カイラル場の揺らぎを取り入れた強結合極限での QCD 相図

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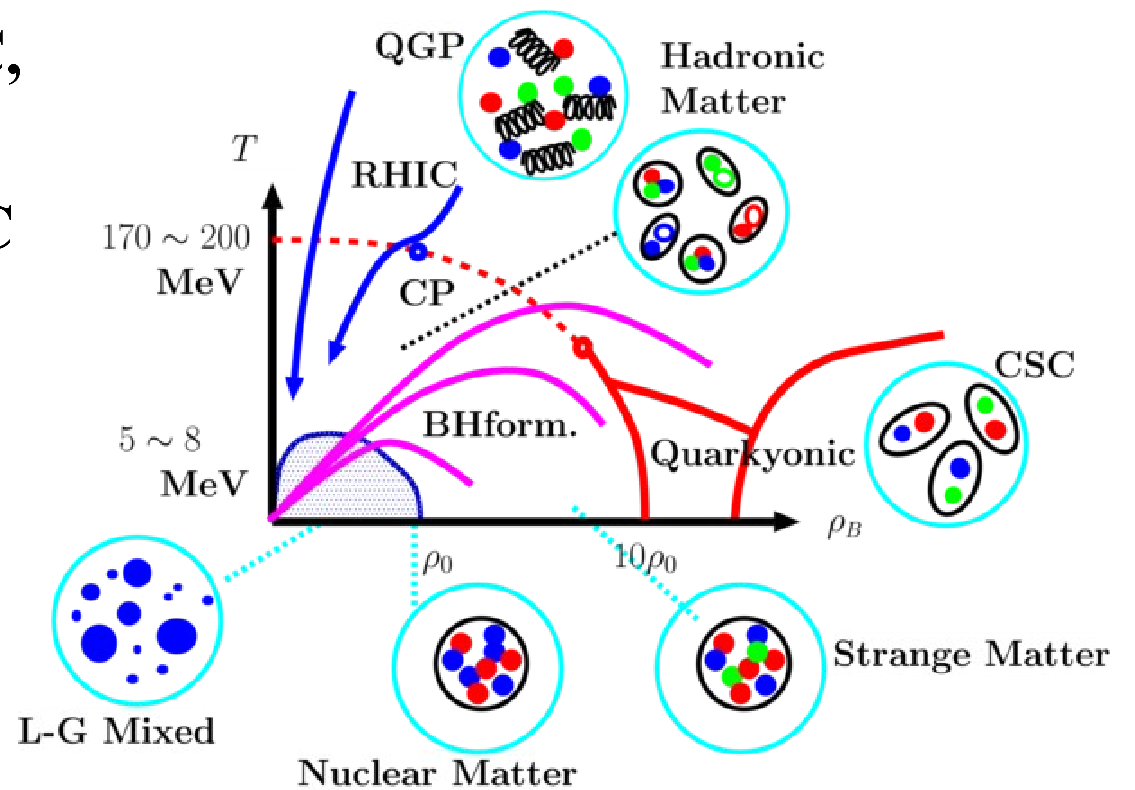
- Introduction
- 強結合極限でのカイラル場有効作用
- 補助場 Monte-Carlo による QCD 相図（強結合極限）
- Summary

AO, T. Z. Nakano, in prep.

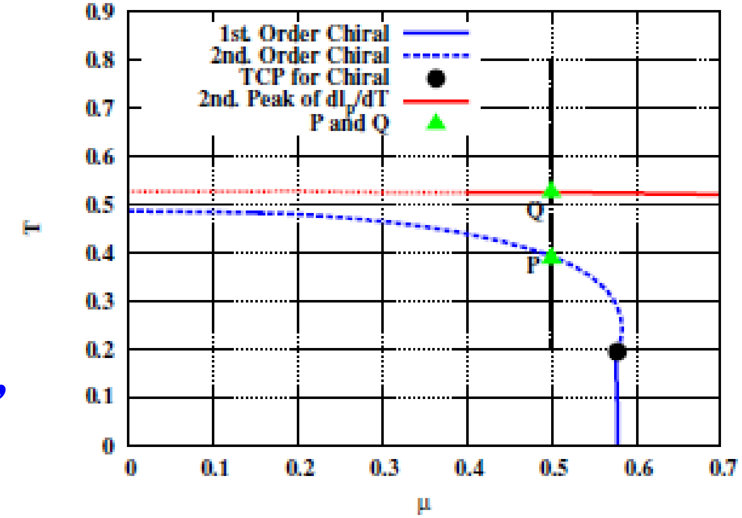
QCD Phase diagram

- Phase transition at high *T*
  - Early universe / RHIC, LHC / Lattice MC, pQCD, ….
- High μ transition
  - Compact Astrophysical Objects (Neutron stars, Supernovae, Black hole formation, ...)
  - RHIC-BES, FAIR, J-PARC, Astro-H, Grav. Wave, …
  - Sign problem in Lattice MC

