Auxiliary field Monte-Carlo study of the QCD phase diagram at strong coupling

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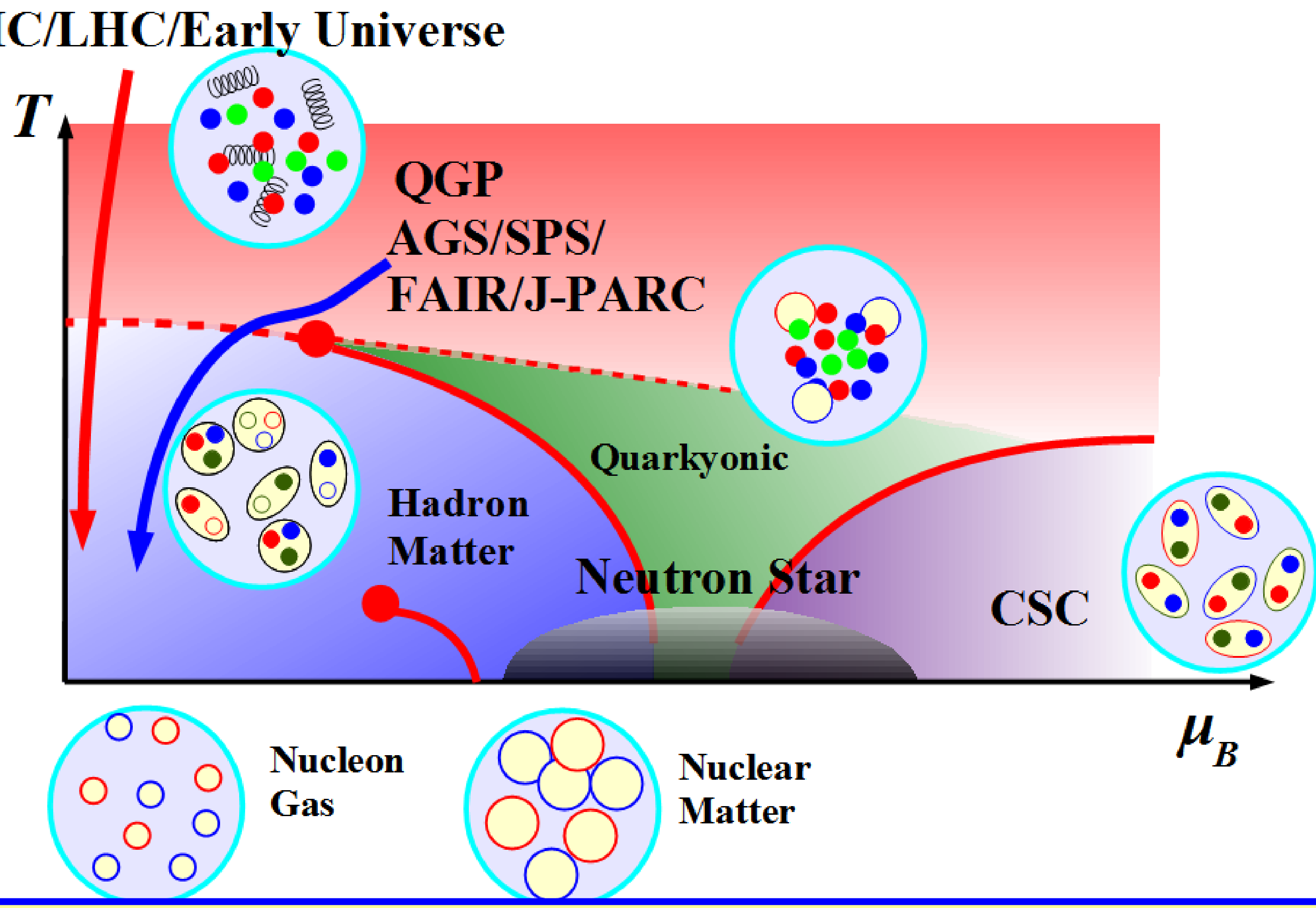
- **Introduction**
- **Auxiliary field effective action in the Strong Coupling Limit**
- **AFMC phase diagram**
- **Summary**

AO, T. Ichihara, T. Z. Nakano, in prep.

QCD phase diagram

- **Various phases, rich structure (conjectured)**
  Related to early universe and compact star phenomena, and CP may be reachable in RHIC.



Can we understand QCD phase diagram in lattice QCD ?

Lattice QCD at Finite Density

- **Dream**
  Ab initio calc. of phase diagram and nuclear matter EOS
- **Unreachable ?**
  Sign prob. is severe at low T & high μ
  - **No go theorem**
    Han, Stephanov ('08), Hanada, Yamamoto ('11), Hidaka, Yamamoto ('11)
    Phase quenched sim. at finite quark μ ~ Finite isospin μ
    (No flavor mixing, as justified in large Nc)
    → Average sign factor vanishes at low T & high μ due to π cond.
- **Hope ?**
  - **Sampling method other than phase quenched simulation ?**
  - **Strong coupling lattice QCD**
    → Mean field approximation
    Monomer-Dimer-Polymer simulation

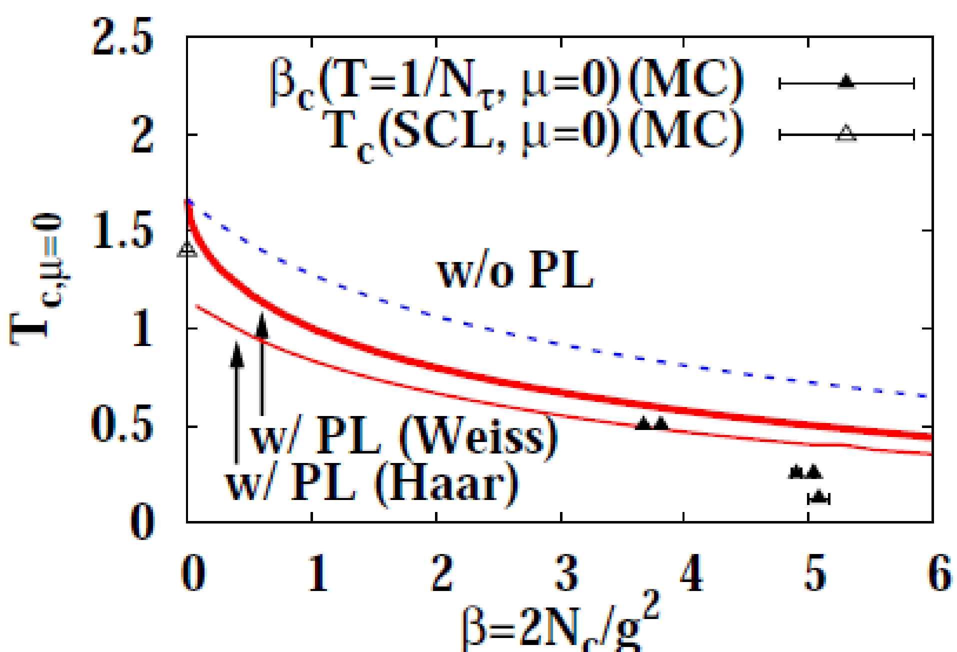
Strong Coupling Lattice QCD

- **Successful from the dawn of lattice gauge theory**
  - **Pure YM: Area Law, MC calc. of string tension, $1/g^2$ expansion**
    Wilson ('74), Creutz ('80), Munster ('80)
- **Strong Coupling Lattice QCD with quarks**
  - **Spontaneous breaking of chiral sym. in vacuum, Chiral transition**
    Kawamoto, Smit ('81), Damgaard, Kawamoto, Shigemoto ('84)
    → Utilized in constructing effective models
    Gocksch, Ogilvie (85), Fukushima ('03), Ratti, Thaler, Weise ('06), ...
  - **Phase diagram in the strong coupling limit (mean field)**
    Bilic, Karsch, Redlich ('92), Fukushima ('04), Nishida ('04)
  - **Finite coupling and Polyakov loop effects (mean field)**
    Faldt, Petersson ('86), Miura, Nakano, AO ('09),
    Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10),
    Nakano, Miura, AO, Kawamoto ('11)
  - **Fluctuation effects via MDP simulation (mean field)**
    Karsch, Mutter('89); de Forcrand, Fromm ('10); de Forcrand, Unger ('11)

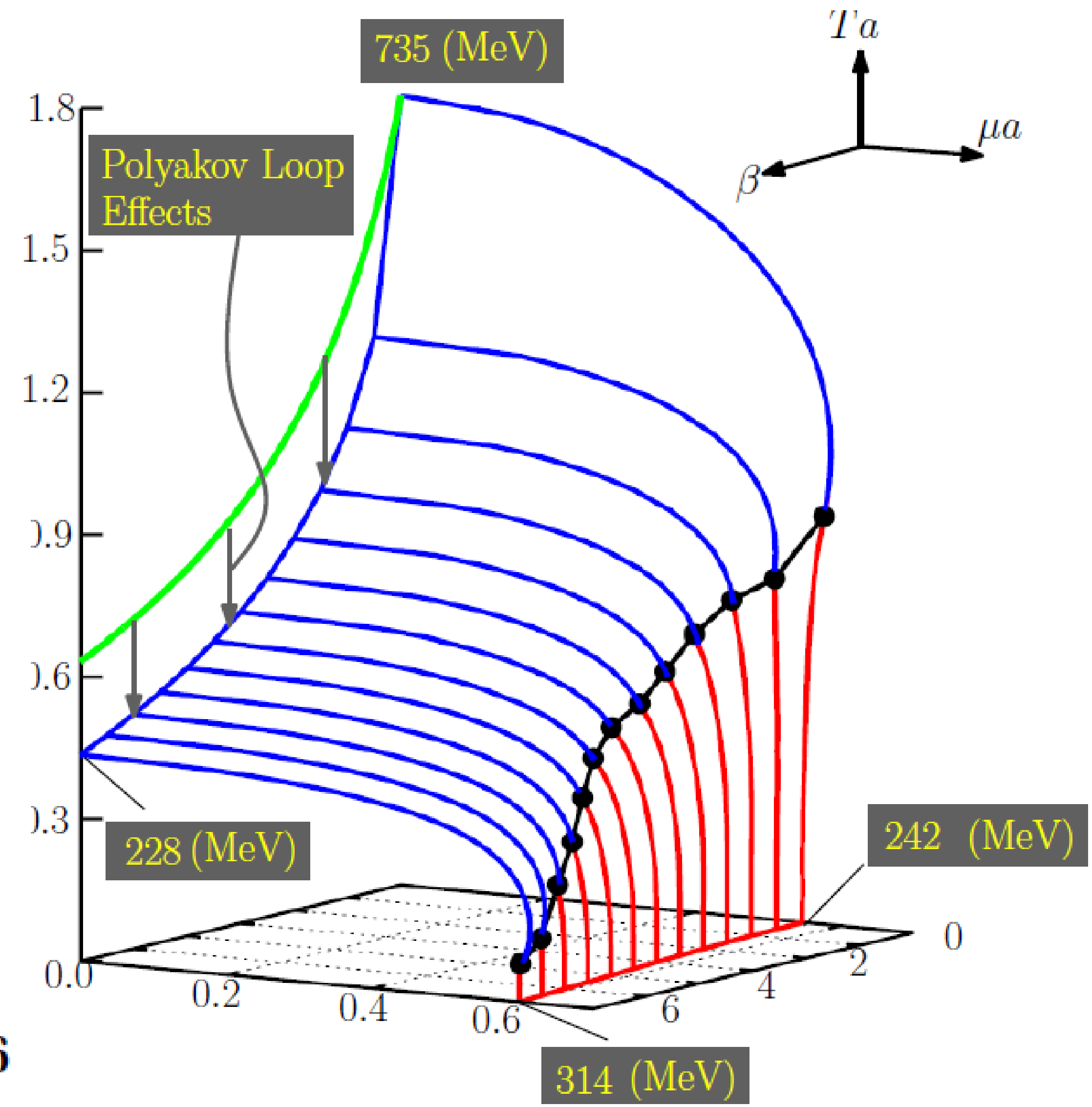
Finite Coupling and Polyakov Loop Effects

