Quantum Pump

Hisao Hayakawa (YITP, Kyoto University)

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YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Collaborators

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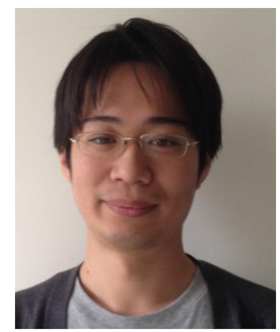
Tatsuro Yuge
(Osaka Univ.)

Ayumu Sugita
(Osaka City Univ.)

Takahiro Sagawa
(Univ. of Tokyo)

Chikako Uchiyama
(Yamanashi Univ.)

Kota Watanabe
(YITP)



Ryoshuke Yoshii
(YITP)

Refs: Yuge et al., PRB86, 23508 (2012)
R. Yoshii & HH, in preparation
C. Uchiyama et al., in preparation

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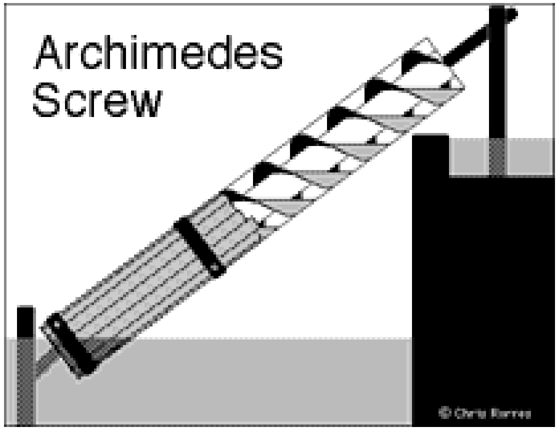
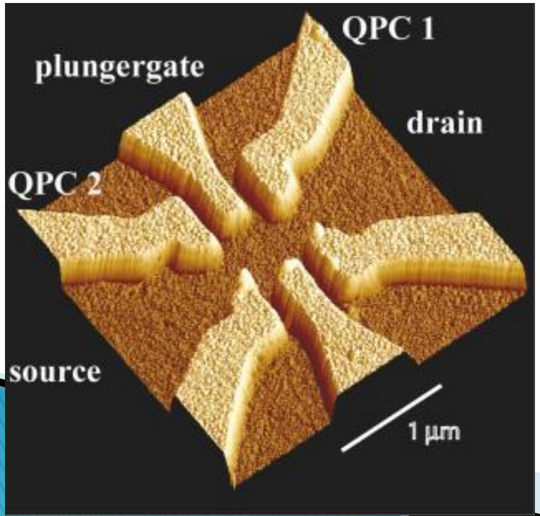
Introduction

- Pump=>We need a bias.

The current can flow in a mesoscopic system without dc bias =>*Quantum pumping*.

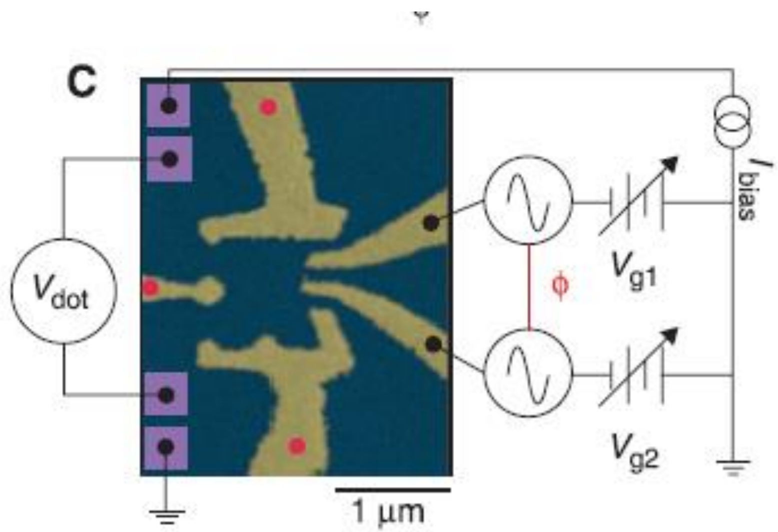


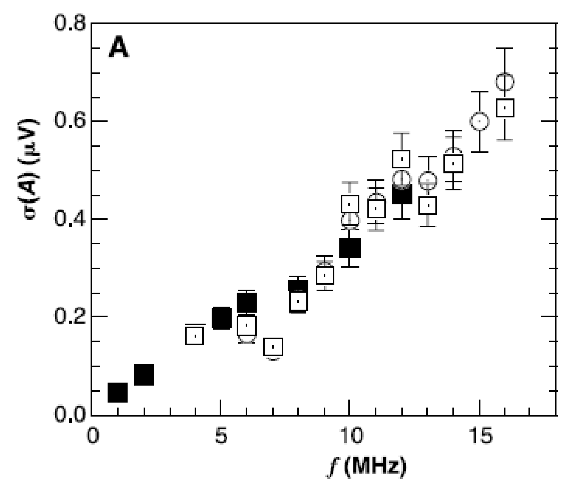


A nano-machine to extract a work

Previous studies

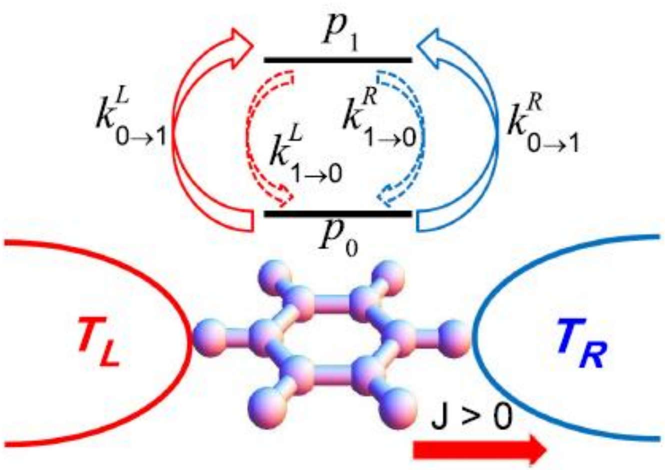
- Experiments
  - Pothier et al. (1992) get a classical pumping for a mesoscopic system.
  - Switkes et al. (1999) control the gate voltage and get 20 electrons current per a cycle in a quantum dot system.

