# Exotic Superconductivity
# A matter of Symmetry and Topology

Feb 2013

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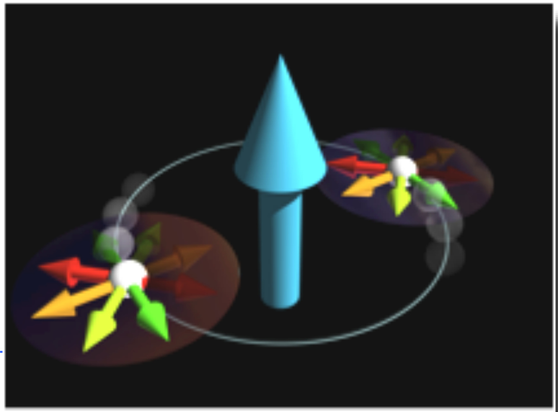
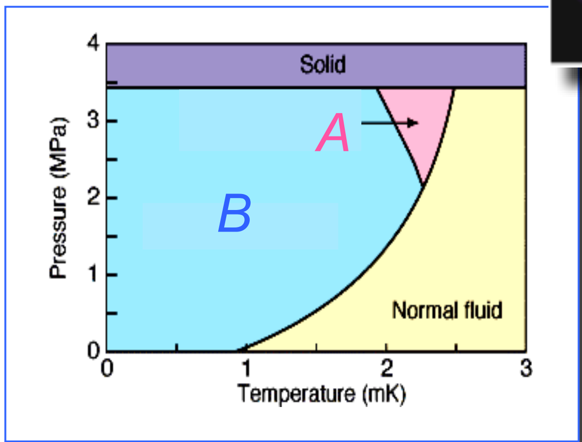
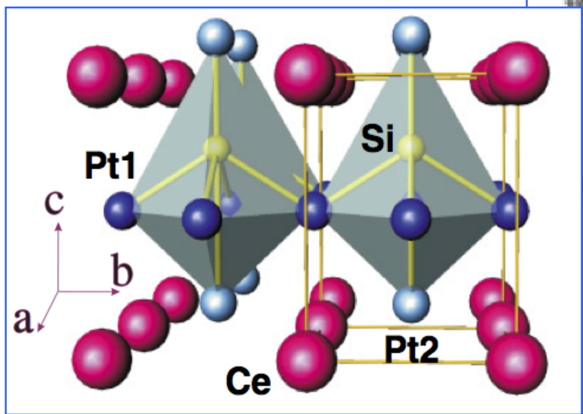
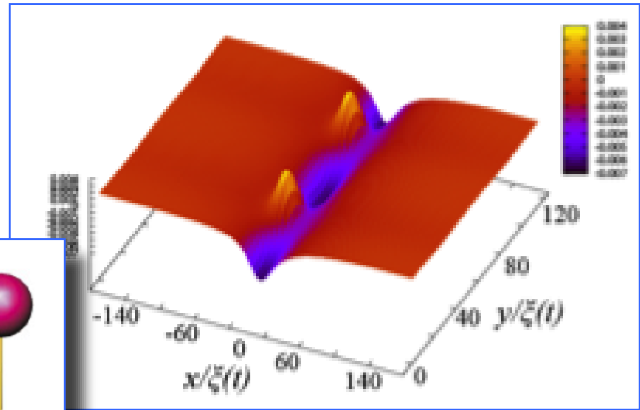
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Global COE Program

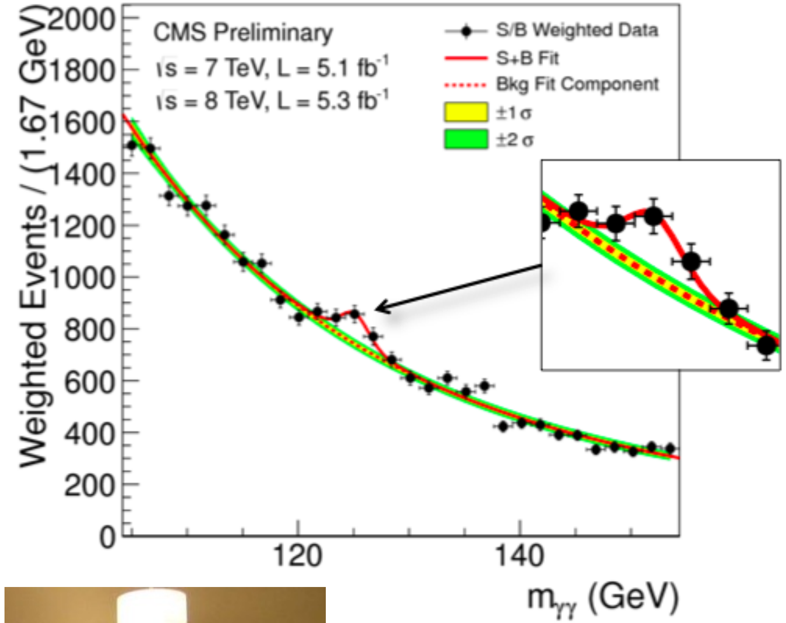


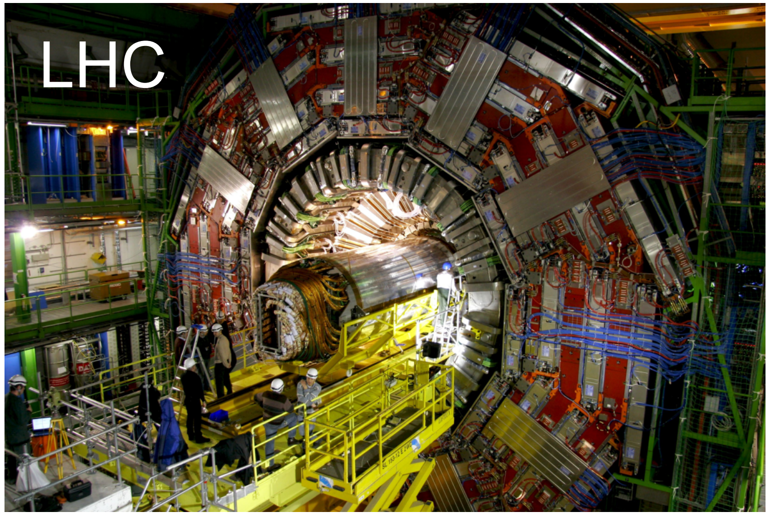


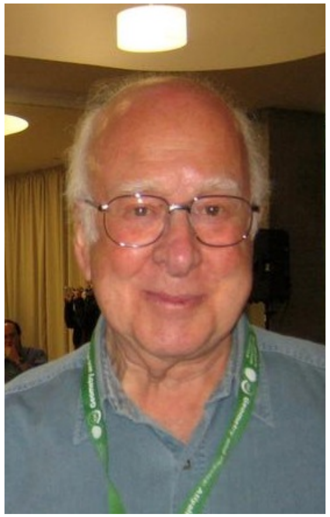



# Hunting the Higgs - finally a happy end

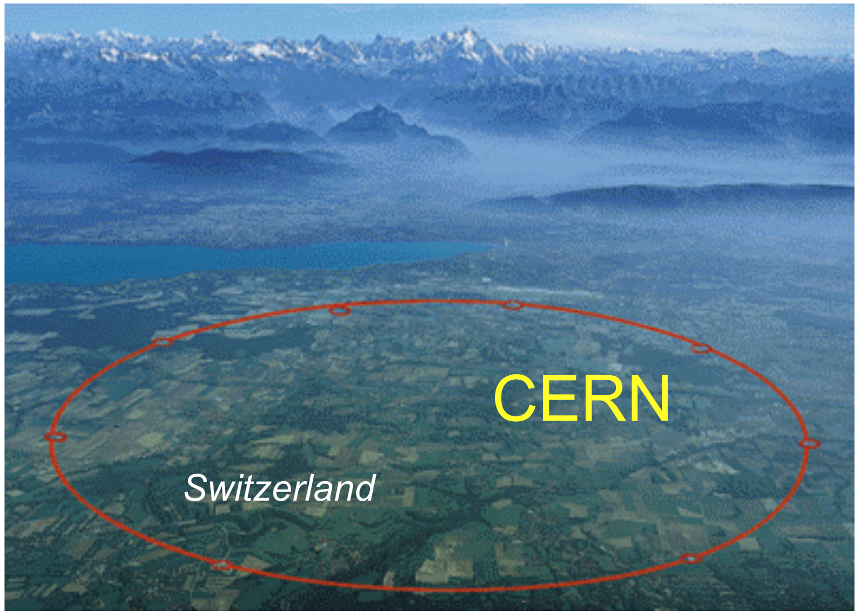


P. Higgs

P.W. Anderson



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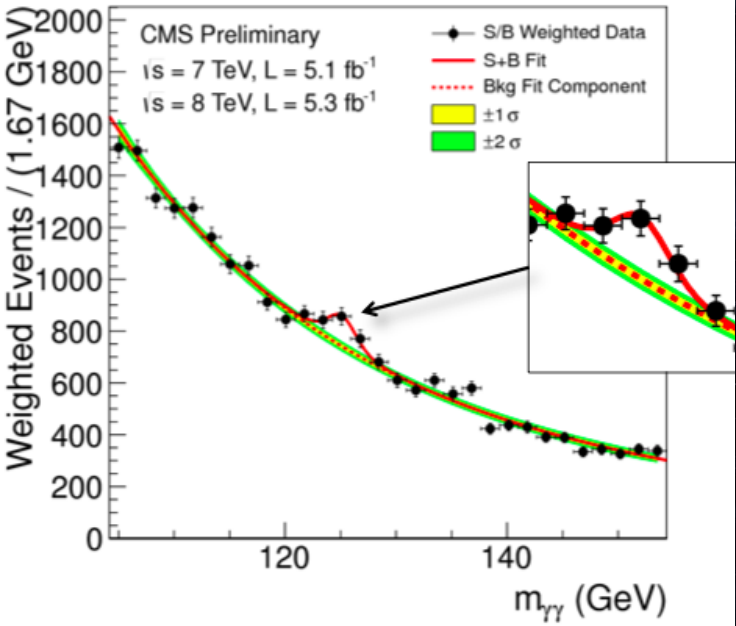


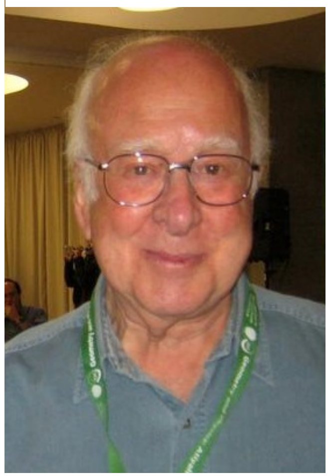

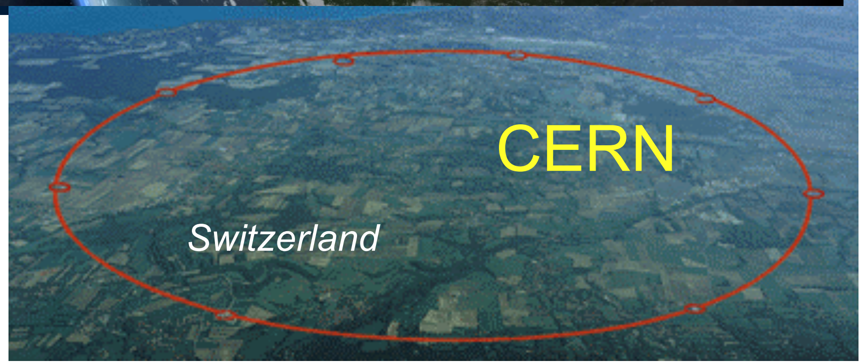


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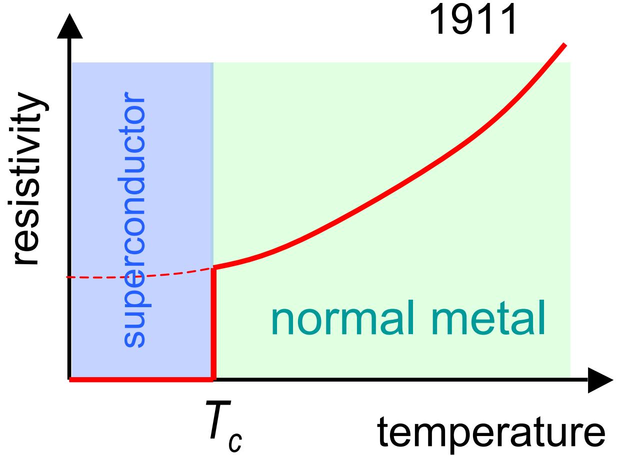
# Phenomenon "Superconductivity"

vanishing electrical resistance



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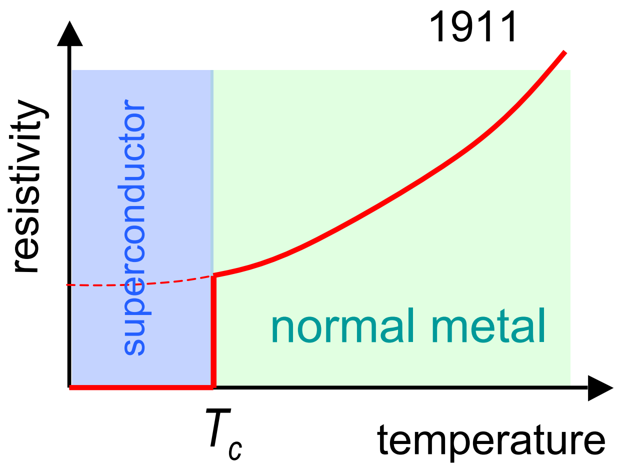
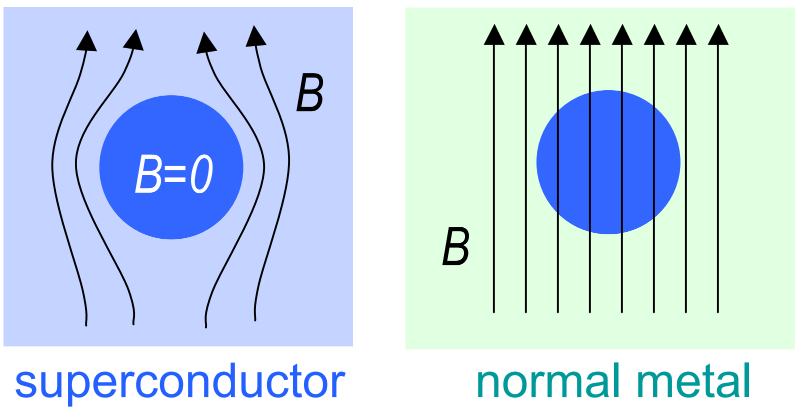



Meissner-Ochsenfeld effect 1933



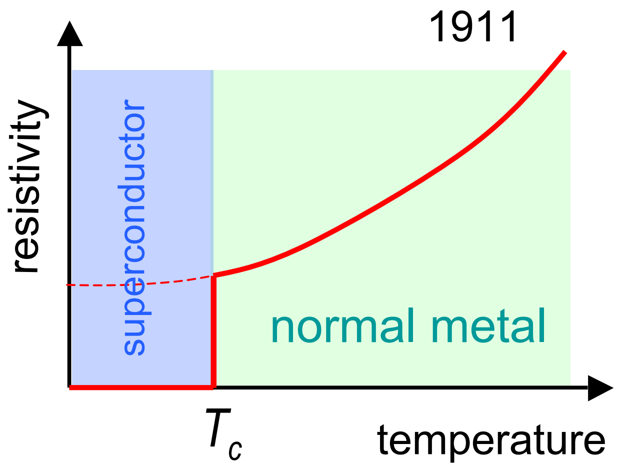

London equation

$$\vec{\nabla}^2 \vec{B} - \lambda^{-2} \vec{B} = 0$$

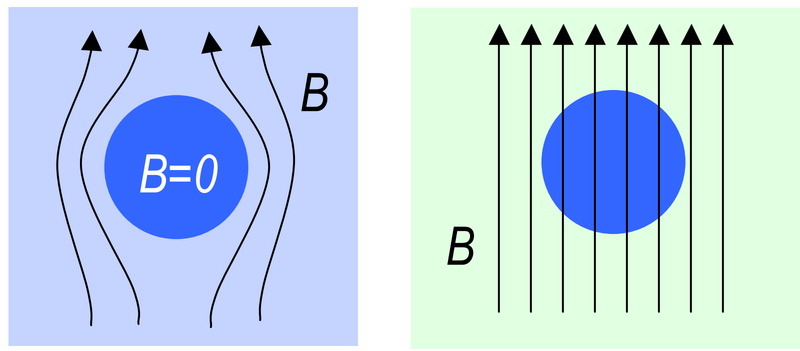
screening of magnetic field

# Phenomenon "Superconductivity"

## vanishing electrical resistance

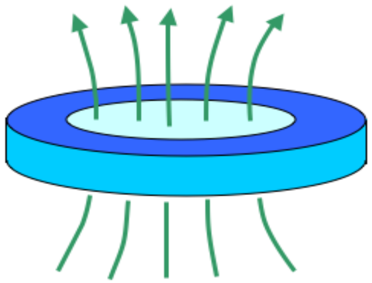



## Meissner-Ochsenfeld effect 1933




superconductor normal metal

## flux quantization



$$\Phi = n\Phi_0 = n\frac{hc}{2e}$$

## London equation

$$\vec{\nabla}^2\vec{B} - \lambda^{-2}\vec{B} = 0$$

screening of magnetic field

## Ginzburg-Landau theory

$$\Psi(\vec{r}) = |\Psi(\vec{r})|e^{i\phi(\vec{r})}$$

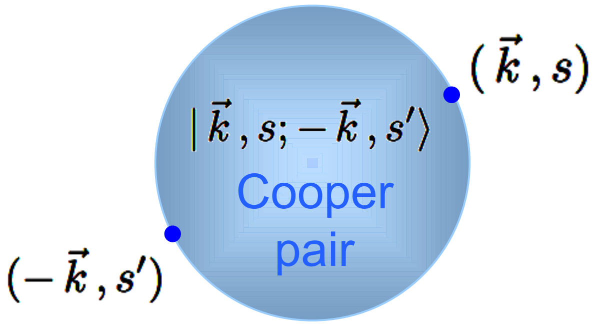
complex macroscopic condensate wavefunction

spontaneously broken *U(1)* - gauge symmetry

# Superconducting Condensate

## Bardeen-Cooper-Schrieffer

*superconductivity as a Fermi surface instability*




electrons of opposite momenta correlate to form a coherent state of Cooper pairs

$$|\Phi_{BCS}\rangle = \bigotimes_{\vec{k}\,,s,s'} \left\{ u_{\vec{k}\,,ss'} + v_{\vec{k}\,,ss'} |\vec{k}\,, s; -\vec{k}\,, s'\rangle \right\}$$

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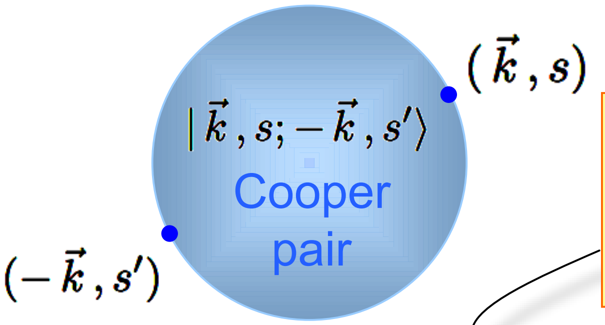


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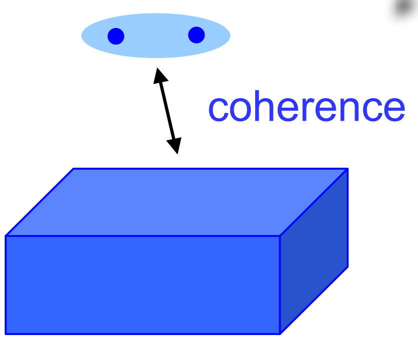
add/remove freely
Cooper pairs



