Quantum Walk

Applications to Topological Quantum Phenomena in Condensed Matter

Norio Kawakami

Condensed Matter Theory

Collaborators

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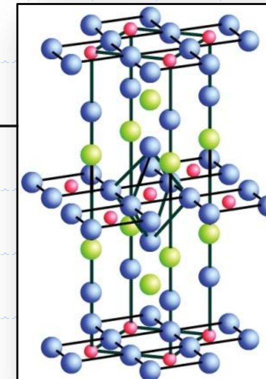
Condensed Matter Physics

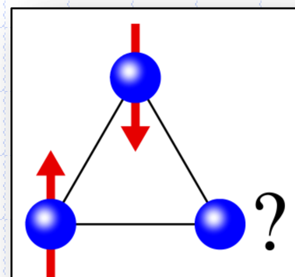
Emergence: Novel Quantum States

Key words: recent topics



Correlation

Mott insulators, Heavy fermions
Exotic superconductors, etc



Frustration

Spin liquid, Entanglement
Unconventinal order, etc

Topology

Topological insulators, superconductors
Majorana fermions, etc

Novel Quantum States in condensed matter

Correlation, Frustration and Topology

Yukawa Institute, Kyoto (2011)

Organizers:* *Kawakami, Tohyama, Totsuka, etc

Nov. 7-11:	**frustration**
Nov. 14-18:	**frustration / topology**
Nov. 21-25:	**topology / correlation**
Nov. 28- Dec.2:	**correlation / superconductivity**
Dec. 5-9:	**all topics**

Condensed Matter Physics

Emergence: Novel Quantum States

Key words: recent topics

Correlation

Mott insulator, Heavy fermions
Exotic superconductors, etc

Frustration

Spin liquid, Entanglement
Unconventinal order, etc

Topology

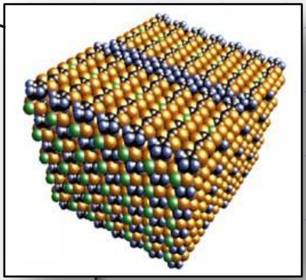
Topological insulators, superconductors
Majorana fermions, etc

M. Sigrist

Y. Matsuda

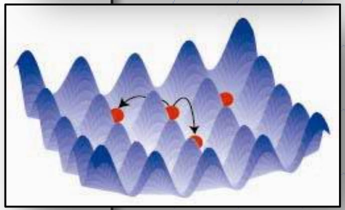
N. Kawakami

Platforms of Topological Phenomena



■ **Solid state** Maeno (MEXT 新学術)

Semiconductors, Superconductors, Correlated systems, Spin systems, etc

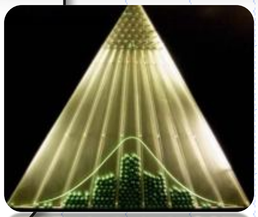


■ **Cold atoms** Takahashi (EXP), Fujimoto (Theory)

Tunable parameters, Synthetic gauge, etc

■ **Quantum walk**

Real Time dynamics



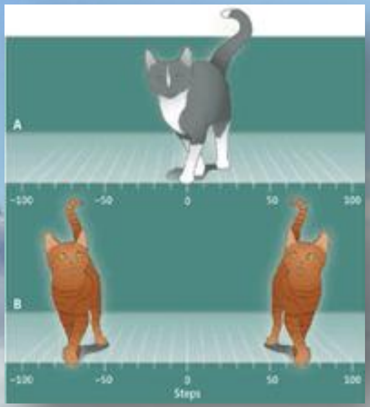
etc

What' going on?
QW
an emergent new field

量子ウォーク？
never heard ?
Condensed matter?

Google Scholar
hits so many !
(since 2009)

What is a quantum walk ?

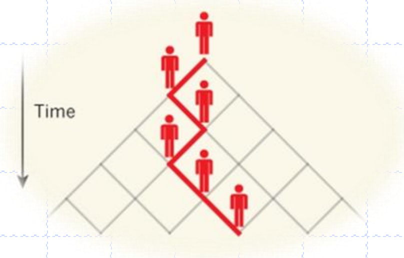


Quantum walk

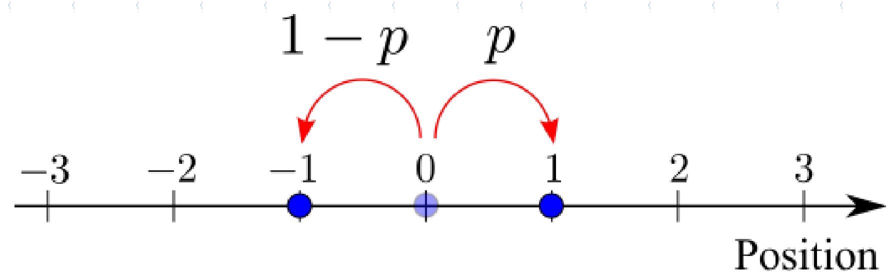
quantum mechanical time-evolution of particles

Quantum version of random walk

Random walk

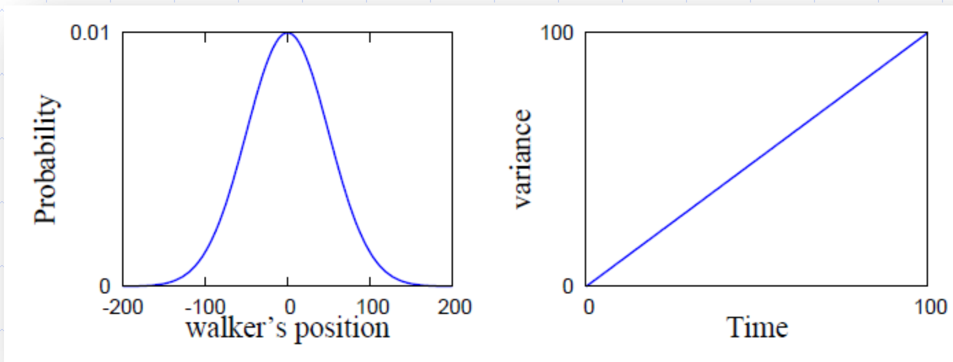


Walkers move to right (left) with probability *p* (*1-p*)

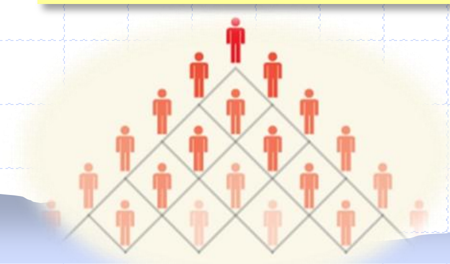


Walker's position at *t* : Gaussian

(Random walks) **variance $\sigma^2 \propto t$**

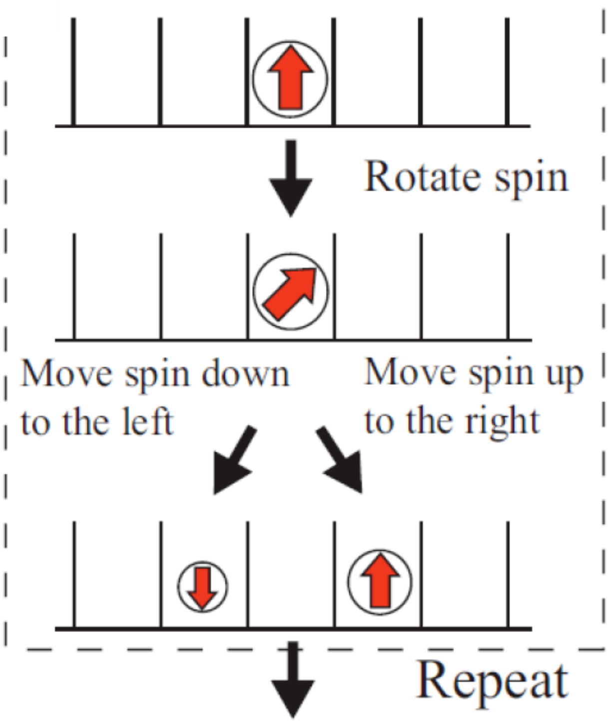


Quantum walk



Quantum walk

Discrete-time QW



A walker at *n*:

internal degrees |*L*>, |*R*>

(|*L*> ↓, |*R*> ↑)

◇Coin operator

rotate spin, mix |L> and |R>

$$C_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}$$

◇Shift operator

spin-selective motion

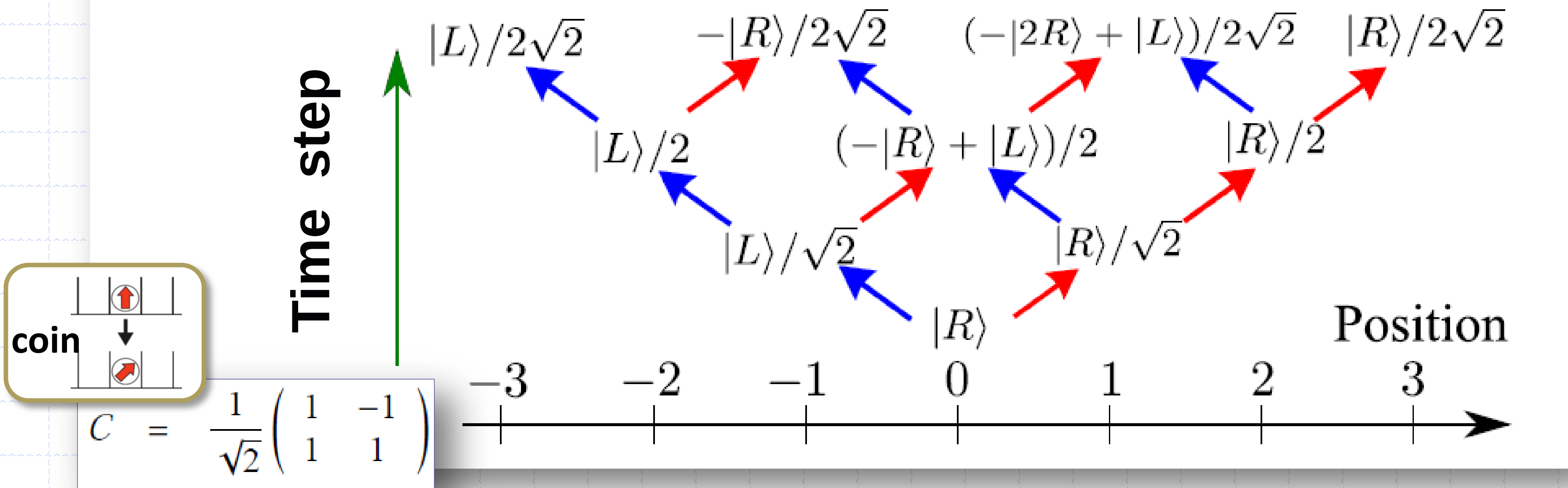
$$W = \sum_n (|n-1, L\rangle\langle n, L| + |n+1, R\rangle\langle n, R|)$$

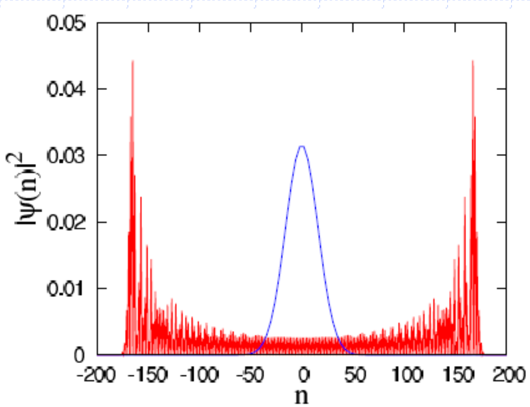
↓ left ↑ right

◇Time evolution operator

$$U = W\left(\sum_n |n\rangle\langle n| \otimes C_n\right)$$

- Time-evolution of QW with $\theta = \pi/4$: ***Hadamard walk***





- QW have two peaks at the edges, and their variance is proportional to t^2.
- Then, QW can evolve faster than the classical random walks.
- QW is useful for quantum computations.

