Superstring Theory, M-theory and the Dynamics of Branes

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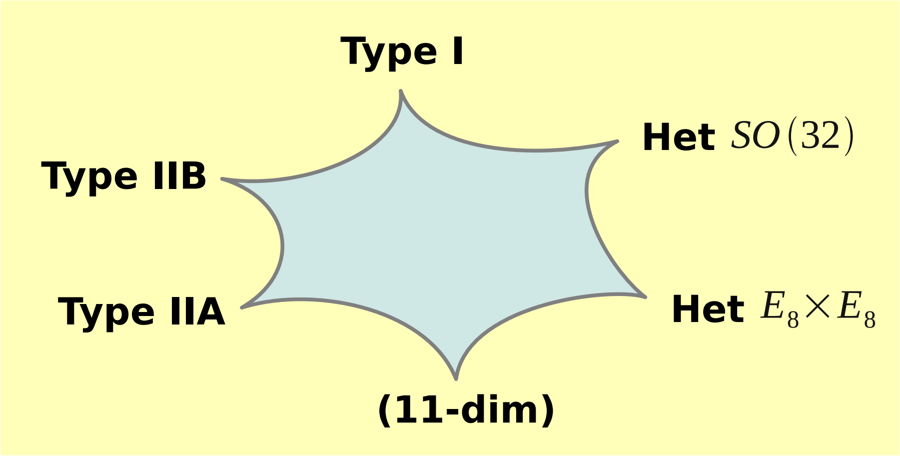
(String) Dualities

-- Quantum equivalence relations among
seemingly different (superstring) theories.

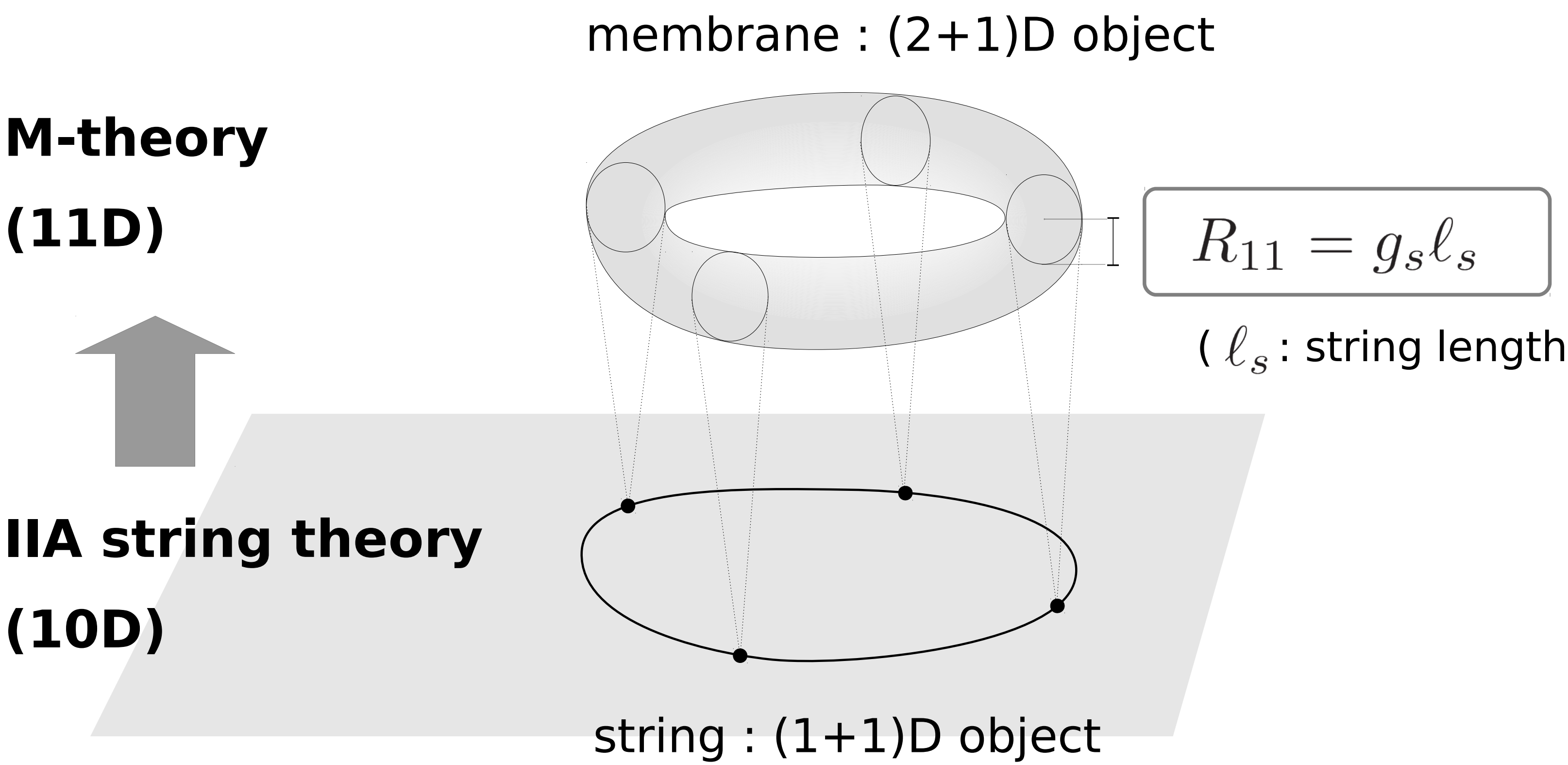
-- Often relates
Strongly-coupled ⟷ weakly-coupled theories