Miura, Nakano, AO ('09), Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10), Nakano, Miura, AO, Kawamoto ('11)

- **Finite coupling & Pol. loop reduces T_c while μ_c is stable.**
  - **MC results of T_c at μ=0 are explained at β_g=2 N_c/g^2 < 4.**
  - **Compatible with empiricals.**



Nakano et al. ('11)



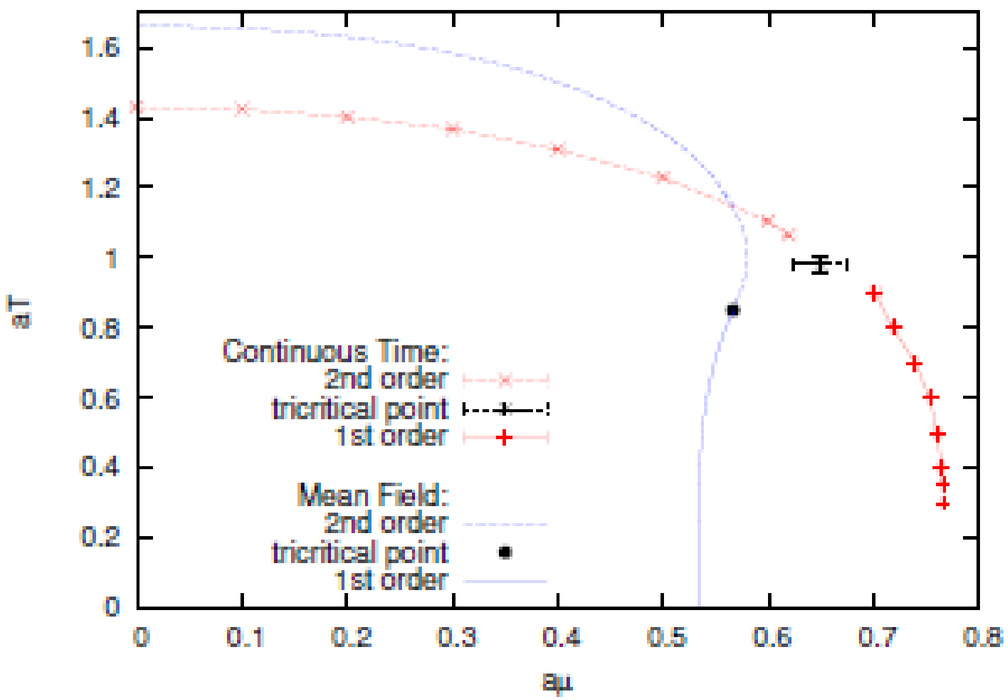
Miura et al., in prep.

Monomer-Dimer-Polymer phase diagram

- **MDP simulation**

  Karsch, Mutter('89); de Forcrand, Fromm ('10); de Forcrand, Unger ('11)

  - **Partition function**
    = sum of config. weights of various loops.
  - **Extension to finite coupling ($1/g^2 \neq 0$) is not straightfoward.**

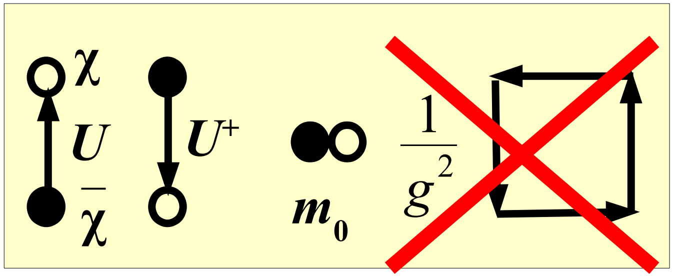


Both finite coupling and fluctuation effects are important.
Is there any way to include both of these ?
→ Auxiliary Field Monte-Carlo method

Auxiliary Field Monte-Carlo in the Strong Coupling Limit

Strong Coupling Effective Action

- **Strong Coupling Limit Lattice QCD action**
  - **no plaquette action, aniso. lattice $a_\tau = a_s/\gamma$, unrooted stag. fermion**

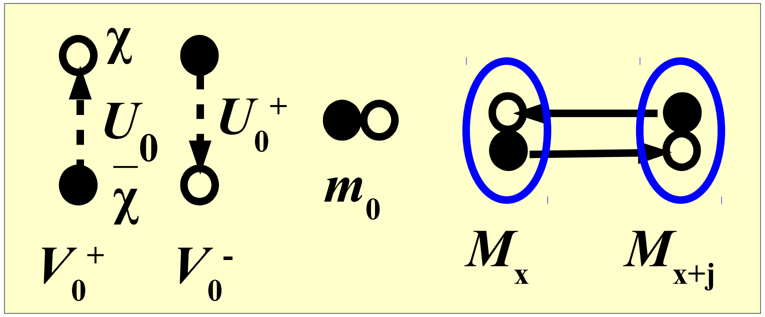


$$S_{\rm LQCD} = \frac{1}{2}\sum_x \left[e^{\mu/\gamma^2} \bar{\chi}_x U_{0,x} \chi_{x+\hat{0}} - e^{-\mu/\gamma^2} \bar{\chi}_{x+\hat{0}} U^+_{0,x} \chi_x \right]$$

$$+ \frac{1}{2\gamma} \sum_{x,j} \eta_j(x) \left[\bar{\chi}_x U_{j,x} \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U^+_{j,x} \chi_x \right] + \frac{m_0}{\gamma} \sum_x \bar{\chi}_x \chi_x$$

anisotropy

- **Strong Coupling Limit Effective Action**
  - **Leading orders in $1/g^2$ and $1/d$ +Spatial link integral**



→ Eff. action of mesonic composites

(d=spatial dim.)

$$S_{\rm eff} = \frac{1}{2}\sum_x \left[V^+(x) - V^-(x) \right] - \frac{1}{4N_c\gamma^2} \sum_{x,j} M_x M_{x+\hat{j}} + \frac{m_0}{\gamma}\sum_x M_x$$

$$V^+(x) = e^{\mu/\gamma^2} \bar{\chi}_x U_{x,0} \chi_{x+\hat{0}}, \quad V^-(x) = e^{-\mu/\gamma^2} \bar{\chi}_{x+\hat{0}} U^+_{x,0} \chi_x, \quad M_x = \bar{\chi}_x \chi_x$$

Introduction of Auxiliary Fields

- **Bosonization of *MM* term in Mean Field Treatment**

$$-\alpha \sum_{j,x} M_x M_{x+\hat{j}} \to \alpha d \sigma^2 + 2 d \alpha \sigma \sum_x M_x$$

Const. quark mass

- **More rigorous treatment**

$$-\alpha \sum_{j,x} M_x M_{x+\hat{j}} = -\alpha \sum_{x,y} M_x V_{x,y} M_y = -\alpha L^3 \sum_{\boldsymbol{k},\tau} f(\boldsymbol{k}) M_{-\boldsymbol{k},\tau} M_{\boldsymbol{k},\tau}$$

  - **Meson hopping matrix has positive and negative eigen values**

$$V_{x,y} = \frac{1}{2} \sum_j (\delta_{x+\hat{j}\,y} + \delta_{x-\hat{j},y}), \quad f_M(\boldsymbol{k}) = \sum_j \cos k_j,$$

$$f_M(\bar{\boldsymbol{k}}) = -f_M(\boldsymbol{k}) \quad [\bar{\boldsymbol{k}} = \boldsymbol{k} + (\pi,\pi,\pi)]$$

  - **Extended Hubbard-Stratotonic (HSMNO) transf.**
    → Introducing " *i* " gives rise to the sign problem.

Miura, Nakano, AO (09), Miura, Nakano, AO, Kawamoto (09)

$$\exp(\alpha A B) = \int d\varphi\, d\phi \exp[-\alpha(\varphi^2 - (A+B)\varphi + \phi^2 - i(A-B)\phi)]$$

Auxiliary Field Effective Action

- **Bosonized effective action**

Const. quark mass

$$S_{\rm eff}(\sigma,\pi,\chi,\bar\chi,U_0)=\frac{1}{2}\sum_x\left[V^+(x)-V^-(x)\right]+\sum_x\bar\chi_x\chi_x\Sigma_x$$

$$+\frac{L^3}{4N_c\gamma^2}\sum_{\boldsymbol{k},\tau,f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k})\left[\sigma_k^*\sigma_k+\pi_k^*\pi_k\right]$$

$$\Sigma_x=\frac{1}{4N_c\gamma^2}\sum_j\left[(\sigma+\underline{i\varepsilon\pi})_{x+\hat{j}}+(\sigma+\underline{i\varepsilon\pi})_{x-\hat{j}}\right]+\frac{m_0}{\gamma}$$

Nearest Neighbor Interaction

Negative mode → high k modes

- **Grassmann and Temporal Link Integral (analytic)**

Faldt, Petersson ('86), Nishida ('04)

$$S_{\rm eff}(\sigma,\pi)=\frac{L^3}{4N_c\gamma^2}\sum_{k,f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k})\left[\sigma_k^*\sigma_k+\pi_k^*\pi_k\right]$$

$$-\sum_{\boldsymbol{x}}\log\left[X_N(\boldsymbol{x})^3-2X_N(\boldsymbol{x})+2\cosh(3N_\tau\mu/\gamma^2)\right]$$

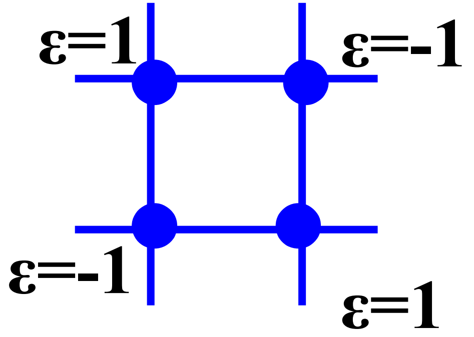
$$X_N(\boldsymbol{x})=X_N[\Sigma(\boldsymbol{x},\tau)]\quad(\text{known function})$$

$$=2\cosh(N_\tau \operatorname{arcsinh}\Sigma/\gamma^2)\qquad(\text{for const.}\ \Sigma)$$

Merits of Auxiliary Field Monte-Carlo

- **Fermion matrix is spatially separated**
  → Fermion det. at each point

- **Imaginary part (π) involves**
  $\varepsilon = (-1)^{x0+x1+x2+x3} = \exp(i\,\pi(x_0+x_1+x_2+x_3))$
  → Phase cancellation of nearest neighbor spatial site det for π field having low k



- **Phase appears only from the log(det) term,**

$$\log\left[\underbrace{X_N(\boldsymbol{x})^3 - 2X_N(\boldsymbol{x})}_{\text{Complex}} + \underbrace{2\cosh\left(3N_\tau\mu/\gamma^2\right)}_{\text{Real}}\right]$$

Complex **Real**

→ Less phase at larger μ !

