How to Reconcile Maxwell's Demon with the Second Law of Thermodynamics?

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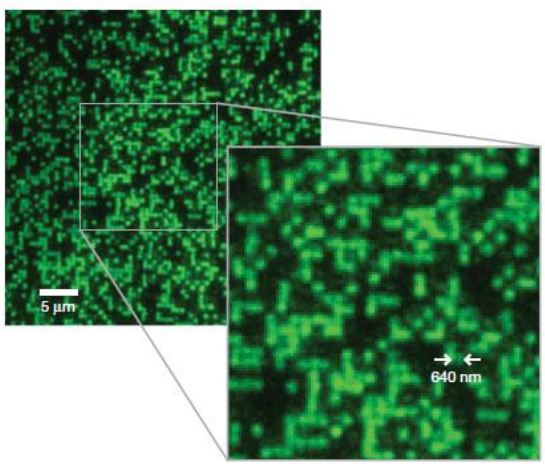
Outline

- **Introduction**
- Information and Entropy
- Second Law with Quantum Feedback
- Second Law with Quantum Measurement
- Conclusion & Discussions

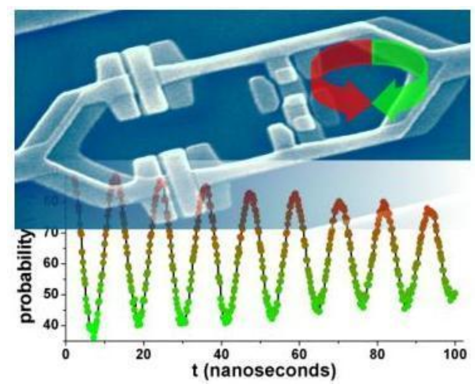
Thermodynamics in the Fluctuating World

Thermodynamics of small systems

Thermodynamic quantities are fluctuating!



Greiner *et al.*, Nature **462**, 74-77 (2009)



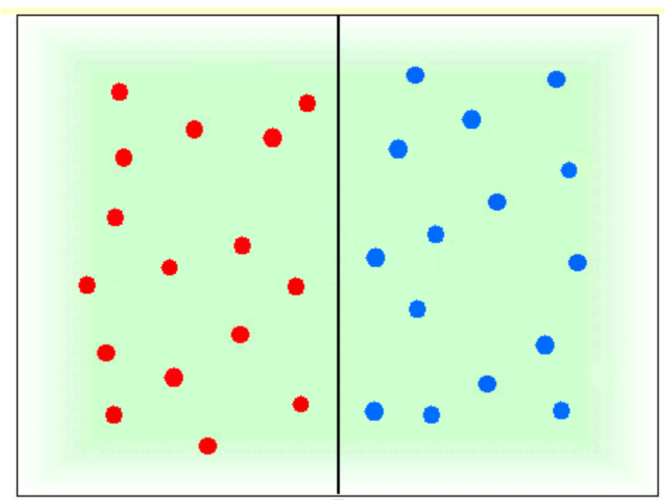
From NEC website

✓ **Second law**

✓ **Nonequilibrium thermodynamics**

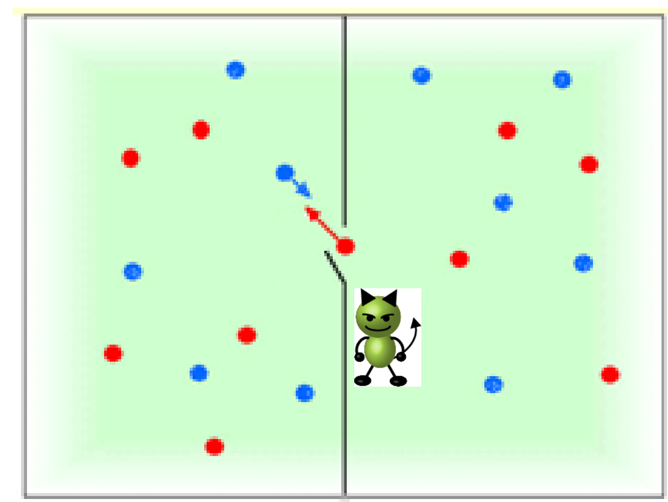
Maxwell's Demon

Fundamental problem on thermodynamics and statistical mechanics since the 19th century



Second law

Maxwell's demon

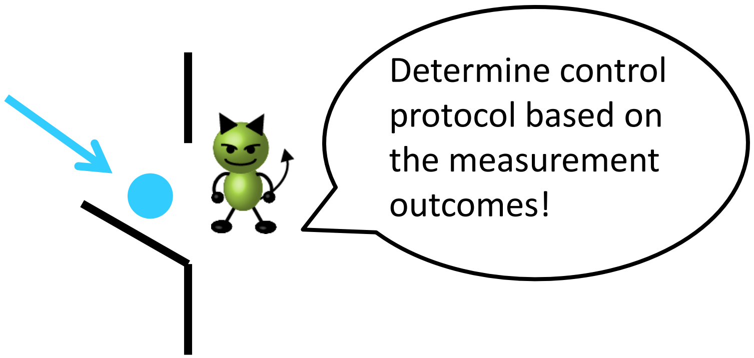


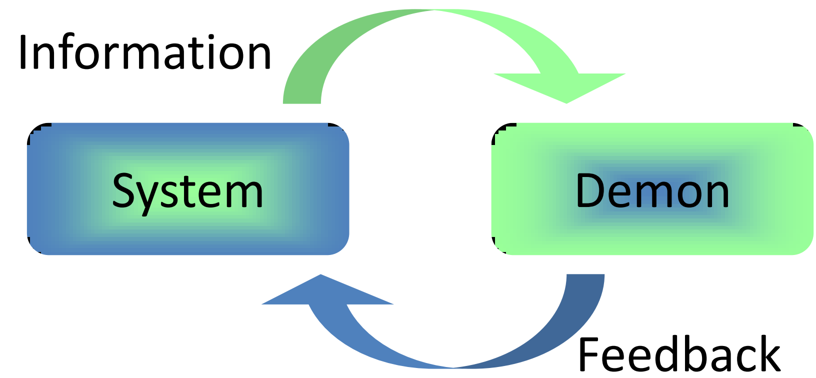
Observe the velocities of each molecules, and open or close the door…

Create a temperature difference?