Unification of 5 different superstring theories

M-theory (Witten 1995)

In the strong coupling limit ($g_s \to \infty$) of IIA string theory, the 11$^{\text{th}}$ dimension shows up.



M-theory (Witten 1995)

-- No intrinsic definition.

-- The only parameter is Newton's constant.

unique (=ultimate) theory.

$$G = \frac{1}{32\pi^2}(2\pi\ell_{\mathrm{P}})^9$$

ℓ_{P} : Planck length

-- Reduce to 11d supergravity at low energy

-- Fundamental objects are

* membranes: (2+1)D object
* M5-branes: (5+1)D object

11D Supergravity

$$g_{mn}, \qquad \psi_{m\alpha}, \qquad C = \frac{1}{6} C_{mnp} \mathrm{d}x^m \mathrm{d}x^n \mathrm{d}x^p \quad (\mathrm{d}C = F)$$

metric gravitino gauge field

$$S \;=\; \frac{1}{16\pi G} \left[\int \mathrm{d}^{11}x \sqrt{-g} R - \frac{1}{2} \int F \wedge *F - \frac{1}{6} \int C \wedge F \wedge F + \cdots \right],$$

Membrane : electric charge of C

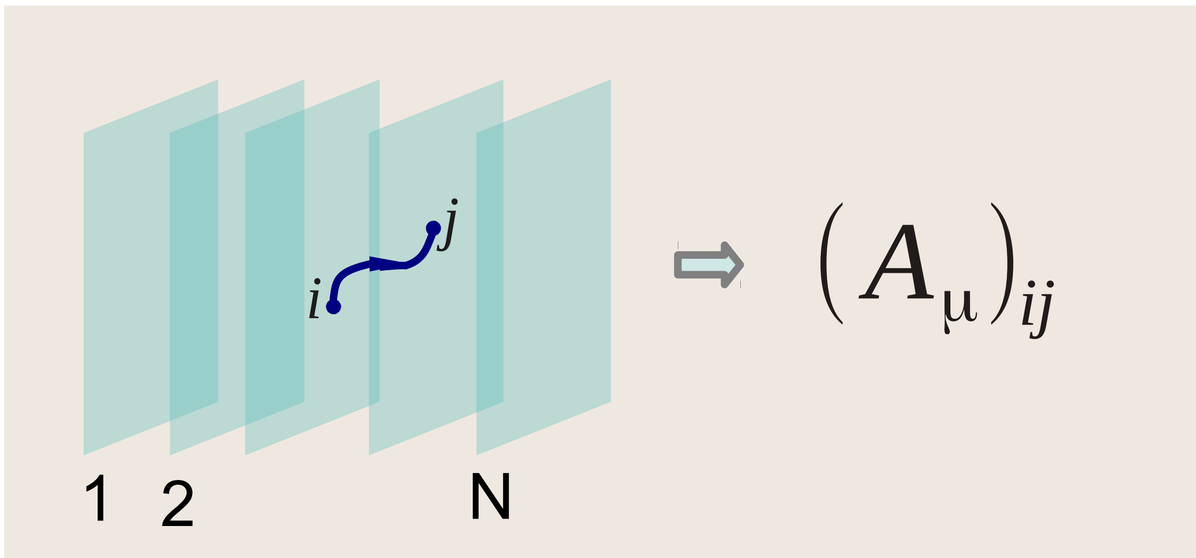
M5-brane : magnetic charge of C

Today's Topic

Recent developments in the study of collective dynamics of **M5-branes**.

Collective Dynamics of Branes

N coincident **D-branes** (=solitonic objects in superstring theory) give rise to U(N) gauge symmetry.



The theory of N coincident **membranes**
= 3D U(N) x U(N) SUSY Chern-Simons gauge theory. (2008)

What about coincident **M5-branes**?

(2,0)-Theory
= Theory of Multiple M5-branes

-- 6D field theory with no Lagrangian

-- labeled by N = number of M5-branes,
(more precisely, by ADE groups)
no other coupling constants.

-- Many low-dim field theories arise
Upon compactification

Compactification of (2,0)-Theory