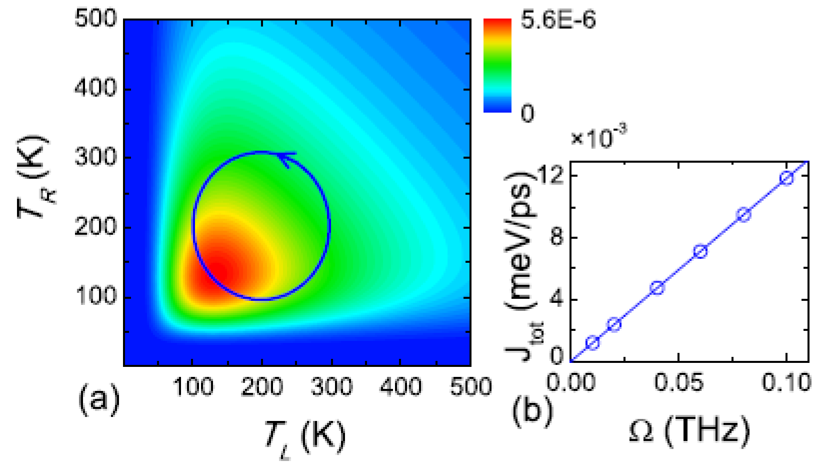




Previous theoretical studies

- Adiabatic quantum pumping (theories)
  - Thouless (1971) for a closed system
  - Open quantum system (P. W. Brower, (1998)).
- Ren-Hänggi-Li (2010) analyzed a spin-boson system and to clarify the role of Berry phase.





Motivation

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- To clarify the general framework of getting adiabatic quantum pumping
- To demonstrate adiabatic pumping for Fermion transport by controlling the bias such as chemical potentials.
- To clarify the role of spins
- To clarify the connection between Berry phase and the path dependent entropy
- To clarify the limitation of the adiabatic treatment

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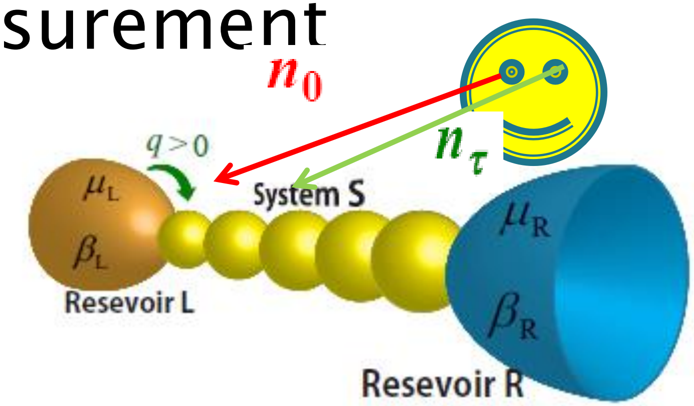
Full-counting statistics(FCS)

- Projection measurement



- Measurement:
  - Number of electrons transfer from left to the system:

$$q = n_\tau - n_0$$

- Distribution $p(q;\tau)$ & cumulant generating function

$$F(\chi;\tau) = \ln \int_{-\infty}^{\infty} \mathrm{d}q \, e^{i\chi q} p(q;\tau)$$

counting field (↑ χ)

→ kth cumulant

$$\langle q^k \rangle_c = \frac{\partial^k}{\partial (i\chi)^k} F(\chi;\tau) \Big|_{\chi=0}$$

How can we compute FCS?

- We assume that the total Hamilitonian satisfies von-Neumann equation.
- We calculate the modified von-Neumann equation via the counting field:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^{\chi}_{\mathrm{tot}}(t) = \frac{1}{i\hbar}\left(\underbrace{\hat{H}^{\chi}_{\mathrm{tot}}}\hat{\rho}^{\chi}_{\mathrm{tot}}(t) - \hat{\rho}^{\chi}_{\mathrm{tot}}(t)\hat{H}^{-\chi}_{\mathrm{tot}}\right)$$

modified total Hamiltonian

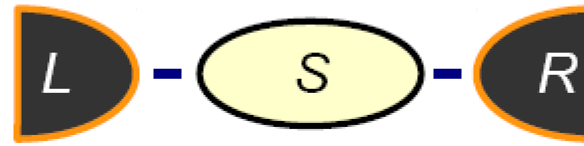
$$\hat{H}^{\chi}_{\mathrm{tot}} = e^{i\chi\hat{N}_L/2}\,\hat{H}_{\mathrm{tot}}\,e^{-i\chi\hat{N}_L/2}$$

- Ref. M. Esposito, *et al.*, RMP **81**, 1665 (2009)

Quantum Master equation

Modified v-N equation for the system + reservoirs

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^{\chi}_{\mathrm{tot}}(t) = \frac{1}{i\hbar}\left(\hat{H}^{\chi}_{\mathrm{tot}}\,\hat{\rho}^{\chi}_{\mathrm{tot}}(t) - \hat{\rho}^{\chi}_{\mathrm{tot}}(t)\hat{H}^{-\chi}_{\mathrm{tot}}\right)$$



Modified Hamiltonian

$$\hat{H}^{\chi}_{\mathrm{tot}} = \hat{H}_S + \hat{H}_L + \hat{H}_R + u\hat{H}^{\chi}_{SL} + u\hat{H}_{\mathrm{SR}}$$

perturbation

eliminate reservoirs' variable $\hat{\rho}^{\chi}(t) = \mathrm{Tr}_{LR}\,\hat{\rho}^{\chi}_{\mathrm{tot}}(t)$

+

Born-Markov approximation

H. P. Breuer & F. Petrucione,
"The Theory of Open Quantum System"

Modified quantum master equation (QME)

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^{\chi}(t) = \frac{1}{i\hbar}\left[\hat{H}_S, \hat{\rho}^{\chi}(t)\right] + u^2\mathcal{D}^{\chi}_L\hat{\rho}^{\chi}(t) + u^2\mathcal{D}_R\hat{\rho}^{\chi}(t)$$

Adiabatic modulation

If modulation of α is adiabatic,

$$F(\chi;\tau) = F_{\text{ex}}(\chi;\tau) + \int_0^\tau \mathrm{d}t\, \underline{\lambda^\chi_{\alpha(t)}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^\chi(t) = \mathcal{K}^\chi_{\alpha(t)}\,\hat{\rho}^\chi(t)$$