J. C. Maxwell (1831-1879)

Demon from the Modern Viewpoint





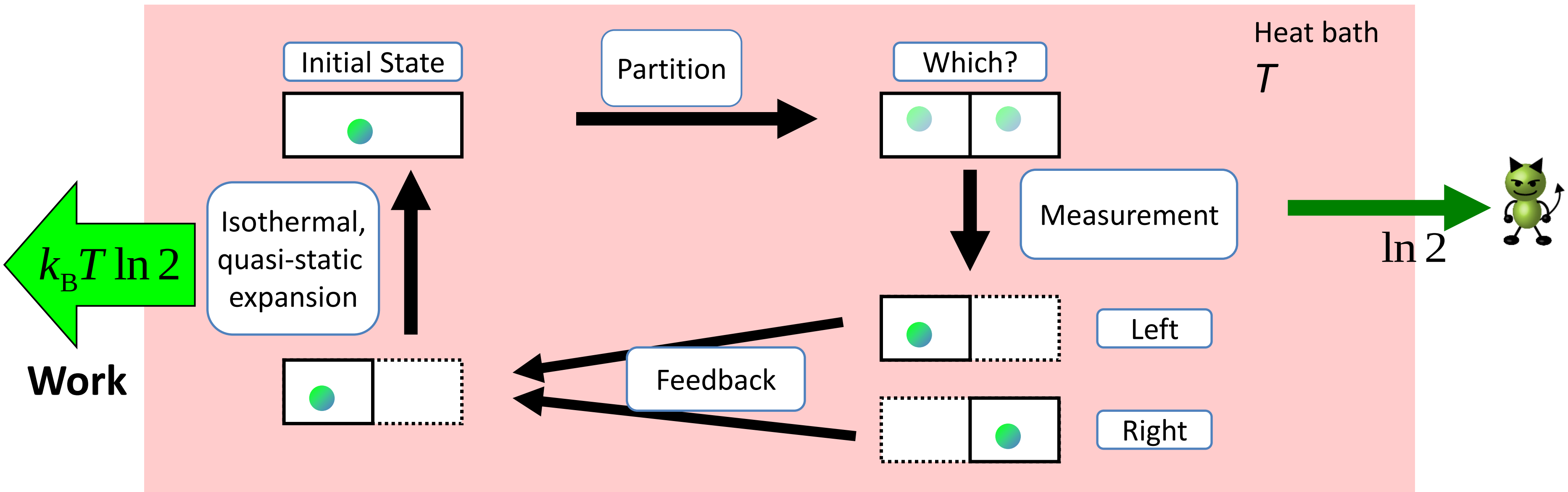
Feedback control at the level of thermal fluctuations

Thermodynamics of information processing

- ✓ Foundation of the second law of thermodynamics
- ✓ Application to nanomachines and nanodevices

L. Szilard, Z. Phys. **53**, 840 (1929)

Szilard Engine (1929)



Free energy: $F = E - TS$

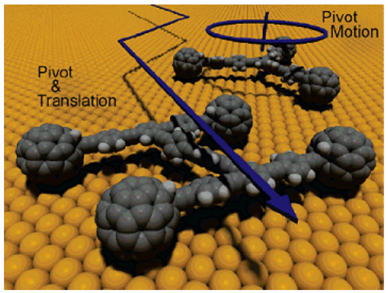
Increase

Decrease by feedback

Can control physical entropy by using information

Information Heat Engine

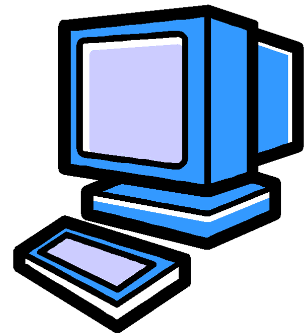
System
(Working engine)



Information

Feedback

Memory
(Controller)



Free energy / work

Entropic cost

✓ **Can increase the system's free energy even if there is no energy flow between the system and the controller**

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Shannon and von Neumann Entropies