Mathematics, Quantum information

今野紀雄 「量子ウォークの数理」(2008)

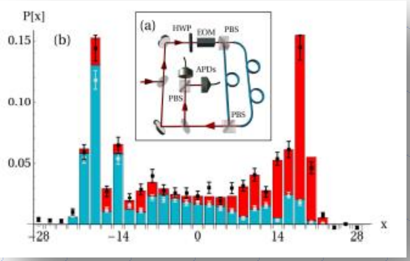

Progress in experiments
so rapid !
since 2009

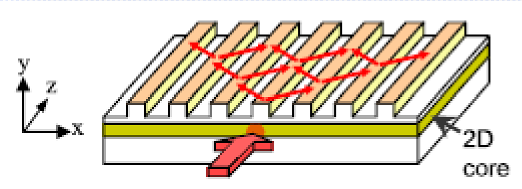
Quantum Walks

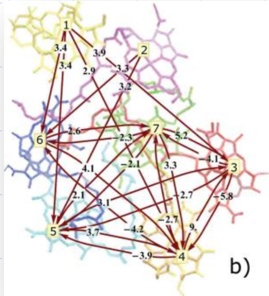
Experiments and proposals

- ◇ Trapped ions
- ◇ Cold atoms
- ◇ Photons
- ◇ NMR
- ◇ Photosynthetic energy transfer (excitons)

etc

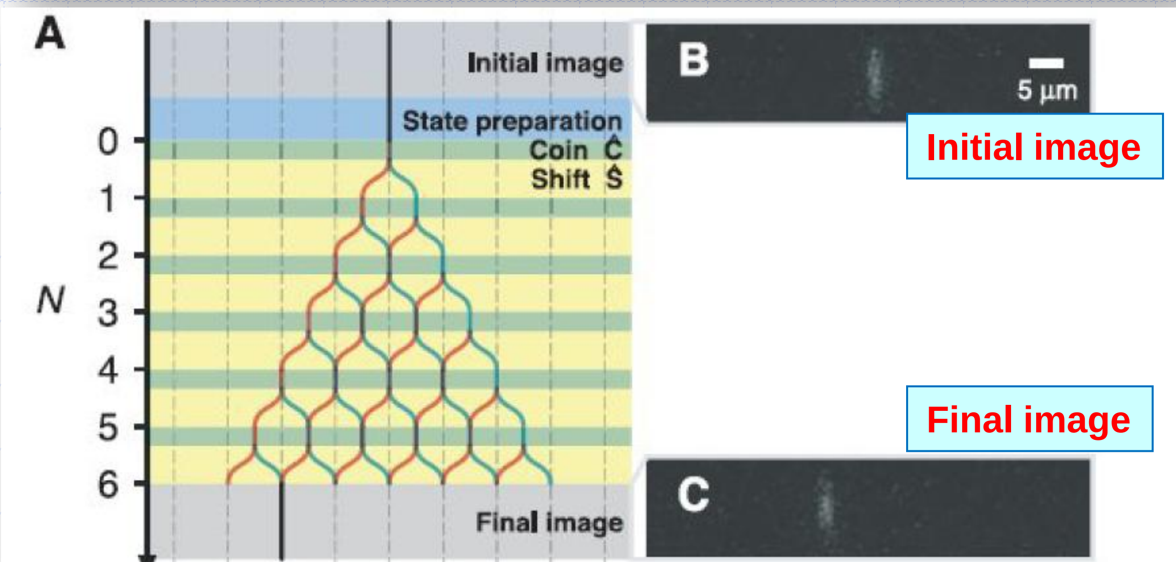


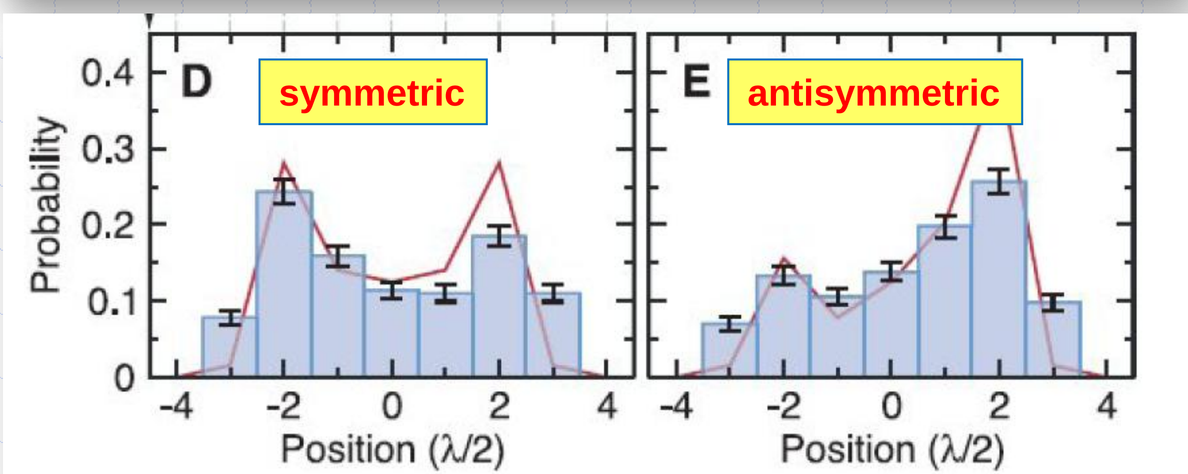




Quantum Walk in Position Space with Single Optically Trapped Atoms
Michal Karski, *et al.*
Science **325**, 174 (2009);

Science 2009





Cold atoms
(Cs atoms)

|L>, |R>
$F=4, m_F=4$
$F=3, m_F=3$

1D
Optical lattice
(Position space)

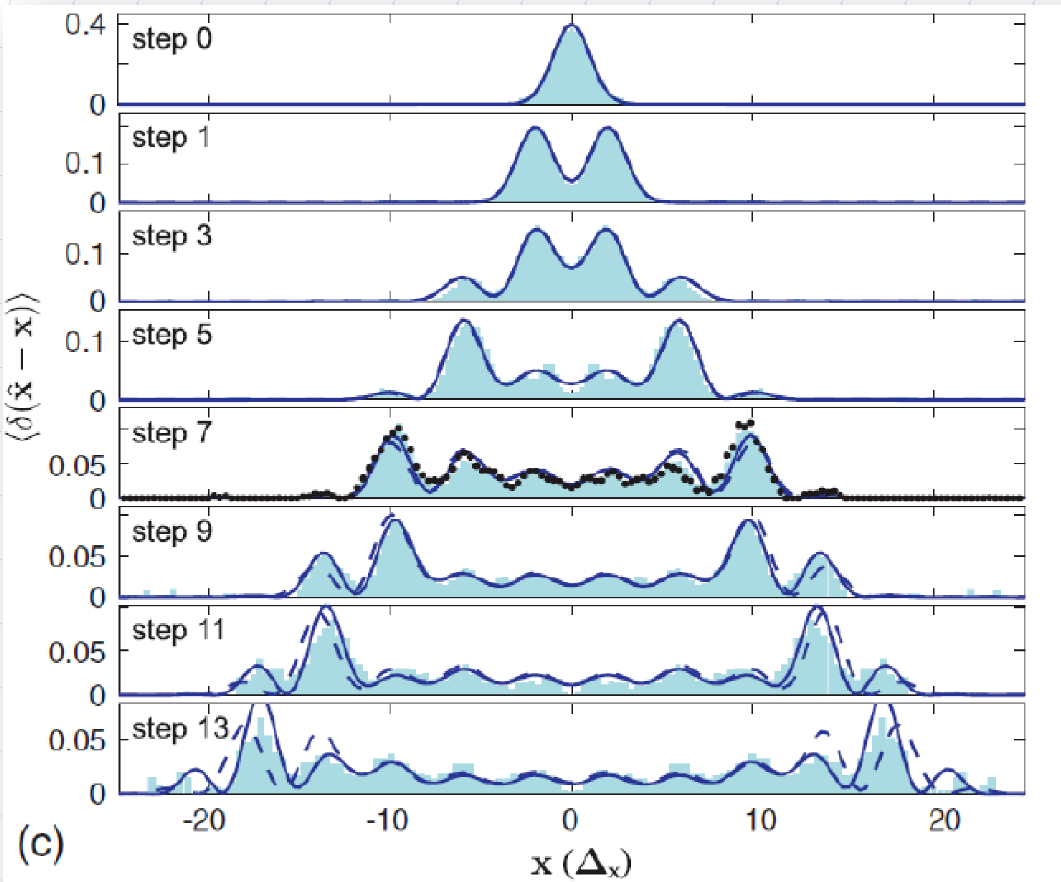
10 steps

$\lambda/2=433$ nm

Realization of a Quantum Walk with One and Two Trapped Ions

F. Zähringer,[1,2] G. Kirchmair,[1,2] R. Gerritsma,[1,2] E. Solano,[3,4] R. Blatt,[1,2] and C. F. Roos[1,2]

PRL (2010)