Control parameters

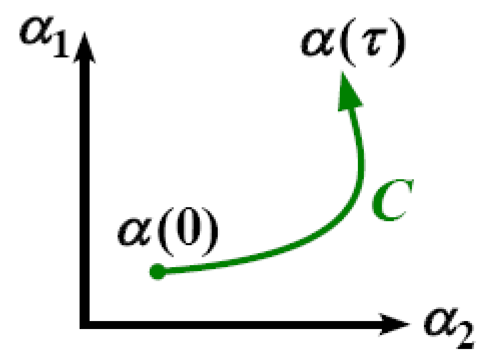
steady-state value at $\alpha(t)$
= eigenvalue of $\mathcal{K}^\chi_{\alpha(t)}$ with max real part

$$F_{\text{ex}}(\chi;\tau) = -\int_C \mathrm{Tr}_S(\hat{l}^{\chi\dagger}_\alpha\, \mathrm{d}\hat{\rho}^\chi_\alpha) + (\text{surface terms})$$

left eigenvector of $\mathcal{K}^\chi_\alpha$
(corresponding to λ^χ_α)

right eigenvector of $\mathcal{K}^\chi_\alpha$
(corresponding to λ^χ_α)

total differential wrt α



extension of Berry-Sinitsyn-Nemenman (BSN) phase in adiabatic pump to quantum systems

N. A. Sinitsyn & I. Nemenman, EPL **77**, 58001 (2007)

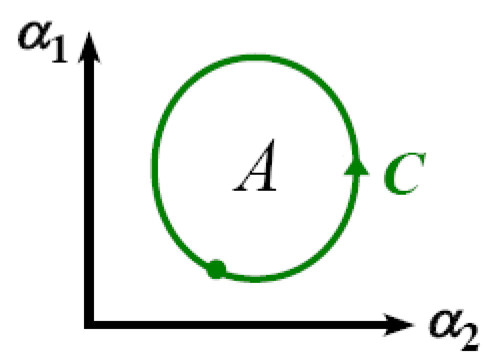
Net current in an adiabatic cycle

$$\langle q\rangle_{\tau_c}^{\mathrm{ex}} = -\oint_C \mathrm{Tr}_S(\hat{l}_\alpha'^{\dagger}\, \mathrm{d}\hat{\rho}_\alpha^{\mathrm{ss}}) = \frac{\partial}{\partial(i\chi)} F_{\mathrm{ex}}(\chi;\tau)\Big|_{\chi=0}$$

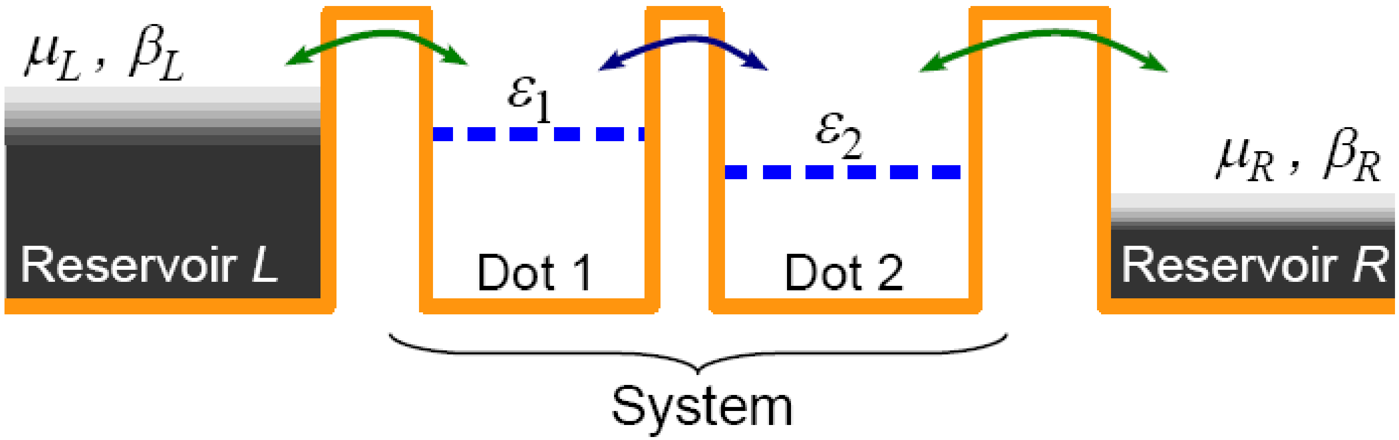
$$= -\frac{1}{2}\sum_{mn}\int_A \mathrm{Tr}_S\left(\frac{\partial \hat{l}_\alpha'^{\dagger}}{\partial\alpha_m}\frac{\partial\hat{\rho}_\alpha^{\mathrm{ss}}}{\partial\alpha_n} - \frac{\partial \hat{l}_\alpha'^{\dagger}}{\partial\alpha_n}\frac{\partial\hat{\rho}_\alpha^{\mathrm{ss}}}{\partial\alpha_m}\right)\mathrm{d}\alpha_m\wedge\mathrm{d}\alpha_n$$

BSN curvature for average $\equiv \Omega_{\alpha_m\alpha_n}$

where $\hat{l}_\alpha' = \frac{\partial}{\partial(i\chi)}\hat{l}_\alpha^{\chi}\Big|_{\chi=0}$



Explicit calculation on double quantum dots (spinless)



$$\hat{H}_{\rm tot} = \hat{H}_L + u\hat{H}_{1L} + \hat{H}_1 + \hat{H}_{12} + \hat{H}_2 + u\hat{H}_{2R} + \hat{H}_R$$

The interaction between dots plays an important role.