Classical probability distribution $\{p_k\}$

Shannon entropy: $$H = -\sum_k p_k \ln p_k$$

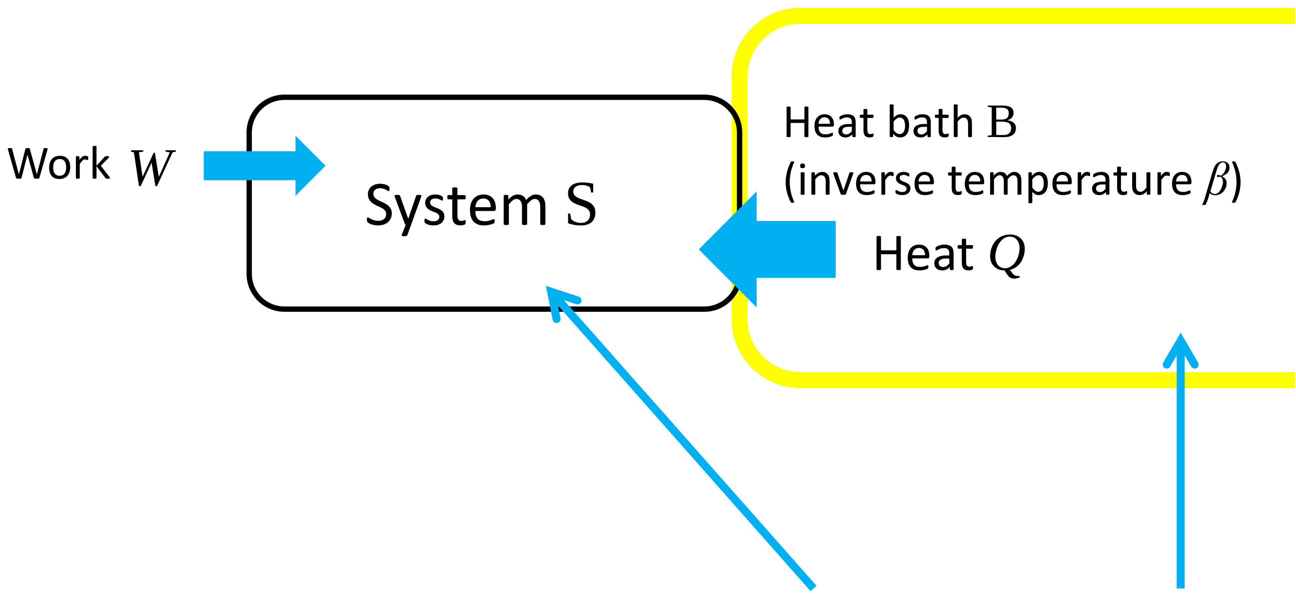
Randomness of the distribution

Quantum version:

Von Neumann entropy: $$S(\rho) = -\mathrm{tr}\left[\rho \ln \rho\right]$$

ρ : density operator

Entropy Production



Entropy production in the total system:

$$\Delta S_{\mathrm{SB}} = \Delta S_{\mathrm{S}} - \beta Q$$

Change in the von Neumann entropy of S

If the initial and the final states are canonical distributions: $\Delta S_{\mathrm{SB}} = \beta\left(W - \Delta F\right)$

Free-energy difference

Second Law of Thermodynamics

$$\Delta S_{\mathrm{SB}} \geq 0$$

Holds true for nonequilibrium initial and final states

A lot of “derivations” have been known
(Positivity of the relative entropy, Its monotonicity,
Fluctuation theorem & Jarzynski equality, …)

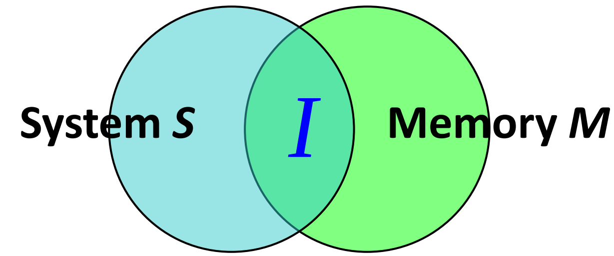
Review article: T. Sagawa, arXiv:1202.0983

If the initial state is the canonical distribution: $W \geq \Delta F$

Mutual Information

System S → **Memory M** (measurement device)

Measurement with stochastic errors

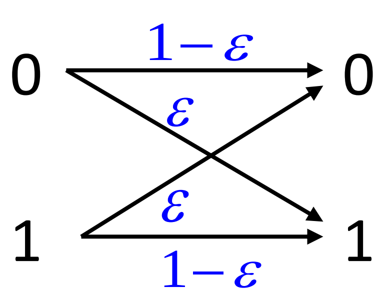


$$I(S:M) \equiv H(S) + H(M) - H(SM)$$

$$0 \leq I \leq H(M)$$

No information

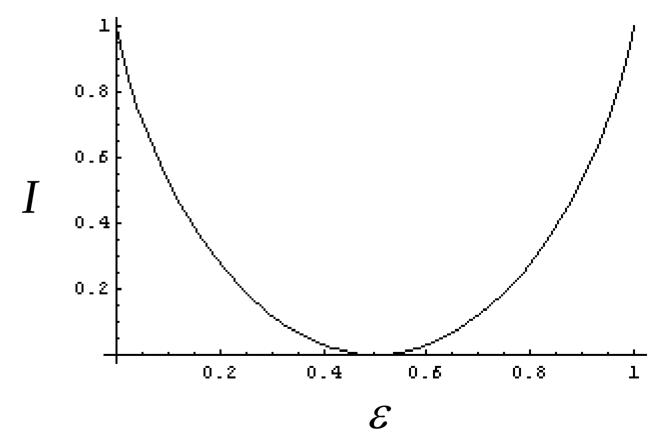
No error



Ex. Binary symmetric channel

Correlation between S and M

$$I = \ln 2 + \varepsilon \ln \varepsilon + (1-\varepsilon)\ln(1-\varepsilon)$$



Quantum Measurement

Projection measurement (error-free)