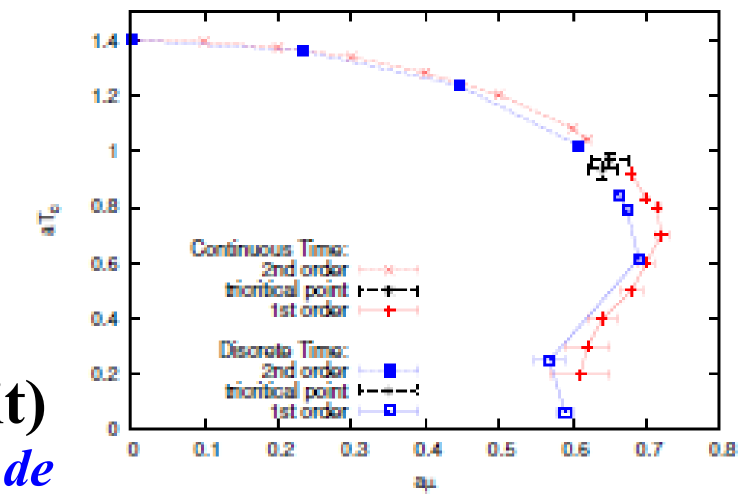


Strong Coupling Lattice QCD for finite μ

- **Effective Models (PNJL, PQM), FRG**
  Fukushima ('11); Fujii, Sano ('10); T. K. Herbst, J. M. Pawlowski, B. J. Schaefer,('11); ...
- **Lattice MC**
  Fodor, Katz ('02); de Forcrand, Philipsen('02); D'Elia, M. Lombardo ('03); Allton et al. ('04); Ejiri ('08); Nagata, Nakamura ('10); Nakagawa, Ejiri, ..(WHOT, '11), ..
- **Strong Coupling Lattice QCD**
  - **Mean Field approaches**
    Damagaard, Hochberg, Kawamoto ('85); Bilic, Karsch, Redlich ('92); Fukushima ('04); Nishida ('04); Kawamoto, Miura, AO, Ohnuma ('07).
  - **MDP simulation (Strong Coupling Limit)**
    Karsch, Mutter('89); de Forcrand, Fromm ('10); de Forcrand, Unger ('11)



Miura, Nakano, AO, Kawamoto, arXiv:1106.1219



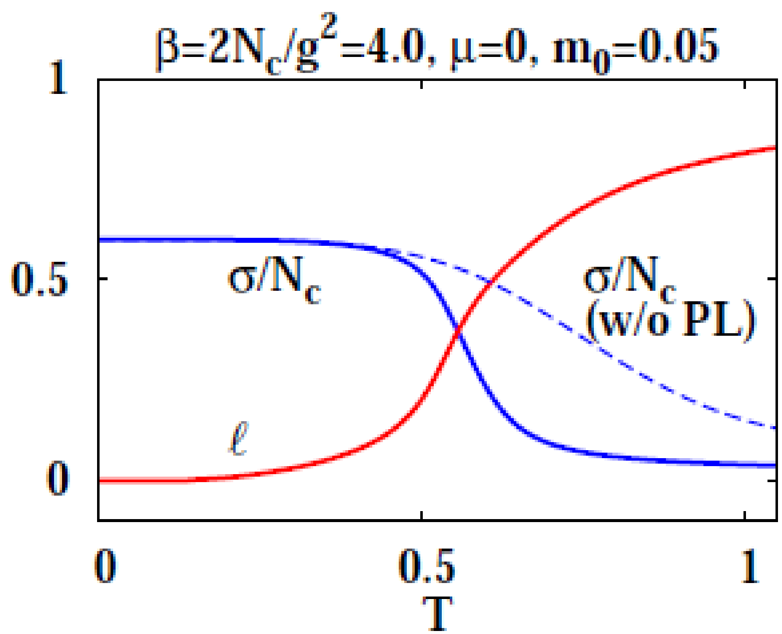
de Forcrand, Unger ('11)

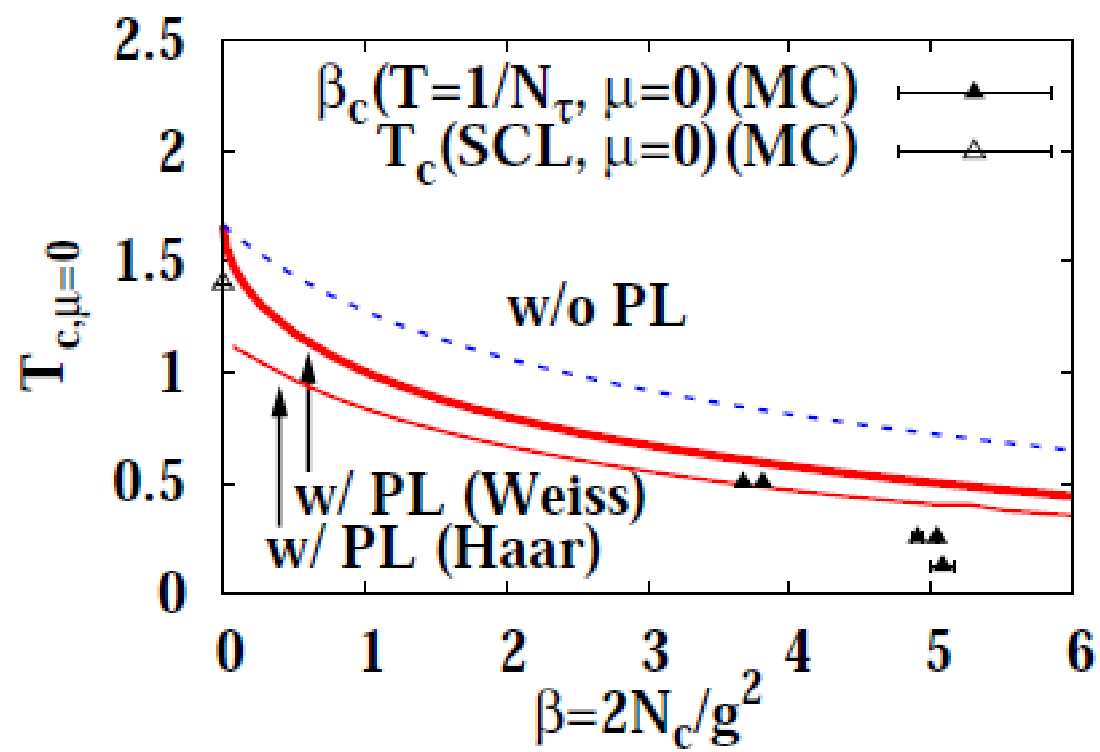
P-SC-LQCD at μ=0

T. Z. Nakano, K. Miura, AO, PRD 83 (2011), 016014 [arXiv:1009.1518 [hep-lat]]

- **P-SC-LQCD reproduces $T_c(\mu=0)$ in the strong coupling region ($\beta= 2N_c/g^2 \leq 4$)**

MC data: SCL (Karsch et al. (MDP), de Forcrand, Fromm (MDP)), N_τ =2 (de Forcrand, private), N_τ=4 (Gottlieb et al.('87), Fodor-Katz ('02)), N_τ =8 (Gavai et al.('90))