Reservoir

$$\hat{H}_\nu = \sum_k \varepsilon_k \hat{c}^\dagger_{\nu k}\hat{c}_{\nu k} \quad (\nu = L, R)$$

System-Reservoir

$$\hat{H}_{1L} = \sum_k \left(V_{Lk}\hat{d}^\dagger_1 \hat{c}_{Lk} + V^*_{Lk}\hat{c}^\dagger_{Lk}\hat{d}_1\right)$$

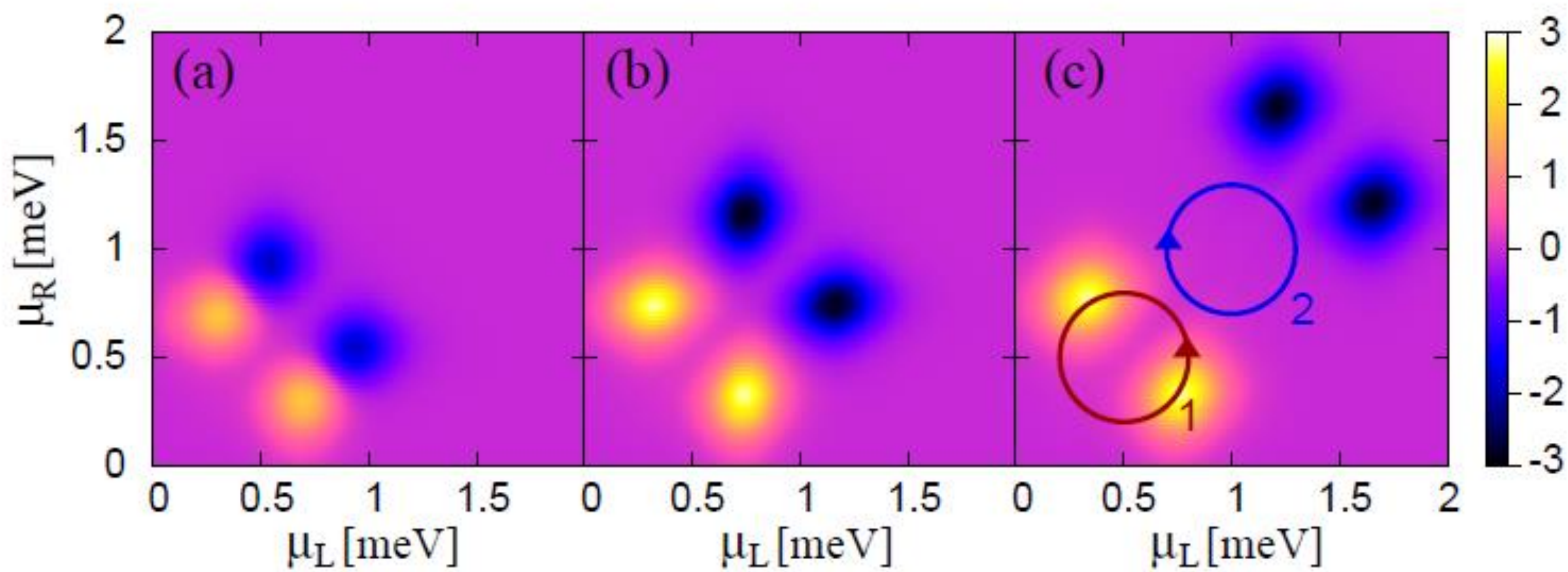
$$\hat{H}_{2R} = \sum_k \left(V_{Rk}\hat{d}^\dagger_2 \hat{c}_{Rk} + V^*_{Rk}\hat{c}^\dagger_{Rk}\hat{d}_2\right)$$

Results for interacting case

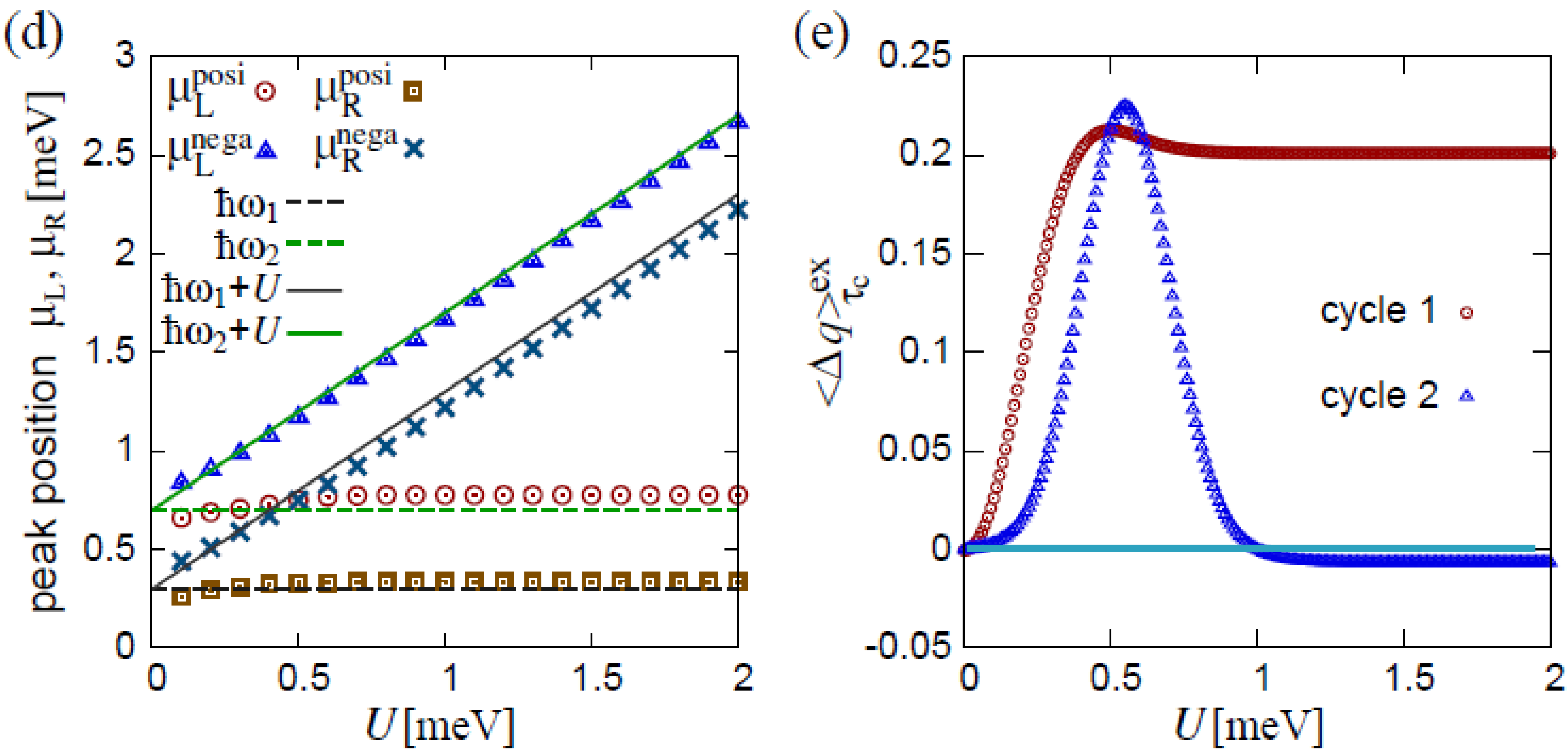
- We introduce the interaction between dots as

$$\hat{H}_{\mathrm{S}} = \sum_{i=1,2} \varepsilon_i \hat{d}_i^\dagger \hat{d}_i + v(\hat{d}_1^\dagger \hat{d}_2 + \hat{d}_2^\dagger \hat{d}_1) + U \hat{d}_1^\dagger \hat{d}_1 \hat{d}_2^\dagger \hat{d}_2.$$

- The results depend on the value of U.