Observable $A = \sum_k \alpha_k P_k$

Projection operators $\{P_k\}$

Probability $p_k = \mathrm{tr}(\rho P_k)$

Post-measurement state $\dfrac{1}{p_k} P_k \rho P_k$

General measurement

Kraus operators $\{M_{k,i}\}$

k : measurement outcome

POVM : $\{E_k\}$

$E_k = \sum_i M_{ki}^\dagger M_{ki}$ $\quad \sum_k E_k = I$

Probability $p_k = \mathrm{tr}(\rho E_k)$

Post-measurement state $\dfrac{1}{p_k} \sum_i M_{ki} \rho M_{ki}^\dagger$

Assume that $E_k = M_k^\dagger M_k$ (a generalization is a future problem)

QC-mutual Information (1)

$$I_{\mathrm{QC}} = H + S(\rho) + \sum_y \mathrm{tr}\left(\sqrt{E_y}\,\rho\sqrt{E_y}\ln\sqrt{E_y}\,\rho\sqrt{E_y}\right)$$
$$= S(\rho) - \sum_y p(y) S\left(\sqrt{E_y}\,\rho\sqrt{E_y}/p(y)\right)$$

ρ : measured state $\qquad E_y$: POVM

$p(y) = \mathrm{tr}[\rho E_y]$: probability of obtaining outcome y

$H = -\sum_y p(y) \ln p(y)$: Shannon information of the outcomes

$S(\rho) = -\mathrm{tr}(\rho \ln \rho)$: von Neumann entropy of the measured state

H. J. Groenewold, Int. J. Theor. Phys. **4**, 327 (1971).
M. Ozawa, J. Math. Phys. **27**, 759 (1986).
TS and M. Ueda, PRL **100**, 080403 (2008).

QC-mutual Information (2)

$$0 \leq I_{\mathrm{QC}} \leq H$$

No information

Error-free & classical

$$E_y = p(y) I_d$$

Identity operator

For any y $[\rho, E_y] = 0$

E_y is a projection

Classical measurement:

For any y $[\rho, E_y] = 0$ $\Rightarrow$ I_{QC} reduces to the classical mutual information

If the measured state is a pure state: $I_{\mathrm{QC}} = 0$

QC-mutual Information (3)

The QC-mutual information gives an upper bound of the accessible classical information x encoded in ρ

F. Buscemi, M. Hayashi, and M.Horodecki, PRL **100**, 210504 (2008).

$$\rho = \sum_x q(x)\rho_x$$: an arbitrary decomposition

ρ_x are not necessarily orthogonal

$$p(x,y) = \mathrm{tr}[E_y \rho_x] q(x) \qquad p(y) = \mathrm{tr}[\rho E_y] = \sum_x p(x,y)$$

$$I = \sum_{xy} p(x,y) \ln \frac{p(x,y)}{q(x)\,p(y)} \qquad I_{\mathrm{QC}} = S(\rho) - \sum_y p(y) S\left(\sqrt{E_y}\rho\sqrt{E_y}/p(y)\right)$$

Theorem: $$I \le I_{\mathrm{QC}}$$

A variant (a "dual") of the Holevo bound

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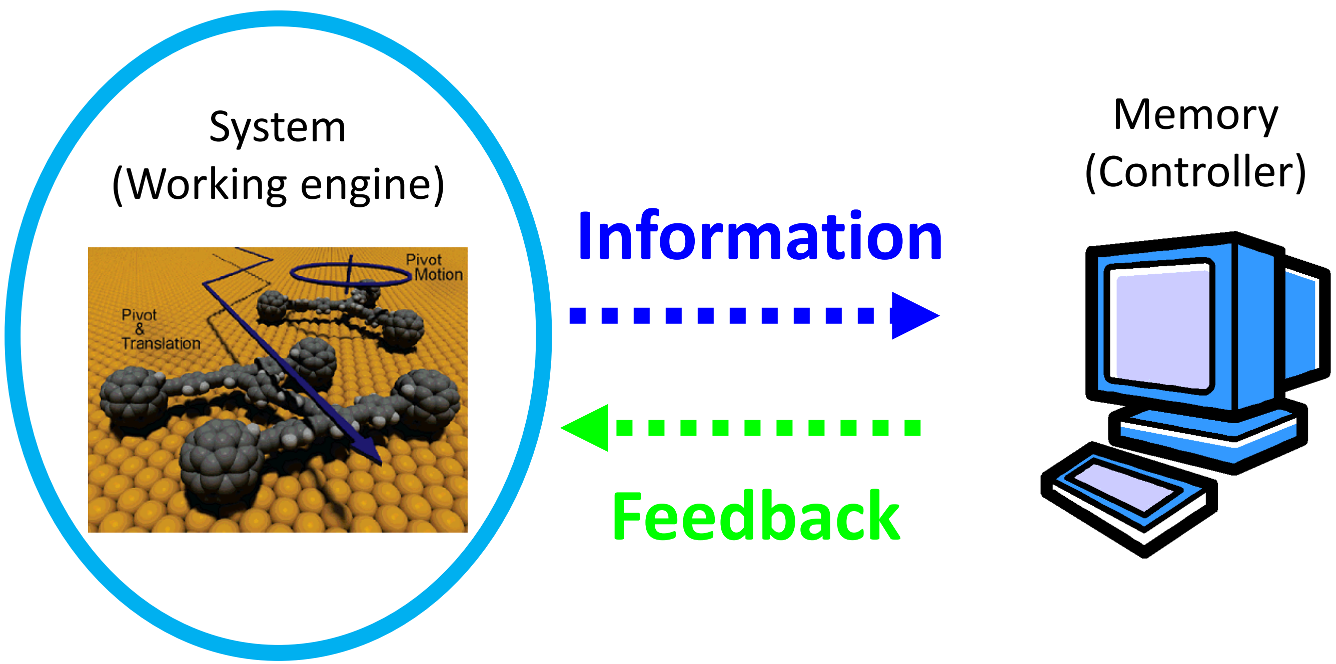
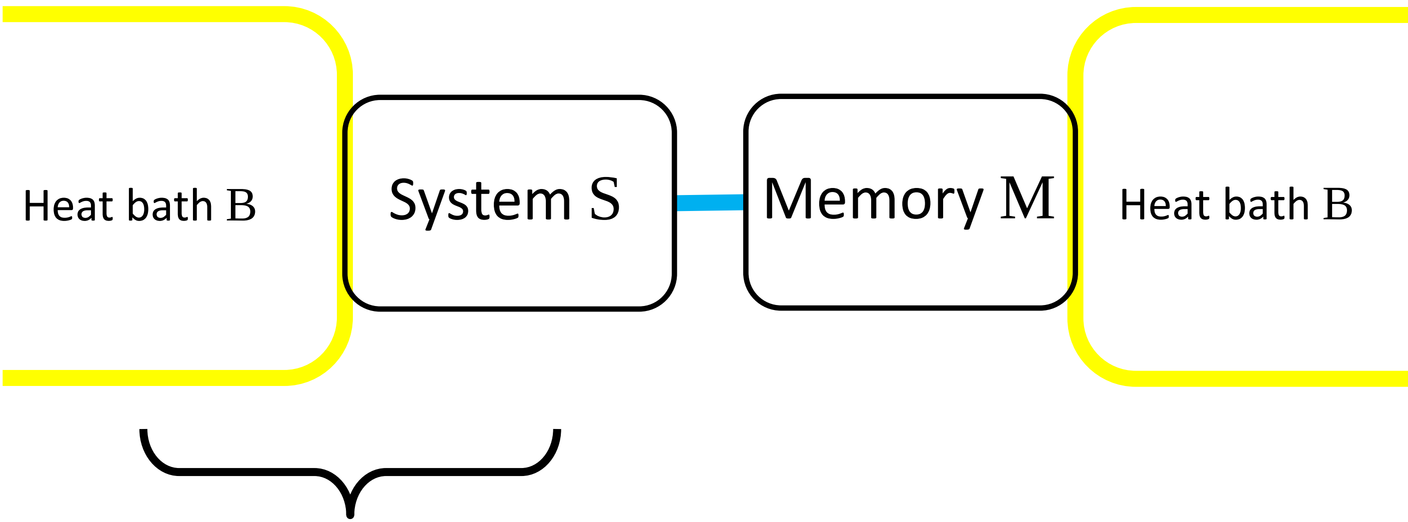
Measurement and Feedback