$\mathbb{R}^{1,4} \times S^1 \longrightarrow$ 5D SUSY Yang-Mills

radius: R_1

coupling: $g^2_{(5)} = 8\pi^2 R_1$

* N M5-branes

* SU(N) gauge symmetry

$$\mathcal{S} \;=\; \frac{1}{2g^2_{(5)}} \int \mathrm{d}^5 x \mathrm{Tr}\left(-F_{\mu\nu}F^{\mu\nu} + \cdots\right)$$

$$F_{\mu\nu} \;\equiv\; \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

Compactification of (2,0)-Theory

$\mathbb{R}^{1,4} \times S^1$ $\longrightarrow$ 5D SUSY Yang-Mills

radius: R_1

coupling: $g_{(5)}^2 = 8\pi^2 R_1$

$\mathbb{R}^{1,3} \times S^1 \times S^1$ $\longrightarrow$ 4D SUSY Yang-Mills

radius: R_1, R_2

coupling: $\frac{1}{g_{(4)}^2} = \frac{2\pi R_2}{g_{(5)}^2} = \frac{R_2}{4\pi R_1}$

Compactification of (2,0)-Theory

$\mathbb{R}^{1,3} \times S^1 \times S^1 \longrightarrow$ 4D SUSY Yang-Mills

radius: R_1, R_2

coupling: $\dfrac{1}{g_{(4)}^2} = \dfrac{R_2}{4\pi R_1}$

What if we change the order of compactification?

$\longrightarrow$ coupling: $\dfrac{1}{g_{(4)}^2} = \dfrac{R_1}{4\pi R_2}$

4D SUSY Y-M at the two different couplings are the same !?

Montonen-Olive Duality

4D (Max SUSY) YM has a hidden equivalence

which inverts the coupling and

exchanges electric/magnetic particles.

cf) electromagnetism $e \cdot m \ = \ 2\pi$

$$\nabla \cdot \mathbf{E} \ = \ \rho \qquad \nabla \times \mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} \ = \ 0$$

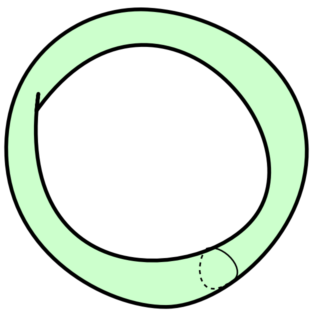
$$\nabla \cdot \mathbf{B} \ = \ 0 \qquad \nabla \times \mathbf{B} - \frac{\partial}{\partial t}\mathbf{E} \ = \ \mathbf{j}$$

M5-branes can explain Montonen-Olive duality

from simple geometry of 2D torus.