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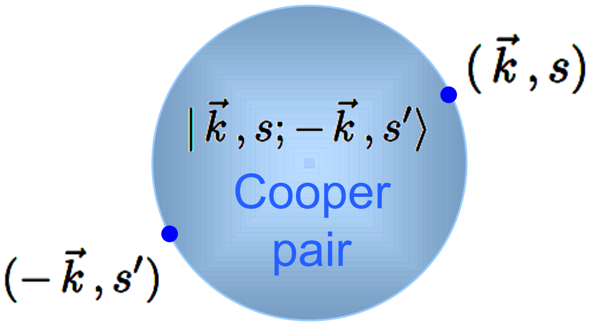


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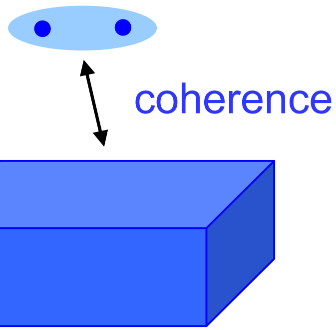


## pair wave function

$$\psi_{\vec{k},ss'} = \langle \Phi_{BCS} | c_{-\vec{k}\, s'} c_{\vec{k}\, s} | \Phi_{BCS} \rangle$$

$$= u^*_{\vec{k},ss'} v_{\vec{k},ss'}$$

# Superconducting Condensate

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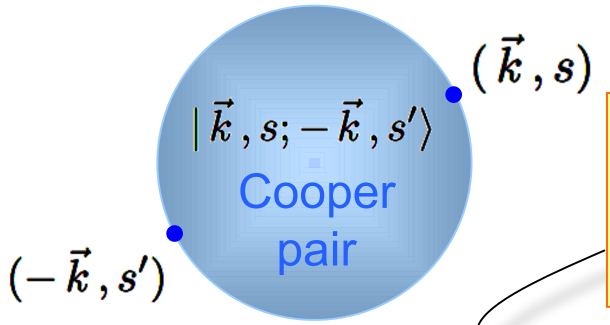


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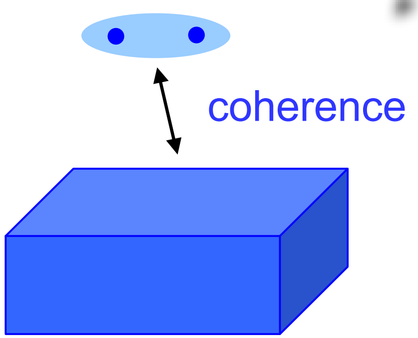
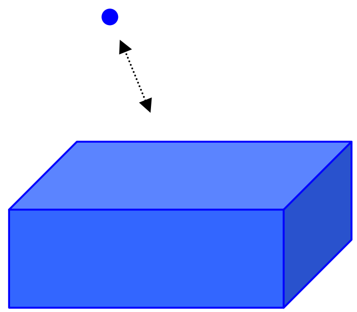
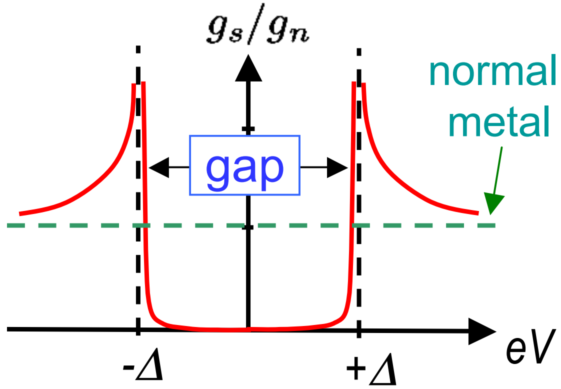



"hard" to add/remove low-energy electron



electron tunneling



# Superconducting Condensate

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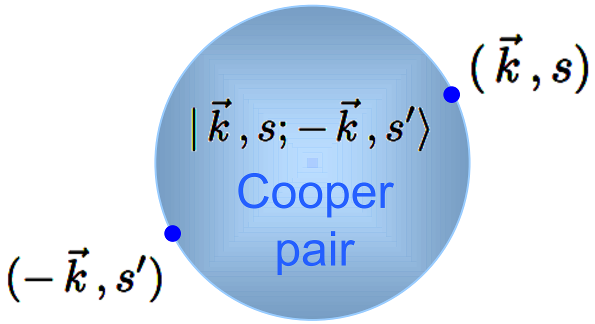


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orbital & spin symmetry

| orbital | spin |
|---|---|
| angular momentum | $|1/2\rangle \otimes |1/2\rangle$ |
| $\ell$ | spin singlet |
| parity $(-1)^\ell$ | spin triplet |

# Superconducting Condensate

Bardeen-Cooper-Schrieffer

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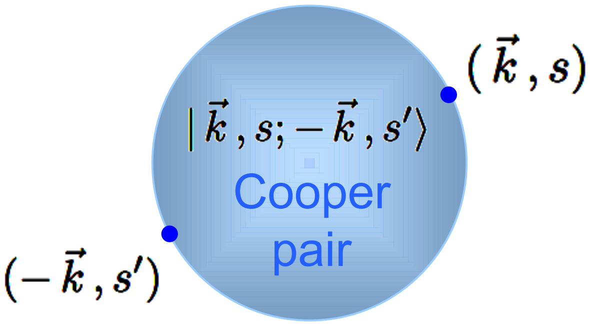


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## Pauli principle

| spin singlet | spin triplet |
| --- | --- |
| ⬍ | ⬍ |
| even parity | odd parity |

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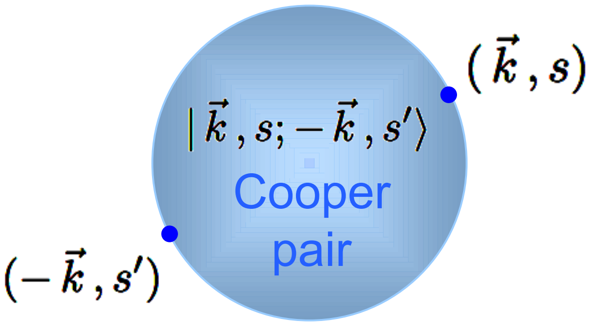


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### Pauli principle

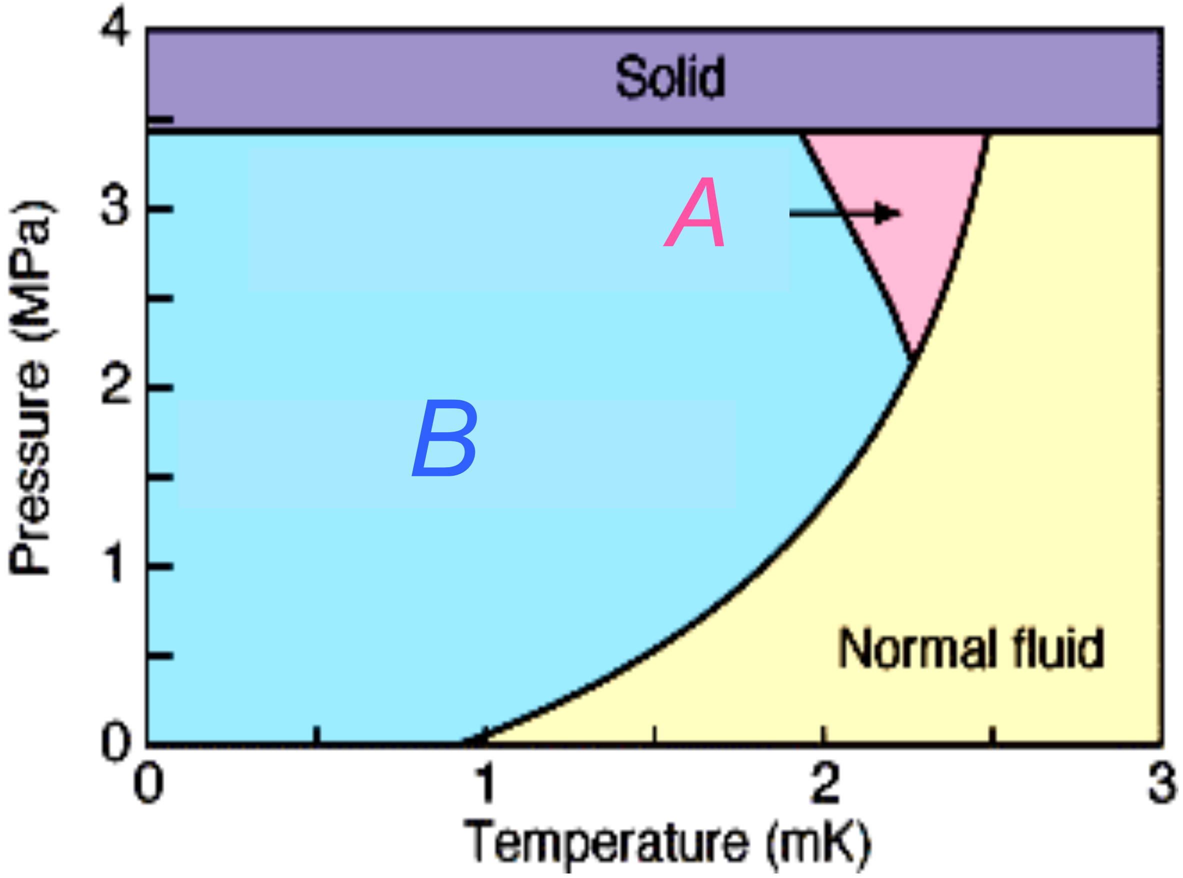
| spin singlet | spin triplet |
|---|---|
| ↕ | ↕ |
| even parity | odd parity |

$\ell = 0$, $S = 0$ — most symmetric "conventional"

$\ell \neq 0$, $S = 0, 1$ — lower symmetry "unconventional"

# Helium-3 - unconventional superfluid

superfluid A- and B- phase

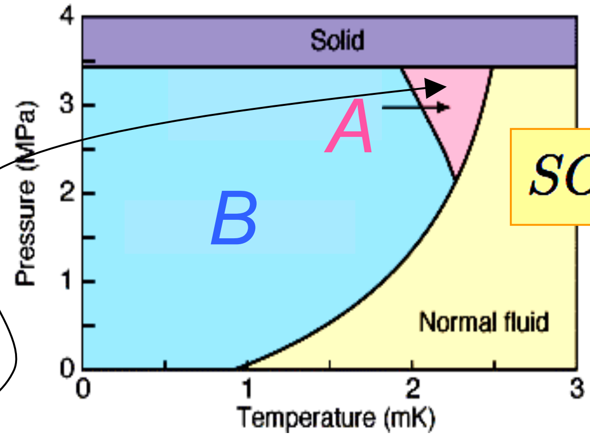


# Helium-3 - unconventional superfluid

pair wave function

$$\hat{\Psi} = \begin{pmatrix} \psi_{\uparrow\uparrow} & \psi_{\uparrow\downarrow} \\ \psi_{\downarrow\uparrow} & \psi_{\downarrow\downarrow} \end{pmatrix}$$

superfluid Helium 3




normal state symmetry

$$SO(3) \times SU(2) \times \mathcal{K} \times U(1)_\phi$$

## A-phase:

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & k_x \pm i k_y \\ k_x \pm i k_y & 0 \end{pmatrix}$$

$$U(1)_{S_z} \times U(1)_{L_z + \phi}$$

broken time reversal symmetry

chiral phase

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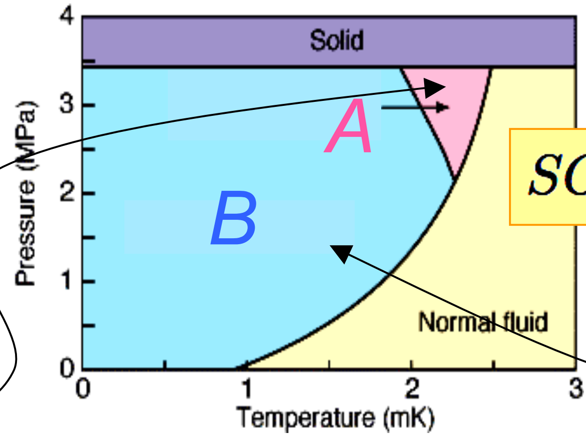


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## B-phase:

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} -k_x + i k_y & k_z \\ k_z & k_x + i k_y \end{pmatrix}$$

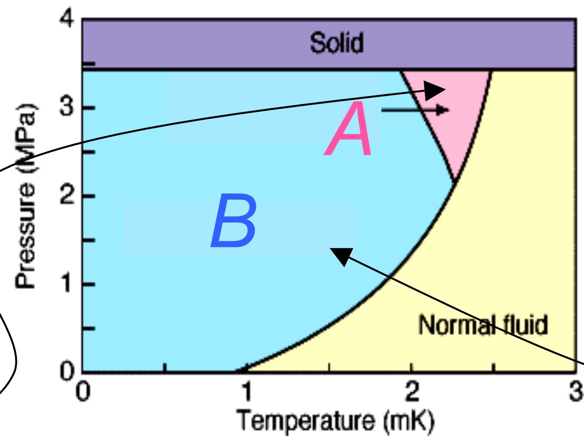
$$SO(3)_{L+S} \times \mathcal{K}$$

dynamical spin-orbit coupling

helical phase

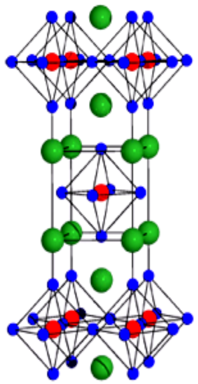
# Helium-3 - unconventional superfluid

superfluid Helium 3




$Sr_2RuO_4$ Maeno et al. (1994)

transition metal oxide



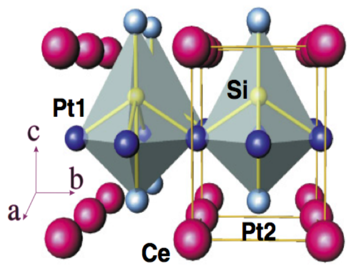
quasi-two-dimensional metal

$$T_c \approx 1.5K$$

*chiral p-wave phase*

$CePt_3Si$ Bauer et al. (2004)

heavy Fermion compound



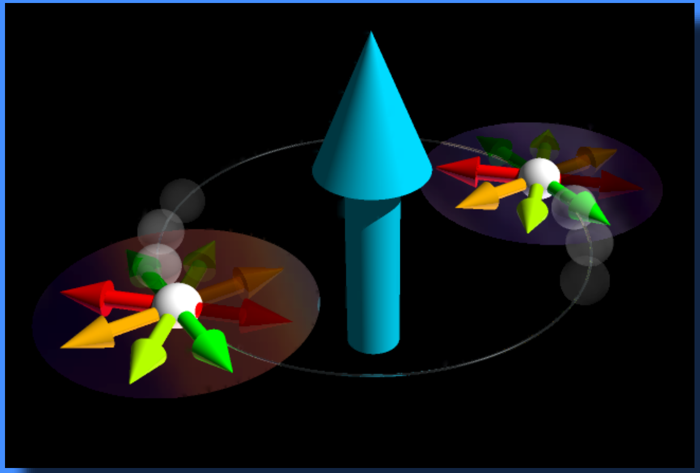

non-centrosymmetric crystal

$$T_c \approx 0.5K$$

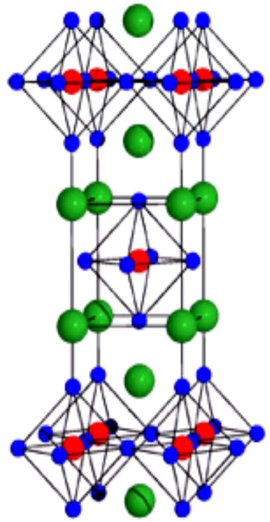
*mixed-parity phase*

$Sr_2RuO_4$

chiral p-wave superconductor



# $Sr_2RuO_4$ - chiral p-wave superconductor



$T_c \approx 1.5K$

Maeno et al 1994

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & k_x \pm i k_y \\ k_x \pm i k_y & 0 \end{pmatrix}$$

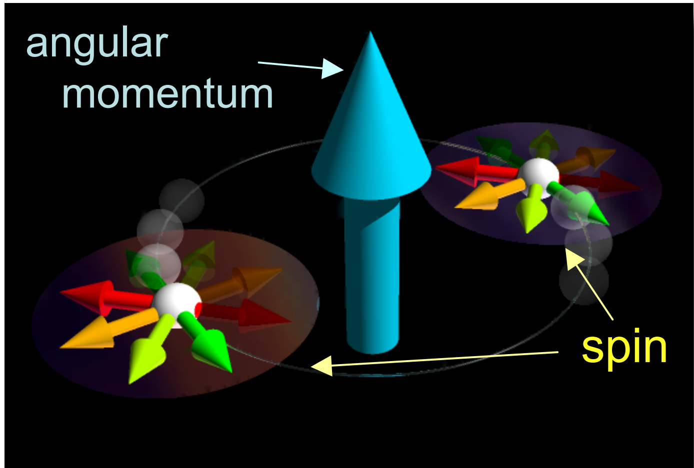
$$k_x + i k_y \overset{\hat{K}}{\longleftrightarrow} k_x - i k_y$$

broken time reversal symmetry $\mathcal{K}$

$$D_{4h} \times SU(2) \times \mathcal{K} \times U(1)_\phi$$

analog to $^3$He A-phase

$$U(1)_{S_z} \times U(1)_{L_z + \phi}$$




Deguchi & Maeno

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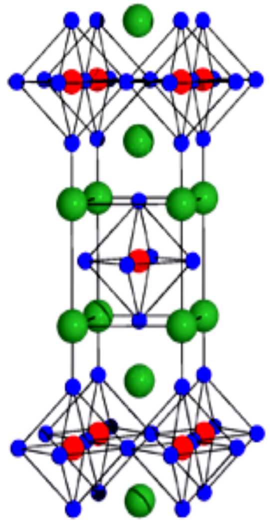

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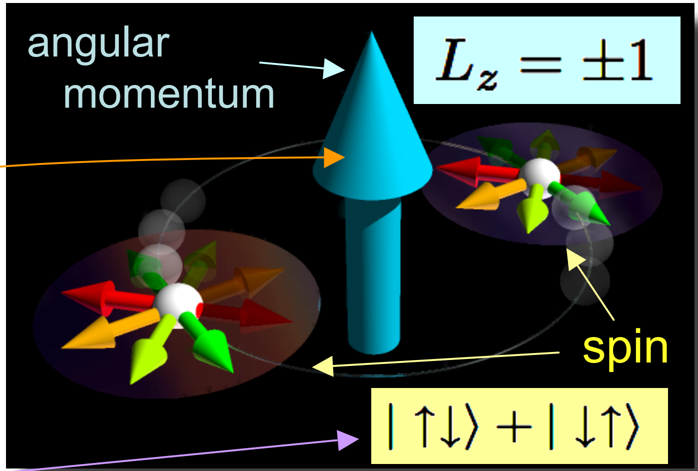
$$\downarrow$$

$$U(1)_{S_z} \times U(1)_{L_z + \phi}$$

orbital rotation

spin rotation

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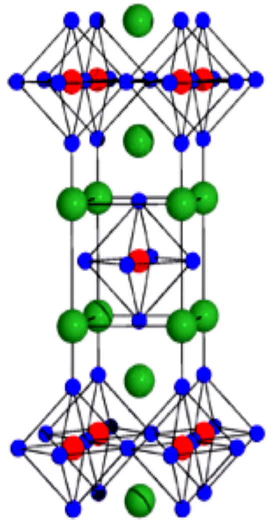

$T_c \approx 1.5K$

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$$e^{+i\theta_{\vec{k}}} \overset{\hat{K}}{\longleftrightarrow} e^{-i\theta_{\vec{k}}}$$