Feedback: Control protocol depends on the measurement outcome

Generalized Second Law: Entropic Balance

Measurement and feedback

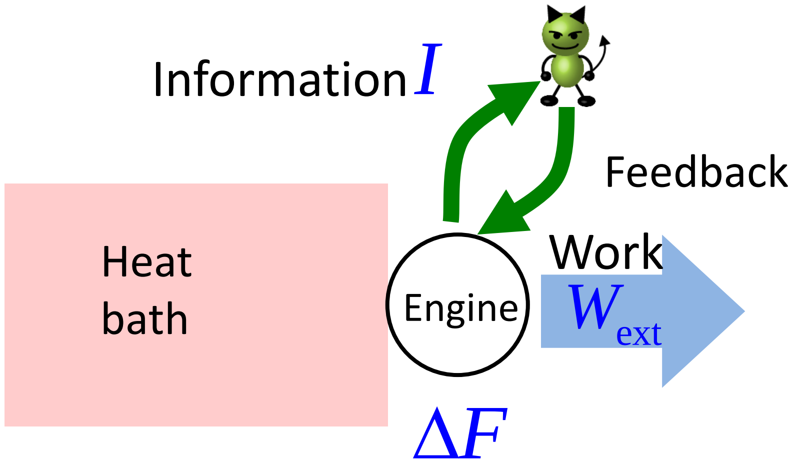


Entropy increase ΔS_{SB}

$$\Delta S_{\mathrm{SB}} \geq -I_{\mathrm{QC}}$$

TS and M. Ueda, PRL **100**, 080403 (2008)

Generalized Second Law: Energetics



TS and M. Ueda, PRL **100**, 080403 (2008)

$$\Delta S_{\text{SB}} \geq -I_{\text{QC}} \quad \Rightarrow \quad W_{\text{ext}} \leq -\Delta F + k_{\text{B}} T I_{\text{QC}}$$

The upper bound of the work extracted by the demon is bounded by the QC-mutual information.

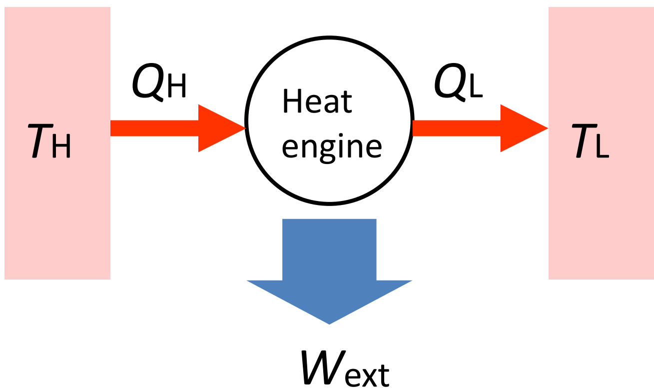
The equality can be achieved:

K. Jacobs, PRA **80**, 012322 (2009)
J. M. Horowitz & J. M. R. Parrondo, EPL **95**, 10005 (2011)
D. Abreu & U. Seifert, EPL **94**, 10001 (2011)
J. M. Horowitz & J. M. R. Parrondo, New J. Phys. **13**, 123019 (2011)
T. Sagawa & M. Ueda, PRE **85**, 021104 (2012)
M. Bauer, D. Abreu & U. Seifert, J. Phys. A: Math. Theor. **45**, 162001 (2012)

Information Heat Engine

Conventional heat engine:

Heat → Work



Heat efficiency

$$e \equiv \frac{W_{ext}}{Q_H} \leq 1 - \frac{T_L}{T_H}$$

Carnot cycle

Information heat engine:

Mutual information → Work and Free energy

$$W_{ext} + \Delta F \leq k_B T I_{QC}$$

Szilard engine

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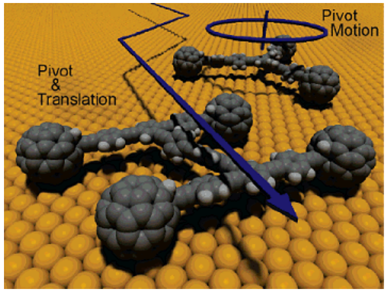
Measurement and Feedback

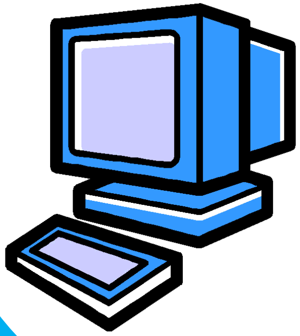
System
(Working engine)



Information

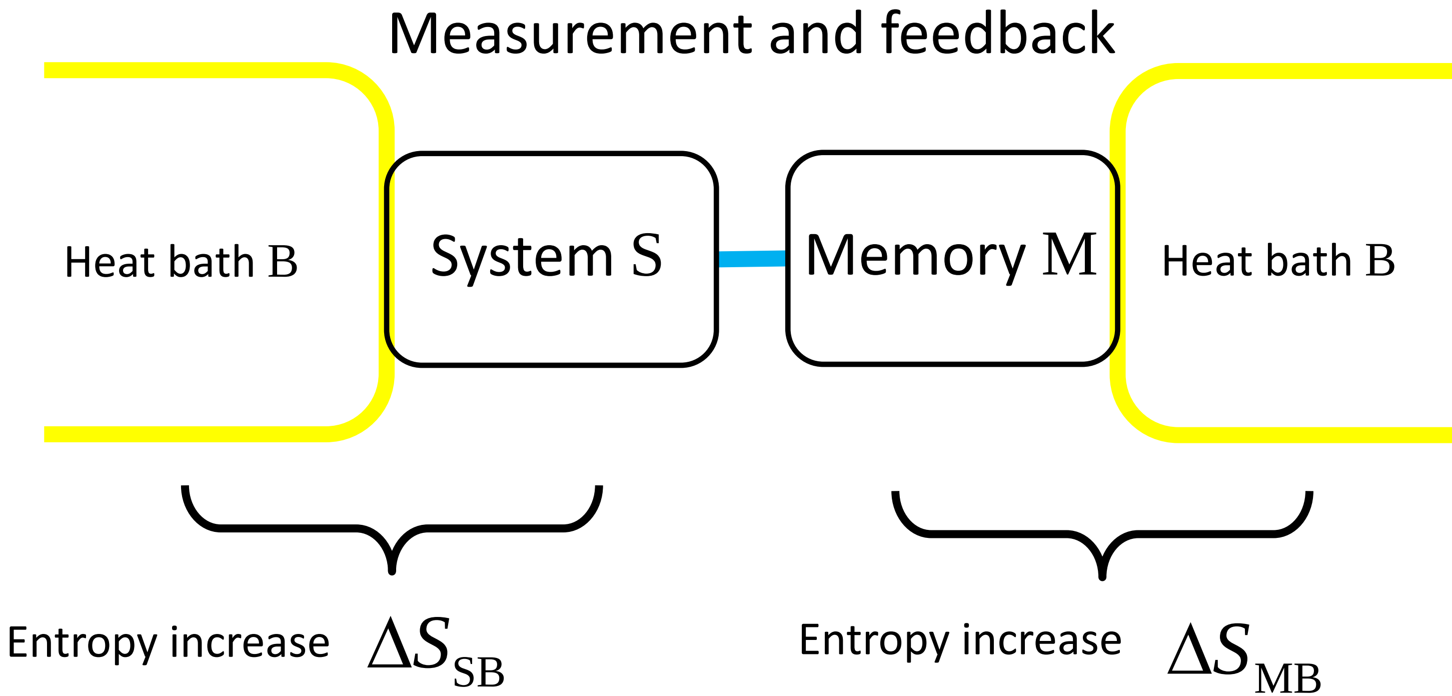
Feedback

Memory
(Controller)



What about thermodynamics for measurement processes?

Generalized Second Law: Entropic Balance



$$\Rightarrow \quad \Delta S_{\mathrm{MB}} \geq +I_{\mathrm{QC}}$$

TS and M. Ueda, PRL **102**, 250602 (2009); **106**, 189901(E) (2011).

Details of Memory

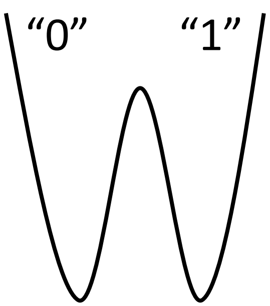
Standard state "0" of the memory with free energy F_0

The memory stores measurement outcome "*k*" with probability p_k.