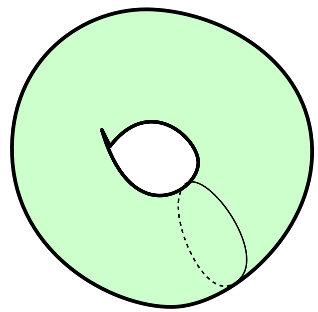
Shape vs. Coupling

coupling: $$\frac{1}{g^2_{(4)}} = \frac{R_2}{4\pi R_1}$$



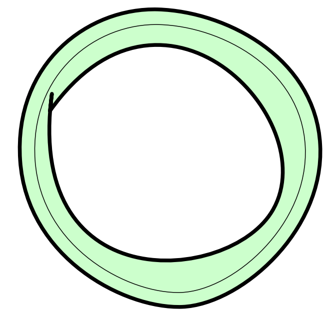
$R_1 \ll R_2$

Weakly coupled



$R_1 \sim R_2$

Strongly coupled



$R_1 \gg R_2$

Weakly coupled
(in dual description)

More general torus

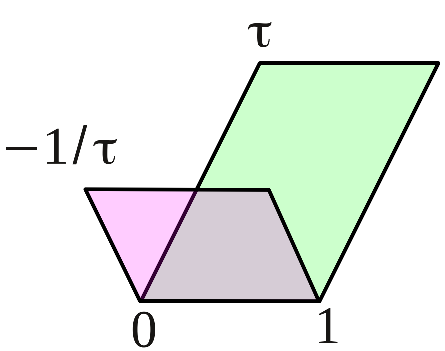
$\mathbb{R}^{1,3} \times T^2$ → 4D SUSY Yang-Mills

shape : $\tau \in \mathbb{C}$

Complex coupling : $\tau = \dfrac{\theta}{2\pi} + \dfrac{4\pi i}{g^2}$

$$\mathcal{S} \;=\; \int \mathrm{d}^4x \mathrm{Tr}\left(\frac{1}{2g^2} F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} + \cdots\right)$$

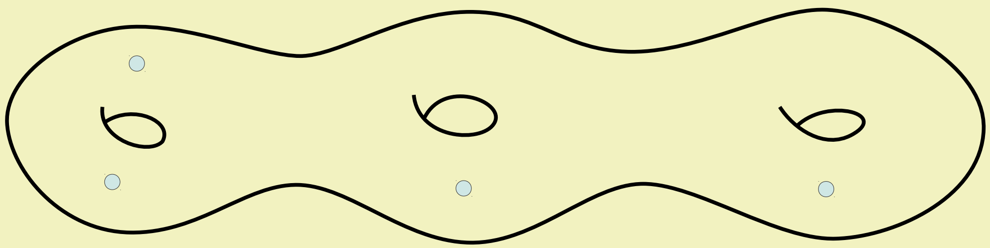
(Instanton density)



Montonen-Olive:

The theory at τ and at $-1/\tau$ are the same.

Further Generalization (Gaiotto 2009)



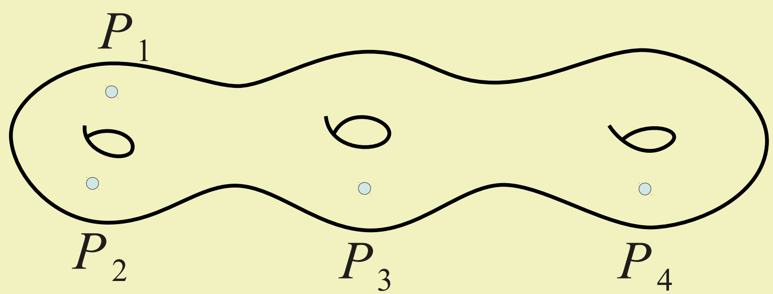
N M5-branes wrapping Riemann surface Σ

(with punctures)

$\longrightarrow$ 4D SUSY gauge theory $T_N(\Sigma)$

(We discuss the case of **2 M5-branes** today)

Wrapping 2 M5-branes on Σ



Shape : τ

* includes the position of punctures

Spike angle at P_a : m_a

→ **4D gauge theory** $T_2(\Sigma)$ (Gaiotto 2009)

UV finite theory with $\mathcal{N}=2$ SUSY.