> ***While we have sign problem, it should be suppressed especially at larger μ. Let's try***

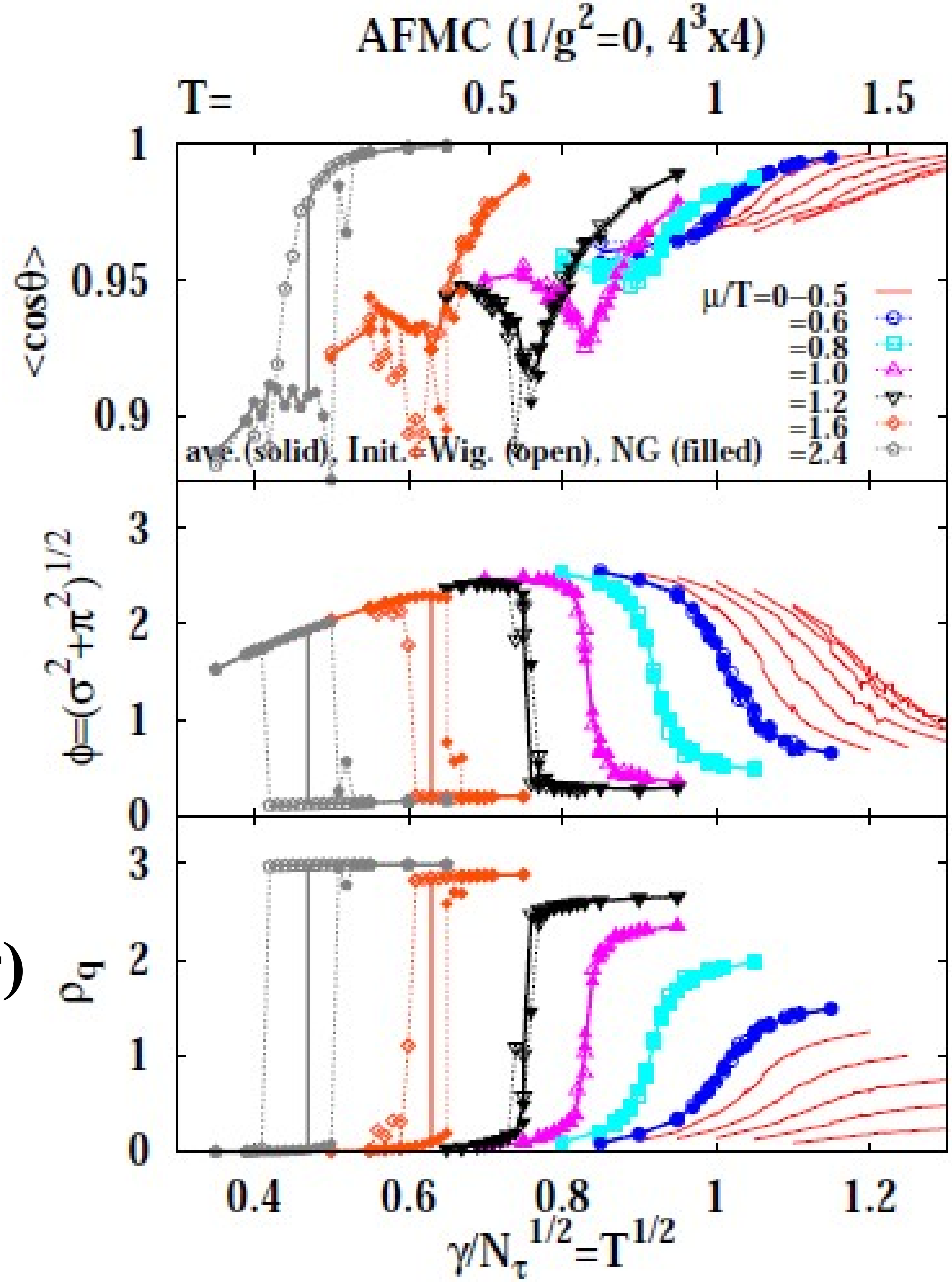
AFMC phase diagram

AFMC Simulations

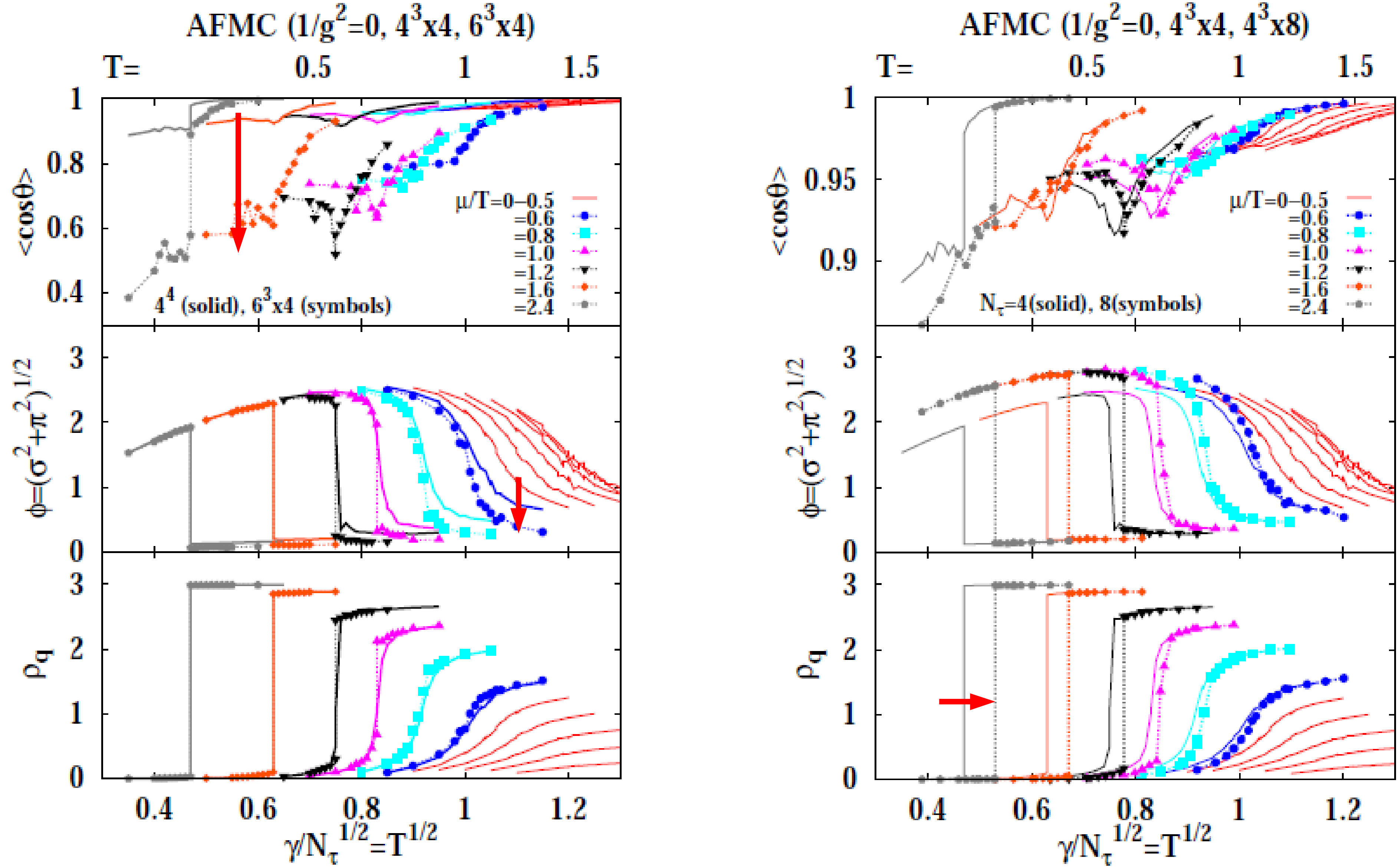
- Unrooted staggered fermion in the chiral limit (m_0=0)
- Lattice size: 4^3 x 4, 6^3 x 4, 4^3 x 8, 4^3 x 12
- Fixed fugasity: μ/T= 0, 0.1, …, 0.5, 0.6, 0.8, 1.2, 1.6, 2.4
- Temperature assignment
  $T = \gamma^2 / N_\tau$ (rather than $T = \gamma / N_\tau$)
  Bilic, Karsch, Redlich ('92), Bilic, Demeterfi, Petersson ('92)
- MC samples : 200 k ~ 1 M sweeps
  - Dynamical var. = σ(k, τ), π(k, τ)
    Det. is evaluated from σ(x, τ), π(k, τ)
    → Generate new σ(k, τ), π(k, τ) for a given τ,
    and Metropolis sampling is carried out.
- Machine = Core i7 PC
- To do: Parallel computing, FFT, Jack knife error estimate,
  larger lattice, ….

Average Sign Factor, Chiral Condensate, Quark Density

- 4^3 x 4 results
- Average sign factor
  $\langle\cos\theta\rangle \geq 0.9$
  in 4^3 x 4 lattice.
  - $\langle\cos\theta\rangle$ becomes small in transition region.
- Chiral condensate quickly decreases around γ_c.
- Quark number denstiy quickly increases around γ_c.
- Results from "NG start" (large σ) and "Wigner start" (small σ) are different with small sampling #.