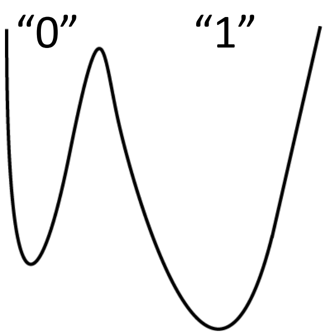
Free-energy difference: $\Delta F \equiv \sum_k p_k F_k - F_0$

Conditional free energy with k

Symmetric memory $\Delta F = 0$



Asymmetric memory $\Delta F \neq 0$



Generalized Second Law: Energetics

$$\Delta S_{\mathrm{MB}} \geq +I_{\mathrm{QC}}$$

$$\Rightarrow \quad W_{\mathrm{meas}} \geq -k_{\mathrm{B}} T H + \Delta F + k_{\mathrm{B}} T I_{\mathrm{QC}}$$

$$H = -\sum_k p_k \ln p_k$$: Shannon entropy of outcomes

$\Delta F=0$ (symmetric memory) & $H=I_{\mathrm{QC}}$ (classical and error-free measurement)

$$\Rightarrow \quad W_{\mathrm{meas}} \geq 0$$ (Bennett's result)

TS and M. Ueda, PRL **102**, 250602 (2009); **106**, 189901(E) (2011).

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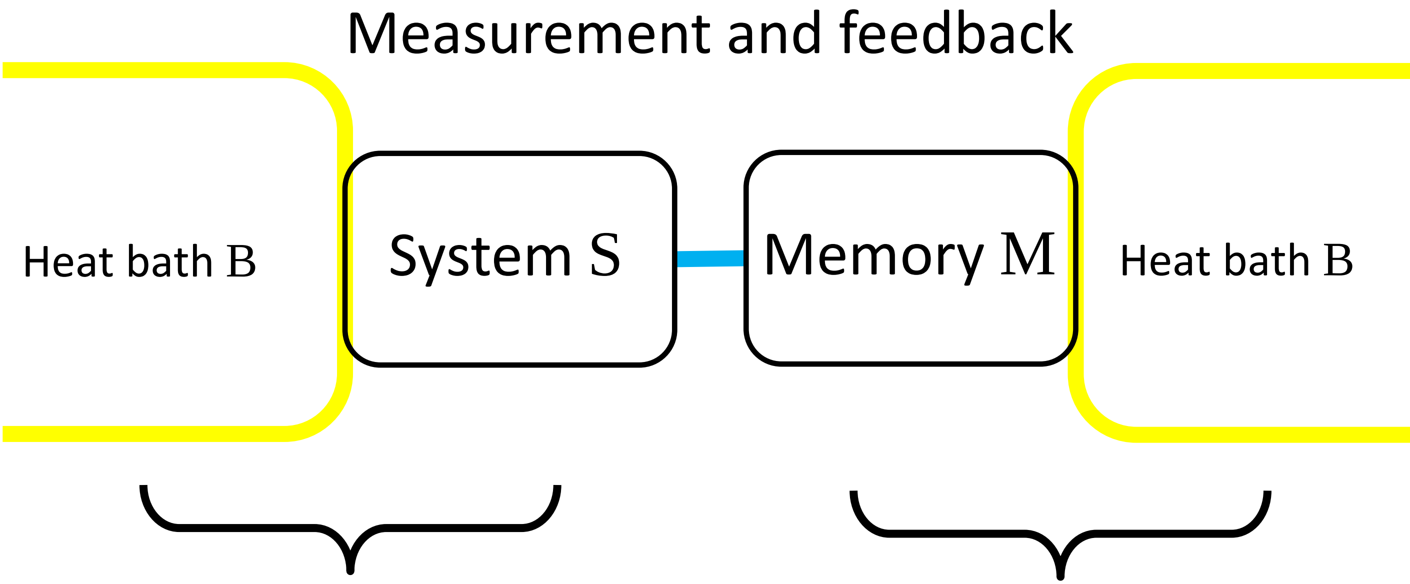
Generalized Second Law: Summary