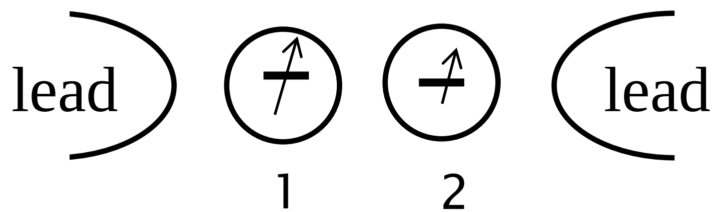
Results (2)



(d)
peak position μ_L, μ_R [meV]
μ_L^{posi} μ_R^{posi}
μ_L^{nega} μ_R^{nega}
$\hbar\omega_1$
$\hbar\omega_2$
$\hbar\omega_1+U$
$\hbar\omega_2+U$
U [meV]
(e)
$<\Delta q>_{\tau_c}^{ex}$
cycle 1
cycle 2
U [meV]

The effect of spins

- Pumping through Intra dot repulsion through spins



- Double dots model

$$H = \sum_{k,\sigma} \varepsilon_k a^\dagger_{k\sigma} a_{k\sigma} + H_{1L} + H_{2R} + H_{12}$$

$$H_{J\beta} = 2J_\beta \sum_{k,k'} \mathbf{S}_\beta \cdot (\mathbf{s}_\beta)_{k,k'}$$ β=1 or 2

$$H_{12} = J_{12} \mathbf{S}_1 \cdot \mathbf{S}_2$$

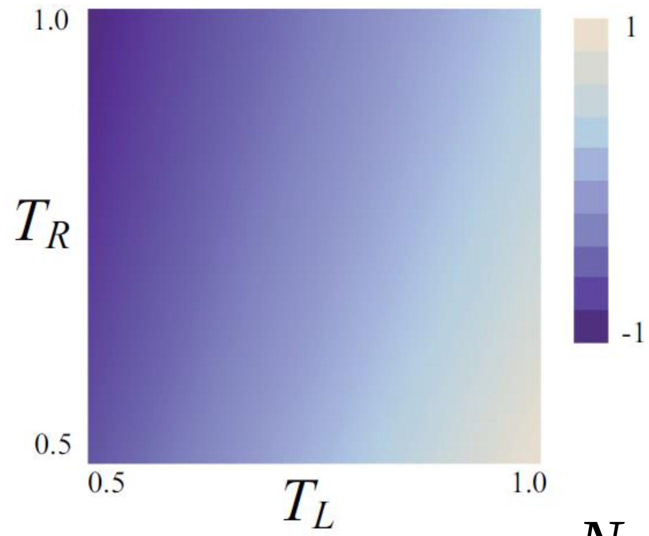
dot spin

$$\mathbf{s}_{\beta,k,k'} = \sum_{s,s'} a^\dagger_{\beta,k,s} \sigma_{s,s'} a_{\beta,k,s'}$$

spin for conduction electrons

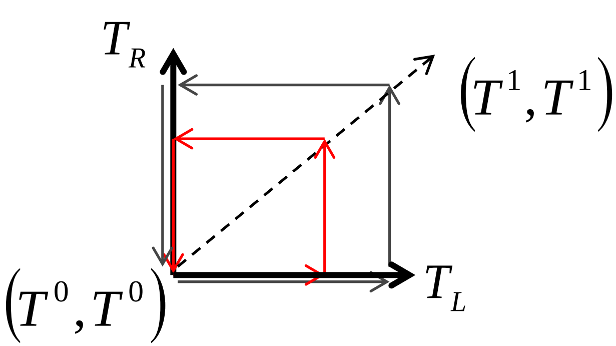
The result of pumping current

- Curvature in the parameter space

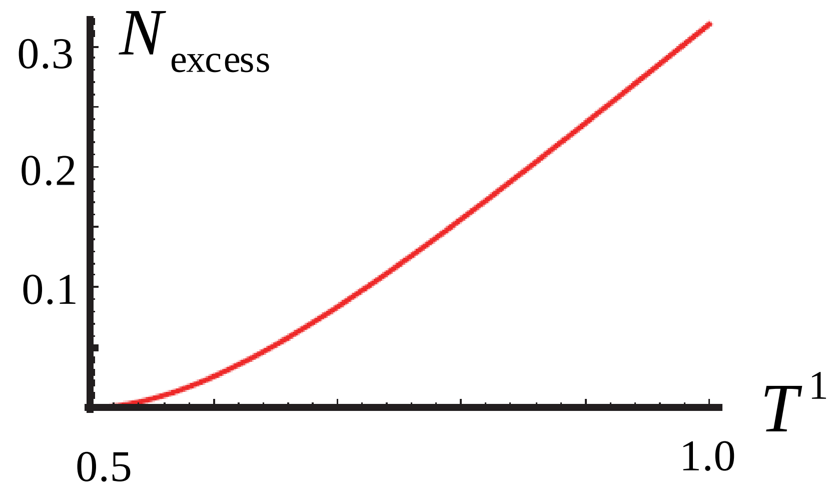


- Pumping current

$$N_{\text{excess}} = J_{12} F_C$$



$J = 1 \quad T^0 = 0.5$



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Application to entropy production

- The calculation of quantum pumping is applicable to entropy production.
- Actually, non-equilibrium entropy is a geometric quantity which depends on the path
- Non-equilibrium entropy is defined by

If $\hat{Z} = \sum_{\nu} \beta_{\nu}(\hat{H}_{\nu} - \mu_{\nu}\hat{N}_{\nu})$

then $\Delta z =$ entropy production σ

Appropriate definition from thermodynamics

- See Sagawa and HH, PRE 84, 051110 (2011).

