$SL(2,R)_L \times U(1)_R$

warped AdS_3

We will consider classical integrable structure of string theory on warped AdS_3.

Classical integrable structure of string theory on warped AdS_3

Global symmetries	$SL(2,R)_L$	$U(1)_R$
Hidden symmetries	Yangian	quantum affine
Class	rational	trigonometric

I.K., K.Yoshida, JHEP 1011,32(2010)　I.K., K.Yoshida, Phys.Lett.B705,251(2011)

I.K., T.Matsumoto, K.Yoshida, 1201.3058

Cherednik, Theor.Math.Phys.47,422(1981); Faddeev, Reshetikhin, Ann.Phys,167,227(1986); etc.

Hybrid integrable structure

- String on warped AdS_3 belongs to two different classes.
- Non-local map between two classes

trigonometric = composite of two rationals

Application to 3D Schrodinger spacetime

[I.K., K. Yoshida, JHEP 1111,94(2011)]

- The analysis here is applicable to 3D Schrodinger spacetime.
- Schrodinger spacetime appears in applications of AdS/CFT to non-relativistic conformal field theories.
- Schrodinger-like spin chain leads to non-hermite Hamiltonian.

Global symmetries	$SL(2,R)_L$	$U(1)_R$
Hidden symmetries	Yangian	????
Class	rational	non-standard deformed rational

Summary

- The integrable structure is important for AdS/CFT.

- String theory on warped AdS_3 has a hybrid integrable structure.

  Yangian (rational) and quantum affine (trigonometric)

- A similar analysis is applicable to 3D Schrodinger spacetime.

Future directions

Hybrid integrable structure

trigonometric = composite of two rationals

Warped AdS_3 would be described as a composite of two AdS_3.

Warped AdS_3/ dipole CFT_2 correspondence
(as a toy model of Kerr/CFT)

[El-Showk, Guica, 1108.6091;
Song, Strominger, 1109.0544]

Can it be described as a composite of AdS_3/CFT_2?

The hidden structure behind Kerr/CFT?

Thank you for your attention

Back up

warped AdS_3

Definition of AdS_3:

$$ds^2 = -(dX^0)^2 + (dX^1)^2 + (dX^2)^2 - (dX^3)^2$$
$$-(X^0)^2 + (X^1)^2 + (X^2)^2 - (X^3)^2 = -L^2$$

Solve the constraint and introduce angle variables:

$$\begin{aligned} X^0 + iX^3 &= L e^{-i\tau/2}\left[\cosh\frac{u}{2}\cosh\frac{\sigma}{2} + i\sinh\frac{u}{2}\sinh\frac{\sigma}{2}\right] \\ X^1 + iX^2 &= L e^{i\tau/2}\left[-\cosh\frac{u}{2}\sinh\frac{\sigma}{2} - i\sinh\frac{u}{2}\cosh\frac{\sigma}{2}\right] \end{aligned}$$

Hopf fiber representation of AdS_3

$$ds^2 = \frac{L^2}{4}\left[-\cosh^2\sigma d\tau^2 + d\sigma^2 + (du + \sinh\sigma d\tau)^2\right]$$

$$ds^2 = \frac{L^2}{4}\left[-\cosh^2\sigma d\tau^2 + d\sigma^2 + (du + \sinh\sigma d\tau)^2\right]$$

one-parameter deformation

$$ds^2 = \frac{L^2}{4}\left[\underbrace{-\cosh^2\sigma d\tau^2 + d\sigma^2}_{\mathrm{AdS}_2} + \boxed{(1+C)}\underbrace{(du + \sinh\sigma d\tau)^2}_{S^1}\right]$$

isometry: $SL(2,R)_L$ $U(1)_R$

This is known as spacelike warped AdS_3.

There is also timelike warped AdS_3.

$$ds^2 = \frac{L^2}{4}\left[\cosh^2\sigma du^2 + d\sigma^2 - (1+C)(d\tau - \sinh\sigma du)^2\right]$$

Yangian algebra

$$\left\{Q^a_{(0)}, Q^b_{(0)}\right\}_{\mathrm{P}} = \varepsilon^{ab}{}_c Q^c_{(0)} ,$$
$$\left\{Q^a_{(0)}, Q^b_{(1)}\right\}_{\mathrm{P}} = \varepsilon^{ab}{}_c Q^c_{(1)} ,$$
$$\left\{Q^a_{(1)}, Q^b_{(1)}\right\}_{\mathrm{P}} = \varepsilon^{ab}{}_c \left[Q^c_{(2)} + \frac{1}{12}\left(Q_{(0)}\right)^2 Q^c_{(0)}\right] ,$$

Serre relation

$$\left\{\left\{Q^a_{(1)}, Q^b_{(1)}\right\}_{\mathrm{P}}, \left\{Q^c_{(0)}, Q^d_{(0)}\right\}_{\mathrm{P}}\right\}_{\mathrm{P}} + ((a,b) \leftrightarrow (c,d)))$$
$$= \frac{1}{4}\varepsilon^a{}_{ei}\varepsilon^b{}_{fj}\varepsilon^h{}_{gk}\varepsilon^{ijk}\varepsilon^{cd}{}_h Q^{\{e}_{(0)}, Q^f_{(0)}, Q^{g\}}_{(1)} + ((a,b) \leftrightarrow (c,d)))$$

Quantum affine algebra

$$\left\{Q^{R,+}, Q^{R,-}\right\}_{\mathrm{P}} = -i\,\frac{q^{Q^{R,3}} - q^{-Q^{R,3}}}{q-q^{-1}}\,,$$

$$\left\{\widetilde{Q}^{R,+}, \widetilde{Q}^{R,-}\right\}_{\mathrm{P}} = -i\,\frac{q^{\widetilde{Q}^{R,3}} - q^{-\widetilde{Q}^{R,3}}}{q-q^{-1}}\,,$$

$$\left\{Q^{R,3}, Q^{R,\pm}\right\}_{\mathrm{P}} = \mp i Q^{R,\pm}\,, \quad \left\{\widetilde{Q}^{R,3}, \widetilde{Q}^{R,\pm}\right\}_{\mathrm{P}} = \mp i \widetilde{Q}^{R,\pm}\,,$$

$$\left\{Q^{R,3}, \widetilde{Q}^{R,\pm}\right\}_{\mathrm{P}} = \pm i \widetilde{Q}^{R,\pm}\,, \quad \left\{\widetilde{Q}^{R,3}, Q^{R,\pm}\right\}_{\mathrm{P}} = \pm i Q^{R,\pm}\,,$$

q-Serre relations

$$\left\{Q^{R,\pm}, \left\{Q^{R,\pm}, \left\{Q^{R,\pm}, \widetilde{Q}^{R,\pm}\right\}_{\mathrm{P}}\right\}_{\mathrm{P}}\right\}_{\mathrm{P}} \;=\; -\gamma^2 \left\{Q^{R,\pm}, \widetilde{Q}^{R,\pm}\right\}_{\mathrm{P}} \left(Q^{R,\pm}\right)^2\,,$$

$$\left\{\widetilde{Q}^{R,\pm}, \left\{\widetilde{Q}^{R,\pm}, \left\{\widetilde{Q}^{R,\pm}, Q^{R,\pm}\right\}_{\mathrm{P}}\right\}_{\mathrm{P}}\right\}_{\mathrm{P}} \;=\; -\gamma^2 \left\{\widetilde{Q}^{R,\pm}, Q^{R,\pm}\right\}_{\mathrm{P}} \left(\widetilde{Q}^{R,\pm}\right)^2\,,$$