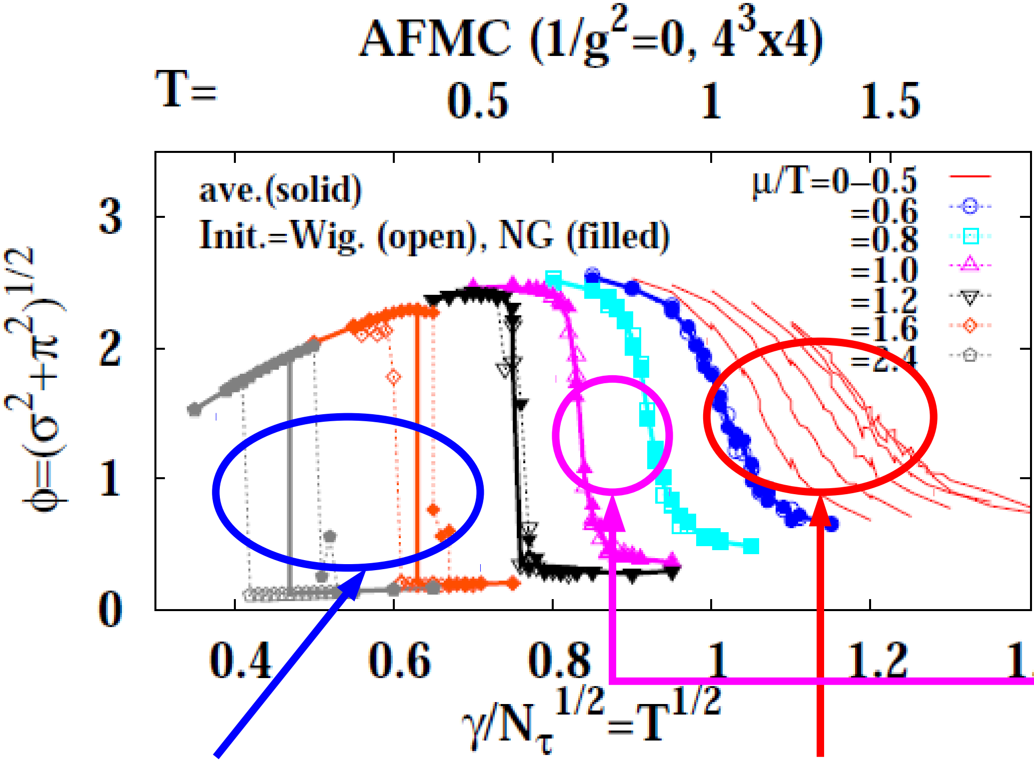
"Larger" Lattice Results

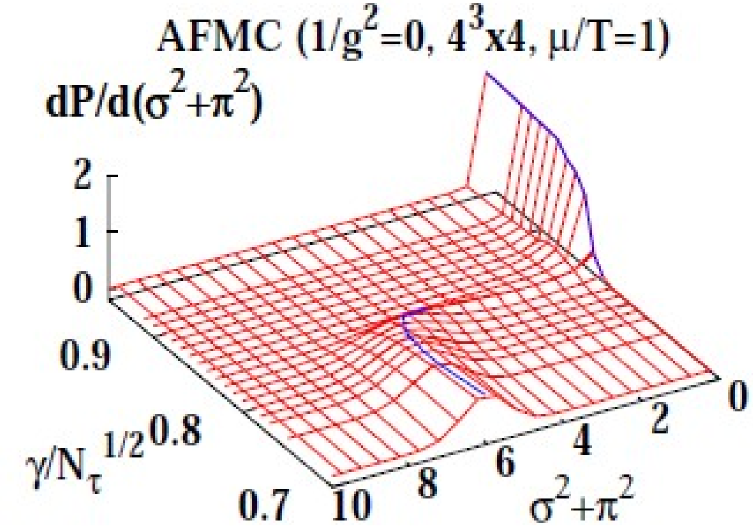


6^3 x 4: Smaller <cos θ>, Sharper trans., small fluc. in Wig. phase

4^3 x 8: Sharper trans., Larger σ, Larger T_c,

How to determine the phase boundary ?

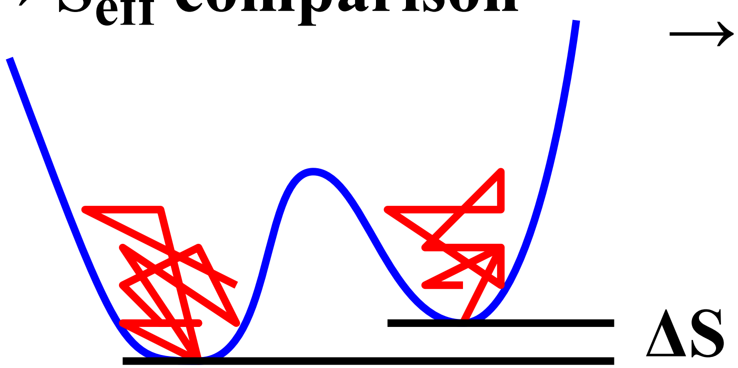


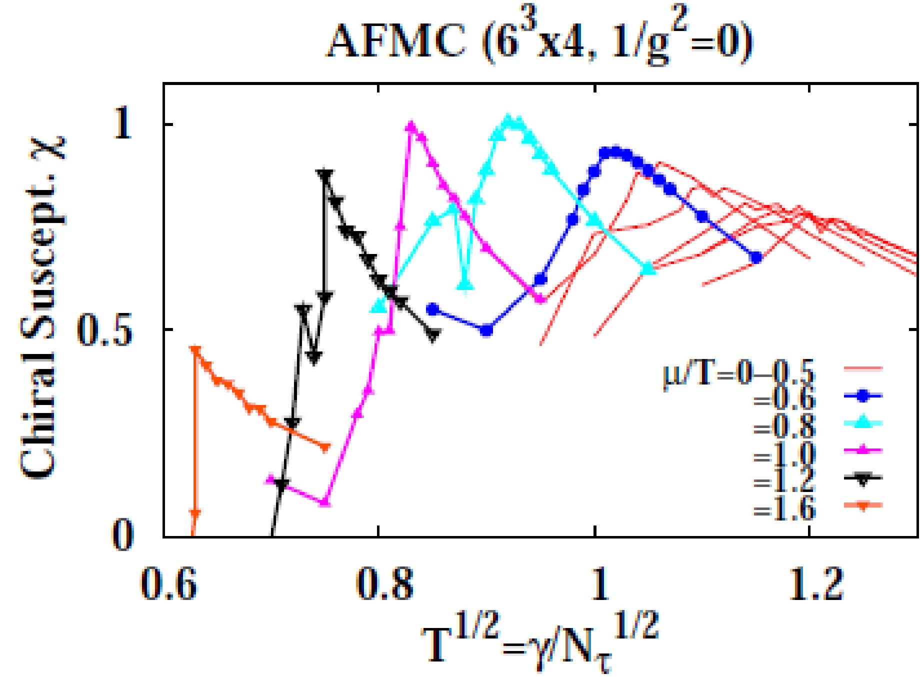


CP region
→ σ distribution

Strong 1st order
→ S_{eff} comparison

(would-be)
2nd order
→ Suscept.
peak

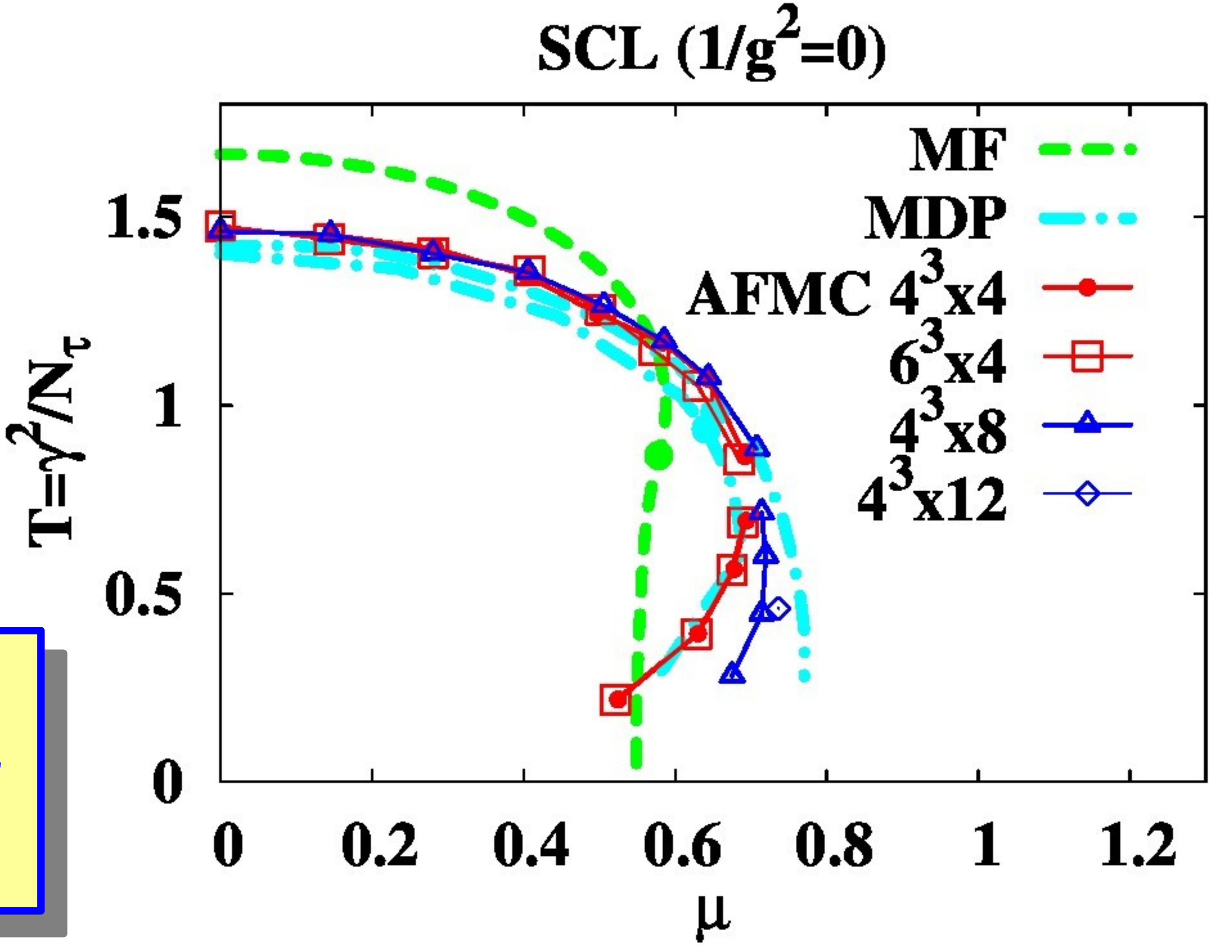




Phase Diagram

- **AFMC predicts smaller T_c (μ=0), and extended Nambu-Goldstone phase at finite μ compared with mean field results.**
- **AFMC results are almost consistent with MDP results.**
  de Focrand, Fromm ('10), de Forcrand, Unger ('11)
  - **N_τ=4 results ~ MDP (N_τ=4)**
  - **$N_\tau = \infty$ Extrapolation ~ Continuous Time MDP**

AFMC can be an alternative to discuss finite density LQCD !