Lattice Unit

How can we include
both fluctuation and finite coupling effects ?
→ Auxiliary field MC

Strong Coupling Limit ($1/g^2=0$) Lattice QCD

Lattice QCD action (staggered fermion)

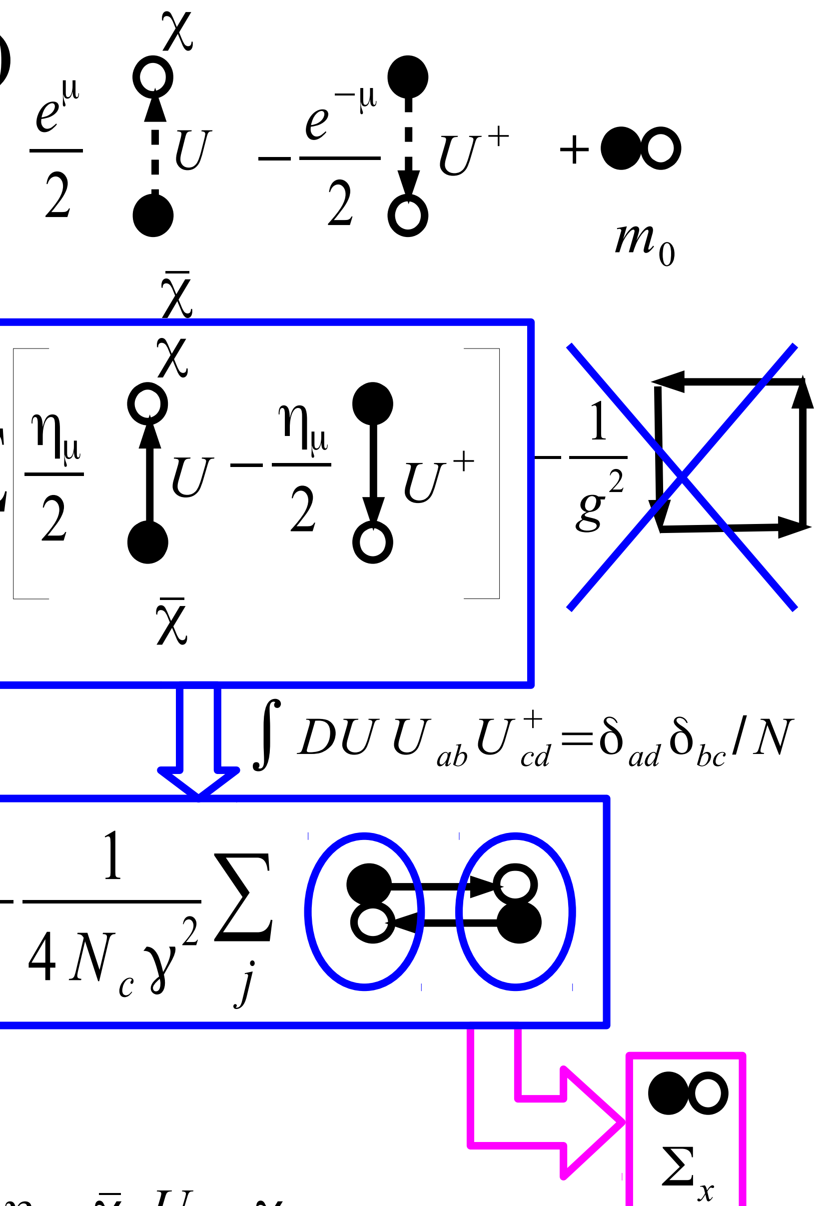
$$S_{\rm LQCD}=\frac{1}{2}\sum_{x,y}\bar{\chi}_x D^{(0)}_{x,y}\chi_y+\frac{m_0}{\gamma}\sum_x M_x+\frac{1}{2\gamma}\sum_{x,j}\left(V_x^{+j}-V_x^{-j}\right)-\frac{1}{g^2}\sum_P \mathrm{tr}\left(U_P+U_P^{+}\right)$$

Spatial link integral in SCL

$$S_{\rm eff}=\frac{1}{2}\sum_{x,y}\bar{\chi}_x D^{(0)}_{x,y}\chi_y+\frac{m_0}{\gamma}\sum_x M_x-\frac{1}{4N_c\gamma^2}\sum_{x,j}M_x M_{x+\hat{j}}$$

Bosonization

$$S_{\rm eff}=\frac{1}{2}\sum_{x,y}\bar{\chi}_x D^{(0)}_{x,y}\chi_y+\sum_x\left(\Sigma_x+\frac{m_0}{\gamma}\right)M_x+\frac{\Omega}{4N_c\gamma^2}\sum_{k,\,f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k})\left[\sigma_k^*\sigma_k+\pi_k^*\pi_k\right]$$



$$D^{(0)}_{x,y}=\delta_{x+\hat{0},y}\delta_{\boldsymbol{x},\boldsymbol{y}}e^{\mu/\gamma^2}U_{x,0}-\delta_{x,y+\hat{0}}\delta_{\boldsymbol{x},\boldsymbol{y}}e^{-\mu/\gamma^2}U^{+}_{y,0},\quad V_x^{+\mu}=\eta_{\mu,x}\bar{\chi}_x U_{\mu,x}\chi_{x+\hat{\mu}},$$

$$V_x^{-\mu}=\eta^{-1}_{\mu,x}\bar{\chi}_{x+\hat{\mu}}U^{+}_{\mu,x}\chi_x,\quad M_x=\bar{\chi}_x\chi_x,\quad \eta_{j,x}=(-1)^{x_0+\cdots+x_{j-1}}$$

Bosonized Effective Action

- **Bosonization of *MM* term (Four Fermi (two-body) interaction)**

$$-\alpha \sum_{j,x} M_x M_{x+\hat{j}} \to \sum_x M_x \Sigma_x + \alpha L^3 N_\tau \sum_{k,\, f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k}) (\sigma_k^* \sigma_k + \pi_k^* \pi_k)$$

$$f_M(\boldsymbol{k}) = \sum_j \cos k_j, \quad \Sigma_x = 2\alpha \sum_{k,\, f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k}) e^{ikx} (\sigma_k + i\varepsilon(x) \pi_k)$$