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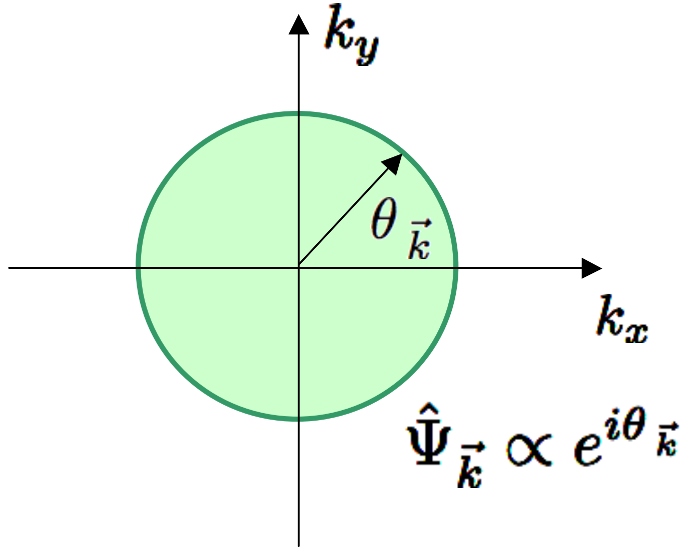

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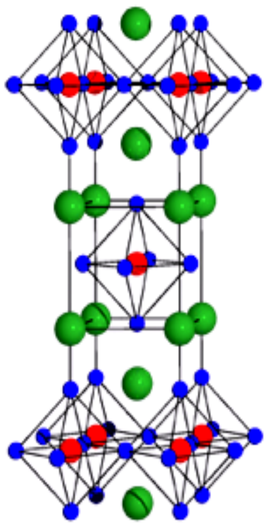

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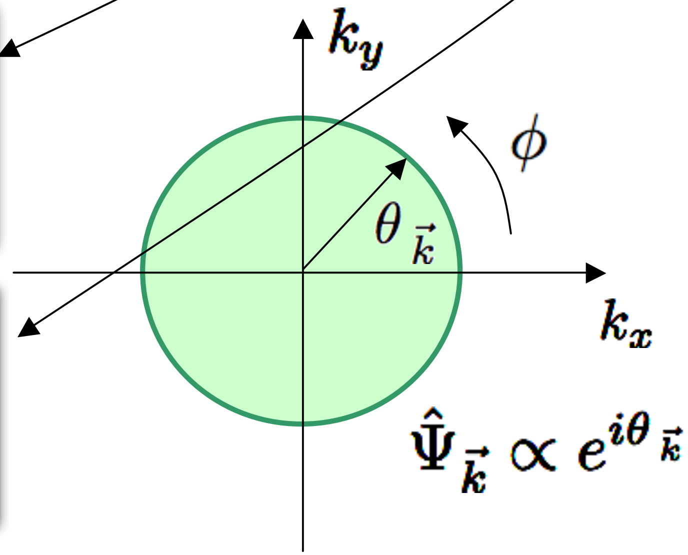
$$U(1)_{S_z} \times U(1)_{L_z + \phi}$$

**rotation**

$$R_\phi \hat{\Psi}_{\vec{k}} = \hat{\Psi}_{\vec{k}} e^{i\phi} \quad \longleftrightarrow \quad \hat{L}_z$$ angular momentum

***U(1)*-gauge**

$$U_\phi \hat{\Psi}_{\vec{k}} = \hat{\Psi}_{\vec{k}} e^{i\phi} \quad \longleftrightarrow \quad \hat{N}$$ charge



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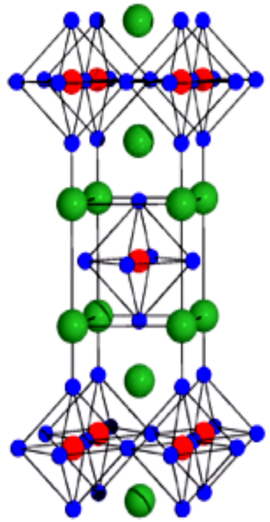

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$$R_\phi \hat{\Psi}_{\vec{k}} = \hat{\Psi}_{\vec{k}} e^{i\phi} \longleftrightarrow \hat{L}_z$$ angular momentum

*U(1)*-gauge

$$U_\phi \hat{\Psi}_{\vec{k}} = \hat{\Psi}_{\vec{k}} e^{i\phi} \longleftrightarrow \hat{N}$$ charge

$$R_\phi U_{-\phi} = 1$$

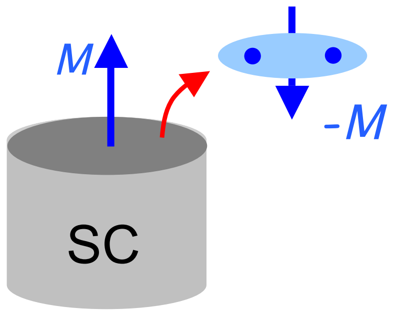
$$\hat{L}_z - \hat{N}$$ conserved "charge"

Volovik

# $Sr_2RuO_4$ - chiral p-wave superconductor

magnetic moment

for charge particles




Cooper pair

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$$U(1)_{S_z} \times U(1)_{L_z+\phi}$$

anomalous electromagnetism

$$\rho = \frac{e^2}{hc} \hat{z} \cdot \vec{B}$$

charge fluctuation generate magnetic flux

$$\vec{j} = \frac{e^2}{h} \vec{E} \times \hat{z}$$

currents generate transverse electric field

$$R_\phi U_{-\phi} = 1$$

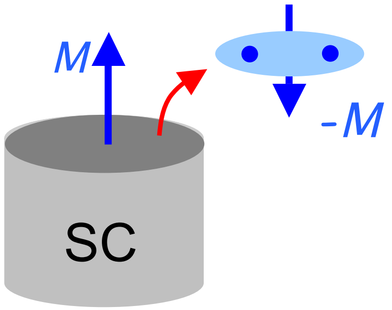
$$\hat{L}_z - \hat{N}$$

conserved "charge"

Volovik

# $Sr_2RuO_4$ - chiral p-wave superconductor

magnetic moment
for charge particles



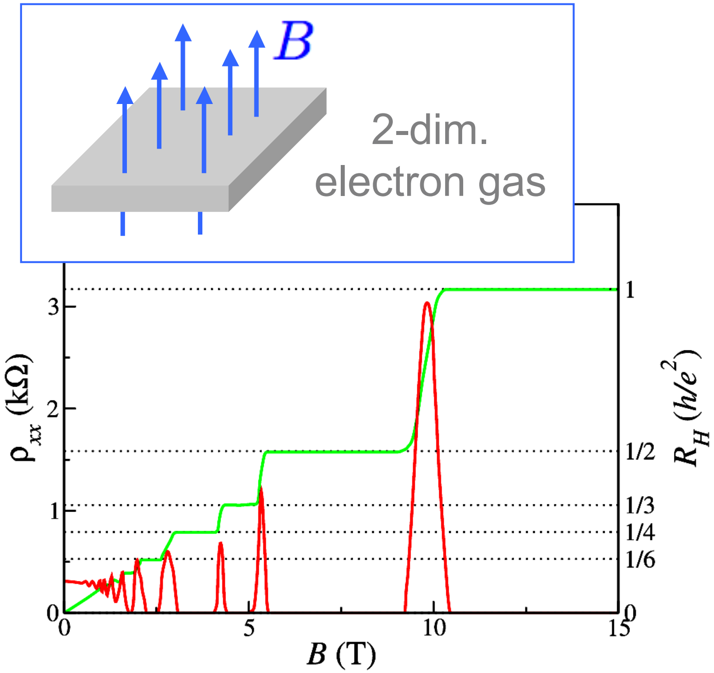

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & k_x \pm i k_y \\ k_x \pm i k_y & 0 \end{pmatrix}$$

analogy to the integer quantum Hall state

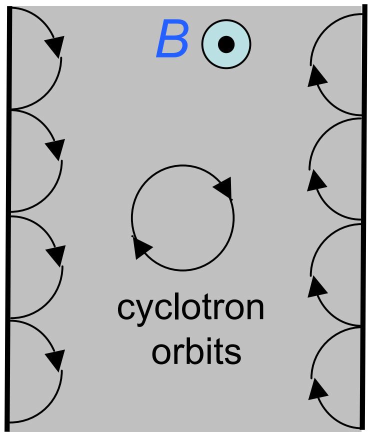
anomalous
electromagnetism

$$\rho = \frac{e^2}{hc}\hat{z}\cdot\vec{B}$$

$$\vec{j} = \frac{e^2}{h}\vec{E}\times\hat{z}$$




$$\nu = 1$$

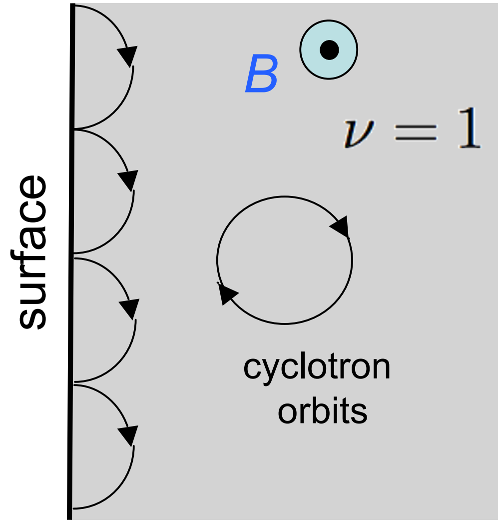


# $Sr_2RuO_4$ - chiral p-wave superconductor

## edge states

- Quantum Hall state

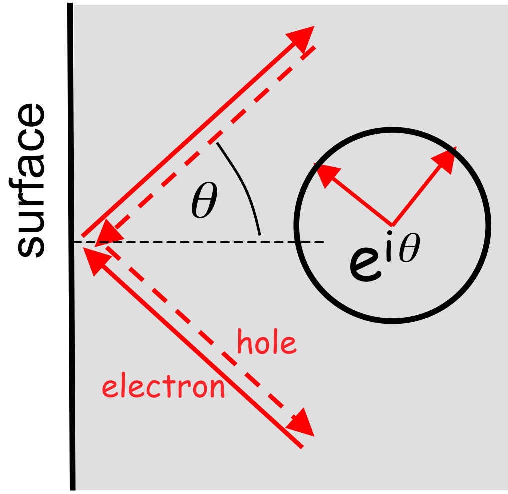
  "bouncing
  cyclotron orbits"

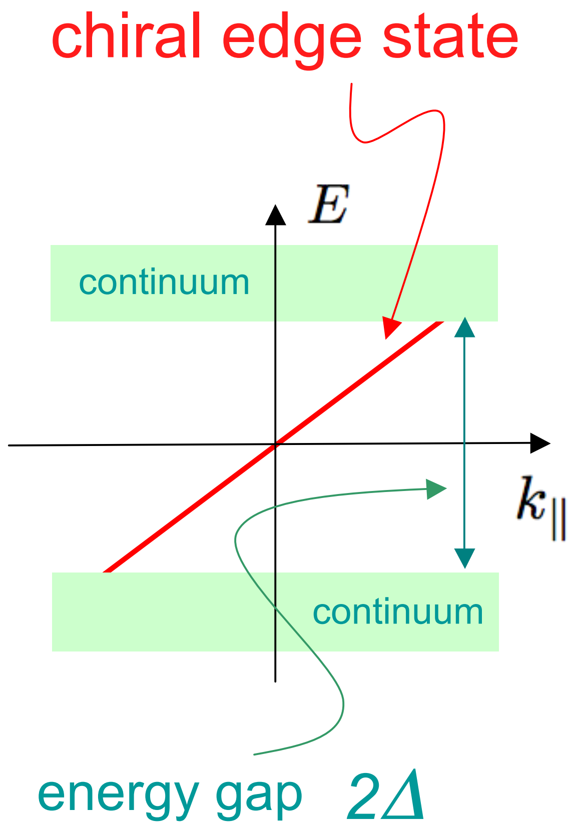



- chiral p-wave SC

  Andreev bound states

  electron-hole hybridized
  Bohr-Sommerfeld-orbits



# $Sr_2RuO_4$ - chiral p-wave superconductor

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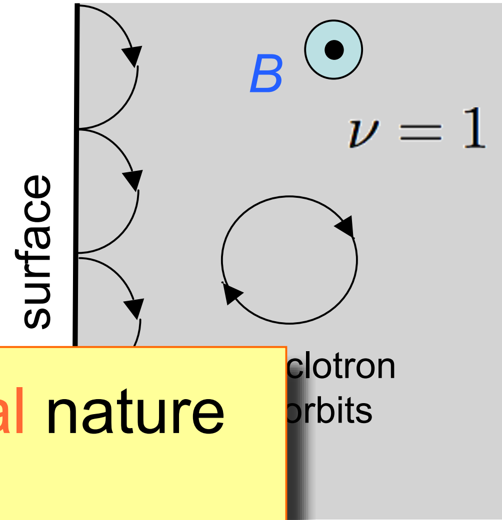
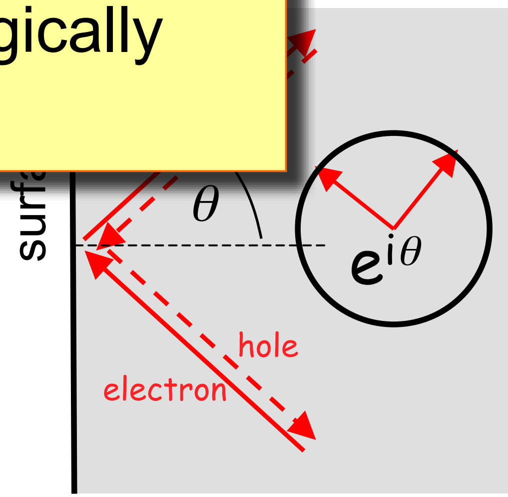


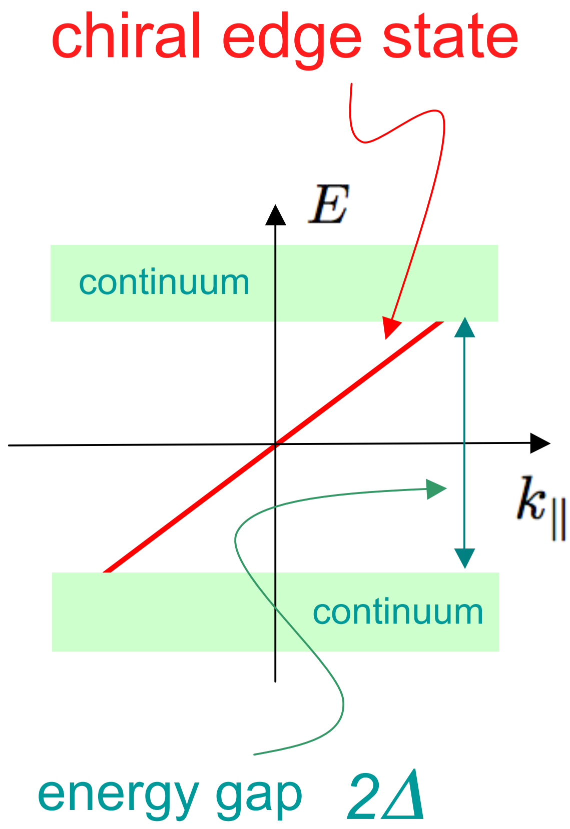

states with topological nature

⬇

edge states topologically protected

electron-hole hybridized
Bohr-Sommerfeld-orbits



# $Sr_2RuO_4$ - chiral p-wave superconductor

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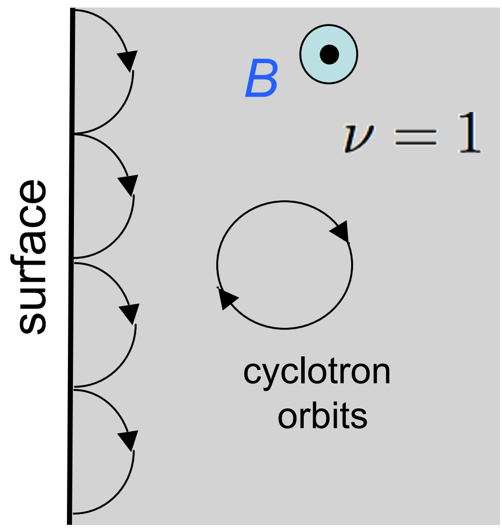
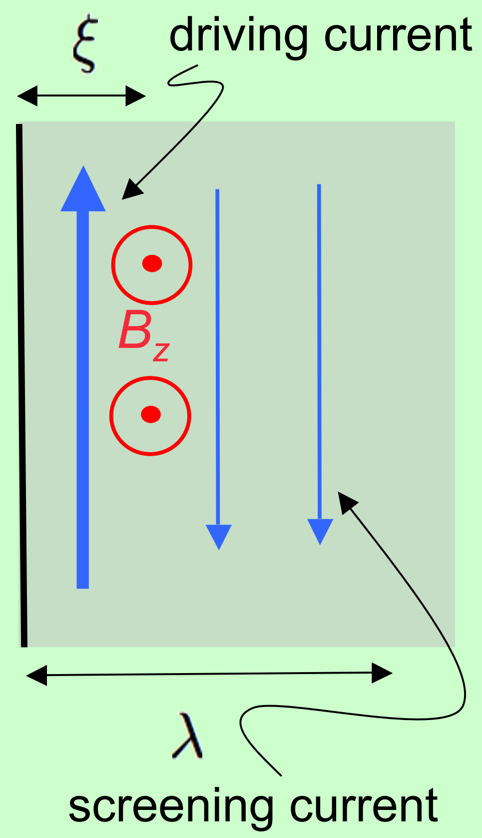


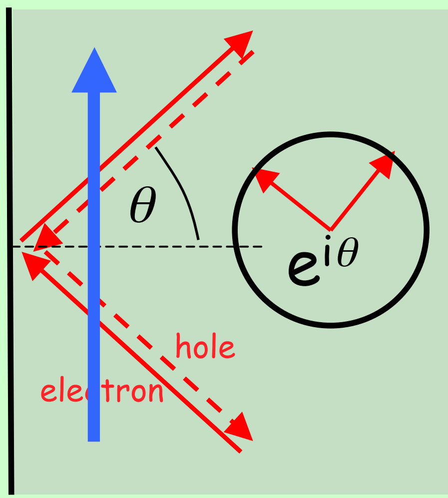

## spontaneous edge currents




- chiral p-wave SC

  Andreev bound states

  electron-hole hybridized Bohr-Sommerfeld-orbits

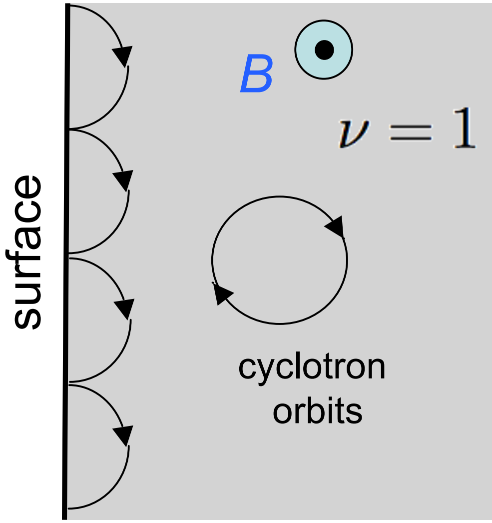


# $Sr_2RuO_4$ - chiral p-wave superconductor

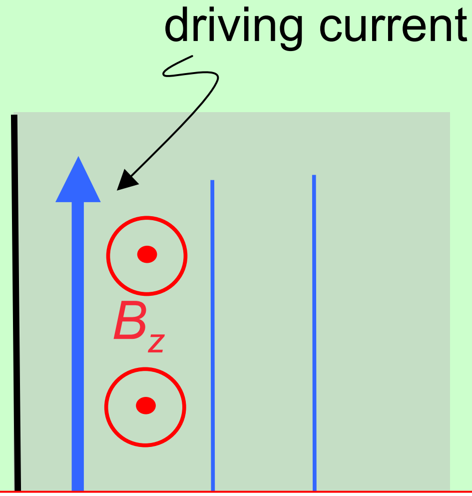
## edge states

- Quantum Hall state

  "bouncing
  cyclotron orbits"