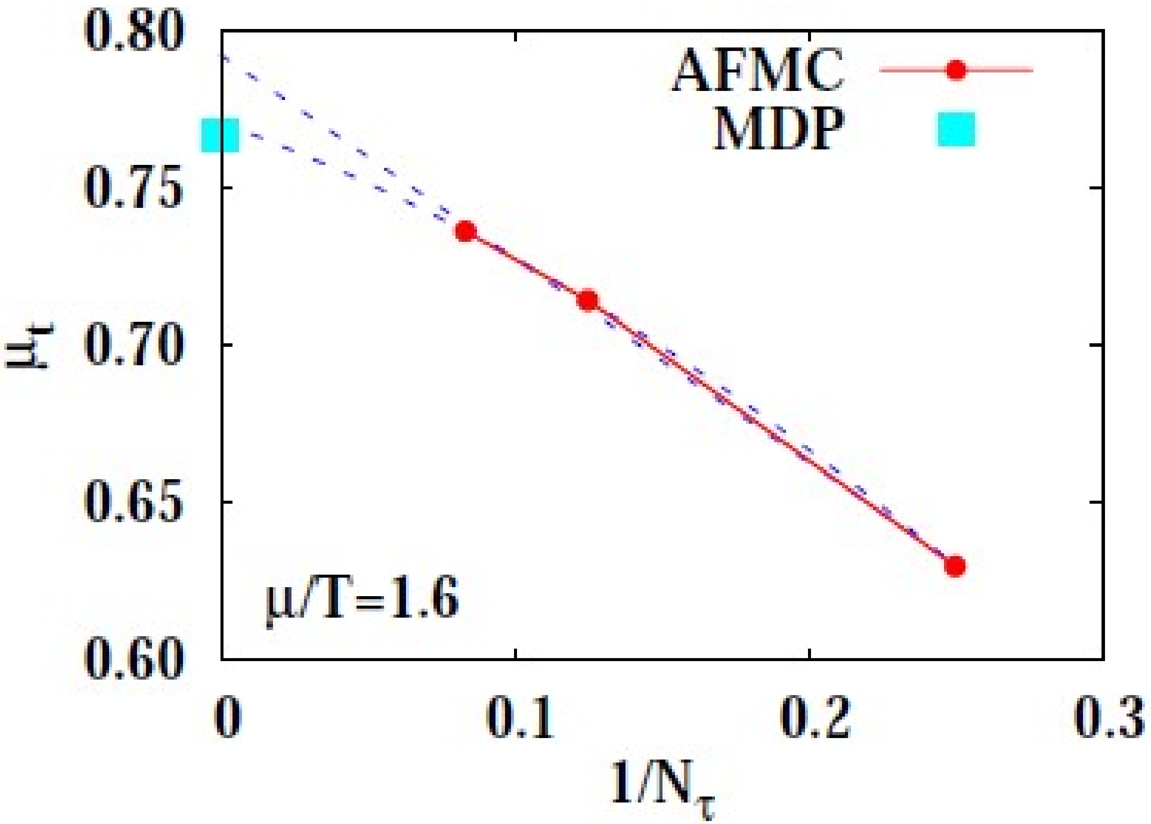
Summary

- **Strong Coupling Lattice QCD has been useful in these 40 years.**
  Misumi (Tue), Kimura (Tue), Nakano (Wed), Unger (Tue, Thu)
- **We have proposed an auxiliary field MC method (AFMC), which simulates the effective action at strong coupling exactly.**
  - **LO in strong coupling ($1/g^0$) and 1/d ($1/d^0$) expansion.**
  - **Phase boundary is moderately modified from MF results by fluctuations, if T= γ^2/N_τ scaling is adopted, as in MDP.**
  - **Sign problem is mild in small lattice (<cos θ> ~ (0.9-1) for 4^4), due to the phase cancellation coming from nearest neighbor interaction.**
  - **Sign problem is less severe at larger μ (except for transition region).**
  - **Extension to NLO SC-LQCD is straightforward.**
    Note: NLO & NNLO SC-LQCD with Polyakov loop effects reproduces MC results of Tc at μ=0.
- **To do: Larger lattice, finite coupling, other Fermion, higher 1/d terms including baryonic action, chiral Polyakov coupling.**

Thank you

Extrapolation to $N_\tau = \infty$ (Continuous Time)

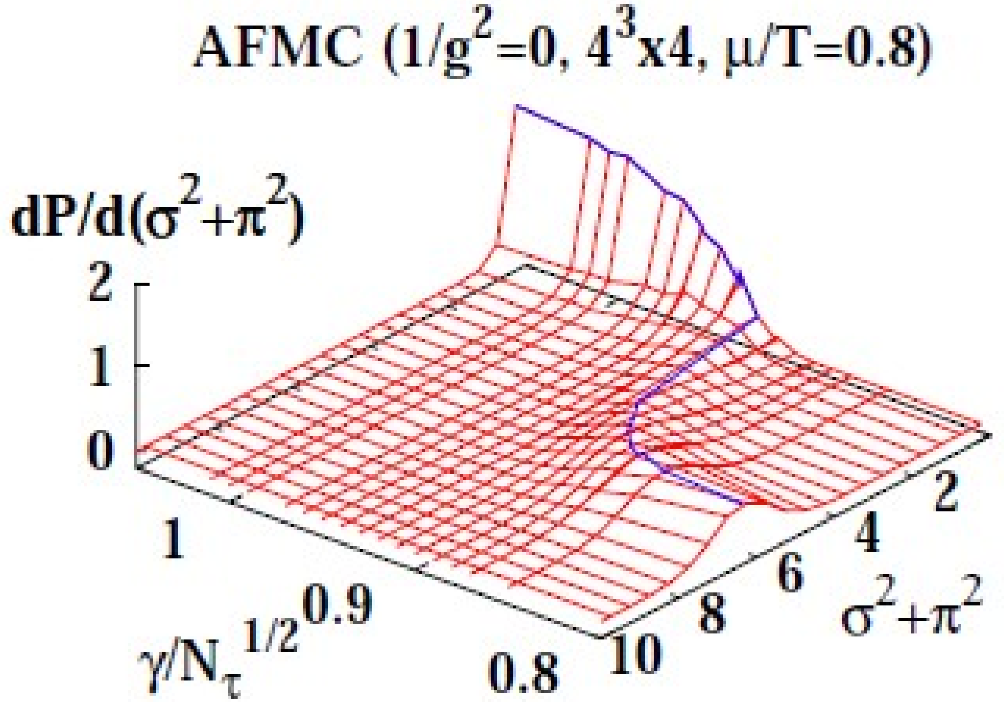
- **Extrapolation of N_τ=4, 8, 12 AFMC results to ∞ agrees with Continuous time MDP results.**
  de Forcrand, Unger ('11)
  → CT-MDP result is confirmed.

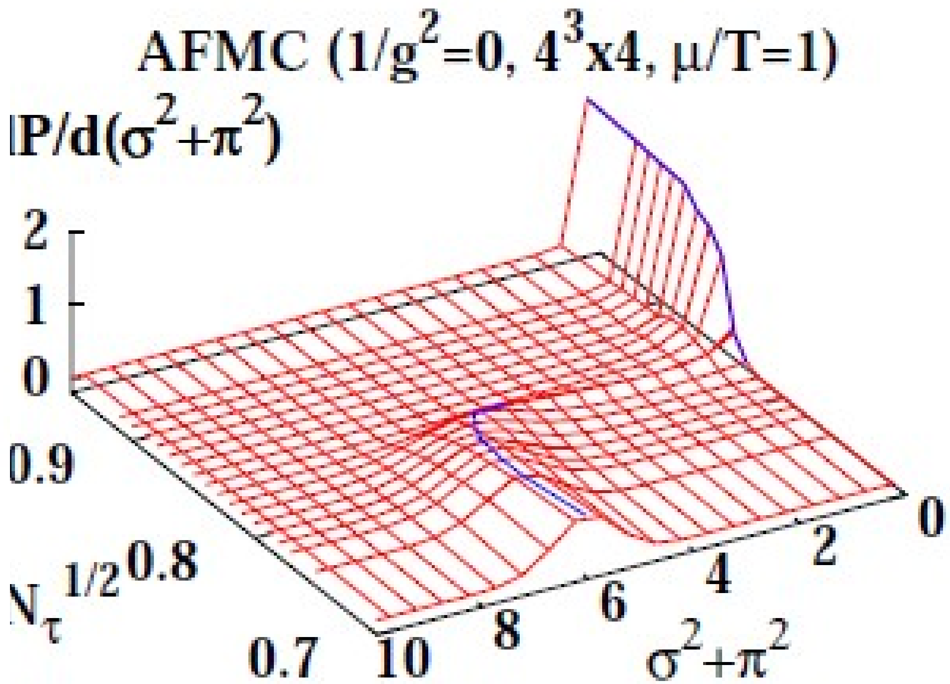


Second or First Order ?

- **Probability distribution in = σ2 + π2**
  → Hint to distinguish 2nd (one peak) and 1st order (two peak) transition

- **AFMC → CP is suggested in the region 0.8 < μ/T < 1.0**
  MDP → CP is around μ/T ~ 0.7