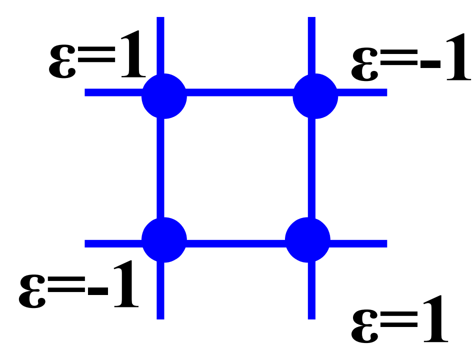
  - **Negative negative eigen values of meson matrix in "High" *k***
    → Involves a factor $i\varepsilon_x$ = i (-1)($x_0+x_1+x_2+x_3$) in x repr.**
    Miura, Nakano, AO (09), Miura, Nakano, AO, Kawamoto (09)

- **Bosonized Effective Action**

$$S_{\mathrm{eff}} = \frac{L^3 N_\tau}{4 N_c \gamma^2} \sum_{k,\, f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k}) \left[\sigma_k^* \sigma_k + \pi_k^* \pi_k \right] - \sum_{\boldsymbol{x}} \log R(x)$$

$$R(x) = X_N(\boldsymbol{x})^3 - 2 X_N(\boldsymbol{x}) + 2\cosh(3 N_\tau \mu)$$

  - **$X_N(x)$ = easily calculated from σ(x) and π(x).**
  - **Imaginary part (π) involves ε_x**
    → Phase cancellation for low k.

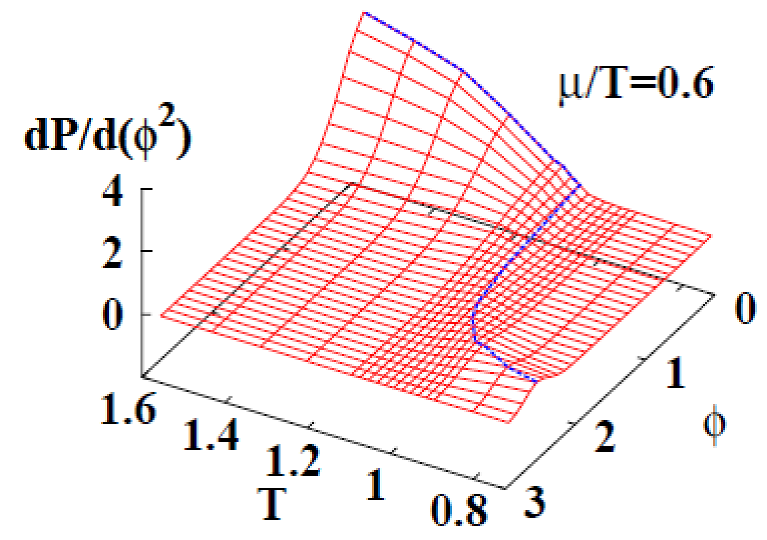


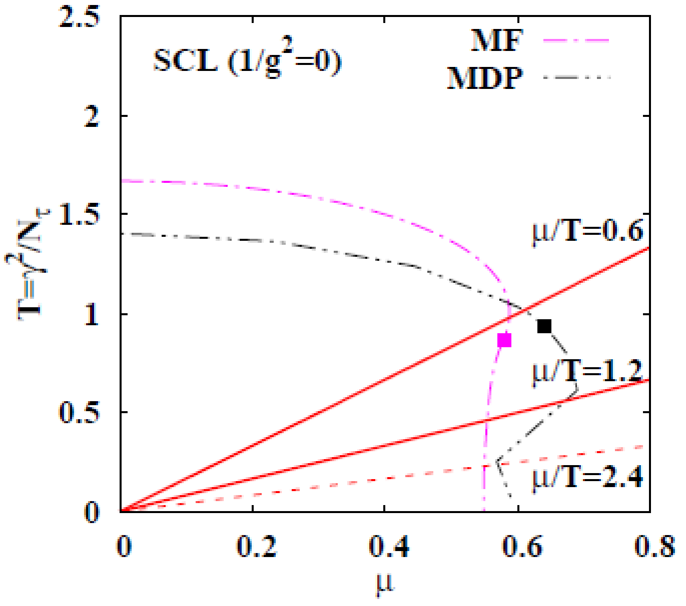
Let's Try !

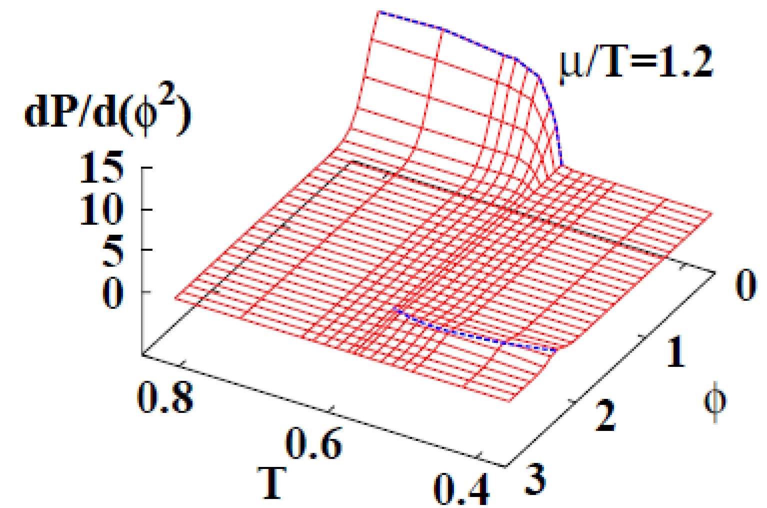
- 4^4 asymmetric lattice + Metropolis sampling of σ_k and π_k.
- Metropolis sampling of full configuration (σ_k and π_k) at a time. (efficient for small lattice)
- Chiral limit (m=0) simulation → Symmetry in $\sigma \leftrightarrow -\sigma$
- Computer: My PC (Core i7).
  (Partially in SR 16000 (single core))