Equilibrium and weakly non-equilibrium cases

- Our entropy reproduces known results for equilibrium entropy.

$$S_{\mathrm{vN}}(\hat{\rho}_{\mathrm{gc}}) := -\mathrm{Tr_S}[\hat{\rho}_{\mathrm{gc}} \ln \hat{\rho}_{\mathrm{gc}}]$$

$$\hat{\rho}_{\mathrm{gc}}(\beta, \beta\mu) := e^{-\beta \hat{H}_{\mathrm{S}} + \beta\mu \hat{N}_{\mathrm{S}}} / Z_{\mathrm{gc}}(\beta, \beta\mu)$$

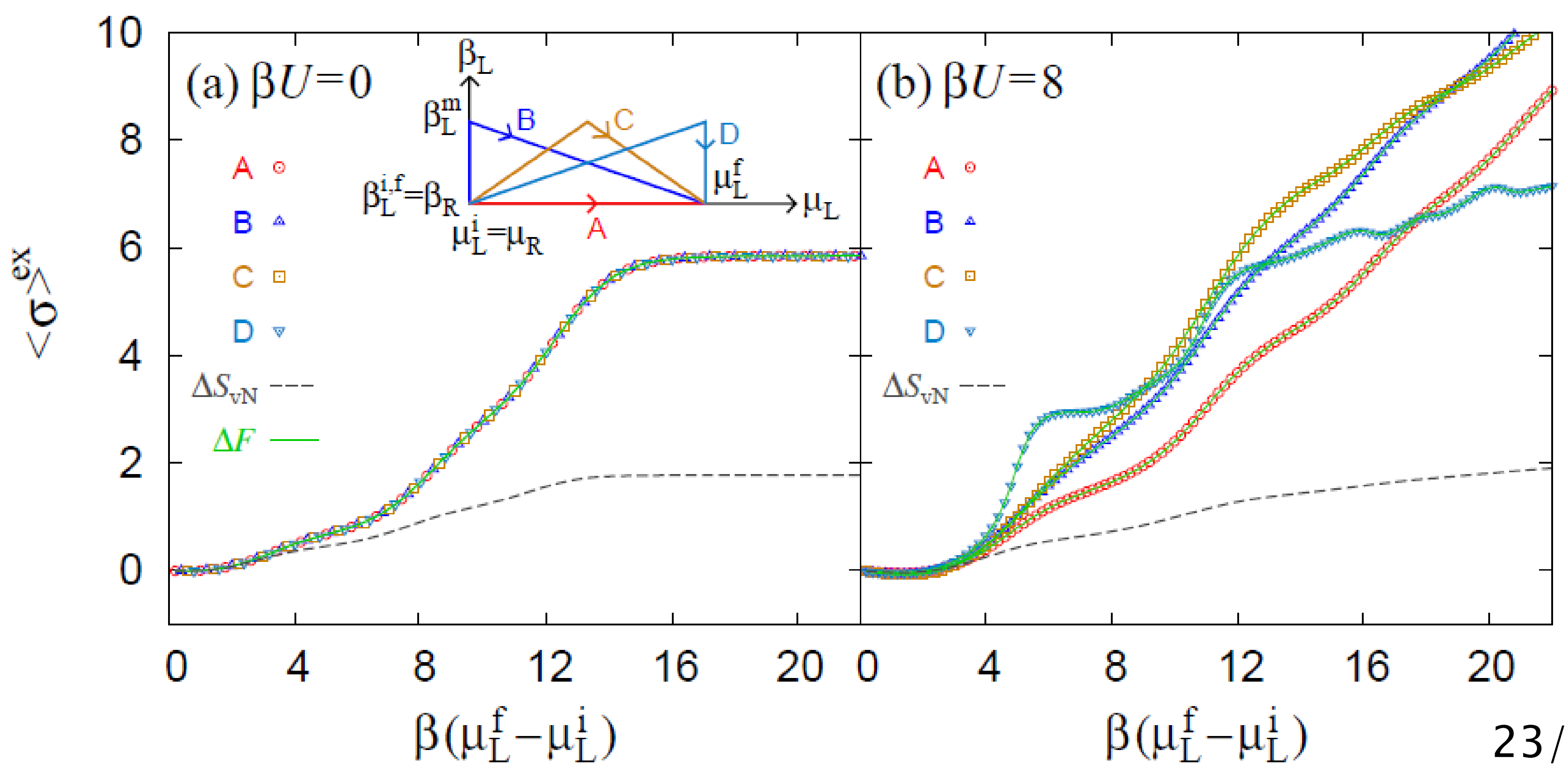
- In weakly non-equilibrium case, the entropy satisfies the symmetrized von-Neumann entropy (by KNST for classical systems and Saito and Tasaki for quantum systems)

$$S_{\mathrm{sym}}(\hat{\rho}) := -\frac{1}{2}\mathrm{Tr_S}[\hat{\rho}(\ln \hat{\rho} + \ln \hat{\mathcal{T}} \hat{\rho} \hat{\mathcal{T}})]$$