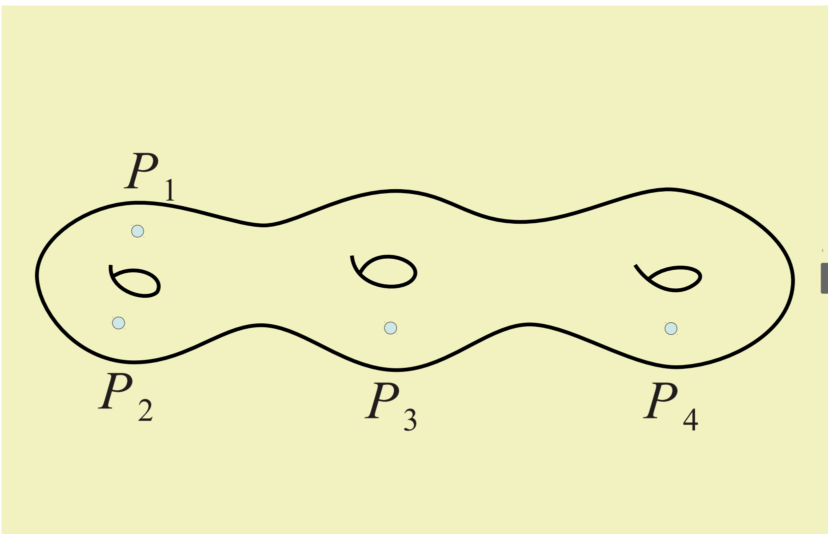
τ (shape) = gauge coupling(s)

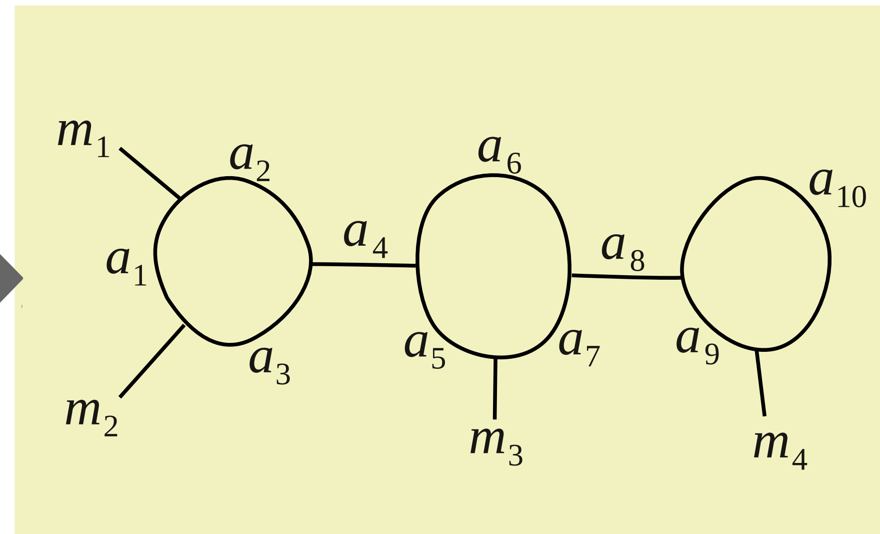
m_a (spike angles) = mass of matter particles

To read off the Lagrangian,

go to the limit where Σ looks like a network of thin tubes.

[example]