Trapped Ions
$^{40}Ca^+$

|*L*>, |*R*>

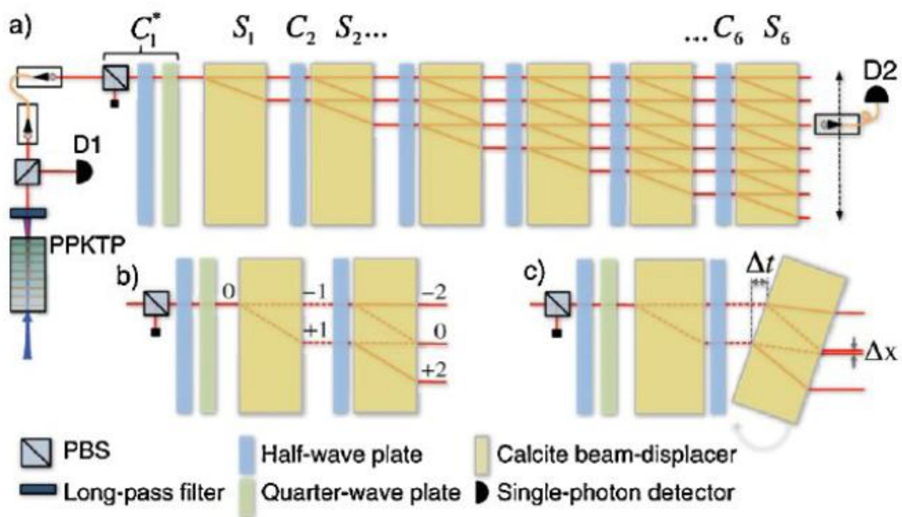
$S_{1/2}$, m=1/2
$D_{5/2}$, m=3/2

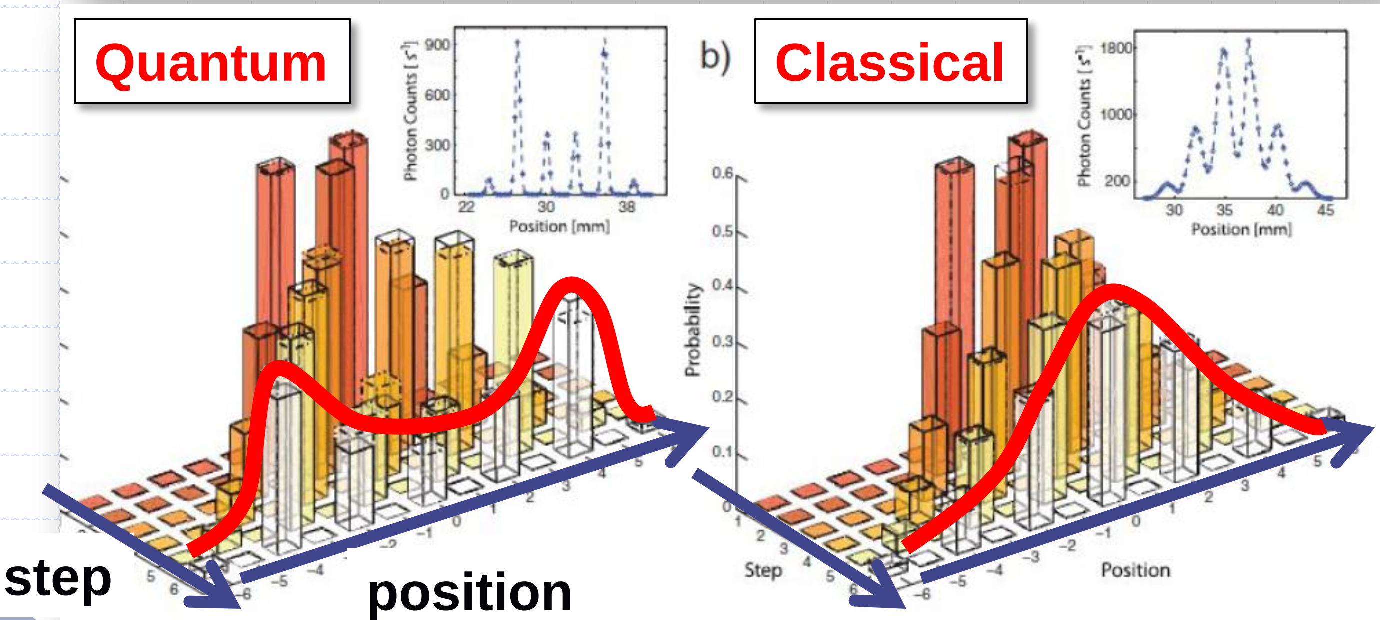
Position:
Phase space
23 steps

Jaynes-Cummings Hamiltonian

Discrete Single-Photon Quantum Walks with Tunable Decoherence

M. A. Broome,[1] A. Fedrizzi,[1] B. P. Lanyon,[1] I. Kassal,[2] A. Aspuru-Guzik,[2] and A. G. White[1]





PRL (2010)

Photons

|L>, |R> **Polarization**

Position:
spatial modes
6 steps
(70 steps, 2011 Erlangen)

Decoherence
temporal- **disorder**
Quantum to Classical

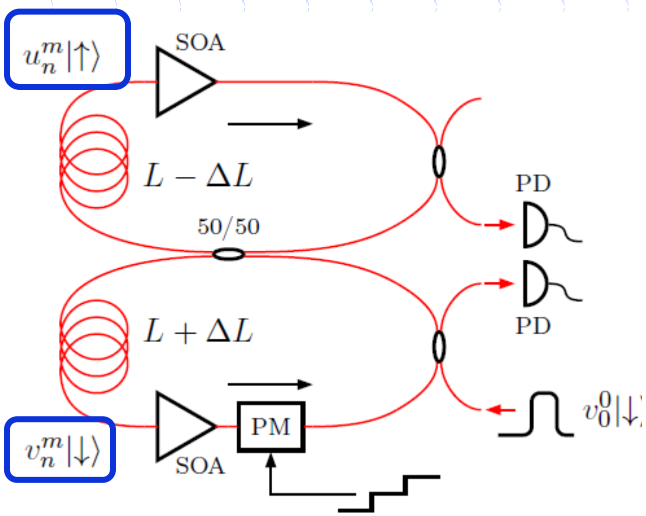
Zitterbewegung, Bloch Oscillations and Landau-Zener Tunneling in a Quantum Walk

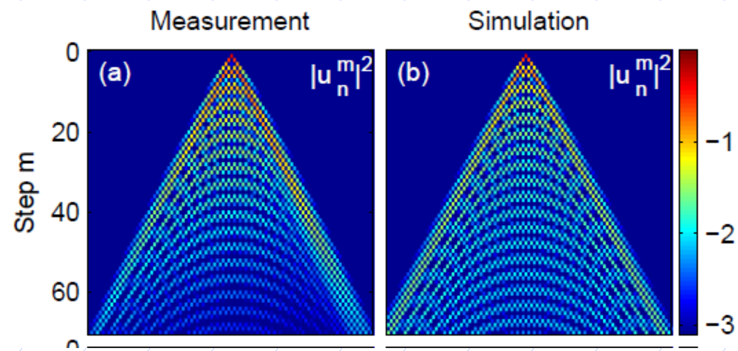
Alois Regensburger,[1,2] Christoph Bersch,[1,2] Benjamin Hinrichs,[1,2] Georgy Onishchukov,[2] Andreas Schreiber,[2] Christine Silberhorn,[2,3] and Ulf Peschel[1,*]

70 steps

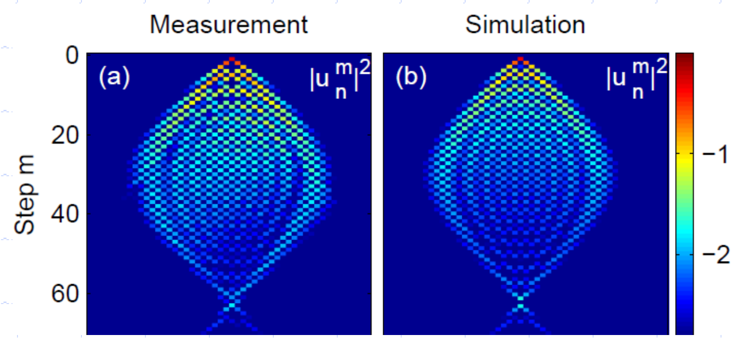
Photonic systems

2011 Erlangen group

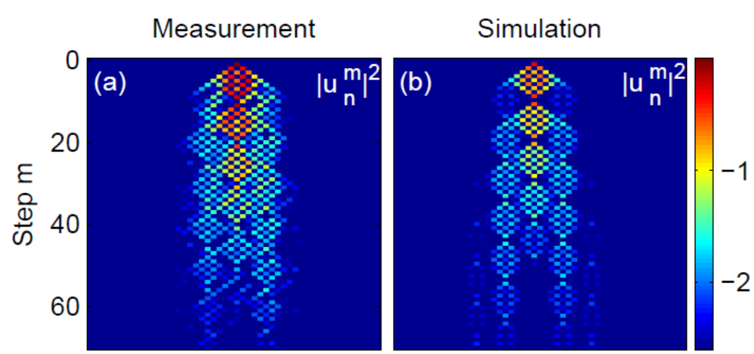




Zitterbewegung



Bloch Oscillation



Landau Zener

50/50 $C = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

PM: Phase modulator

Quantum Walk

Experimental realization

M. Karski et al., Science 325, 174 (2009)

H. Schmitz et al., PRL 103, 090504 (2009)

F. Zahringer et al., PRL 104, 100503 (2010)

M. A. Broome et al, PRL104, 153602 (2010)

A. Peruzzo et al. , Science 329, 1500 (2010)

M. Hilley, Science 329, 1477 (2010)

R. Gerritsma et al., Nature 483, 68 (2010)

A. Schreiber et al, PRL106, 180403 (2011)

Kitagawa, et al., Nature Commun. 3, 882 (2012)

A. Schreiber et al., Science 336, 55 (2012)

A. Schreiber et al., Science 336, 6077 (2012)

J. Matthews et al., Nature 484, 47 (2012)

A. Asupuru-Guzik., Nature Physics 8, 285 (2012)

…. etc

Cold atoms

Trapped ions

Photons

…. etc

Quantum Walk

◇Developed in Quantum Information

Mathematical

e.g. Konno et al.

◇Condensed Matter Physics

1. Toplogical insulators: tuning the { coin / shift } operator

All the possible topological insulators (1D, 2D)

Kitagawa et al 2010

New arena to study topological states

2. Applications to breakdown of Mott insulators

- Zener Tunneling: modeled by **QW**

T.Oka et al 2005

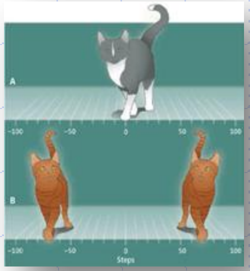
- Non-equilibrium dynamics of Mott phase

cf 1D Non-Hermitian Hubbard: Exact solution

T. Fukui-NK 1998
T.Oka et al. 2010

Correlated electron systems

Quantum Walk



Systematic Studies of Topological Insulators

Complementary to solid state physics

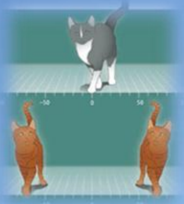
1.Dynamics of 1D Quantum Walks

Topological insulating phases

2. Static and dynamical random defects

How robust topological edge states are ?