Results (1): σ distribution

- **Fixed μ/T simulation: μ/T= 0 ~ 2.4**
- **Low μ region: Second order (Single peak: finite σ → zero)**
- **High μ region: First order (Dist. func. has two peaks)**







Lattice Unit

Results (2): Susceptibility and Quark density

- **Weight factor <cos θ>**

$$\langle \cos\theta \rangle = Z / Z_{\mathrm{abs}}$$

$$Z = \int D\sigma_k D\pi_k \exp(-S_{\mathrm{eff}}) = \int D\sigma_k D\pi_k \exp(-\mathrm{Re}\, S_{\mathrm{eff}}) e^{i\theta}$$

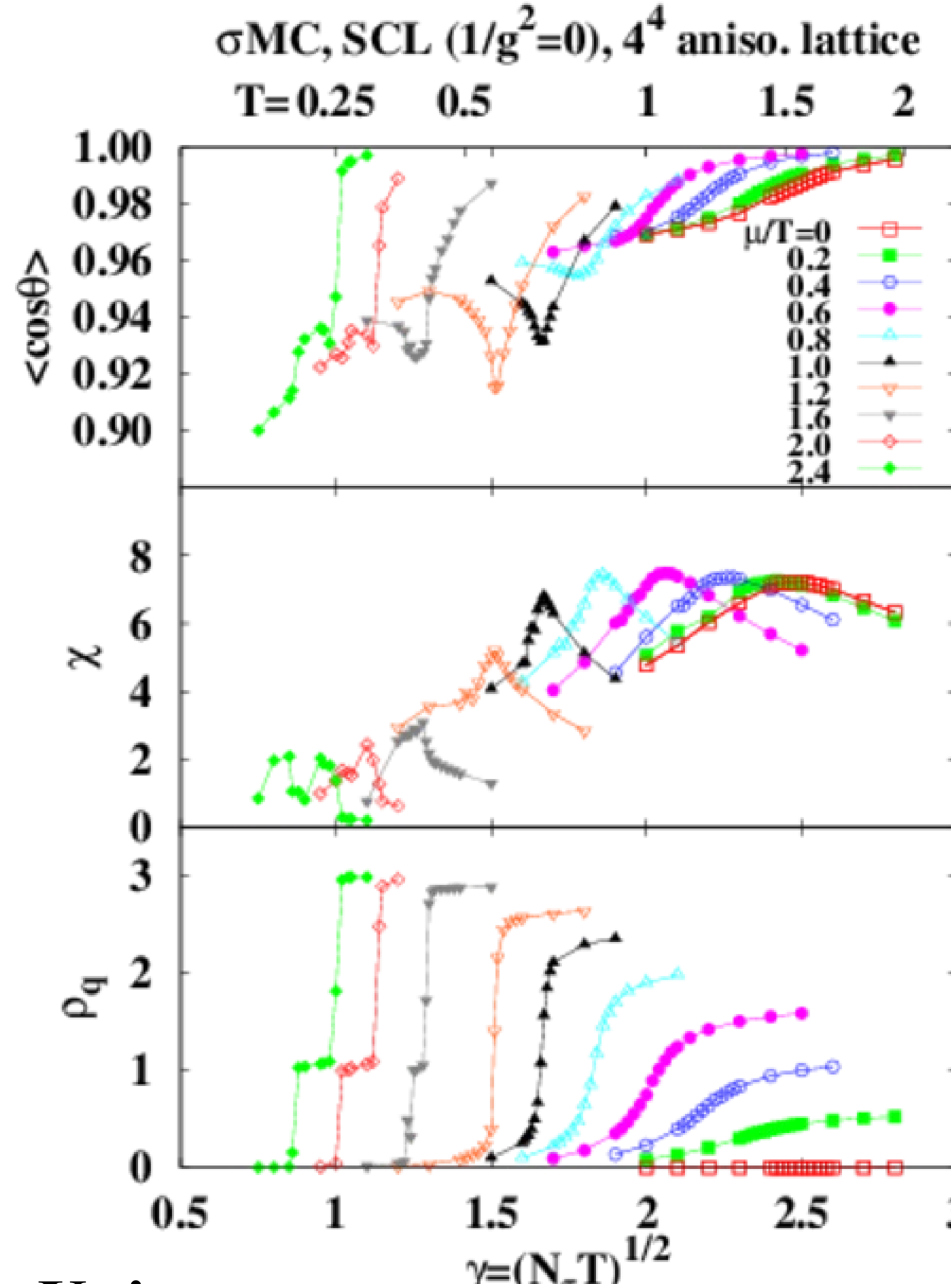
$$Z_{\mathrm{abs}} = \int D\sigma_k \pi_k \exp\left(-\mathrm{Re}\, S_{\mathrm{eff}}\right)$$