## spontaneous edge currents




- chiral p-wave SC

  Andreev bound states

  electron-hole hybridized
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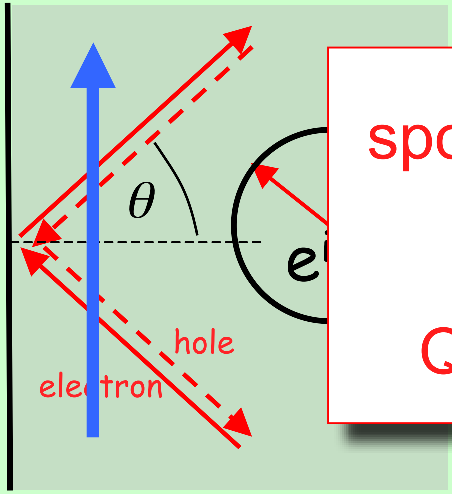



spontaneous Hall effect

but no

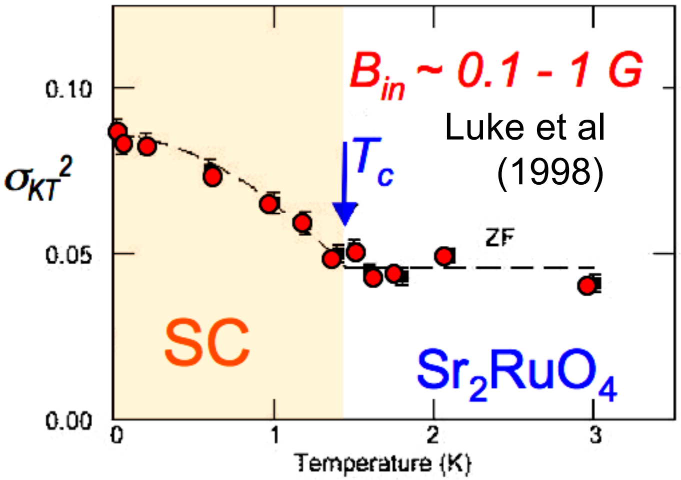
Quantum Hall effect

# intrinsic magnetism ?

## random local magnetism

"edge currents" around
inhomogeneities & defects

*μSR - zero-field relaxtion*

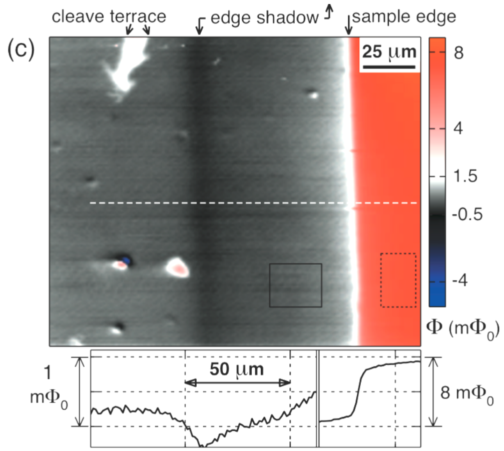



muon-spin depolarization

→ intrinsic magnetism

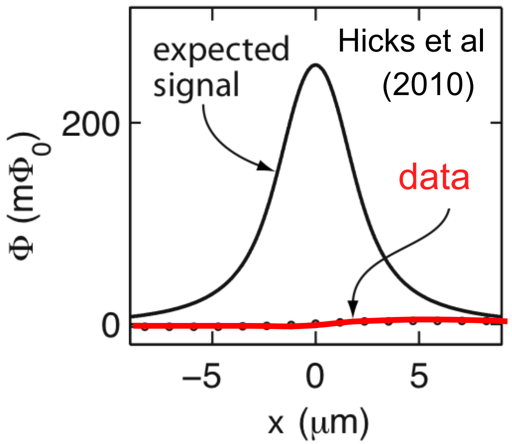
## edge state currents

*scanning probes at sample boundaries*




scanning Hall
Tamegai et al

scanning SQUID
Kirtley, Moler et al



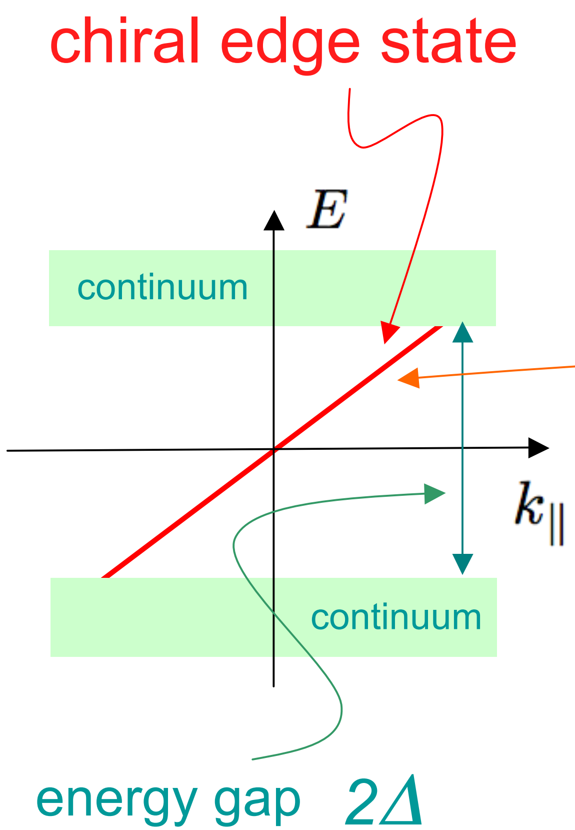

magnetic signal clearly below expected bounds

↓

no edge states ?

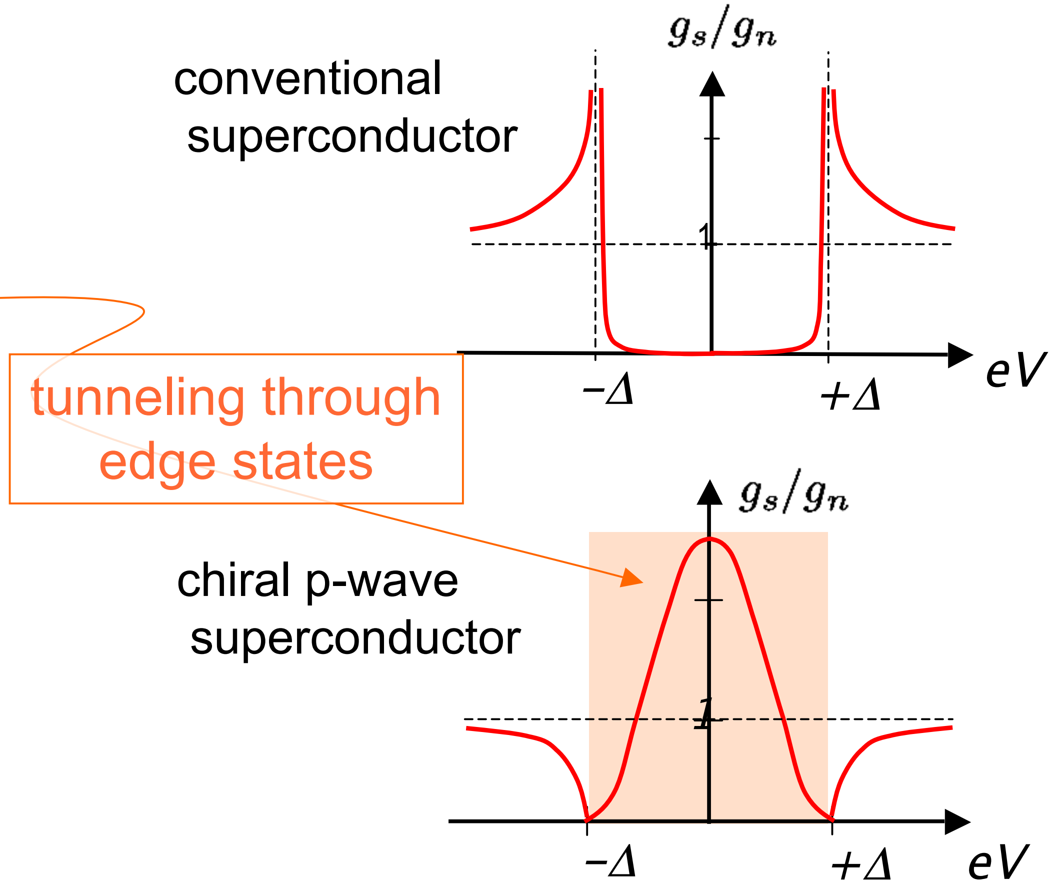
# edge states ?

## local quasiparticle spectrum




## tunneling conductance

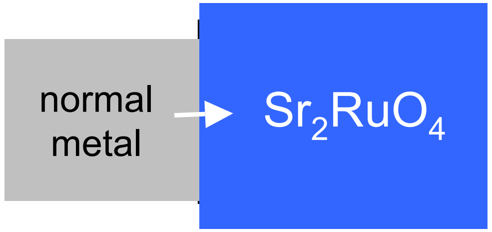
$$g(eV) = \frac{dI}{dV}(eV)$$



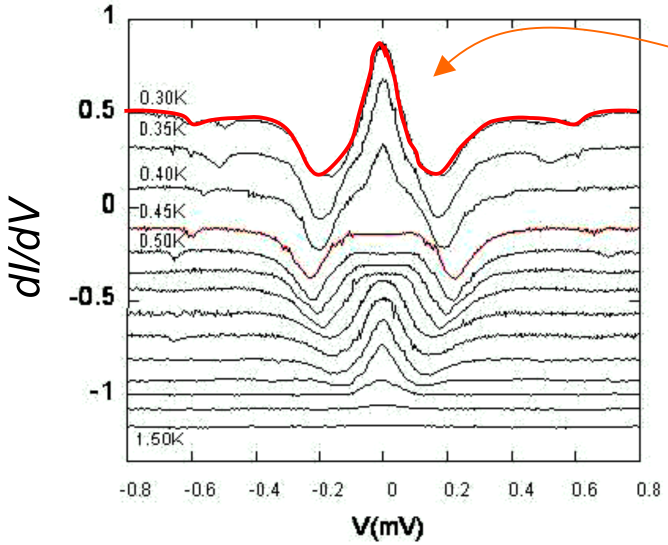
# edge states ?

## local quasiparticle spectrum

tunnel-contact




Mao, Liu et al.

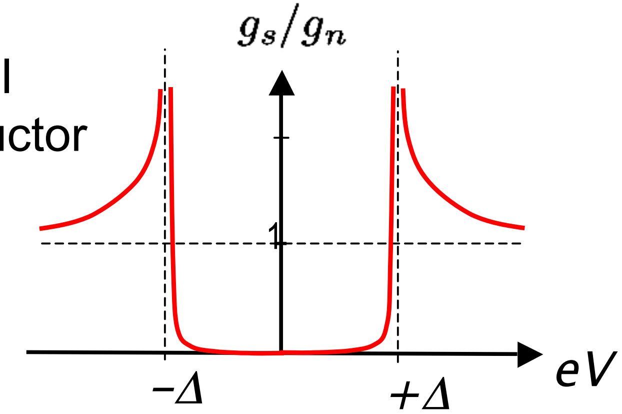



Goll, von Löhneysen et al.
Kashiwaya et al.

## tunneling conductance

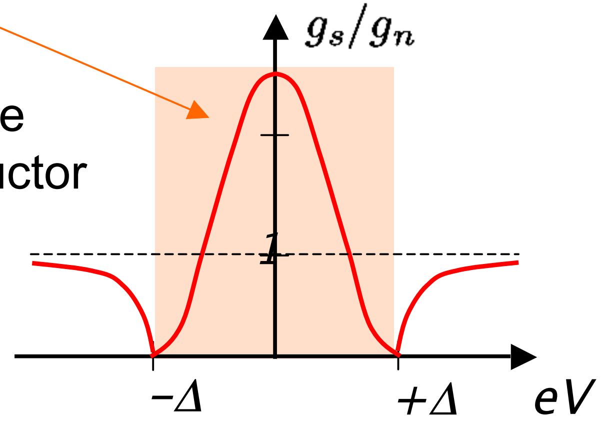
$$g(eV) = \frac{dI}{dV}(eV)$$

conventional superconductor




zero-bias anomaly

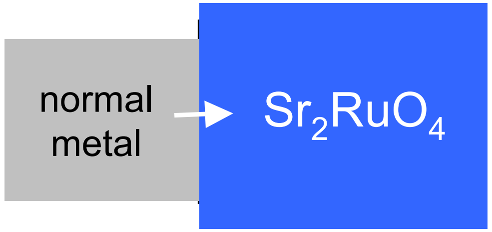
chiral p-wave superconductor



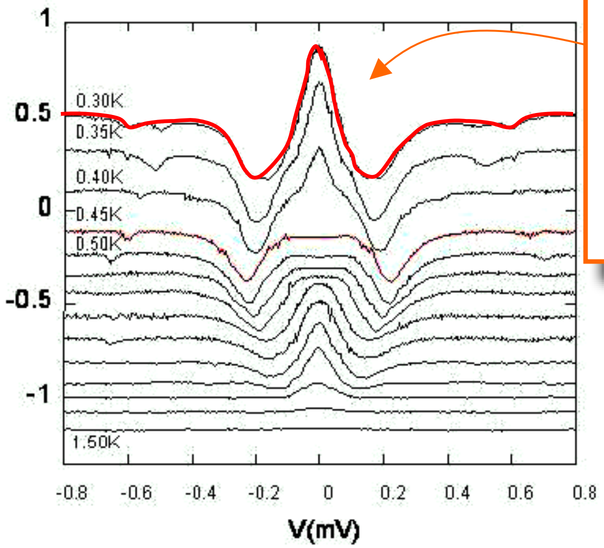
# edge states ?

## local quasiparticle spectrum

tunnel-contact




Mao, Liu et al.

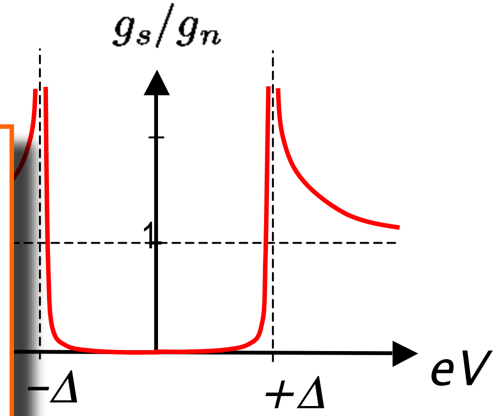



Goll, von Löhneysen et al.
Kashiwaya et al.

## tunneling conductance

$$g(eV) = \frac{dI}{dV}(eV)$$

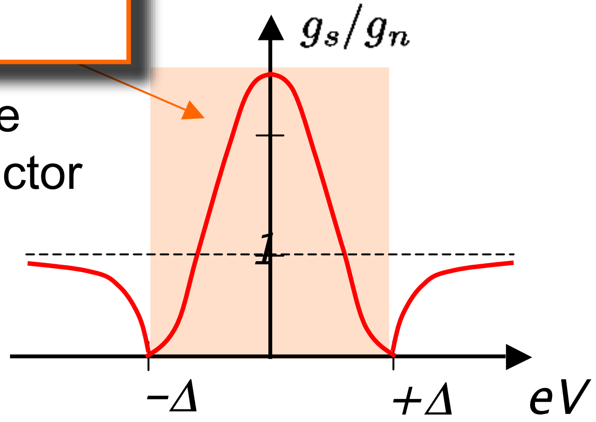
conventional




**chirality ?**

subtle Doppler-shift effects through coupling to a magnetic field

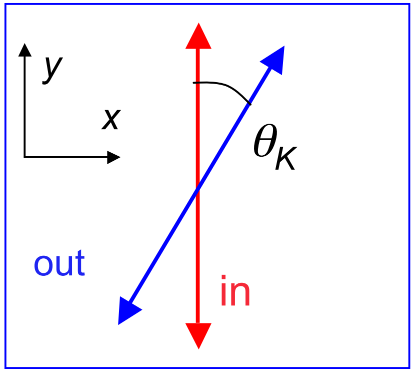
chiral p-wave superconductor

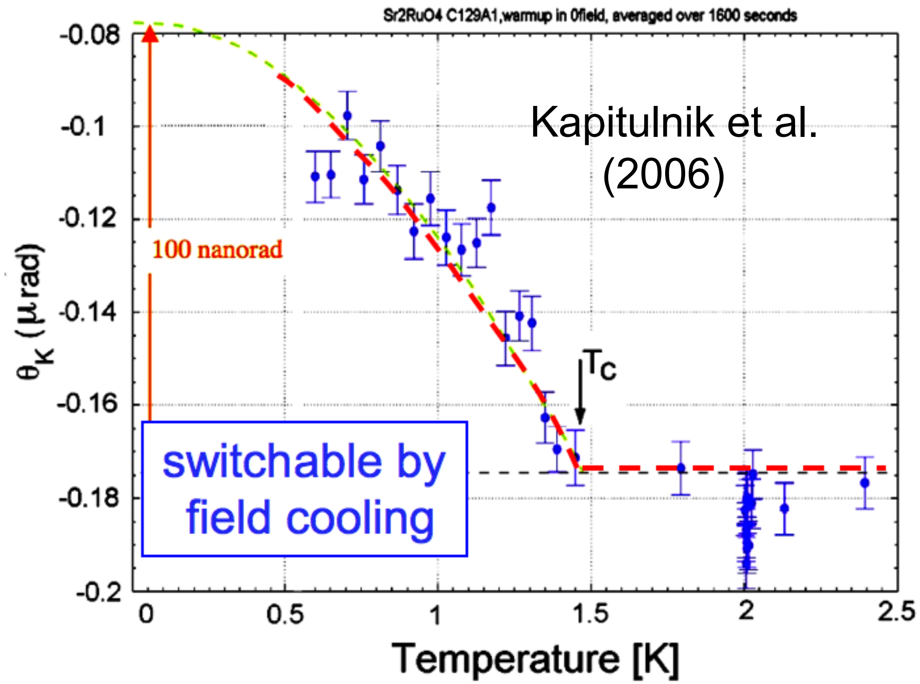


# chirality ?

## polar Kerr effect

polarization axis of
reflected light is rotated



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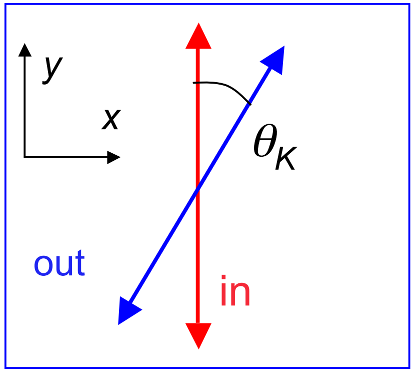
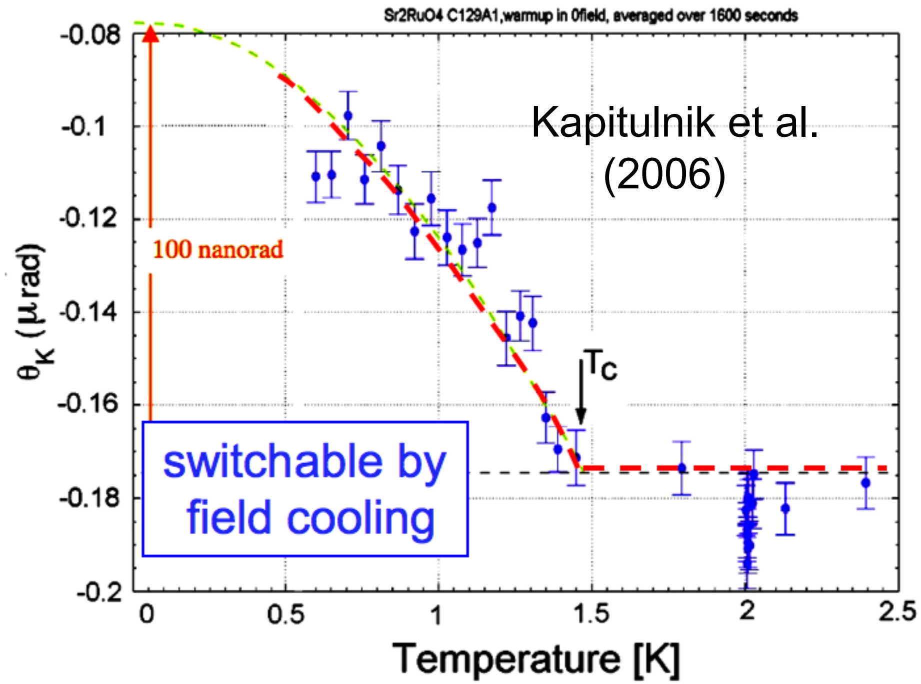



mSR: random intrinsic magnetism — positive

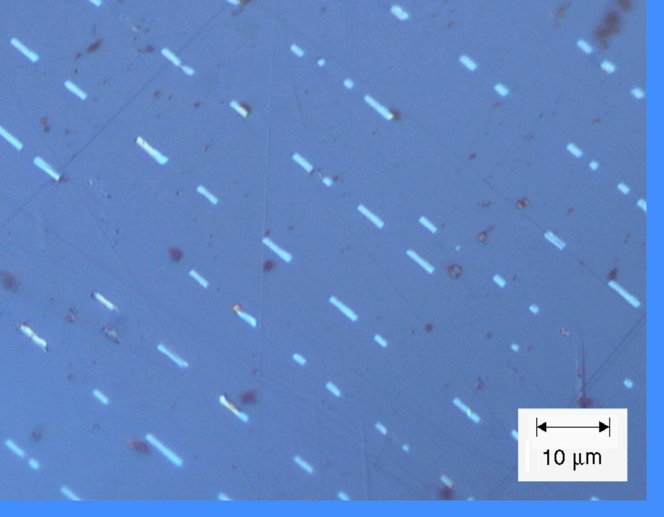
scanning probes: chiral surface currents — negative