Before enjoying a quantum walk,



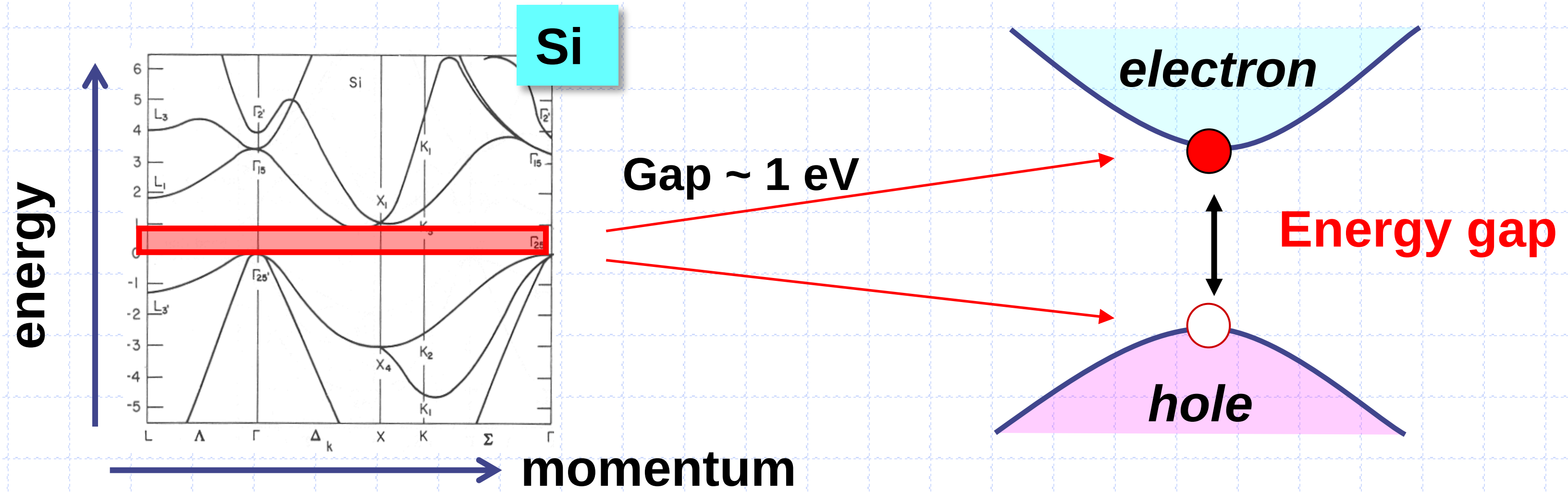
Topological Insulator

minimum

Band Insulators

e.g. semiconductor

Characterized by **energy gap**



Topological Insulators

Characterized by energy gap

Topological number (Z or Z2)

Gapless edge excitations

- ◇ Quantum Hall effect,
- ◇ Polyacetylen,
- ◇ Quantum Spin Hall effect
- ◇ Z_2 topological insulator

Topological number

topological property of the manifold of occupied states

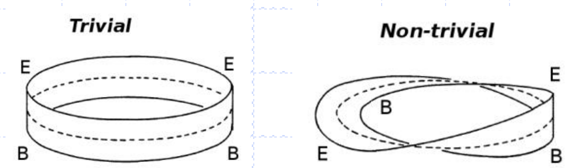
Bloch wave function

$|u(\mathbf{k})\rangle$: Brillouin zone (a torus) $\mapsto$ Hilbert space

Chern number : **Thouless et al, 1982**

$$n = \frac{1}{2\pi i}\int_{BZ} d^2\mathbf{k}\cdot\langle\nabla_{\mathbf{k}}u(\mathbf{k})|\times|\nabla_{\mathbf{k}}u(\mathbf{k})\rangle = \text{integer}$$

Trivial Insulator : $n = 0$
Topol. Insulator : $n \neq 0$



Analogy: Genus of a surface : G = # of holes

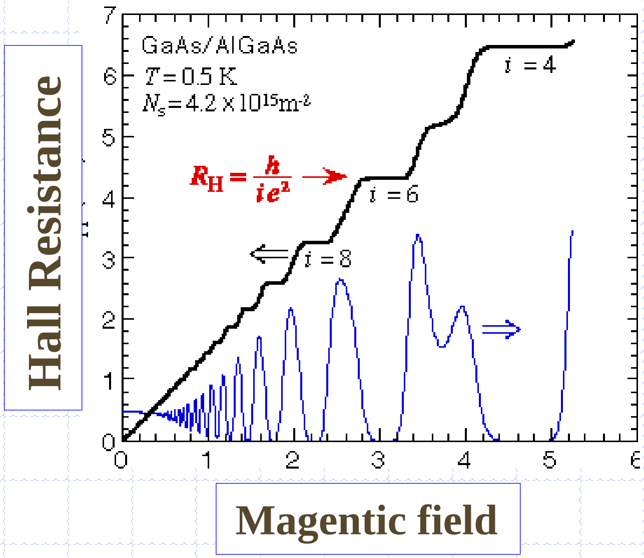
G=0

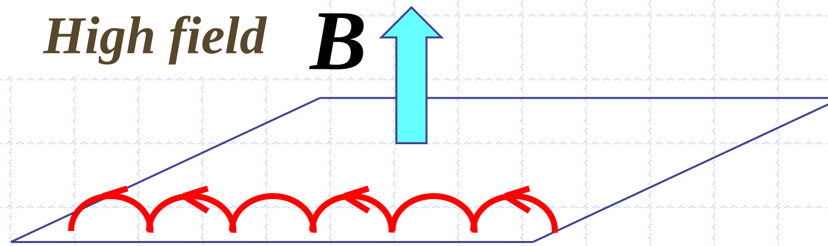

G=1


Quantum Hall effect

2D electrons in high fields

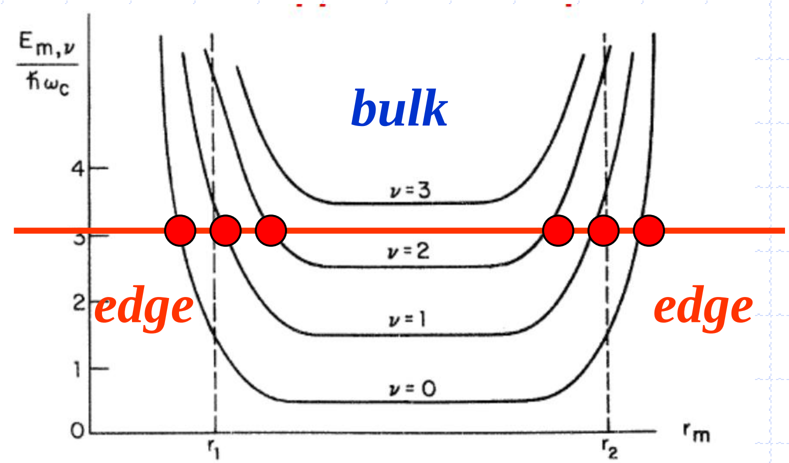
GaAs/AlGaAs





Chiral edge state

Bulk-edge correspondence



Hall conductivity

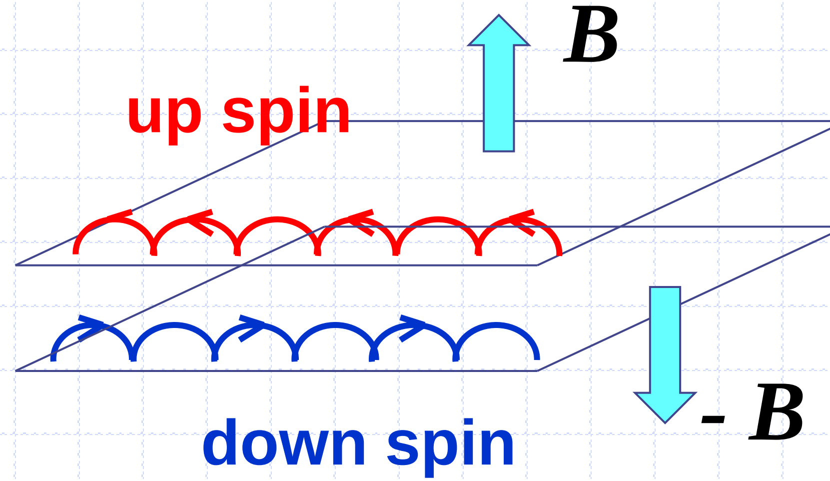
$$\sigma_{xy} = n\, e^2/h$$

***n*: Chern number**
# of edge modes

$$n = \int \frac{d^2k}{(2\pi)^2}\,\varepsilon^{\mu\nu} F_{\mu\nu}(k)$$

Quantum Spin Hall effect

- Time reversal symmetry
- Spin-orbit coupling
- Gapless helical edge state

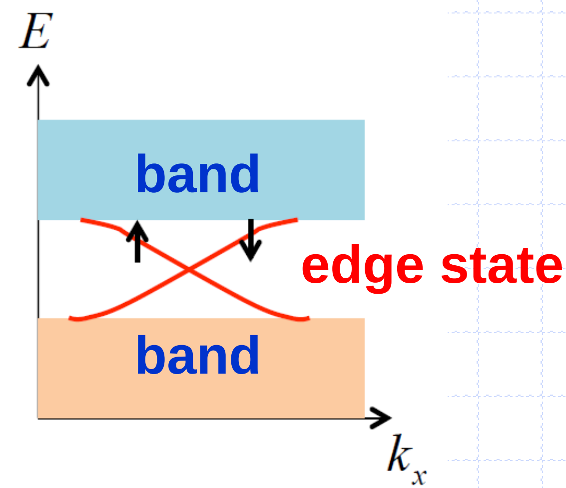


S_z is not conserved:

Topological index

Z → Z_2

Integer → even-odd



Z_2 Topological insulator

Kane-Mele, S. C. Zhang

HgTe/CdTe well (2008)

Topological Insulators

■**Topologically nontrivial**
Topological number

1D: polyacetylene
2D: HgTe/CdTe quantum well
3D: BiSb, BiSe, etc

Topologically protected edge states

Observed !

Spintronics, Quantum information, etc

a unique approach
Real-time dynamics
QW

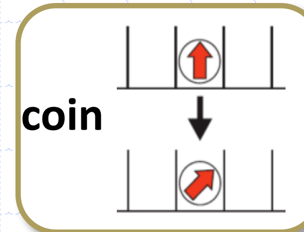
How robust topological ed
Randomness, impurity, etc


Quantum Walks
Clean systems

Topological nature of QW

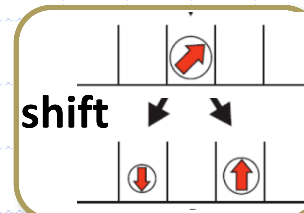
Coin operator

$$C_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}.$$



Shift operator in momentum space

$$W = \sum_n \left(|n+1\rangle\langle n| \otimes |R\rangle\langle R| + |n-1\rangle\langle n| \otimes |L\rangle\langle L| \right)$$
$$= \sum_k \begin{pmatrix} e^{ik} & 0 \\ 0 & e^{-ik} \end{pmatrix} \otimes |k\rangle\langle k|$$



Hamiltonian $U = \exp[-iH\delta t]$

2π periodicity: Floquet energy

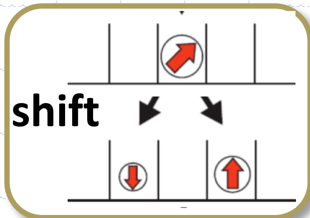
$$H = \int_{-\pi}^{\pi} dk \left[\omega(k) \mathbf{d}(k) \cdot \boldsymbol{\sigma} \otimes |k\rangle\langle k| \right],$$
$$\mathbf{d}(k) = [\sin(\theta)\sin k, \sin(\theta)\cos k, -\cos(\theta)\sin k] / \sin\omega(k),$$
$$\omega(k) = \pm \arccos[\cos(k)\cos(\theta)] + 2n\pi.$$

Dispersion relation

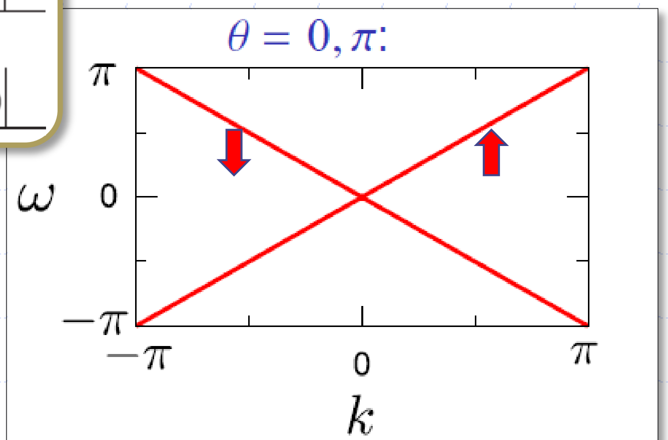
$$H = \int_{-\pi}^{\pi} dk\, [\omega(k)\mathbf{d}(k)\cdot\boldsymbol{\sigma}\otimes|k\rangle\langle k|]\,,$$