Entropy increase ΔS_{SB}

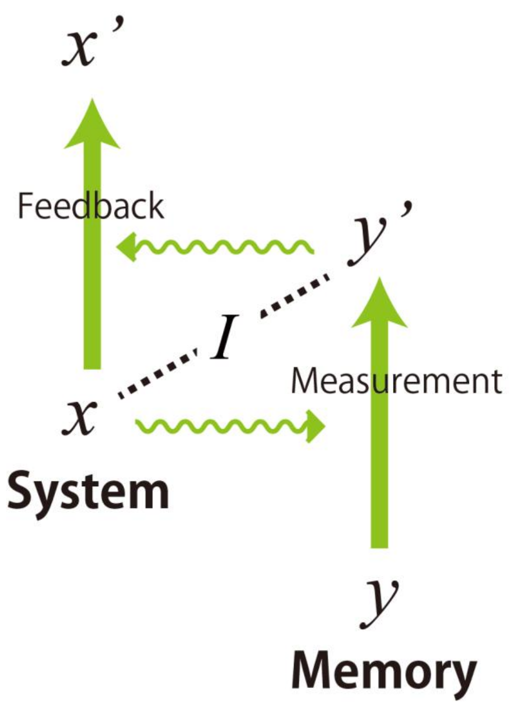
Entropy increase ΔS_{MB}

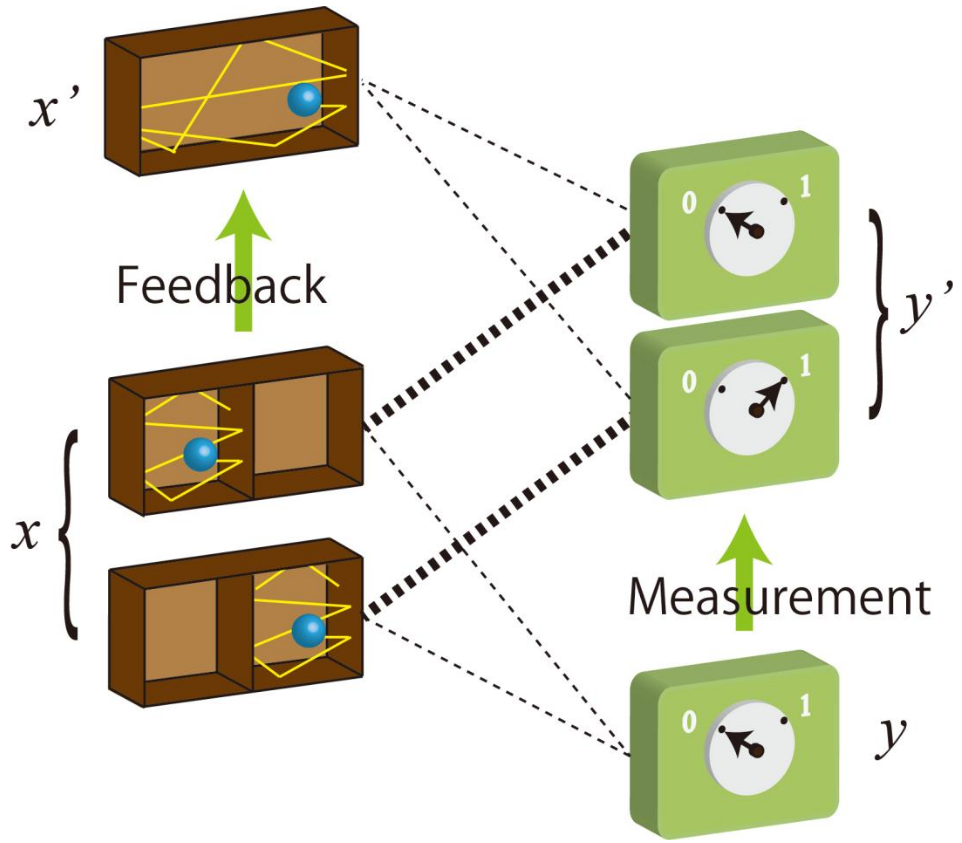
$$\Delta S_{\mathrm{SB}} \geq -I_{\mathrm{QC}}$$

$$\Delta S_{\mathrm{MB}} \geq +I_{\mathrm{QC}}$$

$$\Rightarrow \boxed{\Delta S_{\mathrm{total}} = \Delta S_{\mathrm{SB}} + \Delta S_{\mathrm{MB}} \geq 0}$$

“Duality” between Measurement and Feedback





Time-reversal transformation

Swap system and memory

Measurement becomes feedback (and vice versa)

Resolving the Paradox of Maxwell's Demon

Sagawa-Ueda:

Heat bath **System**

$$\Delta S_{\mathrm{SB}} \geq -I_{\mathrm{QC}}$$

What compensates for the gain (negative entropy production) here?

Memory **Heat bath**

$$\Delta S_{\mathrm{MB}} \geq +I_{\mathrm{QC}}$$

The loss (positive entropy production) here compensates for it during measurement!

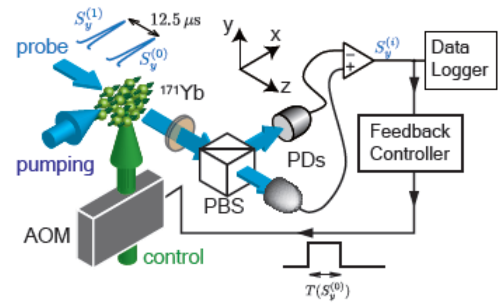
TS and M. Ueda, PRL **100**, 080403 (2008)
TS and M. Ueda, PRL **102**, 250602 (2009); **106**, 189901(E) (2011)

The additional entropy production (accompanied by an excess energy cost) during the **measurement** compensates for the negative entropy production during the feedback!

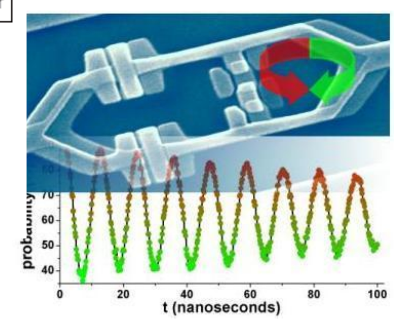
Summary and Future Plans

✓ Generalized second law of thermodynamics with quantum information processing

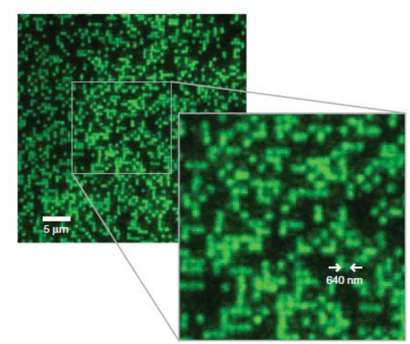
✓ Corollary : Resolved Maxwell's demon paradox



R. Inoue *et al.*, submitted.



- **Ultracold atoms**
- Superconducting qubit
- Etc…



Thank you for your attentions!