$\hat{\mathcal{T}}$ is the time-reversal operator

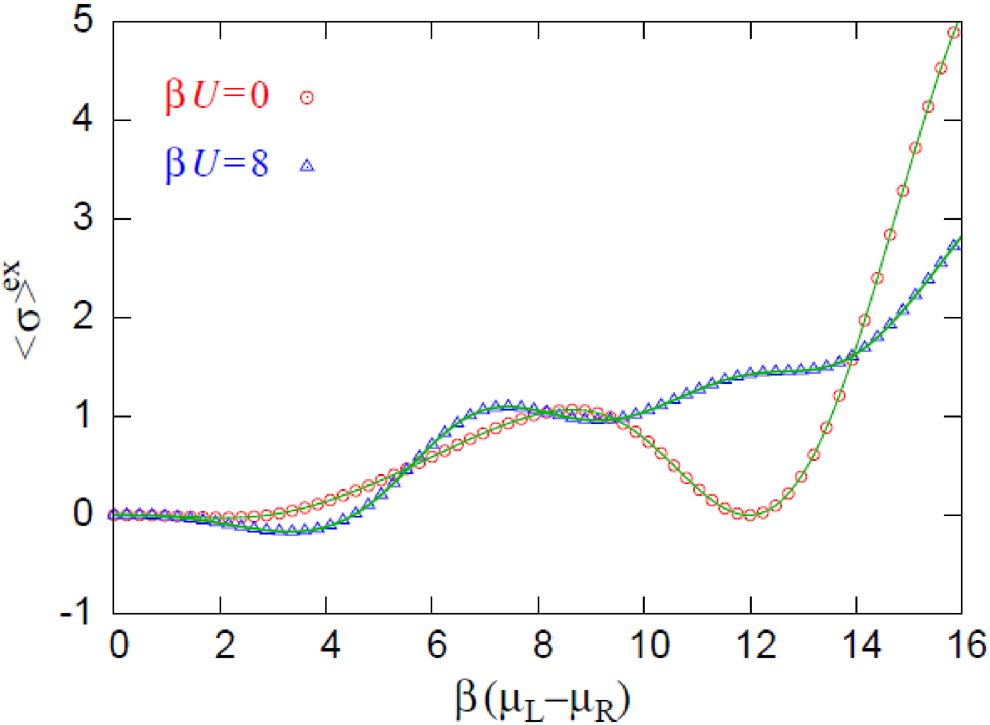
Path dependent entropy

- We demonstrate the existence of path dependent entropy.



Four dots system

- The entropy production exists even for non-interacting model through a cyclic modulation of energy levels.



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Adiabatic and Markovian approximation

- So far, all obtained results are based on adiabatic and Markovian approximation.
- Of course, this is true for very slow modulation, but how can we verify its validity for realistic situations.
- For this purpose, we study the spin-boson system in details as an extension of Ren-Hänggi-Li (2010) for the non-adibatic and non-Markovian case.

Spin-Boson model

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Non-Markovian weak coupling equation

$$\frac{d}{dt}\hat{\rho}(\chi,t) = \frac{1}{i\hbar}[\hat{H}_{\mathrm{S}},\hat{\rho}(\chi,t)] - \frac{1}{\hbar^2}\int_0^t d\tau \mathrm{Tr}_E\left[\hat{H}^{\chi}_{\mathrm{SE}},[\hat{H}^{\chi}_{\mathrm{SE}}(-\tau),\hat{\rho}(\chi,t)\hat{\rho}^{\mathrm{eq}}_{\mathrm{E}}]_{\chi}\right]_{\chi}$$

$$\hat{H}_{\mathrm{S}} = \sum_{n=0,1}\varepsilon_n|n\rangle\langle n| \qquad (\varepsilon_1-\varepsilon_0=\hbar\omega_0)$$

$$\hat{H}_{\mathrm{E},\nu} = \sum_k \hbar\omega_{k,\nu}\hat{b}^{\dagger}_{k,\nu}\hat{b}_{k,\nu}$$

$$\hat{H}_{\mathrm{SE},\nu} = (|0\rangle\langle 1| + |1\rangle\langle 0|)\sum_k \hbar g_{k,\nu}(\hat{b}^{\dagger}_{k,\nu}+\hat{b}_{k,\nu})$$

Spectrum density is the Ohmic model.

$$h_\nu(\omega) = \sum_k g^2_{k,\nu}\delta(\omega-\omega_{k,\nu})$$

$$h_\nu(\omega) = s_\nu\omega e^{-\omega/\omega_{c,\nu}}$$

Non-adiabatic Markovian process

$$\langle \Delta q \rangle_t = \langle \Delta q \rangle_0 + \langle \Delta q \rangle_{\mathrm{Ma}}^{\mathrm{dyn}} - \int_0^t dt' \langle l'_+(t') | \dot{\rho}(0,t') \rangle$$

$$- \int_0^t dt' \langle l'_+(t') | \dot{\rho}(0,t') \rangle$$ <= Excess term

$$= - \int_0^t dt' \left(\underbrace{\underbrace{\sum_{\nu=L,R} \dot{\beta}_\nu \langle l'_+(t') | \frac{\partial}{\partial \beta_\nu} | \rho(0,t') \rangle}_{(2)} + \langle l'_+(t') | \frac{\partial}{\partial t'} | \rho(0,t') \rangle}_{(1)} \right)$$