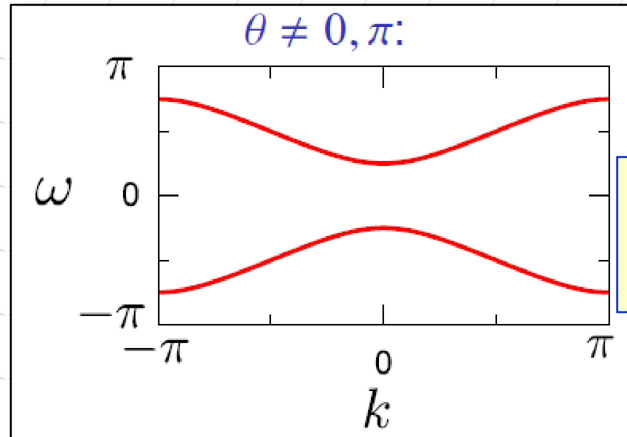
$$\mathbf{d}(k) = [\sin(\theta)\sin k, \sin(\theta)\cos k, -\cos(\theta)\sin k]\,/\,\sin\omega(k)$$

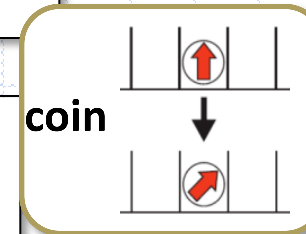
$$\omega(k) = \pm\arccos[\cos(k)\cos(\theta)] + 2n\pi.$$



helical

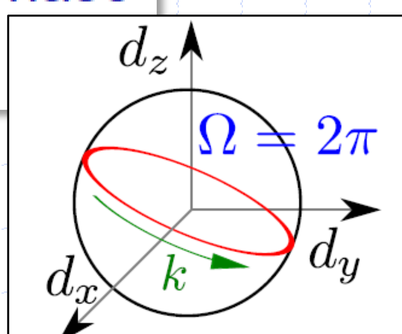






Massive
SO coupling

Berry phase
$d(k)$:



Z=1 Topological Insulator
1D chiral orthogonal class

Topological insulators: d=1, 2, 3

Schnyder, Ryu, Furusaki, Ludwig, PRB '08, NJP '10; Kitaev AIP conf. '08.

System	Cartan nomenclature	TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	**Z**	-
	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	−1	0	0	-	$\mathbf{Z}_2$	$\mathbf{Z}_2$
chiral (sublattice)	AIII (chiral unit.)	0	0	1	**Z**	-	**Z**
	BDI (chiral orthog.)	+1	+1	1	**Z**	-	-
	CII (chiral sympl.)	−1	−1	1	**Z**	-	$\mathbf{Z}_2$
BdG	D	0	+1	0	$\mathbf{Z}_2$	**Z**	-
	C	0	−1	0	-	**Z**	-
	DIII	−1	+1	1	$\mathbf{Z}_2$	$\mathbf{Z}_2$	**Z**
	CI	+1	−1	1	-	-	**Z**

1D Quantum Walk: chiral orthogonal BDI

Time reversal symmetry
Particle- hole symmetry
Sublattice symmetry

Quantum walks

Clean system

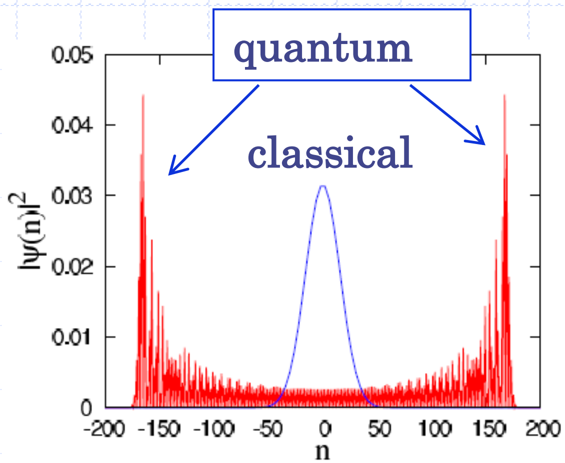
chiral orthogonal

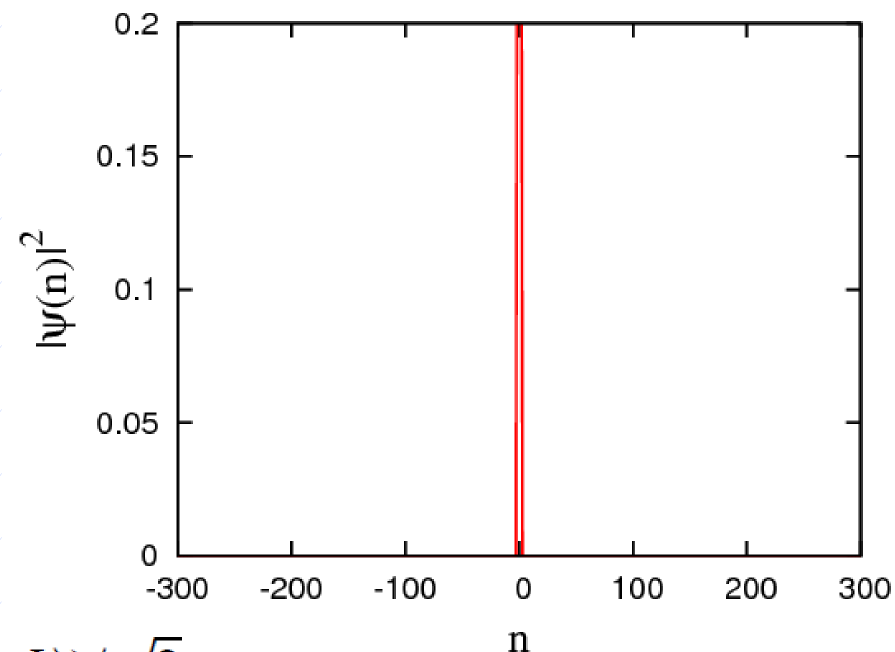
◇Coin operator

$$C_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}$$

Hadamard walk
$\theta_n = \pi/4$

$$C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

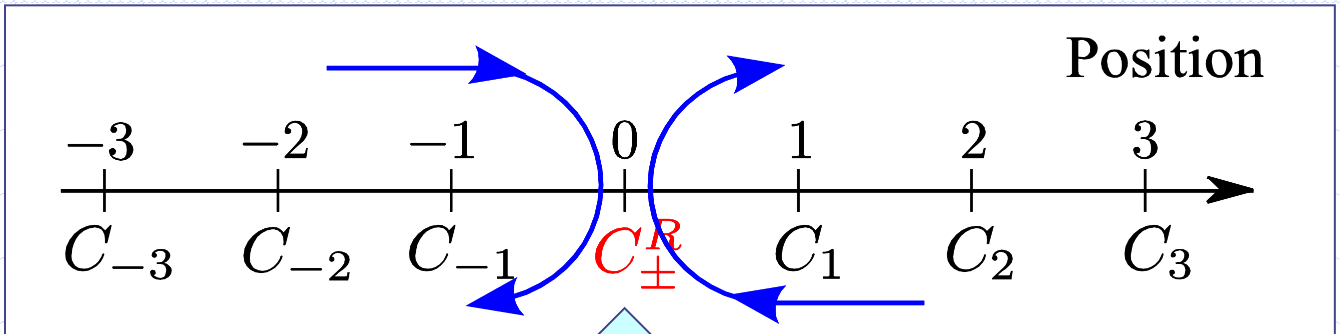




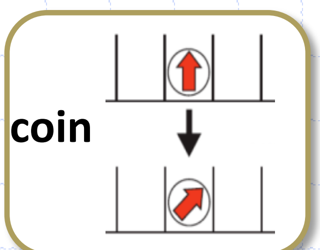
Initial state:

$$\psi(0) = (|0,R\rangle + i|0,L\rangle)/\sqrt{2}.$$

reflecting boundary condition



Completely reflecting (hard wall)



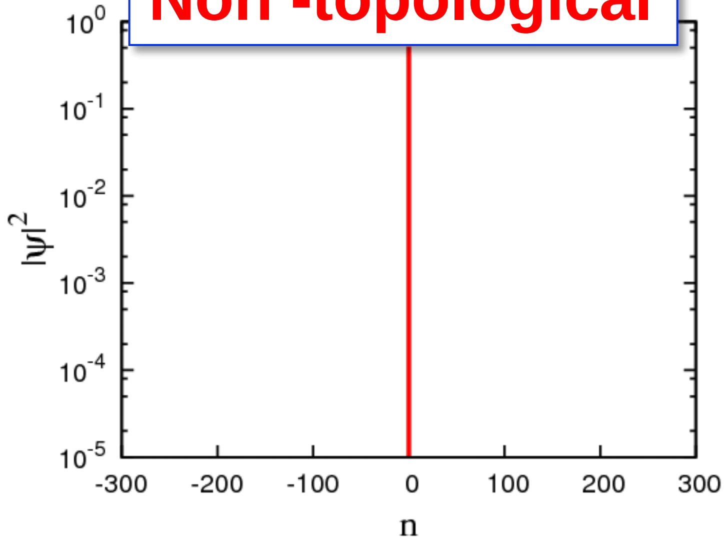
$$C_R^{\pm} \equiv \begin{pmatrix} 0 & \mp 1 \\ \pm 1 & 0 \end{pmatrix}$$

Edge state at x=0 ?

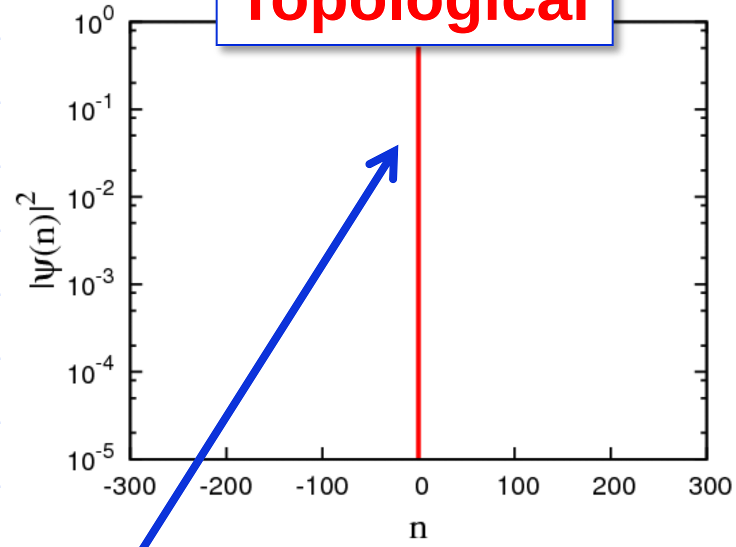
Clean system: with boundary

Initial state: $\psi(0) = (|0, R\rangle + i|0, L\rangle)/\sqrt{2}$

Non -topological



Topological



Clean system

n=0 boundary

C_R^+

Clean system

n=0 boundary

C_R^-

Topological Insulator: edge state

Quantum Walk with Randomness

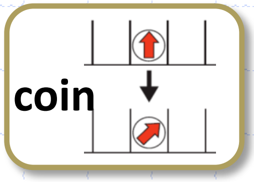
Anderson transition
vs. topological phase