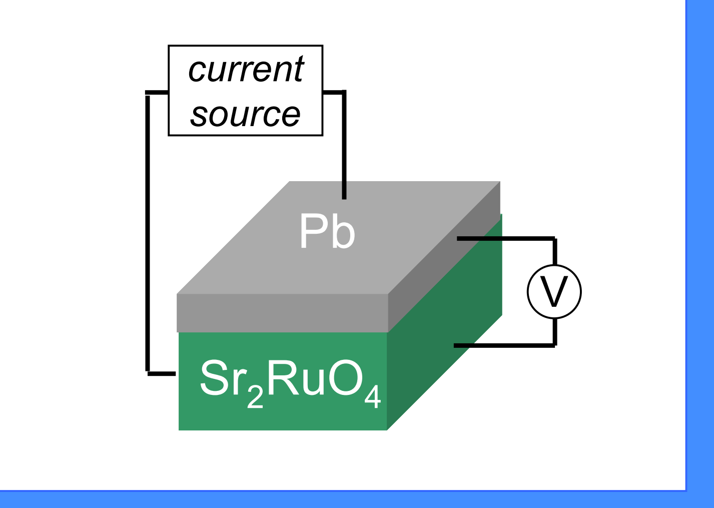
polar Kerr effect: optical property - chirality

Josephson interference effect: chiral domains

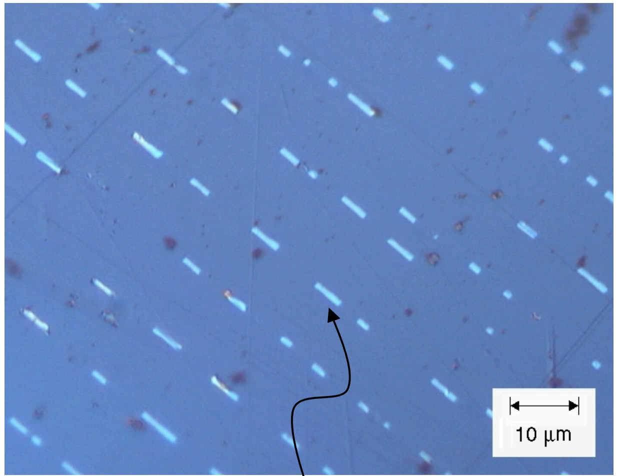
} compatible

# Anomalous Josephson effect



# 3 Kelvin phase - signatures of topology




$\mu m$ - size Ru-inclusion

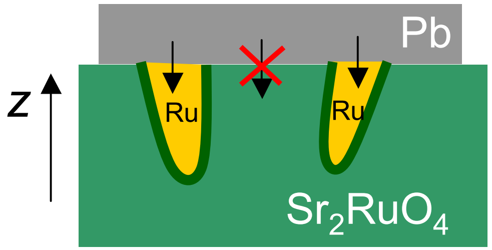
Maeno et al (1997)

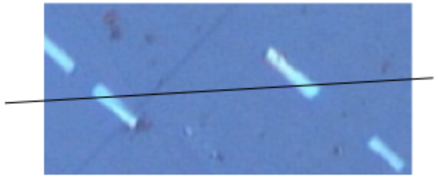
onset of inhomogeneous superconductivity at

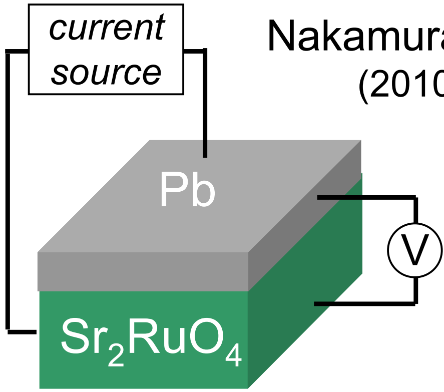
$$T^* \approx 3K$$

"3-Kelvin phase"

## Josephson coupling via Ru-inclusions

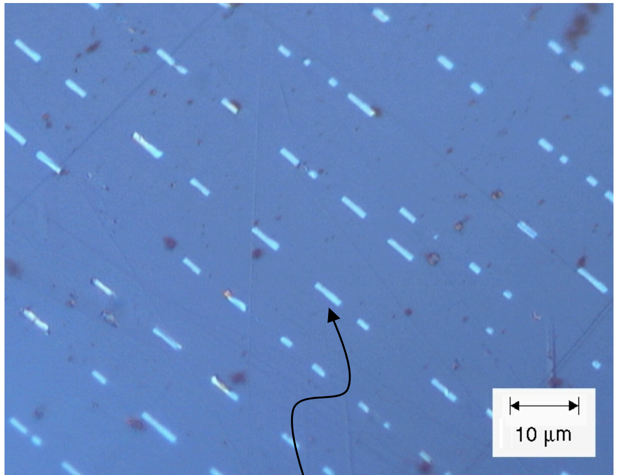





Nakamura et al (2010)

# 3 Kelvin phase - signatures of topology




*µm* - size Ru-inclusion

Maeno et al (1997)

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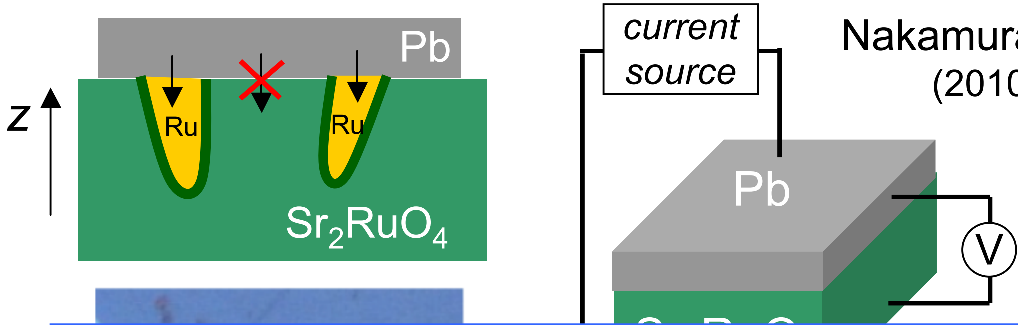
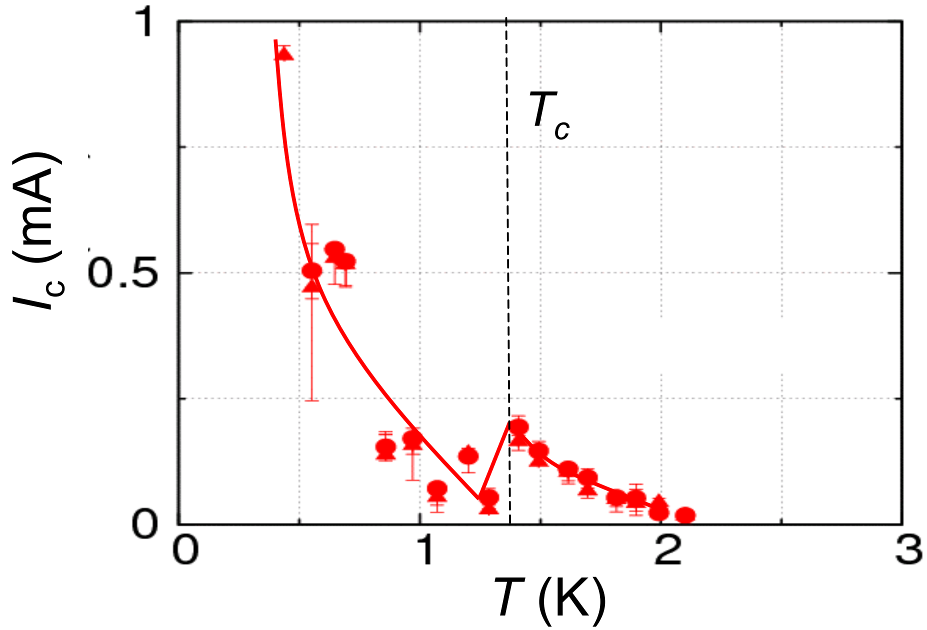



Nakamura et al (2010)

## Josephson critical current



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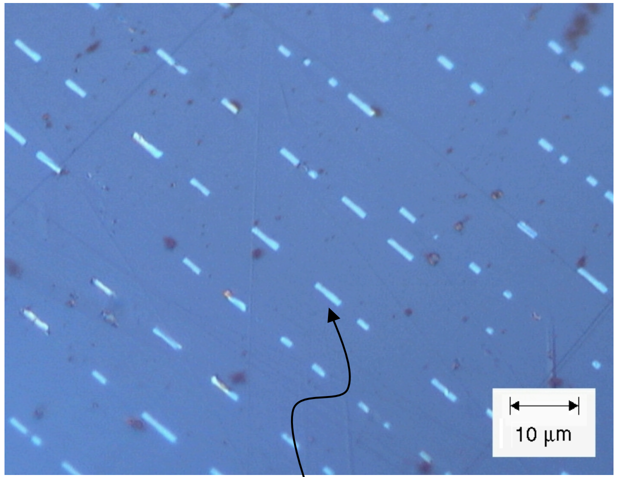


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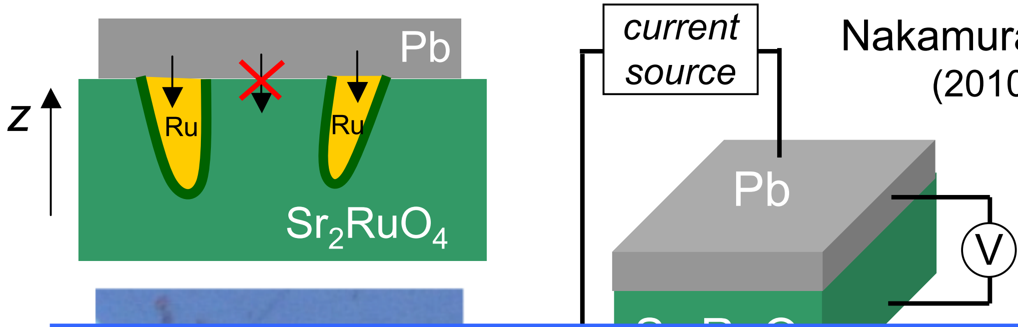


Nakamura et al (2010)

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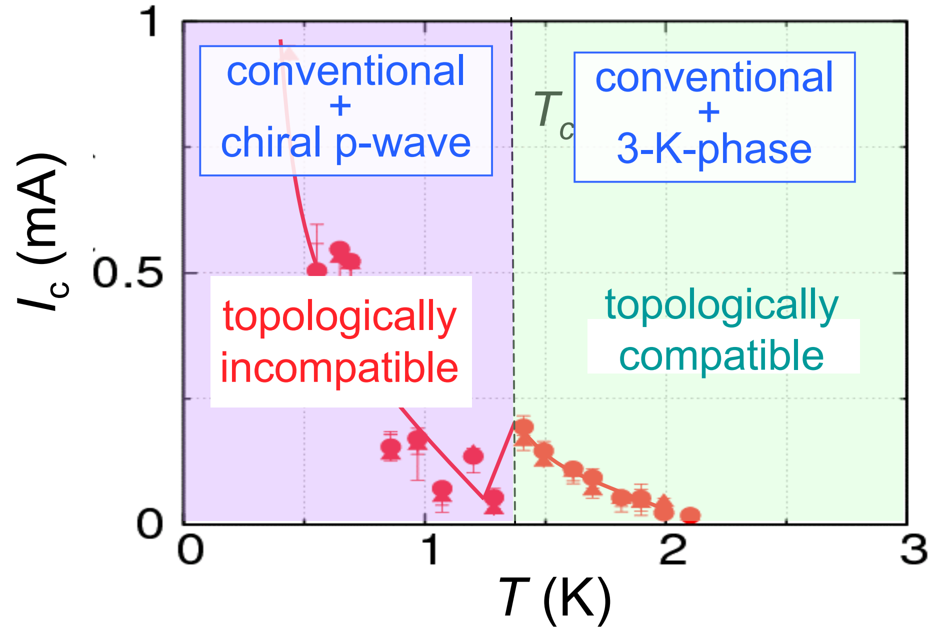

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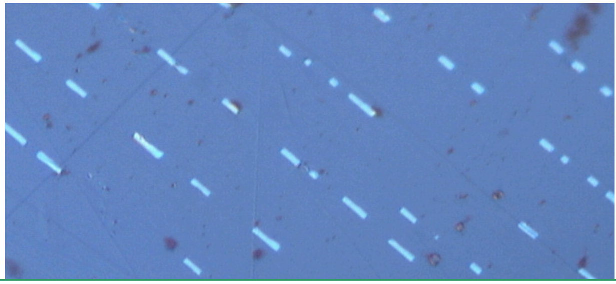

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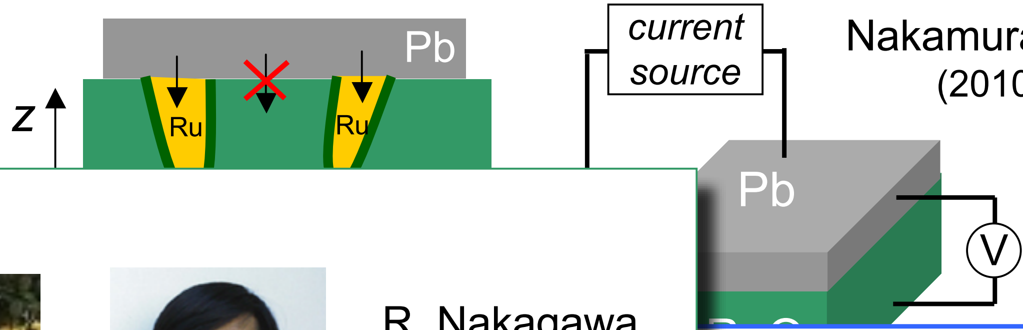
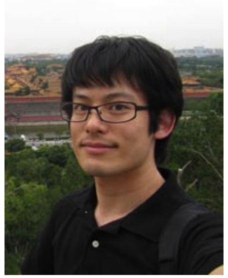



Nakamura et al (2010)

*GCOE team*

Y. Maeno



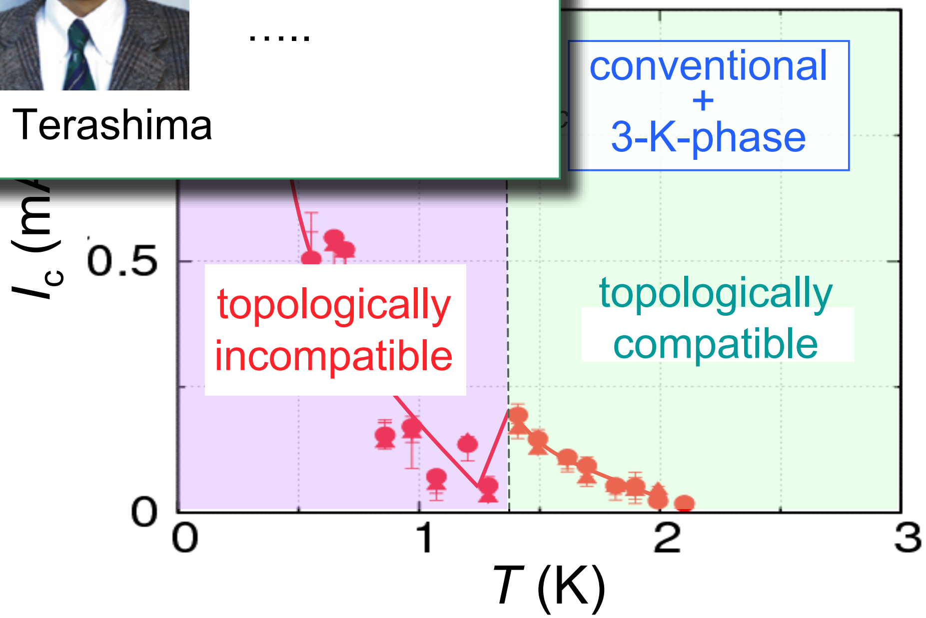
T. Nakamura

S. Yonezawa

T. Terashima

R. Nakagawa
T. Sumi
T. Yamagishi
.....

rrent

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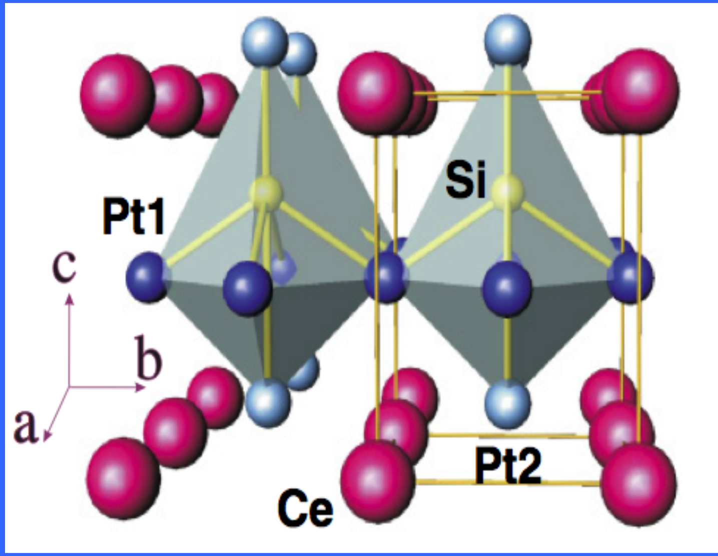
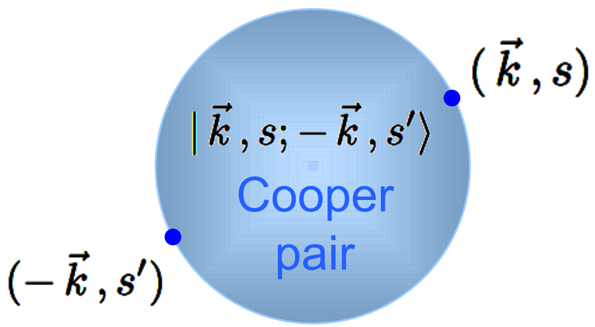
# Non-centrosymmetric Superconductors