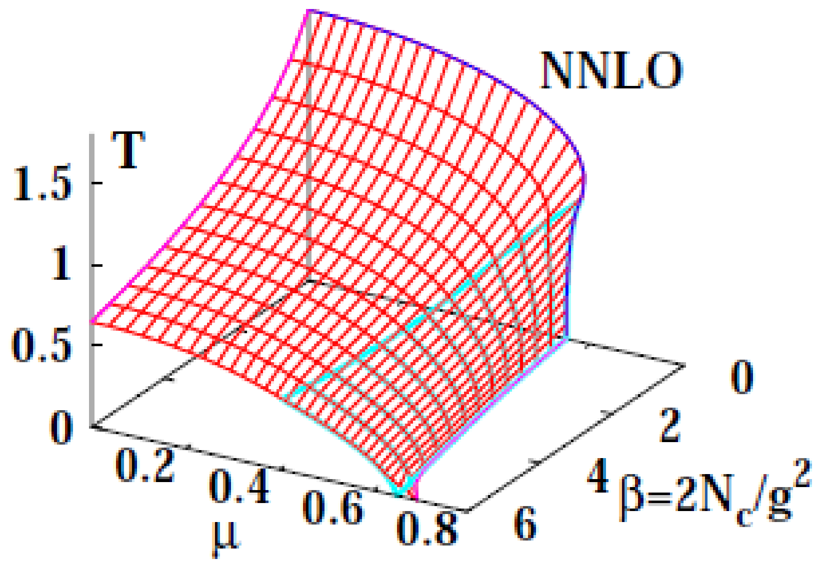
Clausius-Clapeyron Relation

- **First order phase boundary → two phases coexist**

$$P_h = P_q \rightarrow dP_h = dP_q \rightarrow \frac{d\mu}{dT} = -\frac{s_q - s_h}{\rho_q - \rho_h}$$

$$dP_h = \rho_h d\mu + s_h dT, \quad dP_q = \rho_q d\mu + s_q dT$$

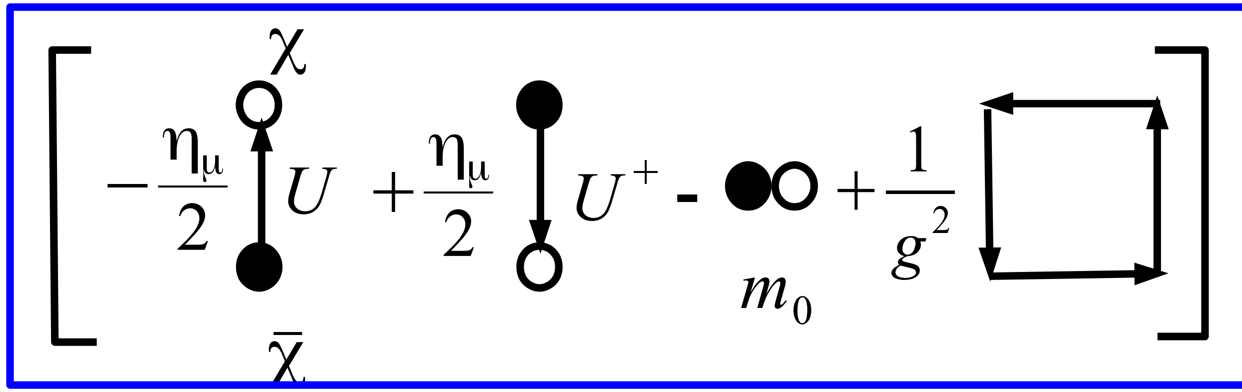
- **Continuum theory**
  → Quark matter has larger entropy and density (dμ /dT < 0)
- **Strong coupling lattice**
  - **SCL: Quark density is larger than half-filling, and "Quark hole" carries entropy → dμ/dT > 0**
  - **NLO, NNLO → dμ/dT < 0**



AO, Miura, Nakano, Kawamoto ('09)

- **Effective Action & Effective Potential (free energy density)**

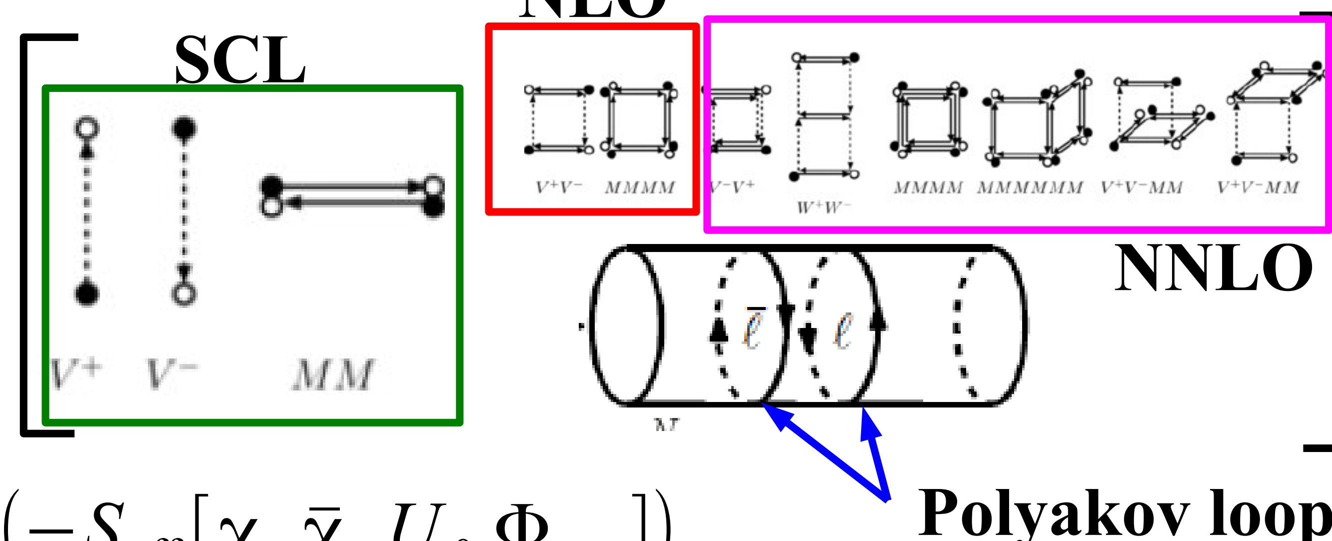
$$Z = \int D[\chi, \bar{\chi}, U_{0,} U_j] \exp \; [\,-S_{LQCD}\,]$$



Spatial link integral

$$\int DU\, U_{ab} U^{+}_{cd} = \delta_{ad}\delta_{bc}/N$$

$$= \int D[\chi, \bar{\chi}, U_0] \exp$$



Bosonization & MF Approx.

$$\approx \int D[\chi, \bar{\chi}, U_0] \exp\left(-S_{\rm eff}[\chi, \bar{\chi}, U_{0,} \Phi_{\rm stat.}]\right)$$

Fermion + U_0 integral

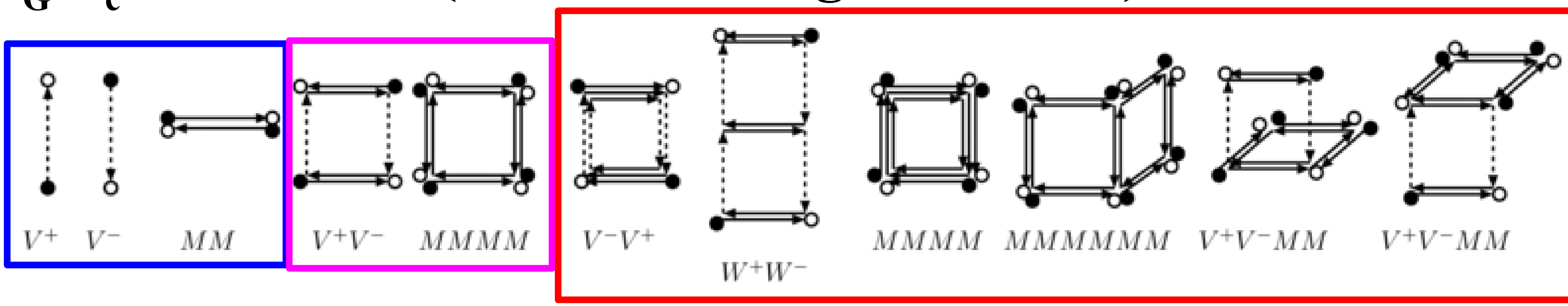
$$\approx \exp\left(-V\, F_{\rm eff}(\Phi_{\rm stat}; T, \mu)/T\right)$$

SC-LQCD with Fermions

- **Effective Action with finite coupling corrections**
  Integral of exp(-S_G) over spatial links with exp(-S_F) weight → S_{eff}

$$S_{\rm eff} = S_{\rm SCL} - \log\langle \exp(-S_G)\rangle = S_{\rm SCL} - \sum_{n=1} \frac{(-1)^n}{n!}\langle S_G^n\rangle_c$$

$\langle S_G^n\rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\rm eff} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$ *SCL (Kawamoto-Smit, '81)*

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$ *NLO (Faldt-Petersson, '86)*

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\ |k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$ *NNLO (Nakano, Miura, AO, '09)*

SC-LQCD Eff. Pot. with Fermions & Polyakov loop

- Effective potential [free energy density, NLO + LO(Pol. loop)]

$$\mathcal{F}_{\rm eff}(\Phi;T,\mu) \equiv -(T\log \mathcal{Z}_{\rm LQCD})/N_s^d = \mathcal{F}_{\rm eff}^{\chi} + \mathcal{F}_{\rm eff}^{\rm Pol}$$

$$\mathcal{F}_{\rm eff}^{\chi} \simeq \left(\frac{d}{4N_c} + \beta_s\varphi_s\right)\sigma^2 + \frac{\beta_s\varphi_s^2}{2} + \frac{\beta_\tau}{2}(\varphi_\tau^2 - \omega_\tau^2) - N_c \log Z_\chi$$
$$- N_c E_q - T(\log \mathcal{R}_q(T,\mu) + \log \mathcal{R}_{\bar{q}}(T,\mu))$$

aux. fields
w.f. ren.
zero point E.
thermal

$$\mathcal{R}_q(T,\mu) \equiv 1 + e^{-N_c(E_q-\tilde{\mu})/T} + N_c\left(L_{p,{\rm x}} e^{-(E_q-\tilde{\mu})/T} + \bar{L}_{p,{\rm x}} e^{-2(E_q-\tilde{\mu})/T}\right)$$

$$\mathcal{F}_{\rm eff}^{\rm Pol} \simeq -2T\,dN_c^2\left(\frac{1}{g^2N_c}\right)^{1/T}\bar{\ell}_p\ell_p - T\log \mathcal{M}_{\rm Haar}(\ell_p,\bar{\ell}_p)$$

quad. coef.
Haar measure

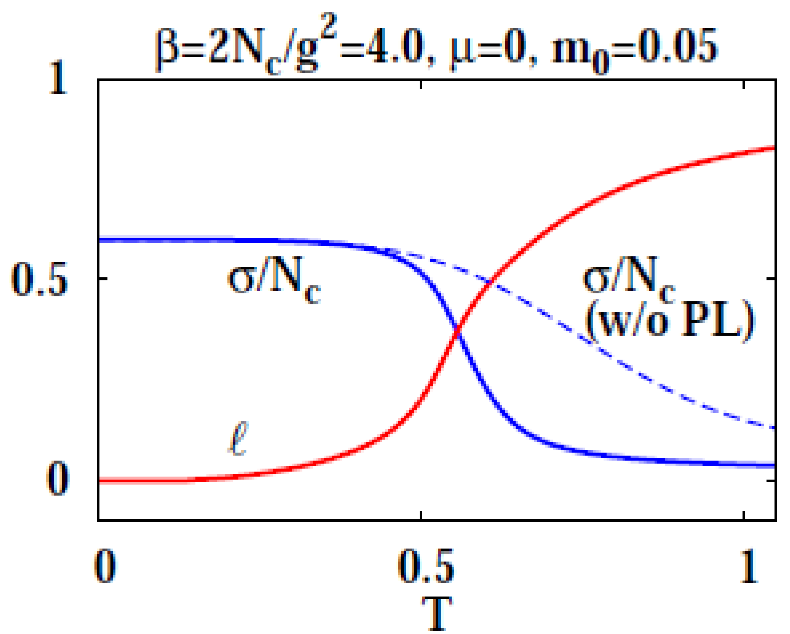
- **Strong coupling lattice QCD with Polyakov loop (P-SC-LQCD)**
  = Polyakov loop extended Nambu-Jona-Lasino (PNJL) model
  (Haar measure method, quadratic term fixed)
  + higher order terms in aux. fields
  - quark momentum integral