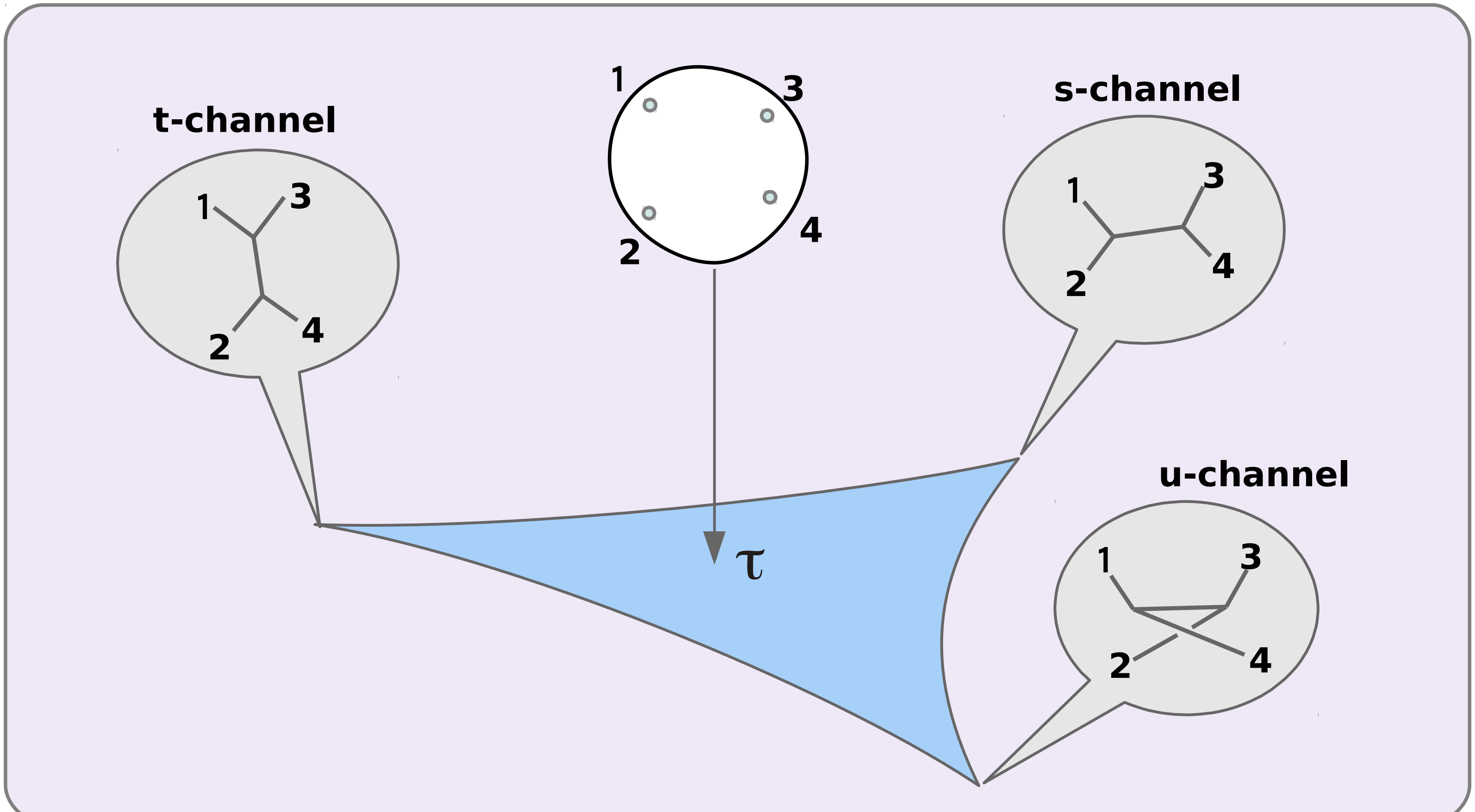
$SU(2)^{10}$ gauge symmetry

from 2 M5-branes wrapping 10 thin tubes.

However,

For any given Σ , there are several weak coupling limits.

[example] 4-punctured sphere



3 different, mutually dual Lagrangians.

Compactifications of (2,0)-Theory

on S^1 $\longrightarrow$ 5D SU(N) Yang-Mills

on $S^1 \times S^1$ $\longrightarrow$ 4D SU(N) Yang-Mills

on Σ $\longrightarrow$ 4D gauge theory $T_N(\Sigma)$

AGT Relation (Alday-Gaiotto-Tachikawa 2009)

Partition function of 4D theory $T_2(\Sigma)$ on 4-sphere	**=**	Correlation function of 2D **Liouville CFT** on Σ (at b=1)

"Mysterious agreement"

cf) Liouville CFT

Coupling : b

Lagrangian : $\mathcal{L} = (\partial_t \phi)^2 + (\partial_x \phi)^2 + e^{2b\phi}$

AGT Relation (Alday-Gaiotto-Tachikawa 2009)

The conjectured relation has been confirmed

through the comparisons of exactly calculable quantities

Correlation functions in 2D CFTs

-- systematic construction have long been known.