# time reversal & inversion symmetry

electrons which can be paired




$$|\Phi_{BCS}\rangle = \bigotimes_{\vec{k}\,,s,s'} \left\{ u_{\vec{k}\,,ss'} + v_{\vec{k}\,,ss'} |\vec{k}\,, s; -\vec{k}\,, s'\rangle \right\}$$

*time reversal & inversion symmetry*

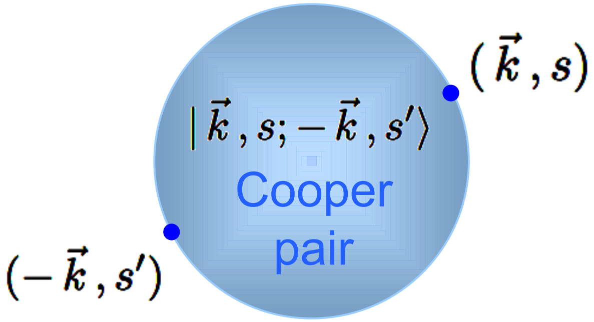
ensure presence of
degenerate pairing partners

$$|\vec{k}\,, s\rangle \longleftrightarrow |-\vec{k}\,, s'\rangle$$

| spin singlet | spin triplet |
| --- | --- |
| ↕ | ↕ |
| even parity | odd parity |

# time reversal & inversion symmetry

electrons which can be paired

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*time reversal & inversion symmetry*

ensure presence of
degenerate pairing partners

$$|\vec{k}, s\rangle \longleftrightarrow |-\vec{k}, s'\rangle$$

| spin singlet | | spin triplet |
|---|---|---|
| ↕ | ⟷ | ↕ |
| even parity | | odd parity |

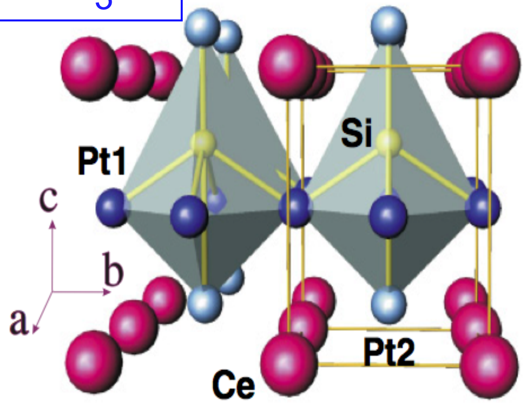
*lack of time reversal or inversion symmetry*

➡ Cooper pairs with mixed parity / spin

# Non-centrosymmetric superconductors

Ce-based heavy Fermion superconductor

$CePt_3Si$

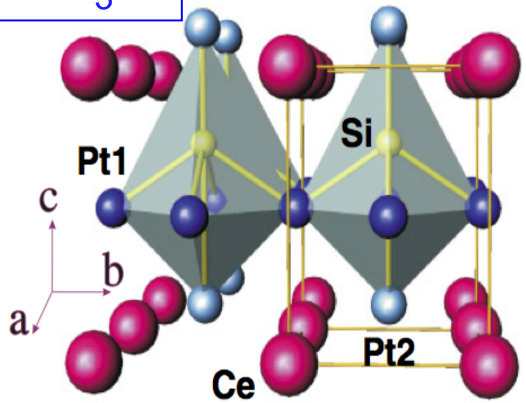



Bauer et al (2004)

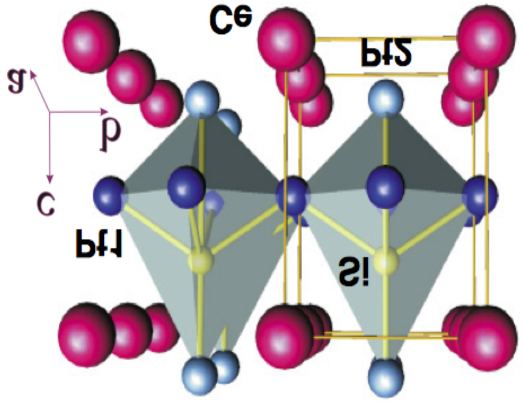
# Non-centrosymmetric superconductors

## Ce-based heavy Fermion superconductor

$CePt_3Si$




inversion →



Bauer et al (2004)

# Non-centrosymmetric superconductors

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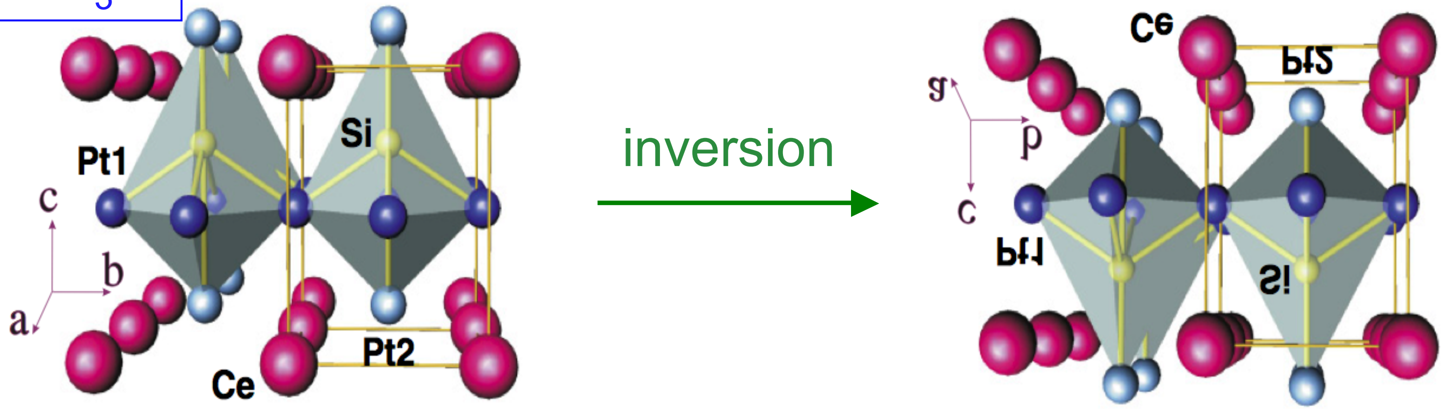
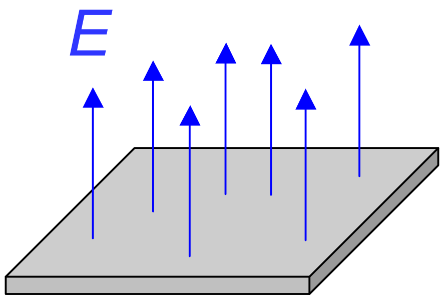


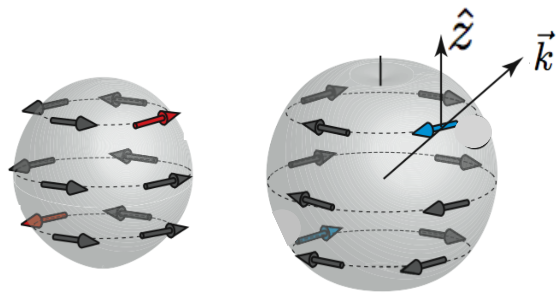

Bauer et al (2004)




consequence: Rashba spin-orbit coupling

$$\mathcal{H}_{so} = \alpha \sum_{\vec{k},s,s'} (\hat{z} \times \vec{k}) \cdot \vec{\sigma}_{ss'} |\vec{k},s\rangle\langle\vec{k},s'|$$

spin splitting of
electron states



# Non-centrosymmetric superconductors

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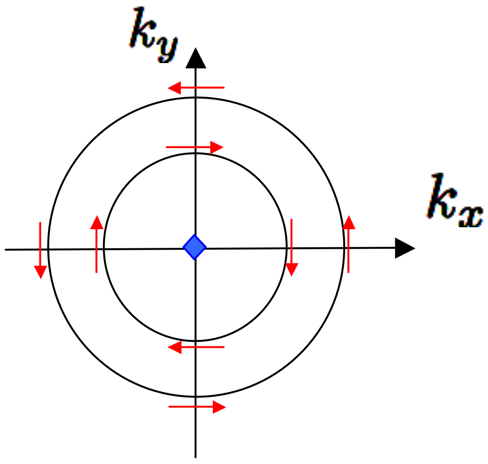
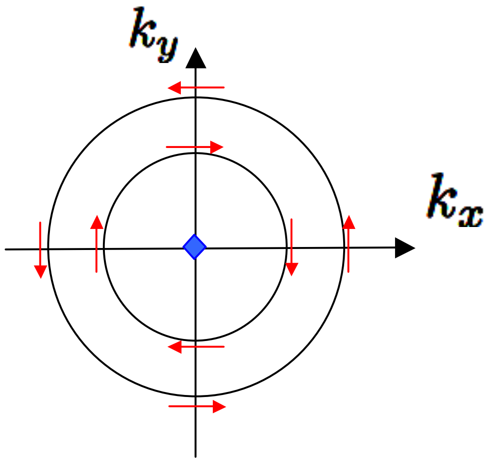



electronic spin structure *determines*
Cooper pairing symmetry

# Non-centrosymmetric superconductors

$$\mathcal{H}_{so} = \alpha \sum_{\vec{k},s,s'} (\hat{z} \times \vec{k}) \cdot \vec{\sigma}_{ss'} |\vec{k}, s\rangle\langle\vec{k}, s'|$$




electronic spin structure *determines*
Cooper pairing symmetry

"conventional" parity-mixed pairing state

odd parity

even parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} i\psi_o(k_x - ik_y) & \psi_e \\ -\psi_e & i\psi_o(k_x + ik_y) \end{pmatrix}$$

"s+p-wave"

analog to $^3$He *B*-phase

$$SO(3)_{L+S} \times \mathcal{K}$$

$$U(1)_{L_z+S_z} \times \mathcal{K}$$

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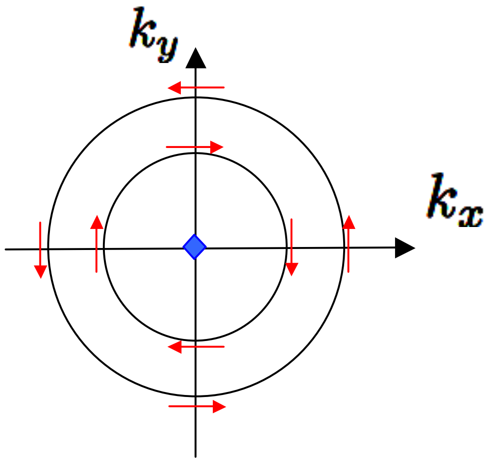


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spin-orbit coupling

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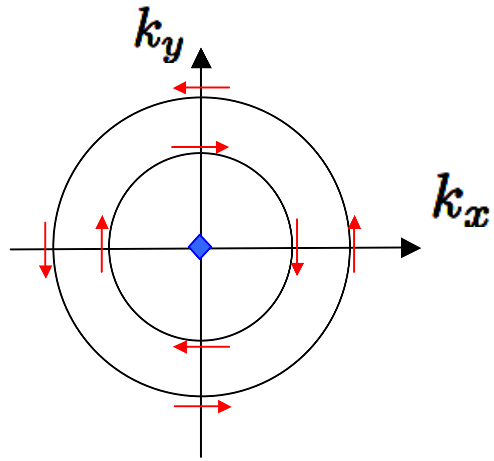


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helical (chirality coupled to spin)

analog to $^3$He *B*-phase

$$SO(3)_{L+S} \times \mathcal{K}$$

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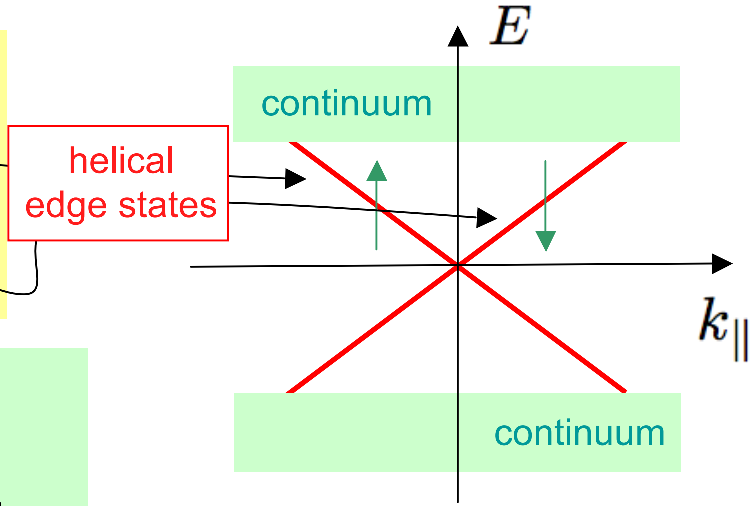
# Helical edge states - Andreev bound states

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} i\psi_o(k_x - ik_y) \uparrow\uparrow & \psi_e \\ -\psi_e & \downarrow\downarrow i\psi_o(k_x + ik_y) \end{pmatrix}$$




$$|\psi_e| > |\psi_o|$$

dominant even parity
topologically trivial

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dominant odd parity
topologically non-trivial

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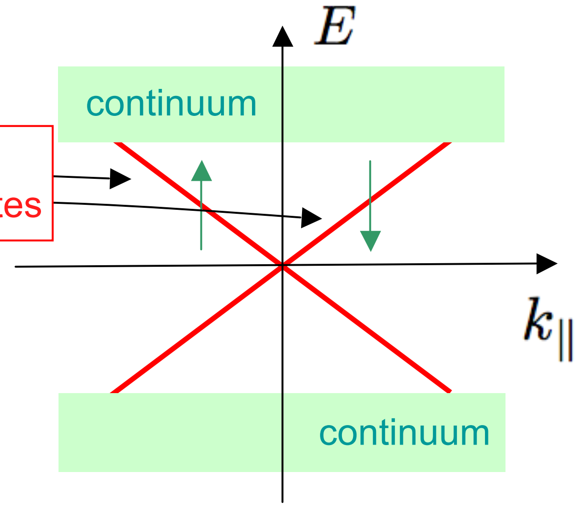
helical edge states

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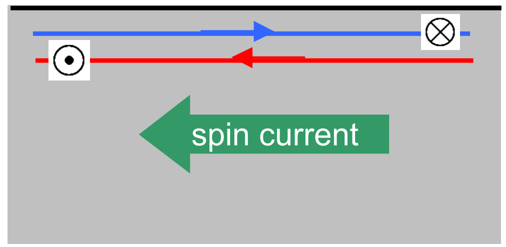


spin current at edges

⬇

*Quantum Spin Hall effect*
for topologically non-trivial state

analogy to topological insulators

# Helical edge states - Andreev bound states

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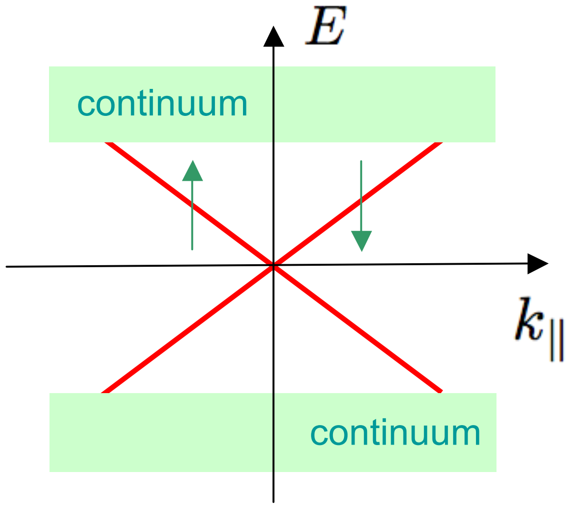


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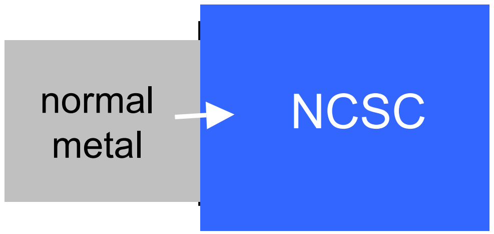
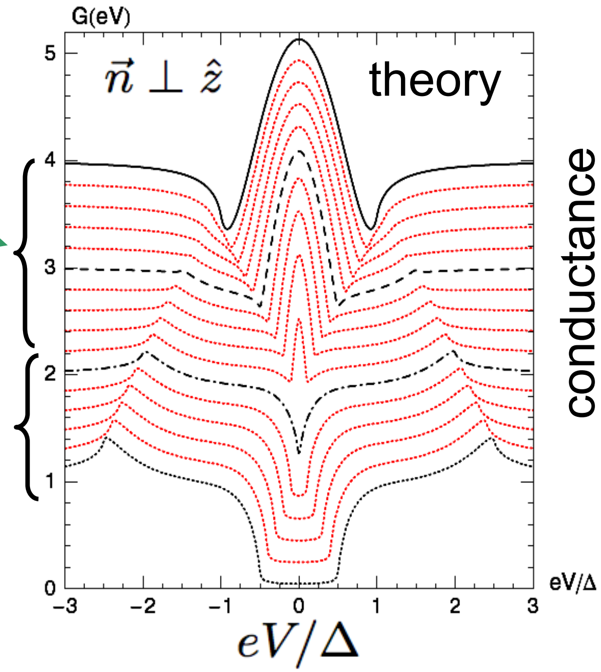
dominant odd parity
topologically non-trivial

quasiparticle spectra - tunneling

tunnel-contact

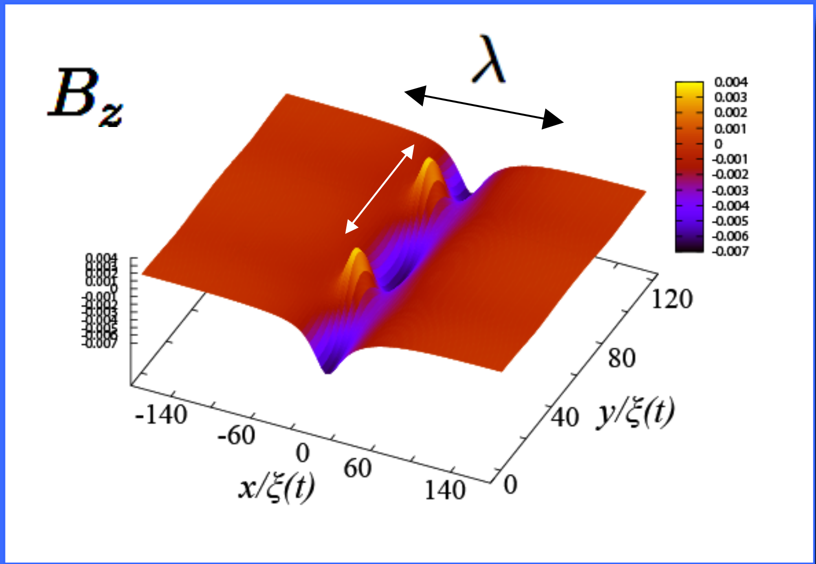



$CePt_3Si$
good candidate for
$|\psi_e| < |\psi_o|$
but no experiments so far



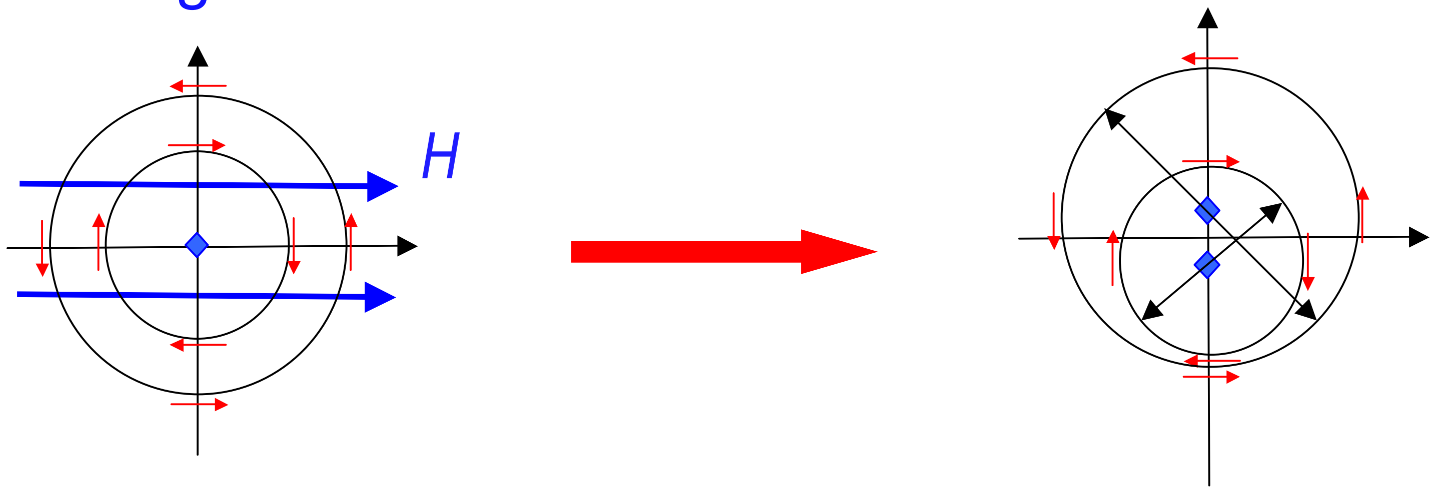
# Helical phase

# magneto-electric effects



# Magneto-electric effects - helical phase

effect of magnetic field on the Fermi surfaces




shift of Fermi surface centers

→ Cooper pairing with finite momentum

*helical phase:* $\Psi(\vec{r}) = \Psi_0(\vec{r})e^{i\vec{q}\cdot\vec{r}}$ with $\vec{q} = K(\hat{z} \times \vec{H})$

analog to Fulde-Ferrel phase (different mechanism)