- **Chiral susceptibility**

$$\chi = -\frac{1}{L^3} \frac{\partial^2 \log Z}{\partial m_0^2}$$

- **Quark number density**

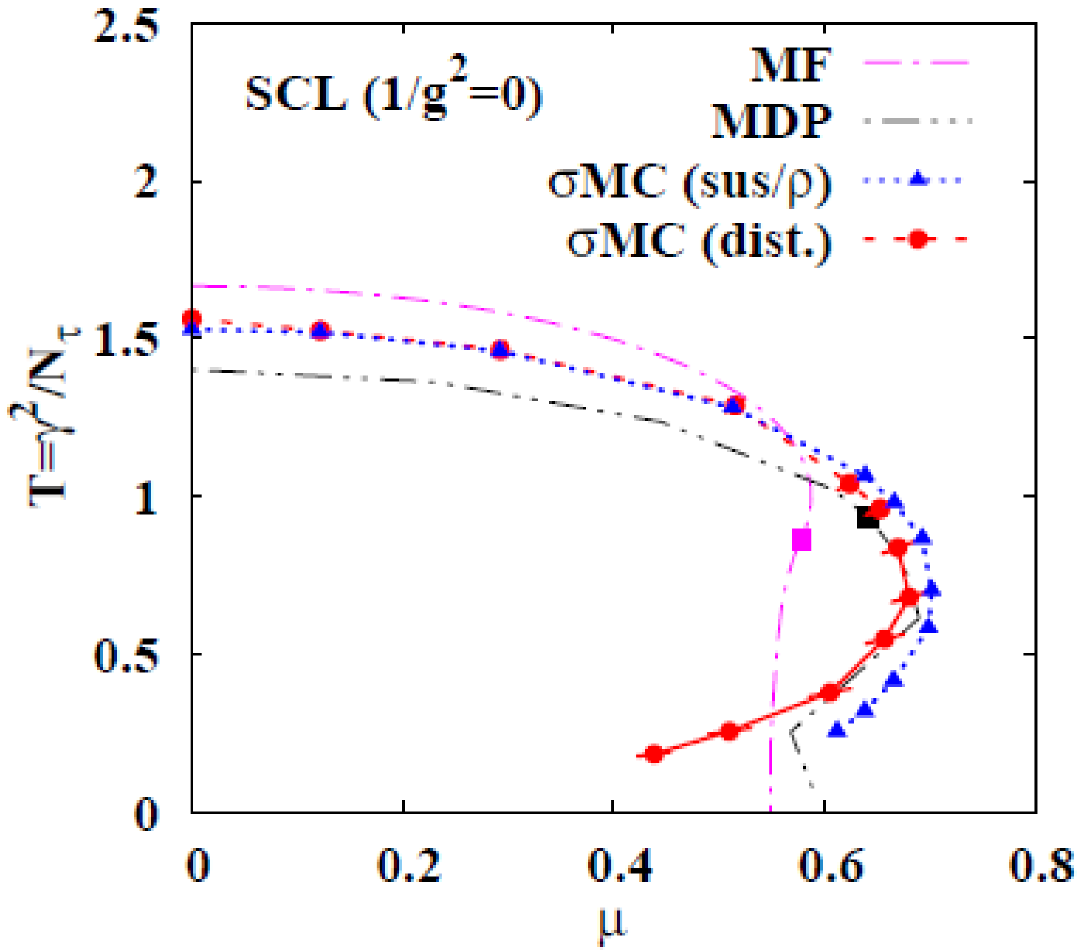
$$\rho_q = -\frac{1}{L^3} \frac{\gamma^2}{N_\tau} \frac{\partial \log Z}{\partial \mu}$$



Lattice Unit

Results (3): Phase diagram

- By taking $T = \gamma^2/N_\tau$,
  γ dep. of the phase boundary becomes small. *Bilic et al. ('92)*
- Phase boundary
  - σ dist.(red) & χ peak (blue)
- Fluctuation effects
  - T_c reduction at μ=0
    μ_c enh. at medium *T*
  - Consistent with MDP
    de Forcrand, Fromm ('09); de Forcrand, Unger ('11)
- Bending at low T: Small size
  de Forcrand, Wenger ('06)