Partition function of SUSY theories on sphere

-- “SUSY localization theorem” allows exact evaluation.

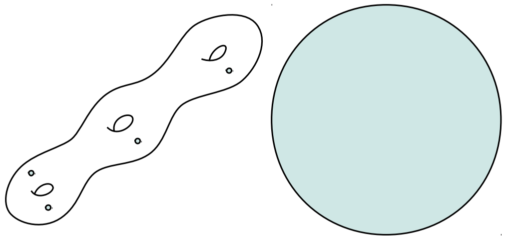
(hot topic in recent years)

Interpretation

$Z(m_a, \tau)$: Partition function of

4D gauge theory $T_2(\Sigma)$ on round 4-sphere

➡ 2 M5-branes wrapping $\Sigma \times$ (4-sphere)



changing the order of compactification,

➡ A 2D field theory on Σ

Correlation function : $\langle V_{m_1}(P_1) \cdots V_{m_n}(P_n) \rangle_\tau$

Since it depends only on the shape τ of Σ ,

the 2D theory should be conformal.

AGT Relation (Alday-Gaiotto-Tachikawa 2009)

Partition function of 4D theory $T_2(\Sigma)$ on 4-sphere	**=**	Correlation function of 2D **Liouville CFT** on Σ (at b=1)

Checking the agreement for different choices of Σ
led to the claim

2 M5-branes wrapping on 4-sphere = Liouville CFT (at b=1)

Compactifications of (2,0)-Theory

on S^1	$\longrightarrow$	5D SU(N) Yang-Mills
on $S^1 \times S^1$	$\longrightarrow$	4D SU(N) Yang-Mills
on Σ	$\longrightarrow$	4D gauge theory $T_N(\Sigma)$
on S^4	$\longrightarrow$	2D A_{N-1} Toda CFT (b=1)

* A_1 Toda CFT = Liouville CFT

Compactifications of (2,0)-Theory

on S^1 $\longrightarrow$ 5D SU(N) Yang-Mills

on $S^1 \times S^1$ $\longrightarrow$ 4D SU(N) Yang-Mills

on Σ $\longrightarrow$ 4D gauge theory $T_N(\Sigma)$

on S^4 $\longrightarrow$ 2D A_{N-1} Toda CFT (b=1)

on 4D Ellipsoid $\longrightarrow$ 2D A_{N-1} Toda CFT (any b)

$$b^2(x_1^2 + x_2^2) + \frac{1}{b^2}(x_3^2 + x_4^2) + c^2 x_5^2 \;=\; 1 \quad \text{in} \quad \mathbb{R}^5$$

Compactifications of (2,0)-Theory

on S^1 $\longrightarrow$	5D SU(N) Yang-Mills
on $S^1 \times S^1$ $\longrightarrow$	4D SU(N) Yang-Mills
on Σ $\longrightarrow$	4D gauge theory $T_N(\Sigma)$
on S^4 $\longrightarrow$	2D A_{N-1} Toda CFT (b=1)
on 4D Ellipsoid $\longrightarrow$	2D A_{N-1} Toda CFT (any b)
on 3D Ellipsoid $\longrightarrow$	3D $SL(N)$ Chern-Simons

Summary

(2,0)-theory = Theory of multiple M5-branes

We cannot write Lagrangian for it

but we are sure it exists.

-- Upon compactification,

it gives rises to various low-dim gauge theories,

and provides geometric explanations of

how dualities of low-dim gauge theories work.

-- It predicts precise correspondences

between field theories in different dimensions.

Summary

Through the study of (2,0)-Theory,

-- We became increasingly aware
there are many important “non-Lagrangian” theories
in different dimensions (not only in 6d).

-- We became interested in
various new “exactly calculable” quantities.
(Collecting such quantities will be as good as
writing down the Lagrangian)