Kaur et al, Dimitrova et al

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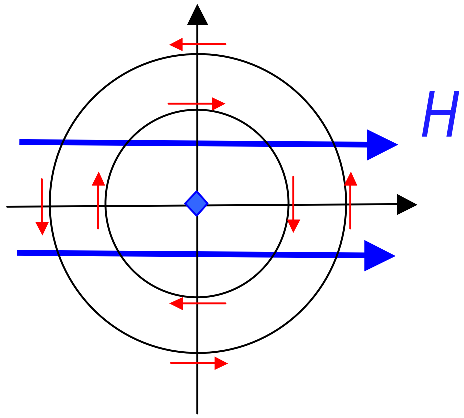


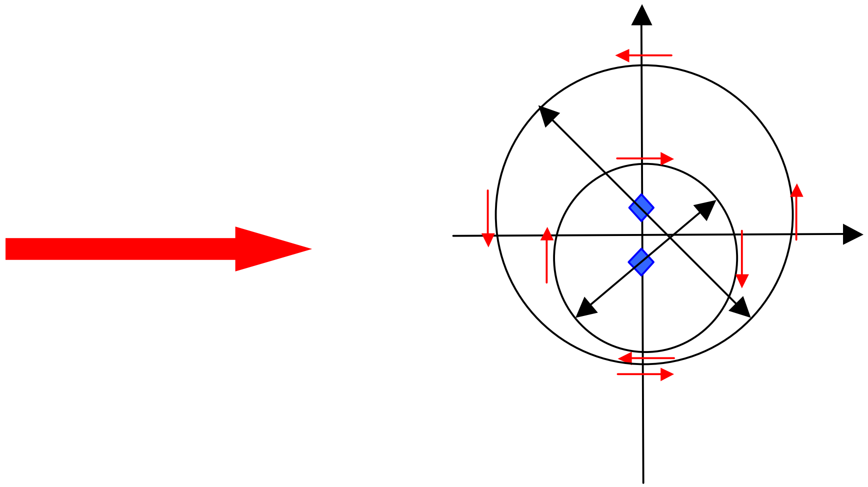

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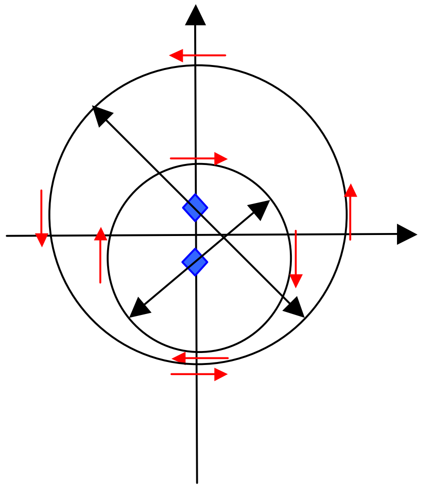
# Magneto-electric effects - helical phase

effect of magnetic field on the Fermi surfaces

gauge freedom

$$\vec{q} = K(\hat{z} \times \vec{H}) - \frac{2e}{\hbar c}\vec{A}$$

phase gradient "removable"
no currents induced



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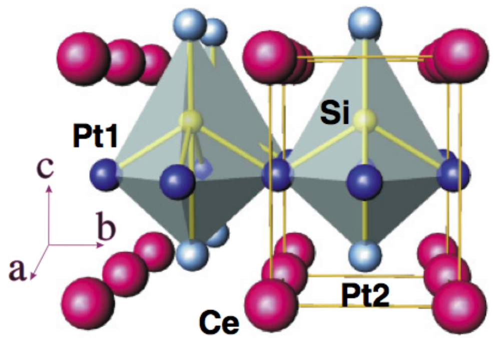
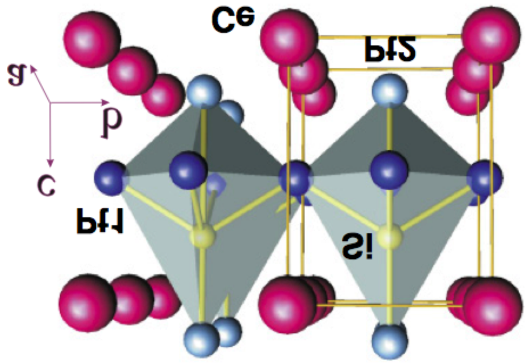
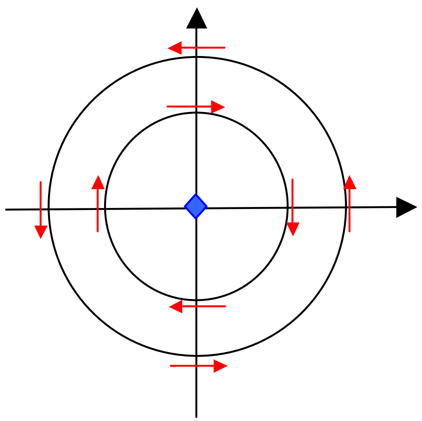
# Crystal twin domains

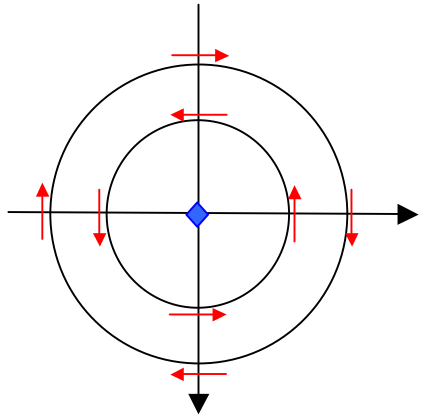
non-centrosymmetric crystals can be twinned




*inversion*







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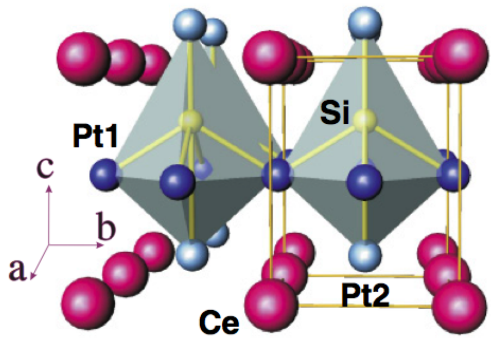
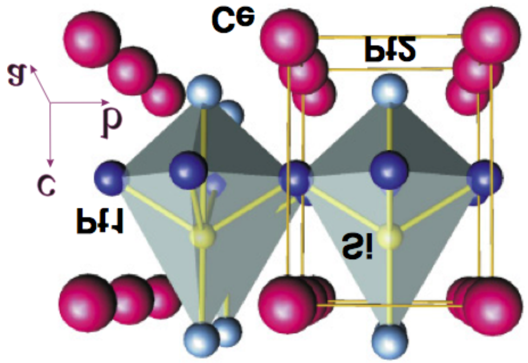
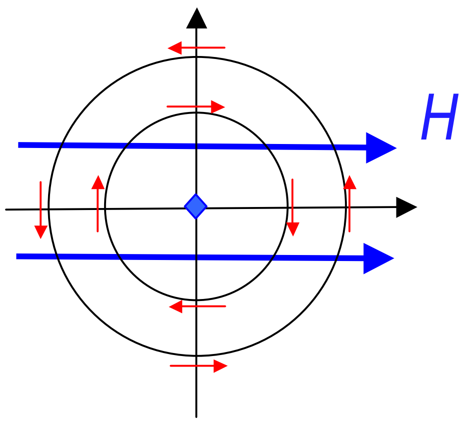
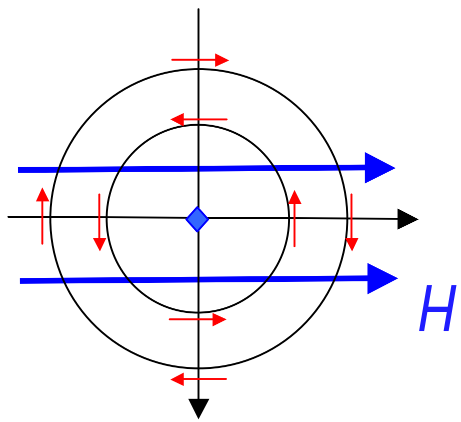



*inversion*

↔





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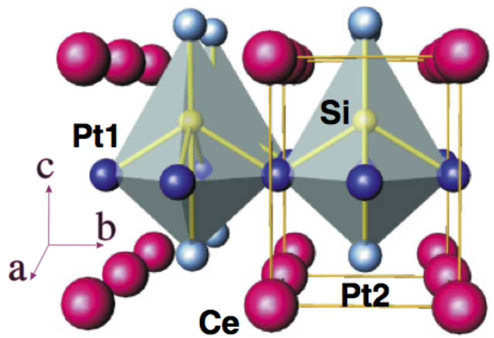


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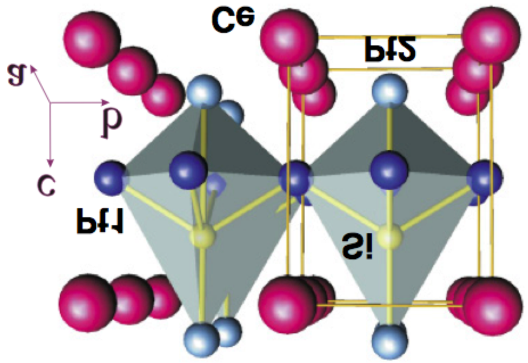
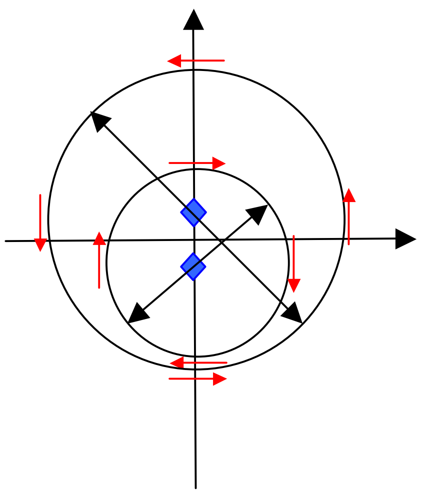
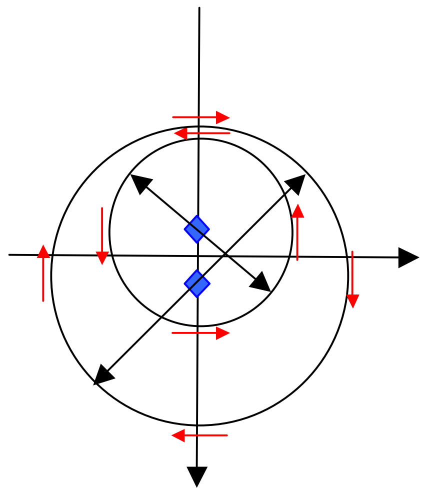




opposite
helical phases

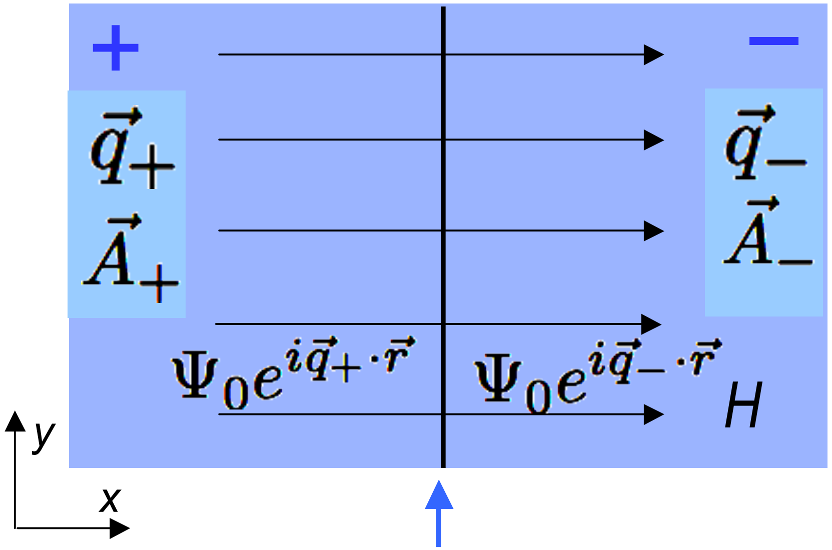
$$\vec{q}_{\pm} = \pm K(\hat{z} \times \vec{H})$$



$$\Psi(\vec{r}) = \Psi_0(\vec{r})e^{i\vec{q}_+ \cdot \vec{r}}$$

$$\Psi(\vec{r}) = \Psi_0(\vec{r})e^{i\vec{q}_- \cdot \vec{r}}$$

# inhomogeneous helical states




matching
at twin boundary

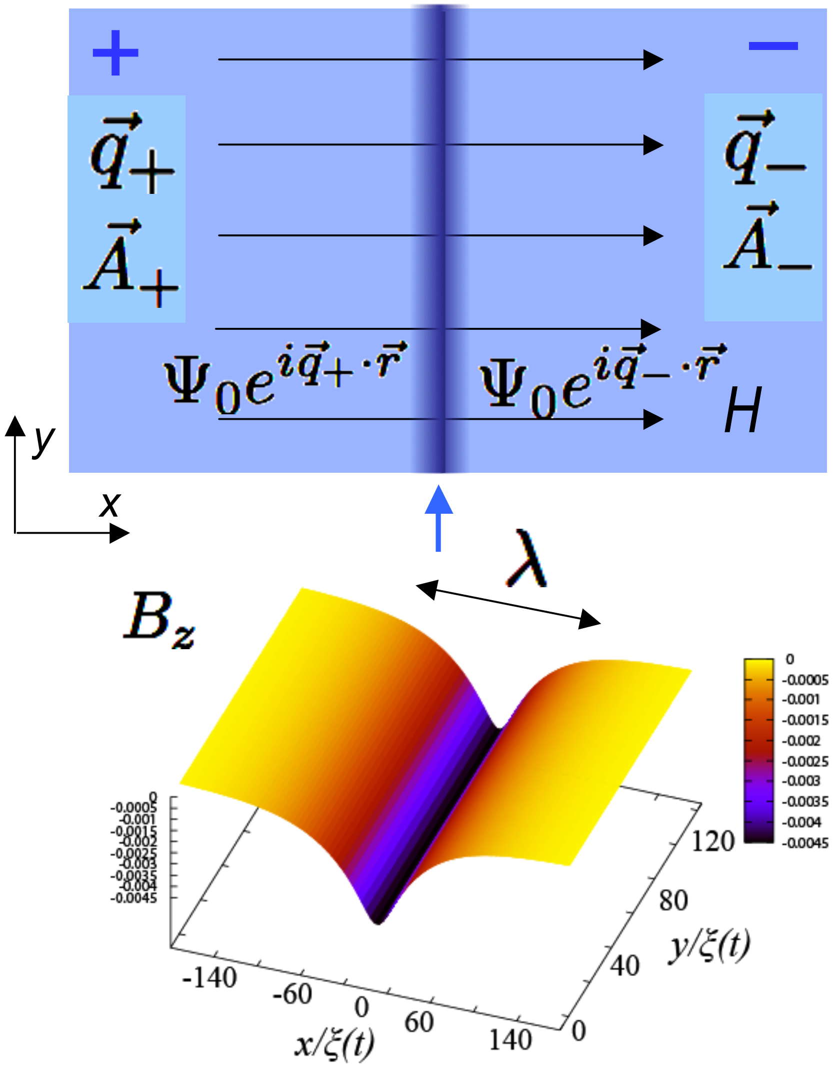
$$\vec{q}_\pm = \pm K(\hat{z}\times\vec{H}) - \frac{2e}{\hbar c}\vec{A}_\pm$$

small fields

➡ "wave function machting"

$$q_{y+} = q_{y-}$$

# inhomogeneous helical states

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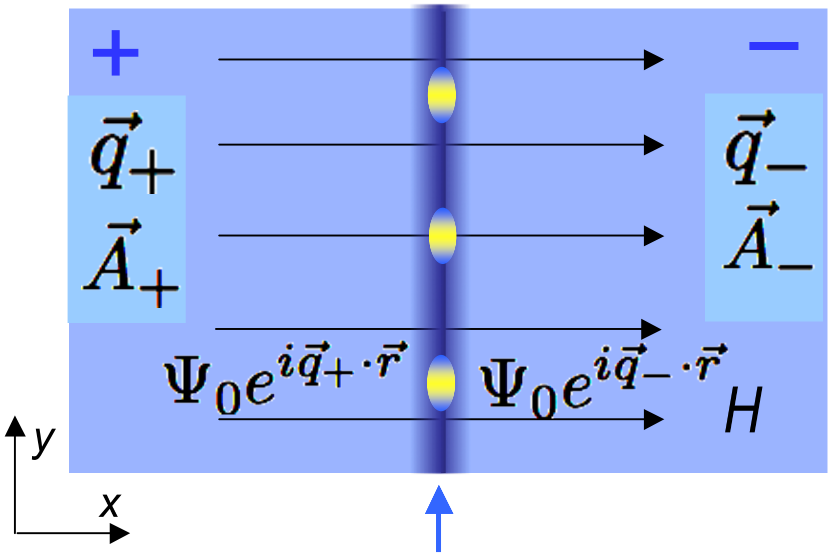
$$q_{y+} = q_{y-}$$