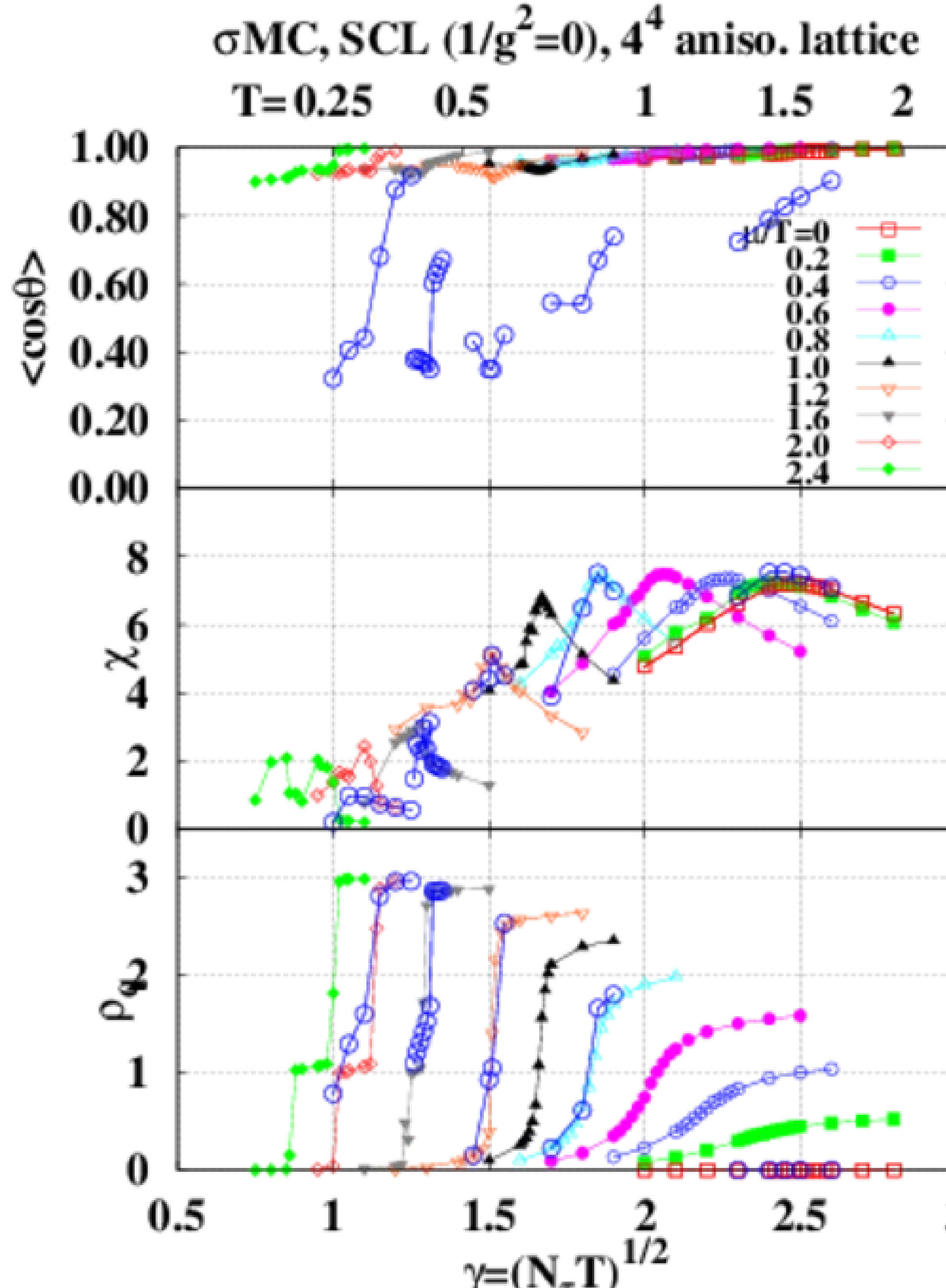
Lattice Unit

Summary

- **We have proposed an auxiliary field MC method (σMC), to simulate the SCL quark-U_0 action (LO in strong coupling ($1/g^0$) and 1/d ($1/d^0$) expansion) without further approximation.**
  c.f. Determinantal MC by Abe, Seki
- **Sign problem is mild in small lattice (<cos θ> ~ (0.9-1) for 4^4), because of the phase cancellation mechanism coming from nearest neighbor interaction.**
- **Phase boundary is moderately modified from MF results by fluctuations, as in MDP simulation.**
- **Future work**
  - **Extension to NLO SC-LQCD is straightforward.**
  - **Larger lattice, finite coupling, different Fermion, higher 1/d terms including baryonic action and chiral Polyakov coupling.**

Thank you

Preliminary Results with 8^3x4 Lattice

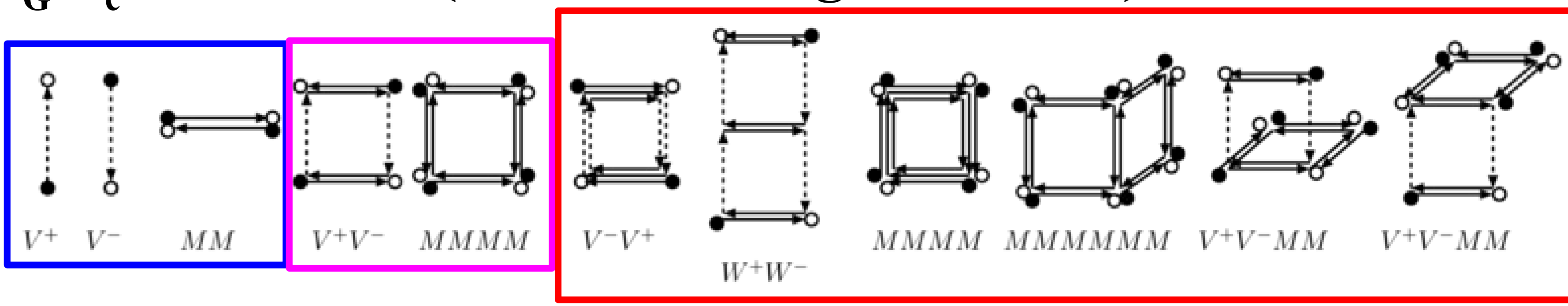


SC-LQCD with Fermions

- **Effective Action with finite coupling corrections**
  Integral of exp(-S_G) over spatial links with exp(-S_F) weight → S_{eff}

$$S_{\text{eff}} = S_{\text{SCL}} - \log\langle \exp(-S_G)\rangle = S_{\text{SCL}} - \sum_{n=1} \frac{(-1)^n}{n!} \langle S_G^n \rangle_c$$

$\langle S_G^n \rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



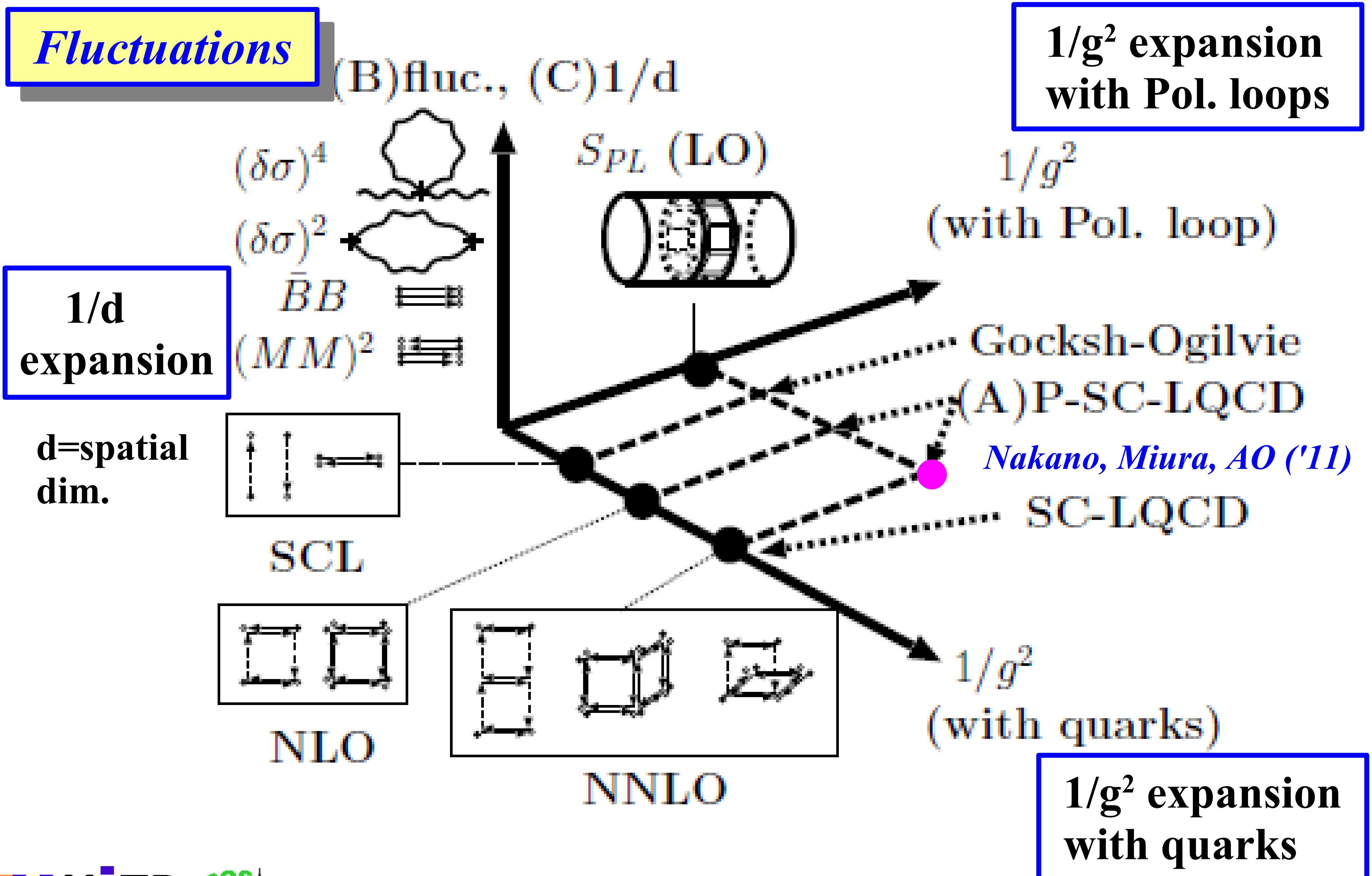
$$S_{\text{eff}} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$ *SCL (Kawamoto-Smit, '81)*