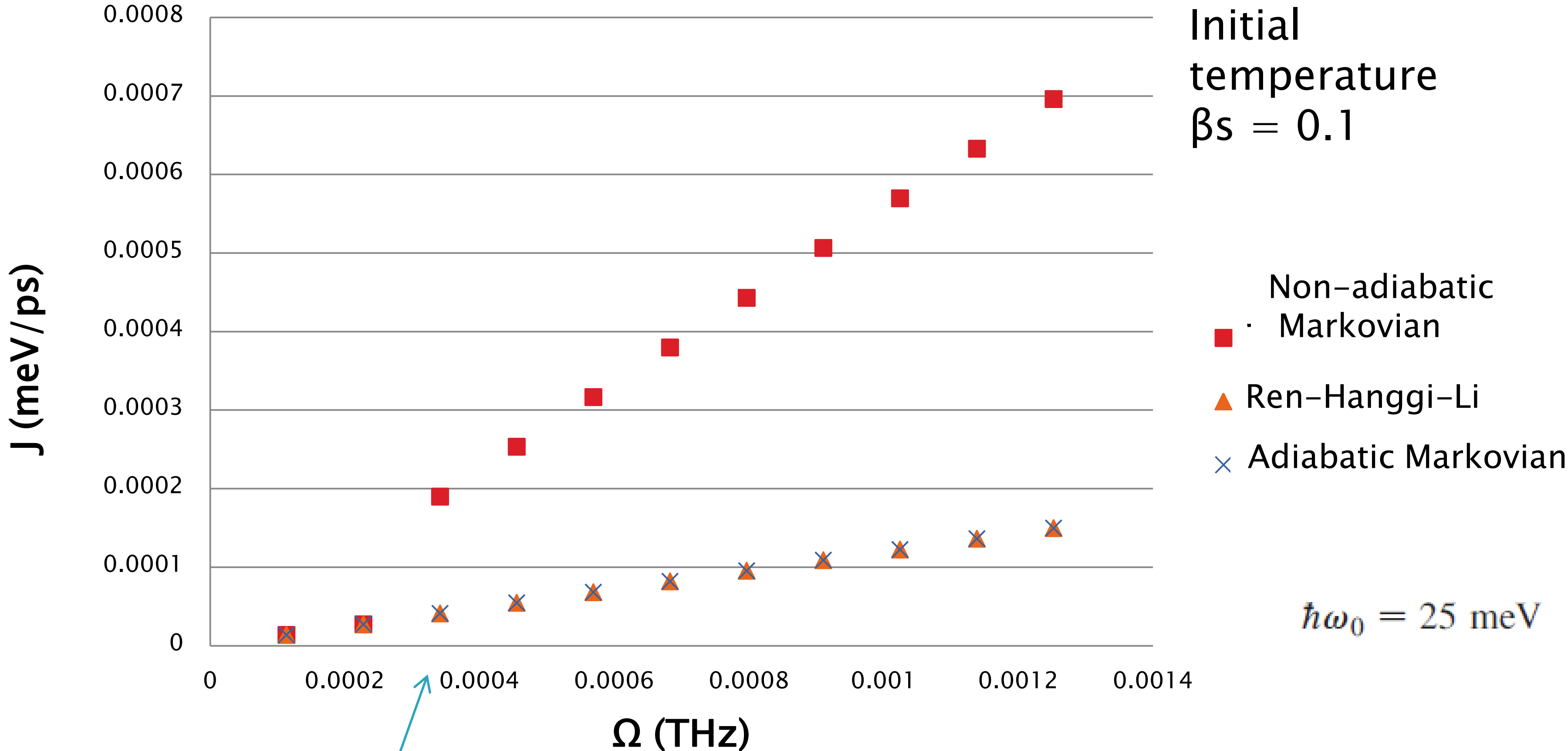
Parameter modulation

$$T_L(t) = 200 + 100 \cos(\Omega t + \pi/4)$$
$$T_R(t) = 200 + 100 \sin(\Omega t + \pi/4)$$

Dynamical phase is not important but
the direct time dependence of
the density matrix is dominant.

Non-adiabatic Markovian process

Note that Ω >0.01 for Ren-Hanggi-Li



Initial temperature βs = 0.1

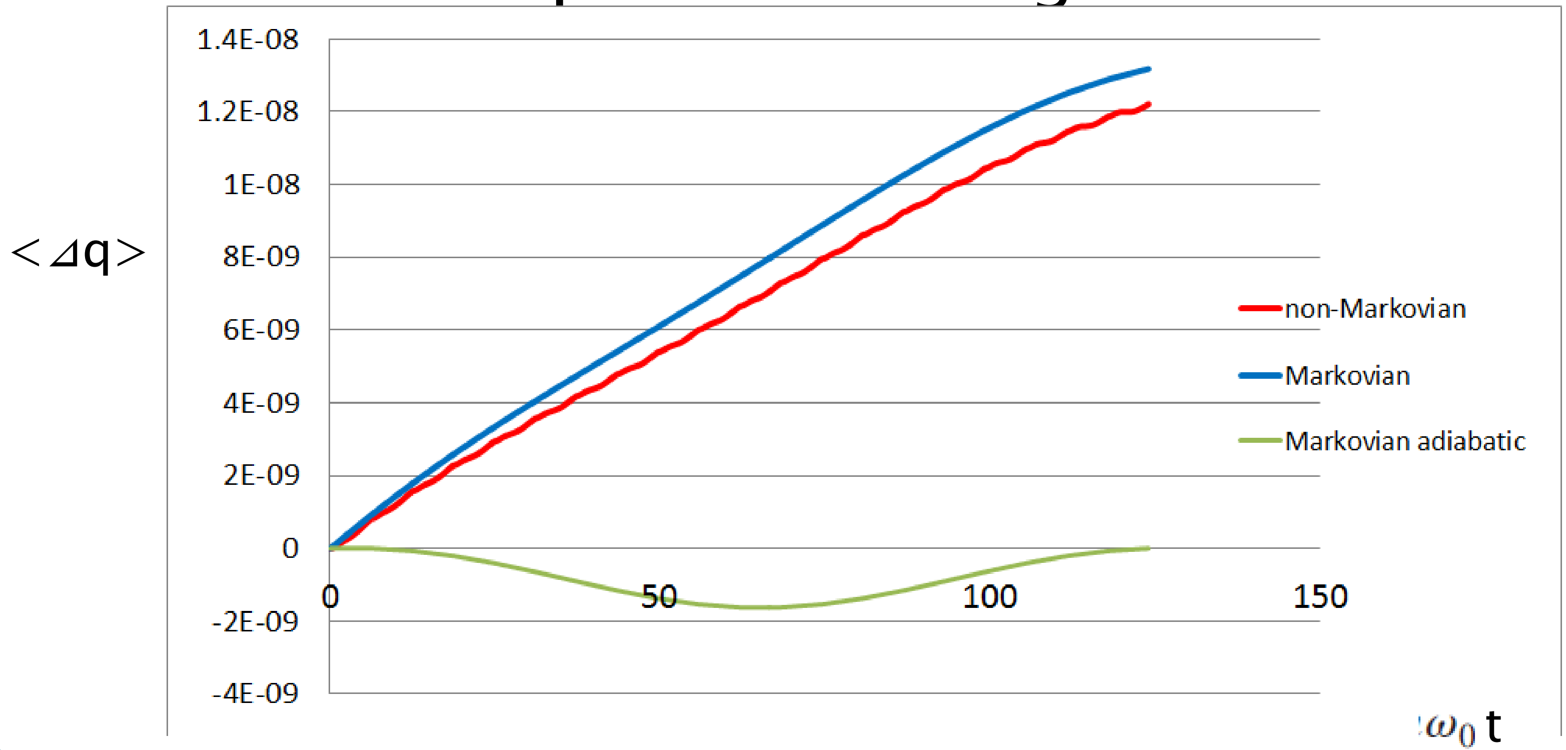
$\hbar\omega_0 = 25$ meV

It looks a discontinuous jump.

The discontinuous change occurs at Ω = 2.32 $\times 10^{-4}$ THz

Non-adiabatic & non-Markovian process

- We can calculate non-adiabatic & non-Markovian process for high Ω



This non-Markovian result is obtained from 20 divided calculation in each period.

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Summary

- We have analyzed a quantum pumping effect on Fermion transport.
- We confirm that there exist geometric effects if there exists the interaction for spinless Fermions.
- Spin effect is also obtained for Kondo problem.
- Such an idea can be used for entropy production.
  - Geometric effects produces the path dependent entropy.
- We have confirmed the region that adiabatic and Marovian approximation can be used from the calculation of a spin-boson system.

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Thank you for your attention.