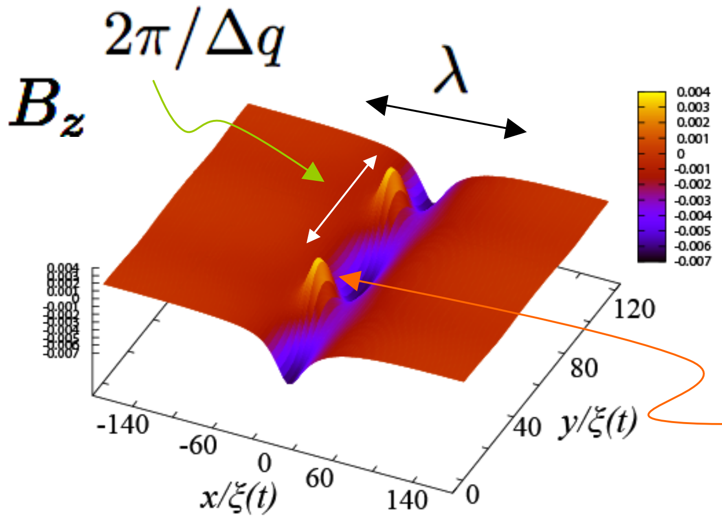
⬇

magnetic field $B_z$ on twin boundary

$$\phi = A_{y+} - A_{y-} = 2\Phi_0 K\hat{x}\cdot\vec{H}$$

# inhomogeneous helical states




$$\vec{q}_\pm = \pm K(\hat{z} \times \vec{H}) - \frac{2e}{\hbar c}\vec{A}_\pm$$

large fields

➡ reduction magnetic flux

$$A_{y+} \approx A_{y-}$$

⬇

wave function phase mismatch

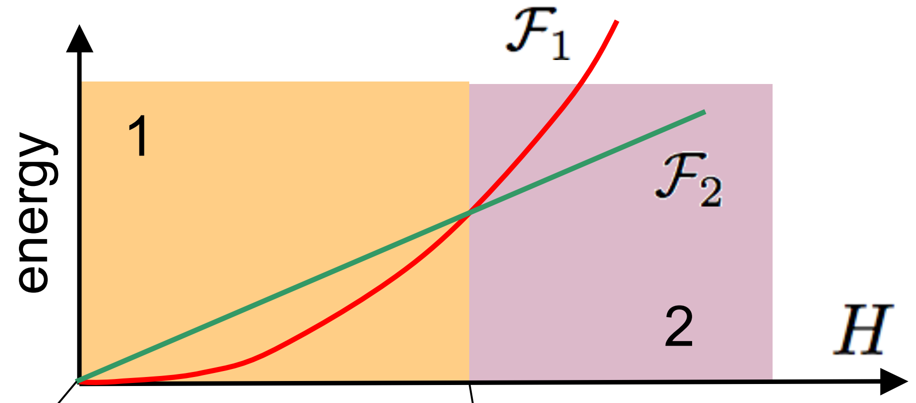
$$\Delta q = q_{y+} - q_{y-} \approx 2K\hat{x}\cdot\vec{H}$$

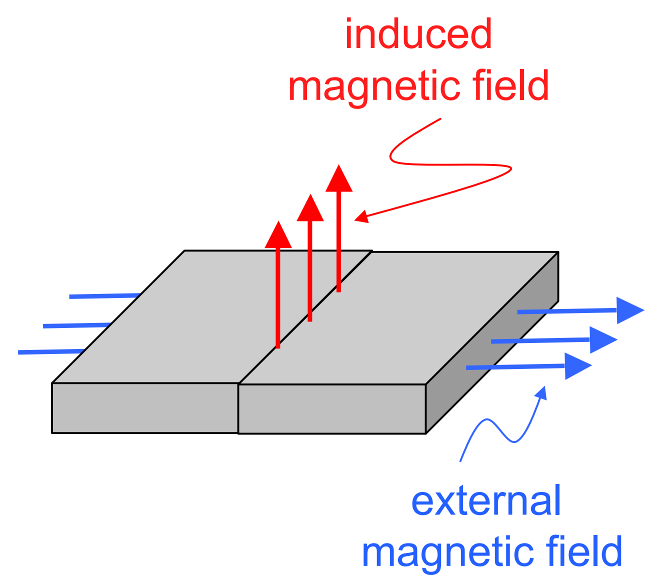
solitons - flux lines

as a field screening effect

# inhomogeneous helical states

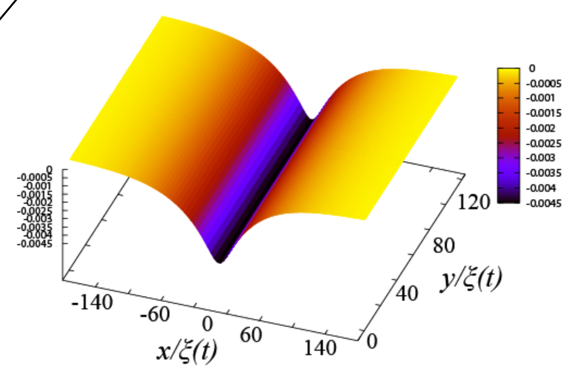
## first-order transition in a magnetic field

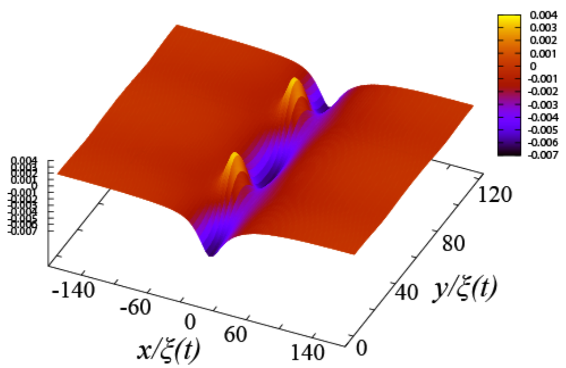



$$\mathcal{F}_1 \propto \phi^2 \propto H^2$$

$$\mathcal{F}_2 \propto \Delta q \propto H$$



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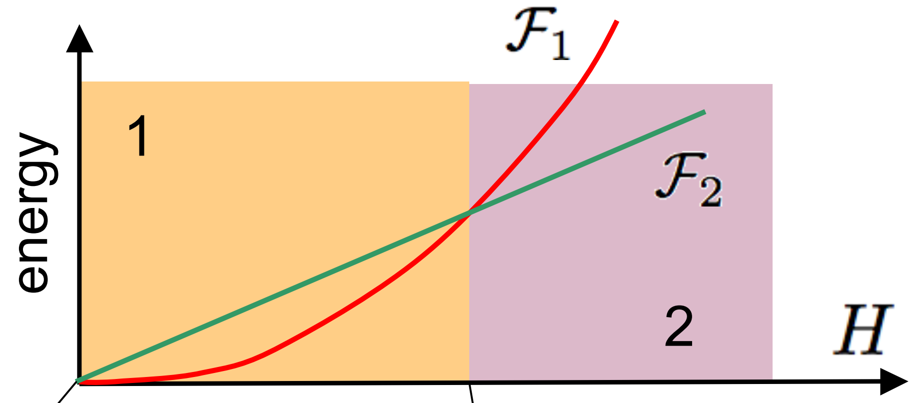


GCOE collaboration

Kazushi Aoyama

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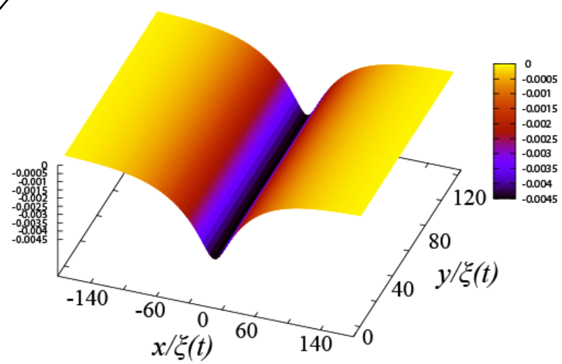


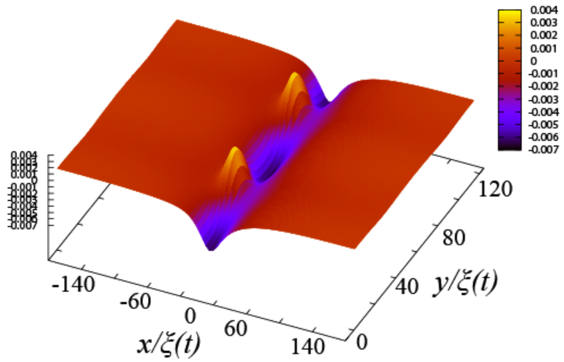

# Conclusion

symmetry and topology classification of superconductors

| class | SU(2) | TRS |
|---|---|---|
| D | no | no |
| DIII | no | yes |
| A | restricted | no |
| AIII | restricted | yes |
| C | yes | no |
| CI | yes | yes |

Schnyder, Ryu, Furusaki & Ludwig (2008)

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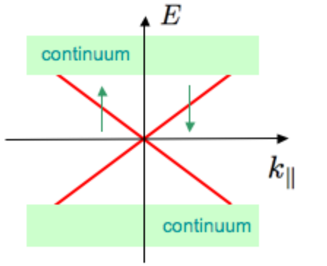
Schnyder, Ryu, Furusaki & Ludwig (2008)

$$U(1)_{L_z+S_z} \times \mathcal{K}$$

mixed-parity "s+p-wave"




helical
edge states

spin current

# Conclusion

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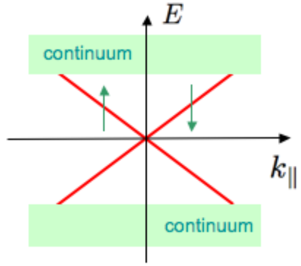
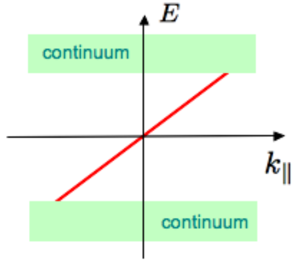



helical edge states

spin current

$U(1)_{S_z} \times U(1)_{L_z+\phi}$

"chiral p-wave"




chiral edge states

charge current

# Collaborators & Acknowledgement


## Theory:

D. Agterberg, K. Aoyama, S. Etter, P. Frigeri, S. Fujimoto,
A. Furusaki, J. Goryo, B. Gut, N. Hayashi, Y. Imai, C. Iniotakis,
H. Kaneyasu, R. Kaur, A. Koga, F. Loder, M. Matsumoto,
D. Perez, T.M. Rice, L. Savary, Y. Tanaka, K. Wakabayashi,
Y. Yokoyama, Y. Yanase

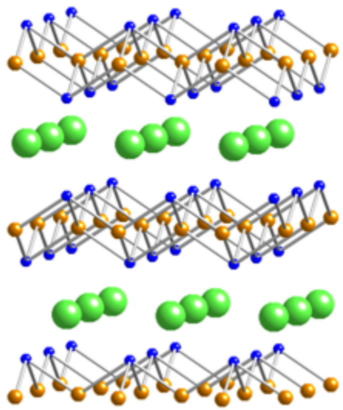
## Experimental groups:

Y. Maeno, E. Bauer, A. Mackenzie, J. Kirtley, K. Moler,
Q. Mao, Y. Liu, H. Yaguchi, T. Nakamura, Y. Onuki,
R. Settai, N. Kimura, I. Bonalde, A. Kapitulnik, …

# Unconventional Superconductors

## strongly correlated electron systems

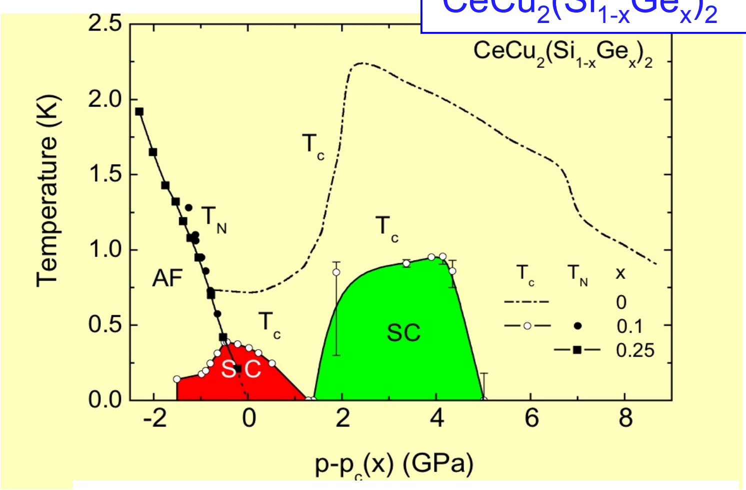
**heavy Fermion compounds**



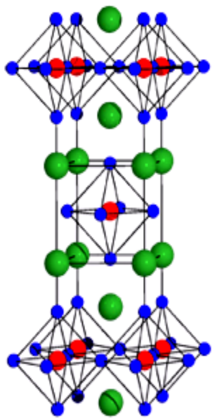
since 1979

*4f*-electrons

$CeCu_2(Si_{1-x}Ge_x)_2$



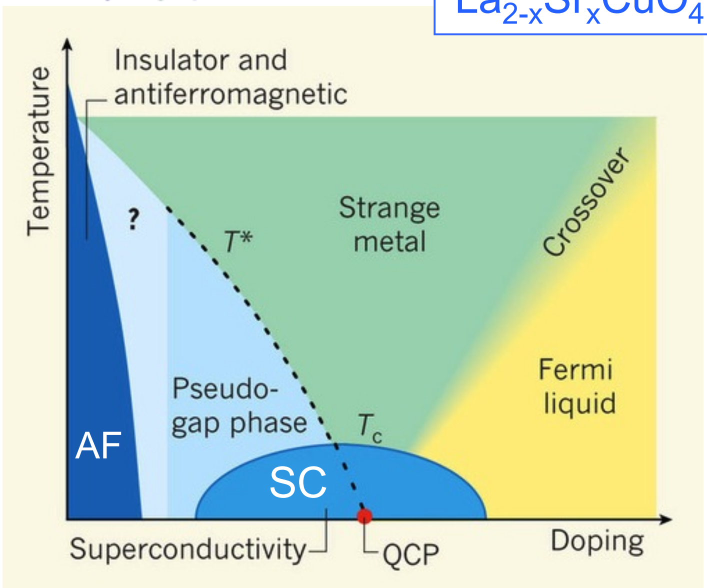

**cuprate high-$T_c$ superconductors**



since 1986

*3d*-electrons

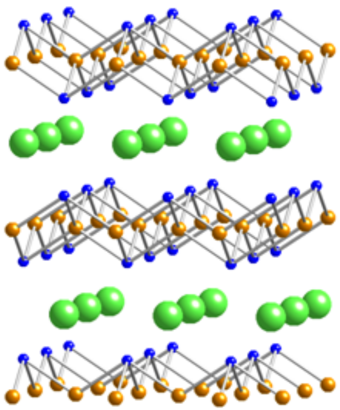
$La_{2-x}Sr_xCuO_4$

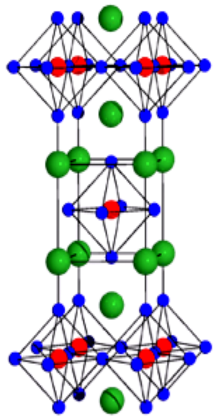


# Unconventional Superconductors

## strongly correlated electron systems

| heavy Fermion compounds | cuprate high-$T_c$ superconductors |
| --- | --- |
| since 1979 | since 1986 |
| *4f*-electrons | *3d*-electrons |





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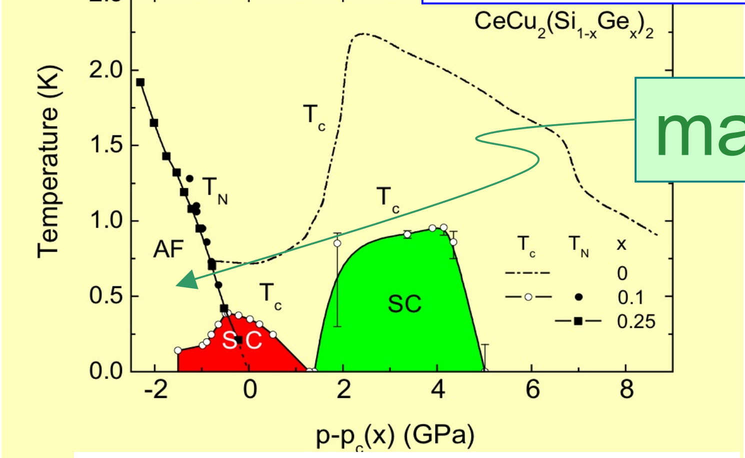


magnetism

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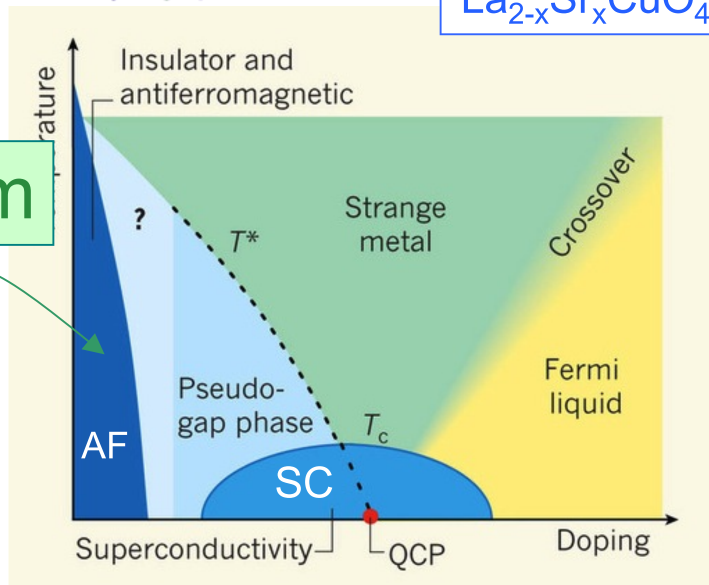
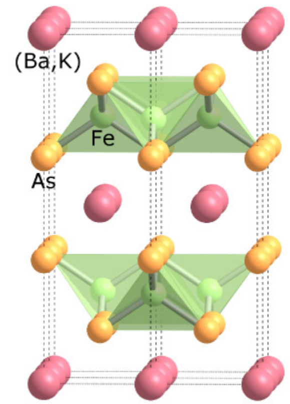


# Unconventional Superconductors

strongly correlated electron systems

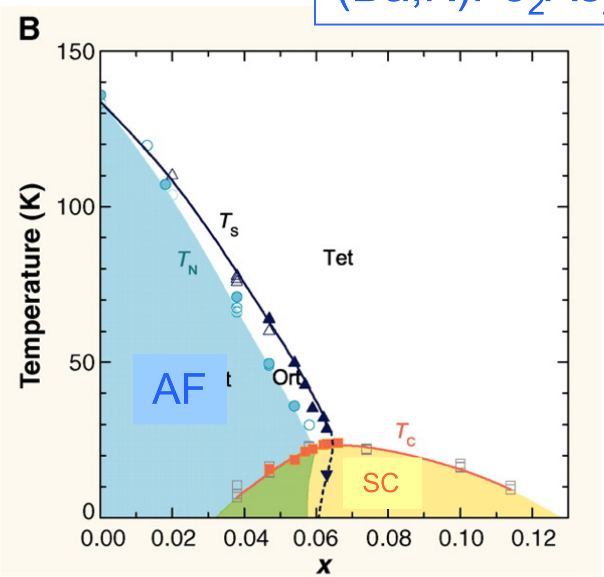
pnictide superconductors




since 2008

*3d*-electrons

$(Ba,K)Fe_2As_2$




"essential !?"

superconductivity
connected with magnetism

Competion

Cooperation

Coexistence