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$ *NLO (Faldt-Petersson, '86)*

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\ |k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$ *NNLO (Nakano, Miura, AO, '09)*

Approximations in Pol. loop extended SC-LQCD



Approximations in Pol. loop extended SC-LQCD

- **Strong coupling expansion**
  - **Fermion terms: LO($1/g^0$, SCL), NLO($1/g^2$), NNLO ($1/g^4$)**
  - **Plaquette action: LO ($1/g^{2N\tau}$)**
- **Large dimensional approximation**
  - **1/d expansion (d=spatial dim.)**
    → Smaller quark # configs. are preferred.
    $\Sigma_j M_x M_{x+j} = O(1/d^0) \rightarrow M \propto d^{-1/2} \rightarrow \chi \propto d^{-1/4}$
  - **Only LO ($1/d^0$) terms are mainly evaluated.**
- **(Unrooted) staggered Fermion**
  - **Nf=4 in the continuum limit.**
- **Mean field approximation**
  - **Auxiliary fields are assumed to be constant.**

⇒ ***This work: Auxiliary Field MC in SCL***

Faldt, Petersson, 1986

- **Fermion action is separated to each spatial point and bi-linear → Determinant of Nτ x Nc matrix**

$$\exp(-V_{\rm eff}/T) = \int dU_0 \begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^+ \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}$$

Nc x Nτ

$$= \int dU_0 \det\left[X_N[\sigma] \otimes \mathbf{1}_c + e^{-\mu/T} U^+ + (-1)^{N_\tau} e^{\mu/T} U \right]$$

Nc

$$= X_N^3 - 2X_N + 2\cosh(3N_\tau\mu)$$

$$I_\tau/2 = [\sigma(x) + i\varepsilon(x)\pi(x)]/2N_c\gamma^2 + m_0/\gamma$$

$$X_N = B_N + B_{N-2}(2;N-1)$$

$$B_N = I_N B_{N-1} + B_{N-2}$$

$$B_N = \begin{vmatrix} I_1 & e^{\mu} & 0 & & \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ & & & -e^{-\mu} & I_N \end{vmatrix}$$

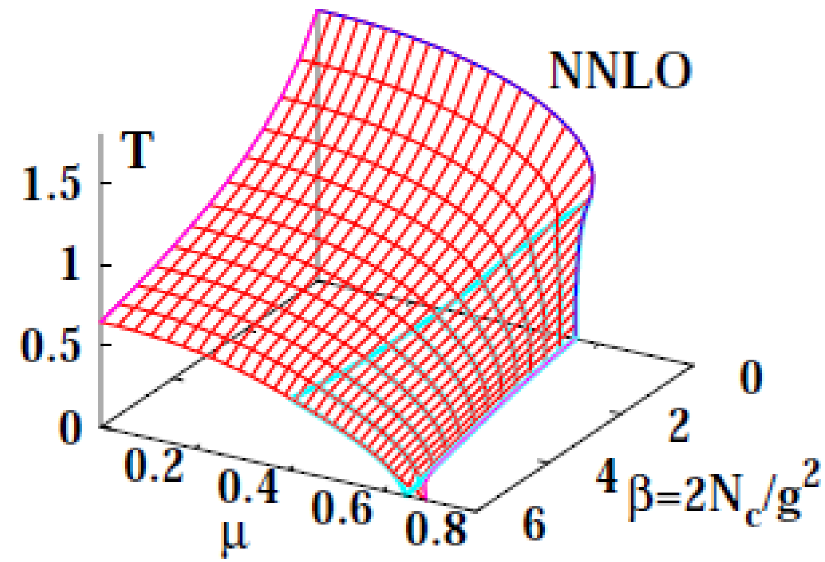
Clausius-Clapeyron Relation

- **First order phase boundary → two phases coexist**

$$P_h = P_q \rightarrow dP_h = dP_q \rightarrow \frac{d\mu}{dT} = -\frac{s_q - s_h}{\rho_q - \rho_h}$$

$$dP_h = \rho_h d\mu + s_h dT, \quad dP_q = \rho_q d\mu + s_q dT$$

  - **Continuum theory**
    → Quark matter has larger entropy and density (dμ /dT < 0)
  - **Strong coupling lattice**
    - **SCL: Quark density is larger than half-filling, and "Quark hole" carries entropy → dμ/dT > 0**
    - **NLO, NNLO → dμ/dT < 0**



AO, Miura, Nakano, Kawamoto ('09)