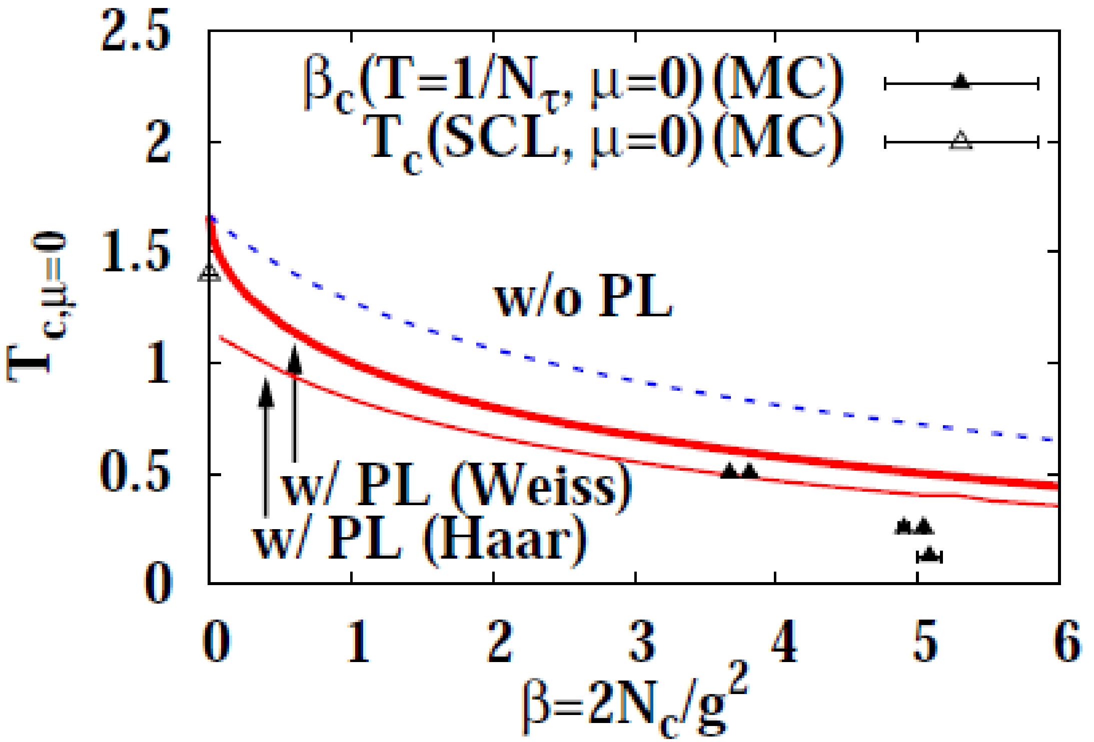
P-SC-LQCD at μ=0

T. Z. Nakano, K. Miura, AO, PRD 83 (2011), 016014 [arXiv:1009.1518 [hep-lat]]

- **P-SC-LQCD reproduces $T_c(\mu=0)$ in the strong coupling region ($\beta = 2N_c/g^2 \le 4$)**

  MC data: SCL (Karsch et al. (MDP), de Forcrand, Fromm (MDP)), $N_\tau=2$ (de Forcrand, private), $N_\tau=4$ (Gottlieb et al.('87), Fodor-Katz ('02)), $N_\tau=8$ (Gavai et al.('90))





Lattice Unit

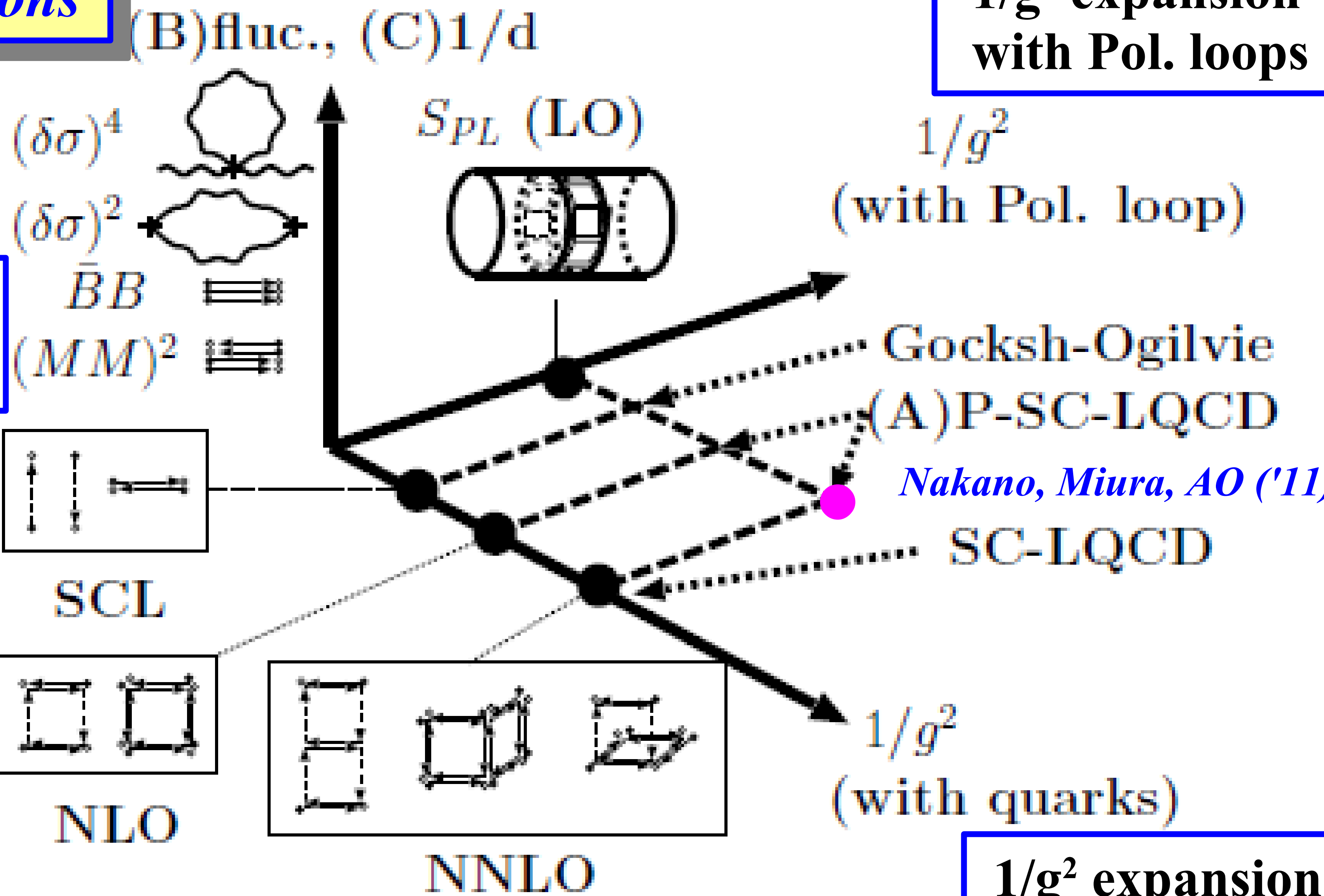
Approximations in Pol. loop extended SC-LQCD

Fluctuations

1/g² expansion with Pol. loops

1/d expansion

d=spatial dim.



1/g² expansion with quarks

Approximations in Pol. loop extended SC-LQCD

- **Strong coupling expansion**
  - **Fermion terms: LO($1/g^0$, SCL), NLO($1/g^2$), NNLO ($1/g^4$)**
  - **Plaquette action: LO ($1/g^{2N\tau}$)**
- **Large dimensional approximation**
  - **1/d expansion (d=spatial dim.)**
    → Smaller quark # configs. are preferred.
    $\Sigma_j M_x M_{x+j} = O(1/d^0) \rightarrow M \propto d^{-1/2} \rightarrow \chi \propto d^{-1/4}$
  - **Only LO ($1/d^0$) terms are mainly evaluated.**
- **(Unrooted) staggered Fermion**
  - **Nf=4 in the continuum limit.**
- **Mean field approximation**
  - **Auxiliary fields are assumed to be constant.**

Introduction of Auxiliary Fields

$$
\begin{aligned}
S^{(s)} &= -\frac{1}{4N_c\gamma^2}\sum_{x,j} M_x M_{x+\hat{j}} = -\frac{L^3}{4N_c\gamma^2}\sum_{\tau,\mathbf{k}} f_M(\mathbf{k})\,\tilde{M}_{\mathbf{k}}(\tau)\,\tilde{M}_{-\mathbf{k}}(\tau) \\
&= \frac{L^3}{4N_c\gamma^2}\sum_{\tau,\mathbf{k},f_M(\mathbf{k})>0} f_M(\mathbf{k})\Big[\varphi_{\mathbf{k}}(\tau)^2 + \phi_{\mathbf{k}}(\tau)^2 + \varphi_{\mathbf{k}}(\tilde{M}_{\mathbf{k}} + \tilde{M}_{-\mathbf{k}}) - i\phi_{\mathbf{k}}(\tilde{M}_{\mathbf{k}} - \tilde{M}_{-\mathbf{k}}) \\
&\qquad + \varphi_{\bar{\mathbf{k}}}(\tau)^2 + \phi_{\bar{\mathbf{k}}}(\tau)^2 + i\varphi_{\bar{\mathbf{k}}}(\tilde{M}_{\bar{\mathbf{k}}} + \tilde{M}_{-\bar{\mathbf{k}}}) + \phi_{\bar{\mathbf{k}}}(\tilde{M}_{\bar{\mathbf{k}}} - \tilde{M}_{-\bar{\mathbf{k}}})\Big] \\
&= \frac{\Omega}{2N_c\gamma^2}\sum_{k,f_M(\mathbf{k})>0} f_M(\mathbf{k})\,[\sigma_k^*\sigma_k + \pi_k^*\pi_k] + \frac{1}{2N_c\gamma^2}\sum_x M_x\,[\sigma(x) + i\varepsilon(x)\pi(x)]
\end{aligned}
$$

$$\Omega = L^3 N_\tau$$

$$\sigma(x) = \sum_{k,f_M(\mathbf{k})>0} f_M(\mathbf{k}) e^{ikx}\sigma_k\ ,\quad \pi(x) = \sum_{k,f_M(\mathbf{k})>0} f_M(\mathbf{k}) e^{ikx}\pi_k$$

$$\sigma_k = \varphi_k + i\phi_k\ ,\ \pi_k = \varphi_{\bar{k}} + i\phi_{\bar{k}}$$

$$V_{x,y} = \frac{1}{2}\sum_j\left(\delta_{x+\hat{j},y} + \delta_{x-\hat{j},y}\right)\ ,\quad f_M(\mathbf{k}) = \sum_j \cos k_j\ ,\quad \bar{\mathbf{k}} = \mathbf{k} + (\pi,\pi,\pi)$$