Static disorder: with boundary

(spatial disorder)

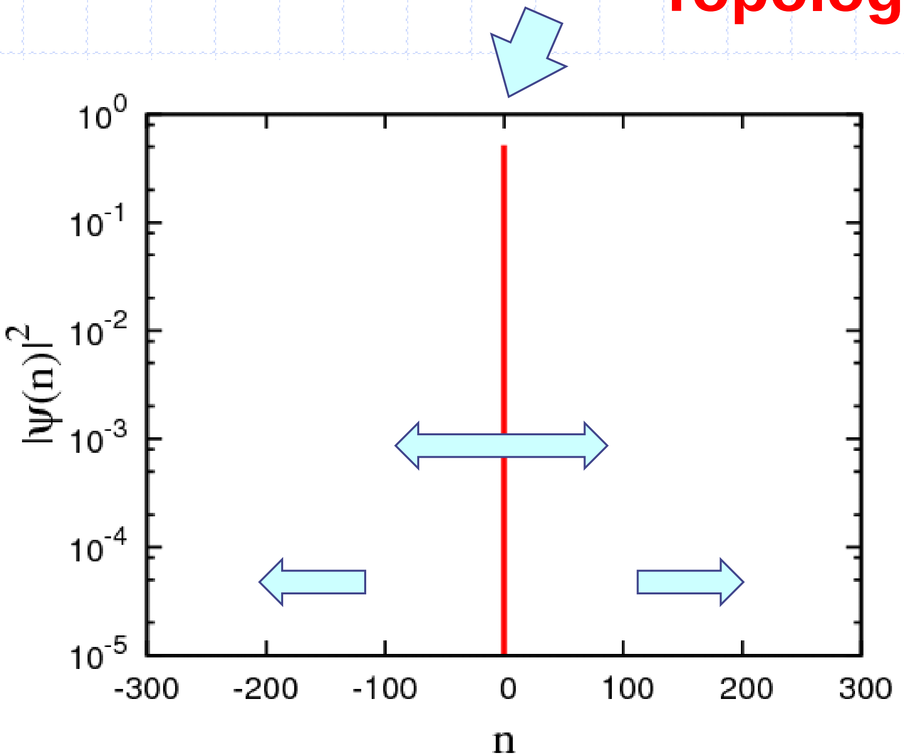
Topological Phase

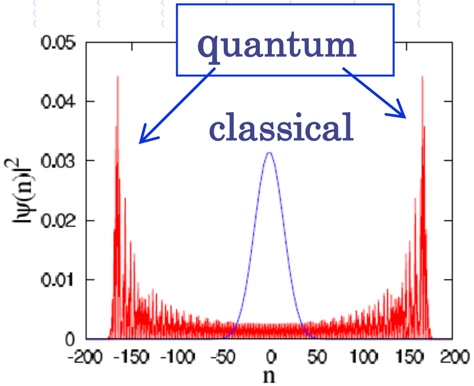
Random Coin



$$C_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}$$

$$\begin{cases} \theta_0 = \pi/4 \\ \delta\theta = \pi/4 \end{cases}$$





- ◇ Edge state: robust ?
- ◇ Anderson localization occurs ?
- ◇ Extended state exists ?

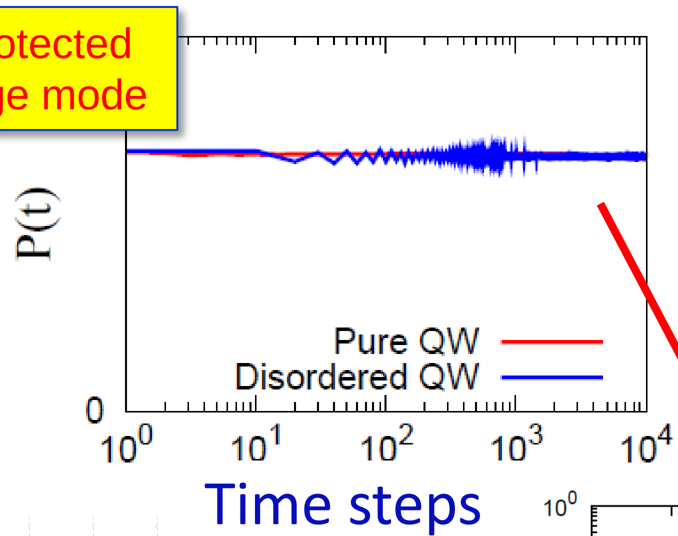
Static disorder: with boundary

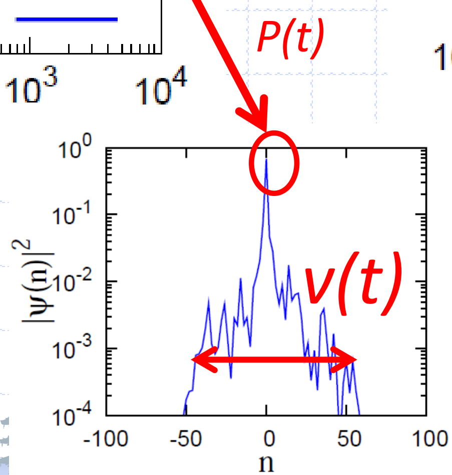
Recurrence probability *P(t)*:

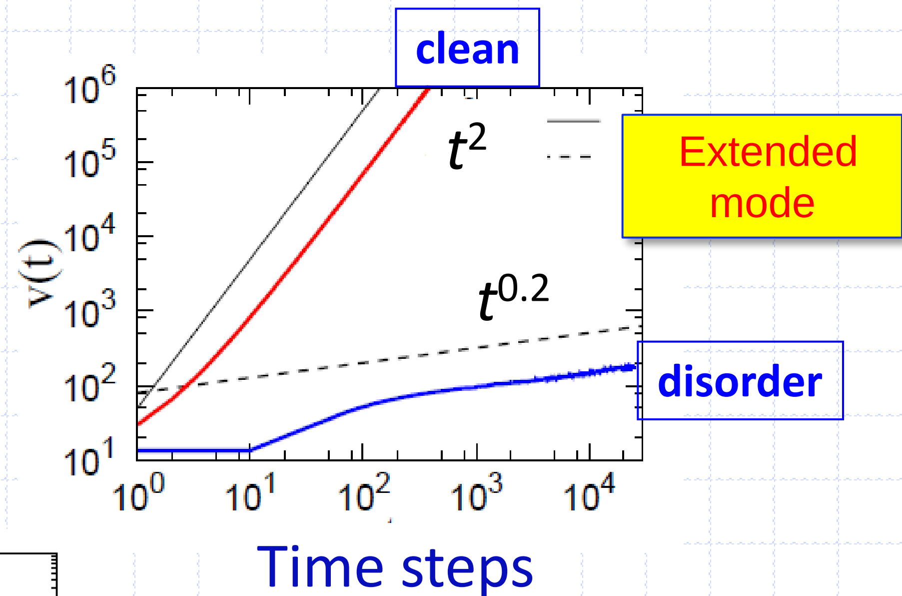
$$P(t) = \sum_{\sigma=R,L} \langle 0,\sigma || \psi(t)\rangle\langle\psi(t) || 0,\sigma\rangle$$

Variance *v(t)*:

$$v(t) = \int n^2 |\psi(t)|^2 dn - \left(\int n|\psi(t)|^2 dn \right)^2$$

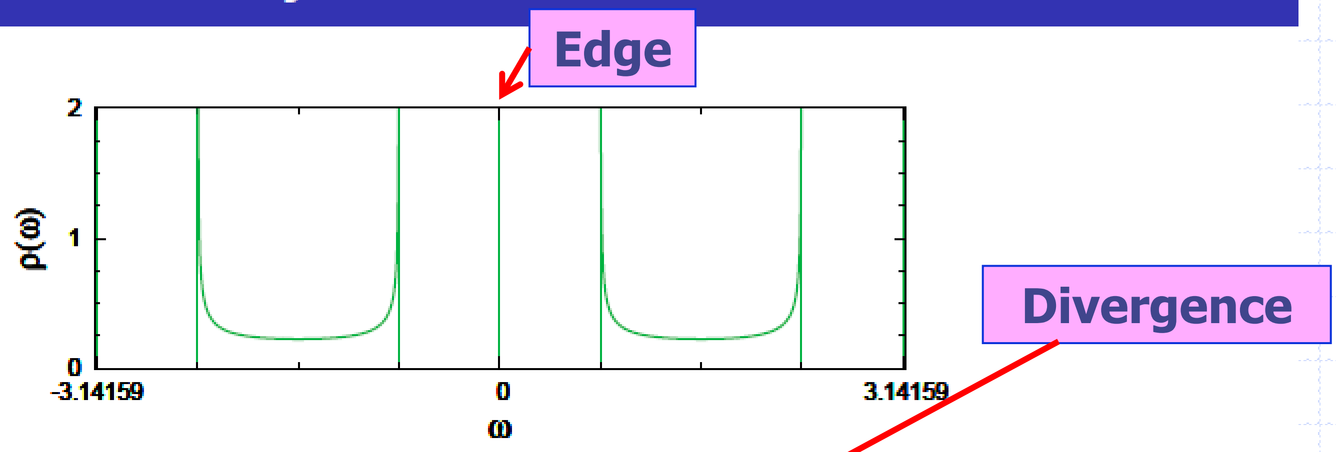




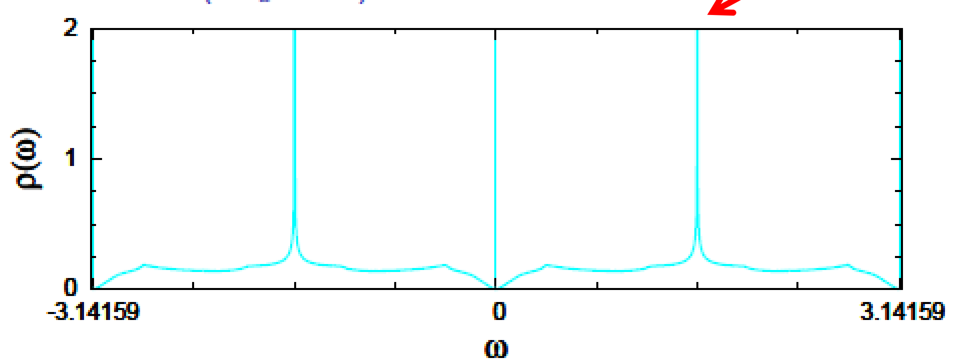


Density of States of QW

- Clean QW:



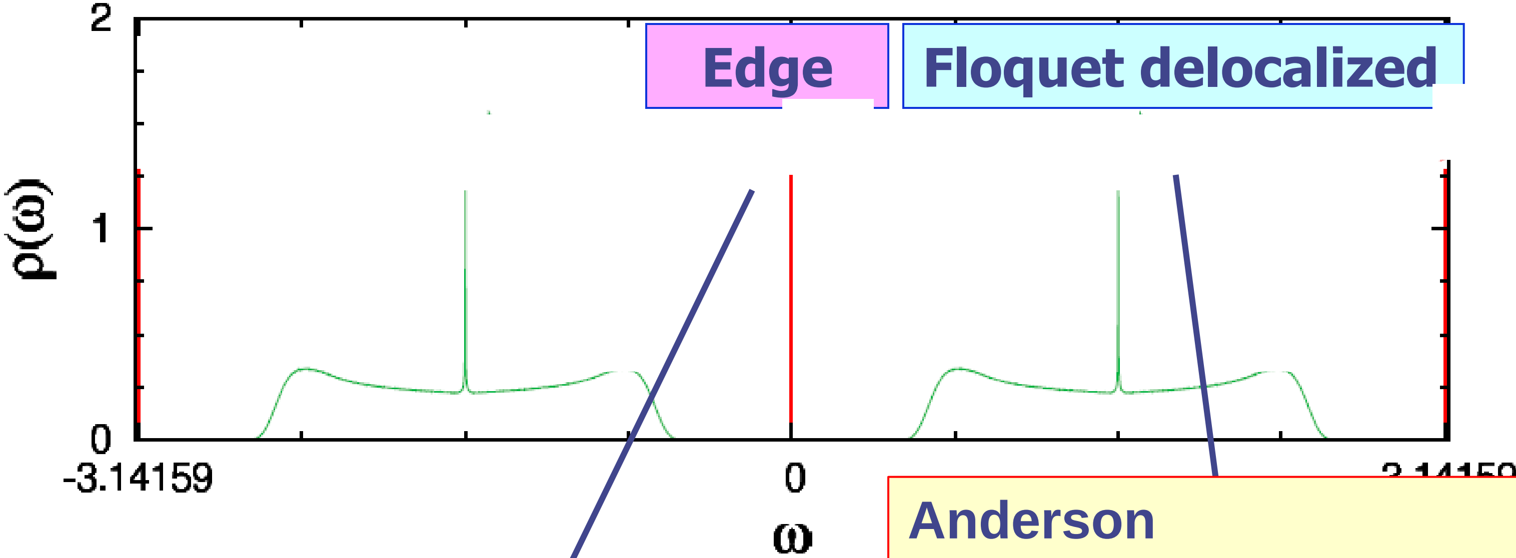
- QW with spatial disorder ($\delta\theta_s = \pi$):

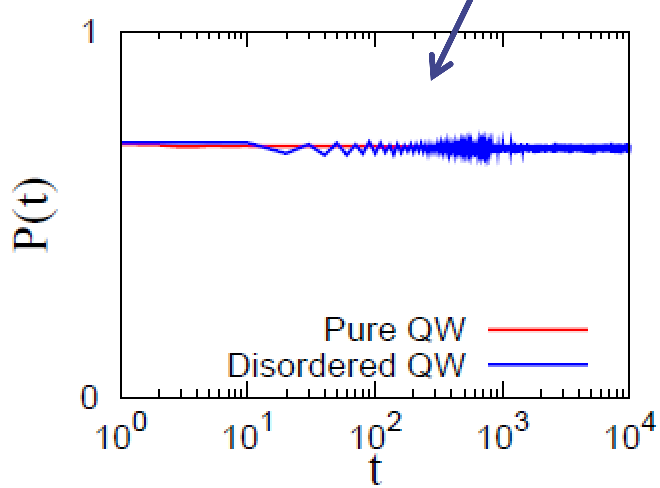


◆Edge state: robust ⇒ topological edge state.
◆Divergence in DOS: $\omega = \pm\pi/2$

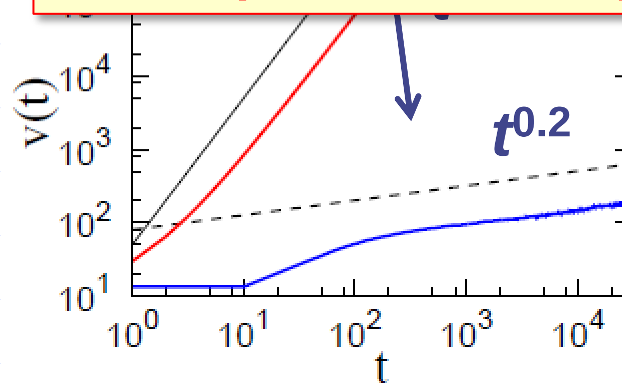
Spatially disordered QW:

edge modes + localized modes + critical modes

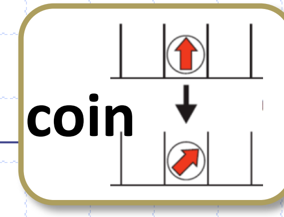




Anderson
Localization-delocalization
Transition
(1D chiral class)



Other classes in one dimension



Chiral Orthogonal
BDI

Topological

$$C_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}$$

Chiral Unitary
AIII

Topological

$$C_n = \begin{pmatrix} e^{i\phi_{1n}}\cos\theta_n & -\sin\theta_n \\ \sin\theta_n & e^{-i\phi_{1n}}\cos\theta_n \end{pmatrix}$$

Unitary
A

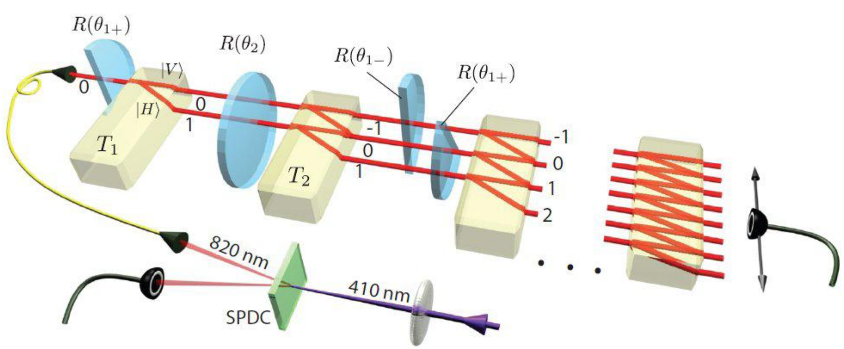
Trivial

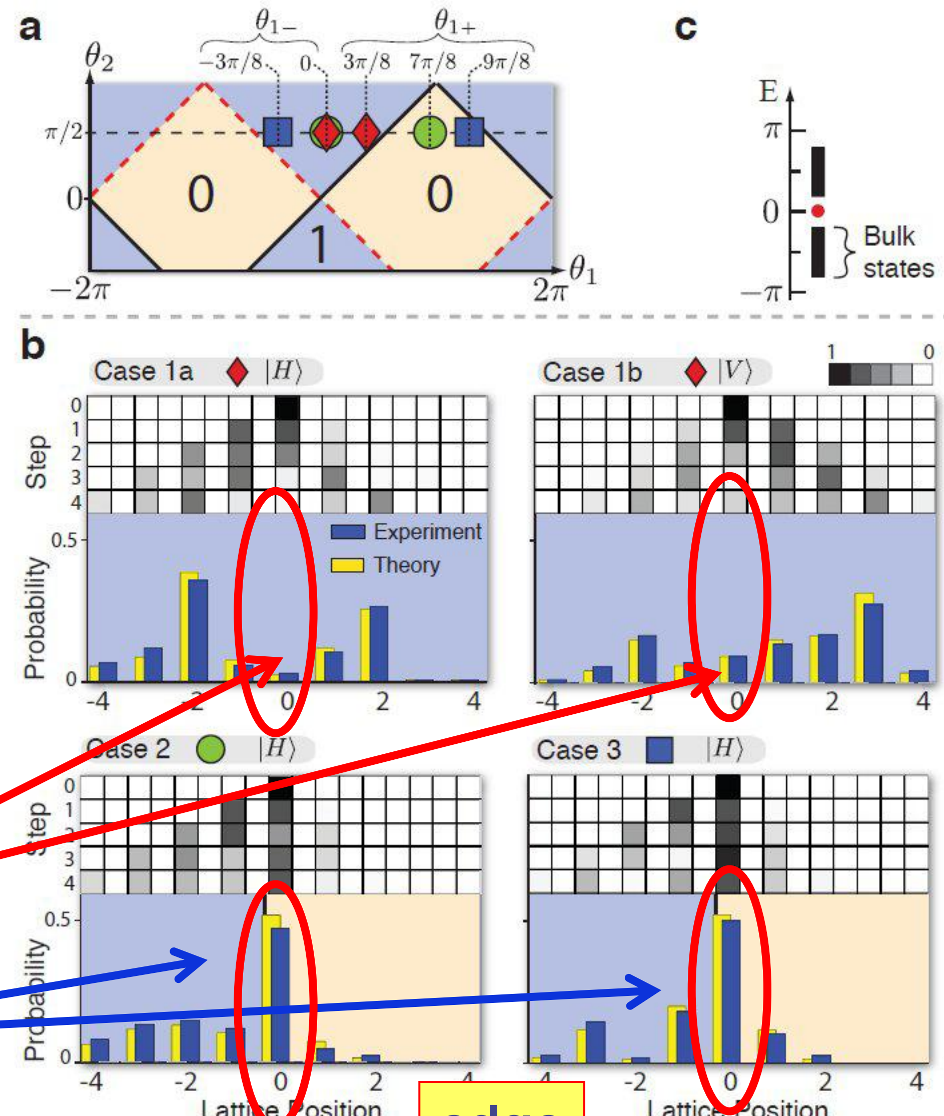
$$C_n = \begin{pmatrix} e^{i\phi_{1n}}\cos\theta_n & -e^{i\phi_{2n}}\sin\theta_n \\ e^{-i\phi_{2n}}\sin\theta_n & e^{-i\phi_{1n}}\cos\theta_n \end{pmatrix}$$

universality class		TRS	PHS	CS	$d=1$	$d=2$	$d=3$
Standard	A	0	0	0	-	$\mathbb{Z}$	-
(Wigner-Dyson)	AI	$+1$	0	0	-	-	-
	AII	-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
Chiral	AIII	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI	$+1$	$+1$	1	$\mathbb{Z}$	-	-
	CII	-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	-1	0	-	$\mathbb{Z}$	-
	DIII	-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	$+1$	-1	1	-	-	$\mathbb{Z}$

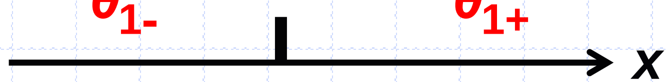
Observation of Edge States

Kitagawa, Broome *et al.*, Nature Commun. (2012)





Topological # θ_{1-} Topological # θ_{1+}



$(\theta_{1-}, \theta_{1+})$ => 1, 1

=> 0, 1

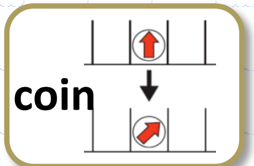
edge


Temporal Disorder
Quantum vs. Classical

Temporal disorder: with boundary

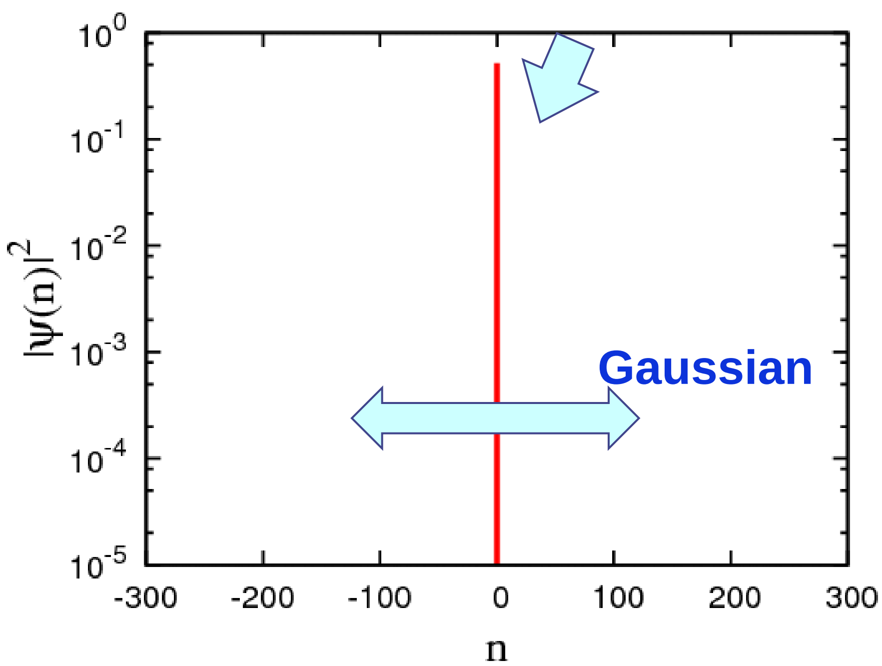
Topological Phase

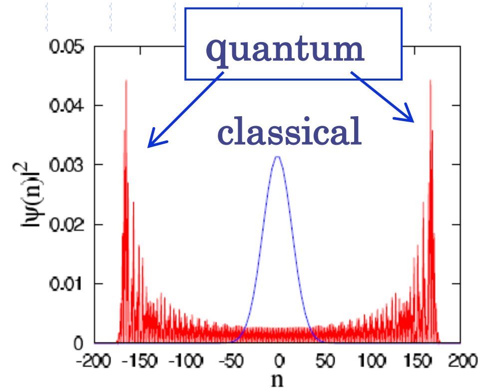
Random Coin



$$C_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}$$

$$\begin{cases} \theta_0 = \pi/4 \\ \delta\theta = \pi/4 \end{cases}$$

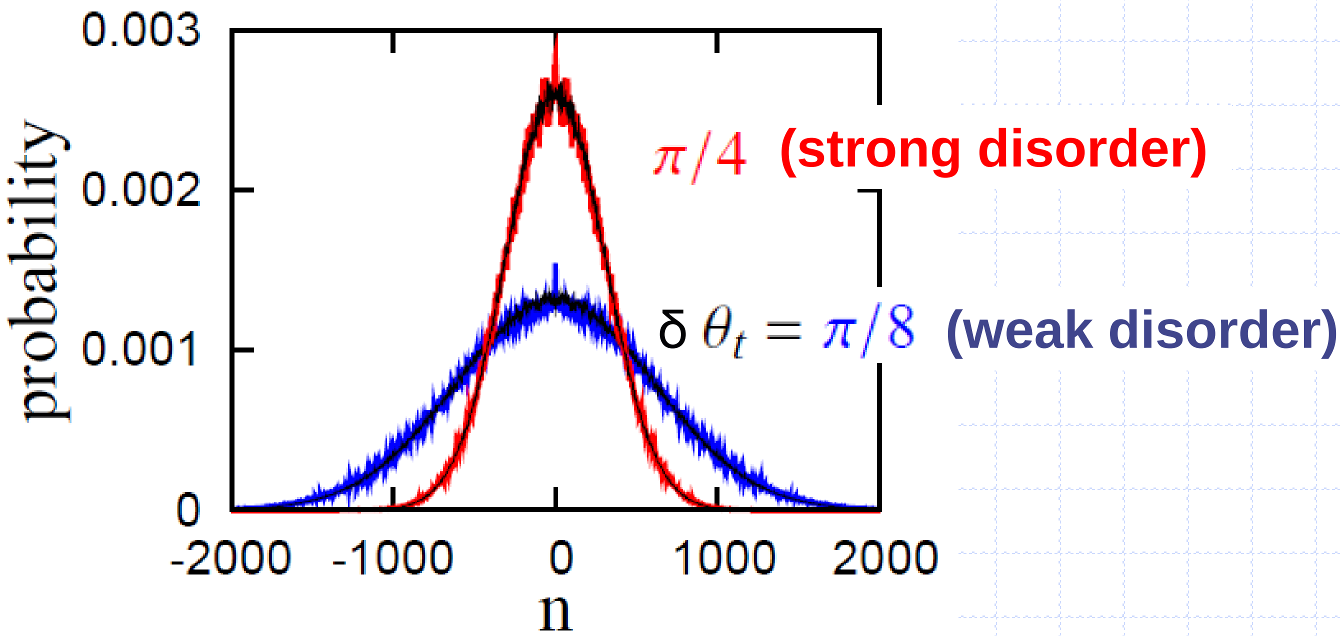




◇ Edge state: disappears
◇ approach Random walk

Dephasing
Decoherence

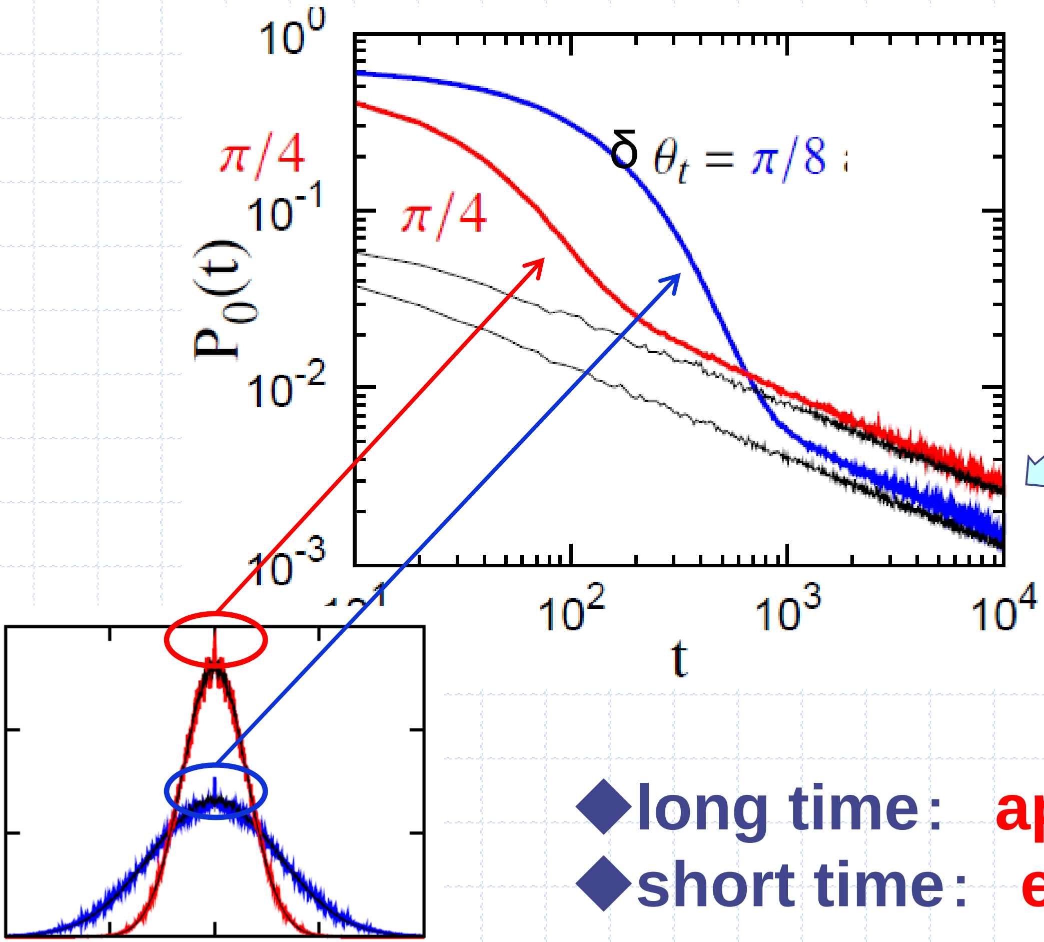
Averaged Probability Distribution
at 10^4 time steps



- ◆**Gaussian distributions => classical random walk.**
- ◆**Small peaks => remnants of edge states.**

Reccurence Probability (edge state)

for $\delta\theta_t$ = *π/8* and *π/4*



Asymptotic behavior without edge state

- **long time: approach Random walk**
- **short time: edge state survives !**

Summary

Real Time Dynamics of Quantum Walks

- ◆Realization of topological states
- ◆Robustness of edge states

  Topologically protected

- ◆Anderson transition

  Flouquet delocalized state

- ◆Quantum to Classical
- ◆Nonequilibrium quantum phenomena

Rich !

Interdisciplinary research arena

Condensed Matter , Laser Physics
Atom Physics, Statistical Physics
etc

Further progress: Related topics

◆ Strongly Driven Systems

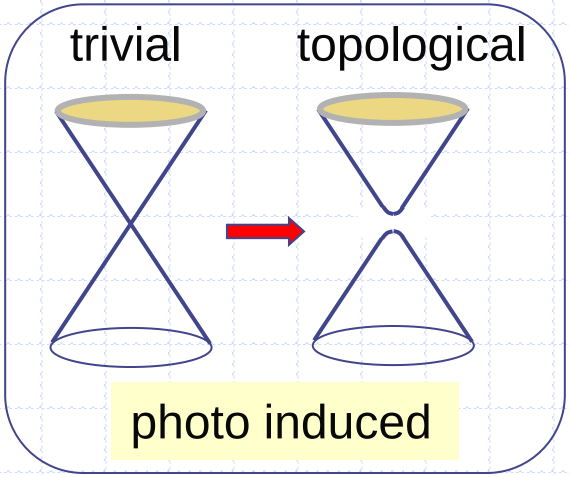
Periodic-time-dependent field

Floquet Topological States

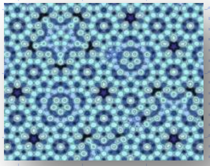
e.g. photo induced topological insulator

⇨ ***M. Nakagawa, Poster 63***

Oka et al. 2009



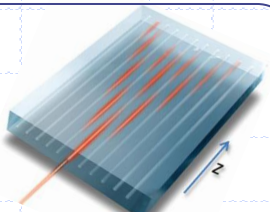
◆Quasi crystals（準結晶）



Topological classification

Observation of topological edge states

Y. E. Kraus et al. PRL (2012)
Photonic crystal



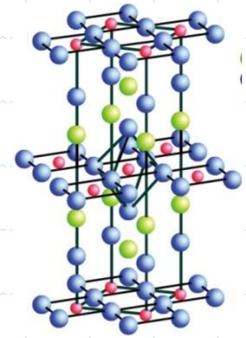
⇨ ***F. Matsuda, Poster 52***

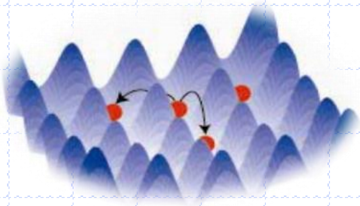
Platforms of Condensed Matter Physics

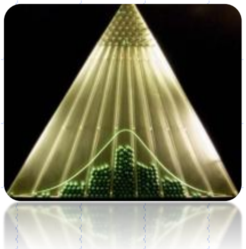
- **Solid state, fluid**
  Semiconductors, Superconductors, Correlated systems, Spin Systems, etc
- **Cold**
  Tunabl
- **Quantum walk**
  Real time dynamics

Etc

So exciting everyday !







What comes next ?

- Novel phenomena ?
- New concept ?
- New platform ?


Thank you for your attention !