Fermion Determinant

Faldt, Petersson, 1986

- **Fermion action is separated to each spatial point and bi-linear → Determinant of Nτ x Nc matrix**

$$\exp(-V_{\rm eff}/T)=\int dU_0 \begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^{+} \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}$$

Nc x Nτ

$$=\int dU_0 \det\left[X_N[\sigma]\otimes \mathbf{1}_c + e^{-\mu/T}U^{+} + (-1)^{N_\tau} e^{\mu/T} U\right]$$

Nc

$$=X_N^3 - 2X_N + 2\cosh(3N_\tau \mu)$$

$$I_\tau/2=[\sigma(x)+i\varepsilon(x)\pi(x)]/2N_c\gamma^2+m_0/\gamma$$

$$X_N=B_N+B_{N-2}(2;N-1)$$

$$B_N=I_N B_{N-1}+B_{N-2}$$

Constant I : $X_N=2\cosh(\text{arcsinh}(I/2)/T)$

$$B_N=\begin{vmatrix} I_1 & e^{\mu} & 0 & & \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ & & & -e^{-\mu} & I_N \end{vmatrix}$$

Results (2): Susceptibility and Quark density

- **Weight factor <cos θ>**

$$\langle \cos\theta \rangle = Z / Z_{\rm abs}$$

$$Z = \int D\sigma_k D\pi_k \exp(-S_{\rm eff}) = \int D\sigma_k D\pi_k \exp(-{\rm Re}\, S_{\rm eff}) e^{i\theta}$$

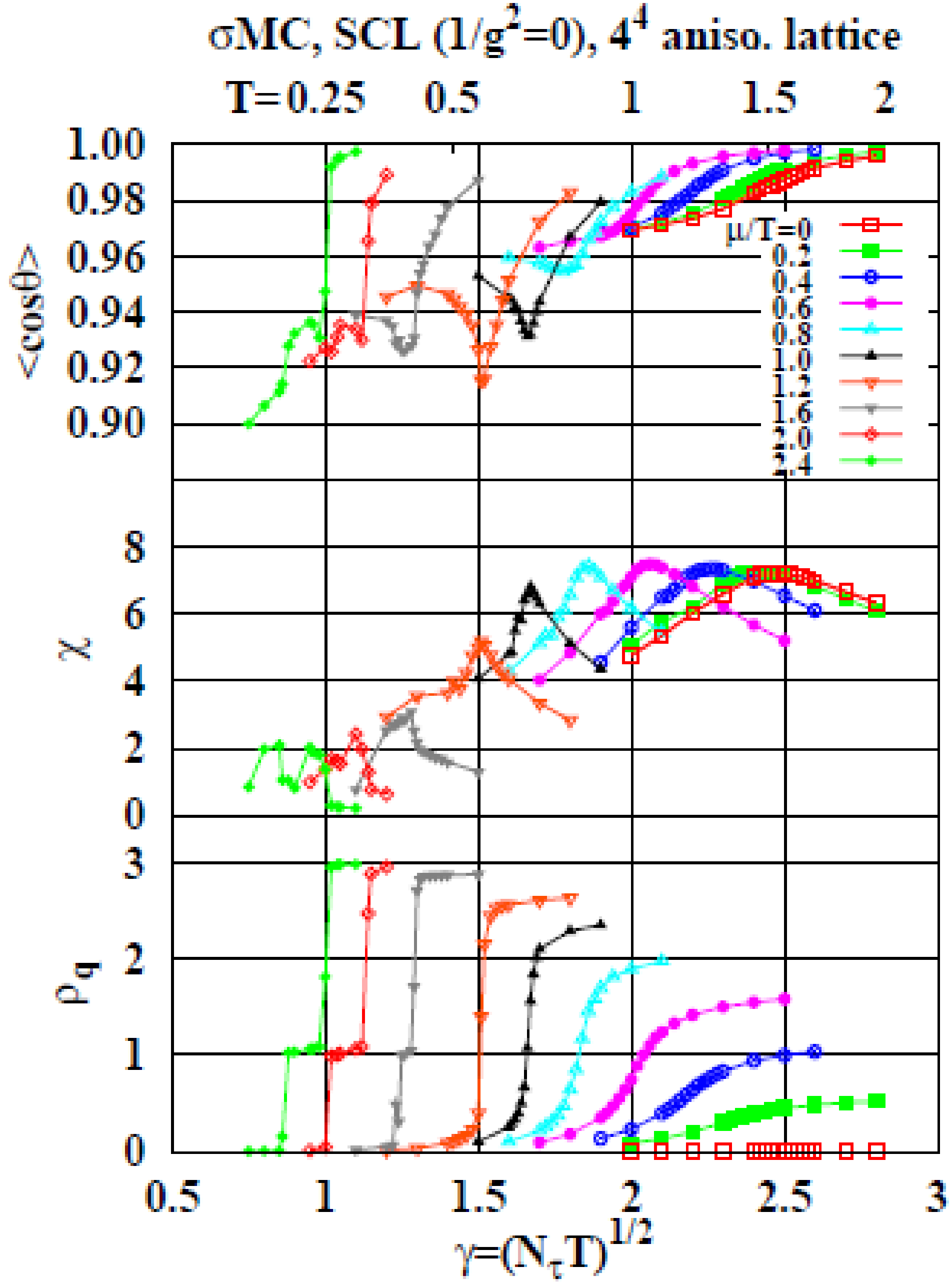
$$Z_{\rm abs} = \int D\sigma_k \pi_k \exp\left(-{\rm Re}\, S_{\rm eff}\right)$$

- **Chiral susceptibility**

$$\chi = -\frac{1}{L^3} \frac{\partial^2 \log Z}{\partial m_0^2}$$

- **Quark number density**

$$\rho_q = -\frac{1}{L^3} \frac{\gamma^2}{N_\tau} \frac{\partial \log Z}{\partial \mu}$$

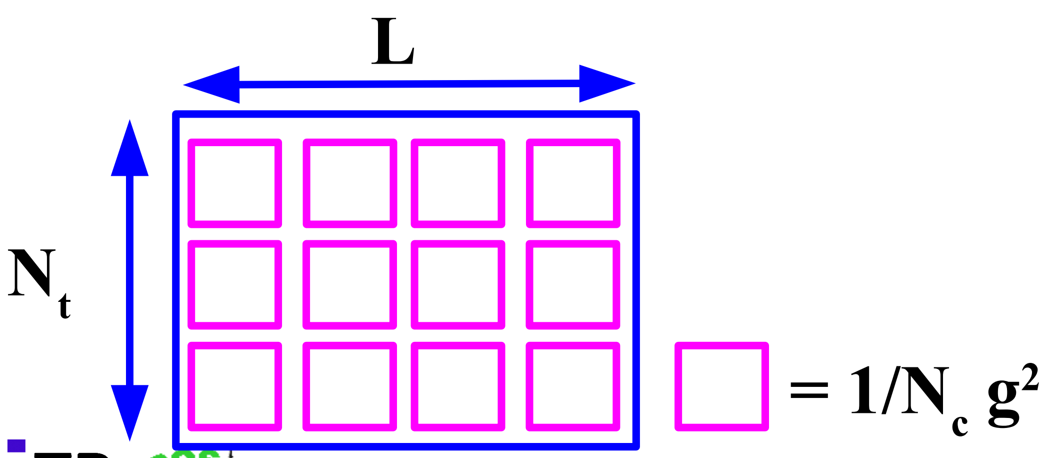


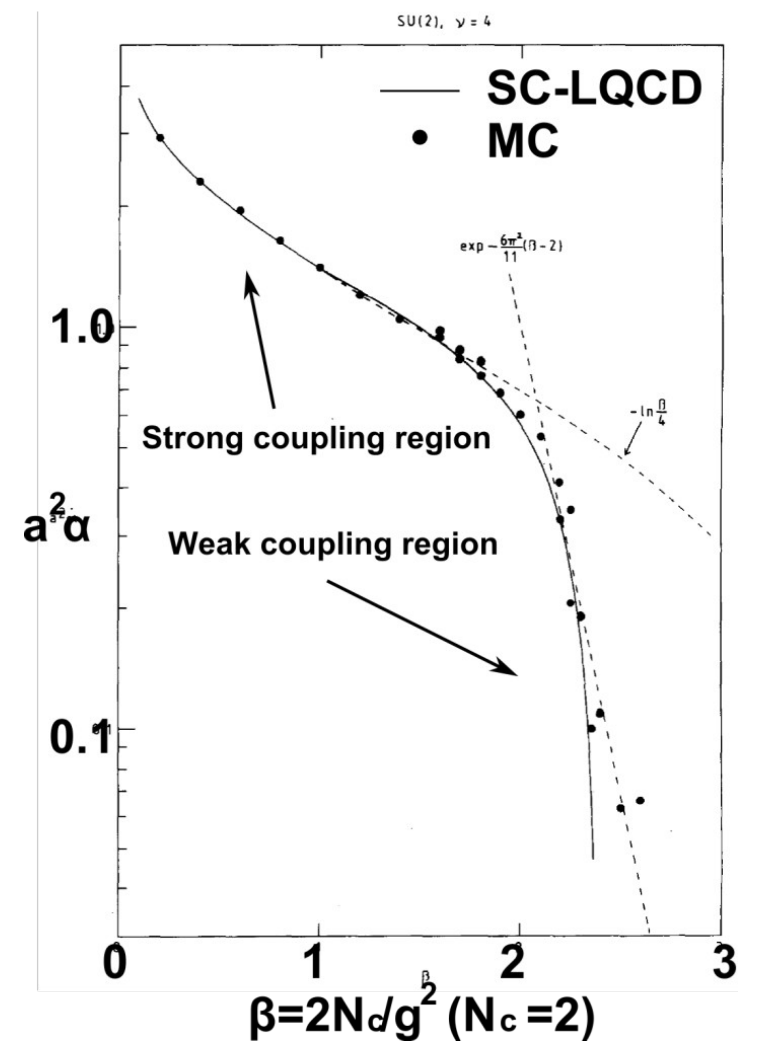
K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, (1980, 1981)

- **Quarks are confined in Strong Coupling QCD**
  - **Strong Coupling Limit (SCL)**
    → Fill Wilson Loop with Min. # of Plaquettes
    → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\Box} {\rm tr}\left[U_\Box + U_\Box^\dagger\right]$$

  - **Smooth Transition from SCL to pQCD in MC (Creutz, 1980; Munster 1980)**

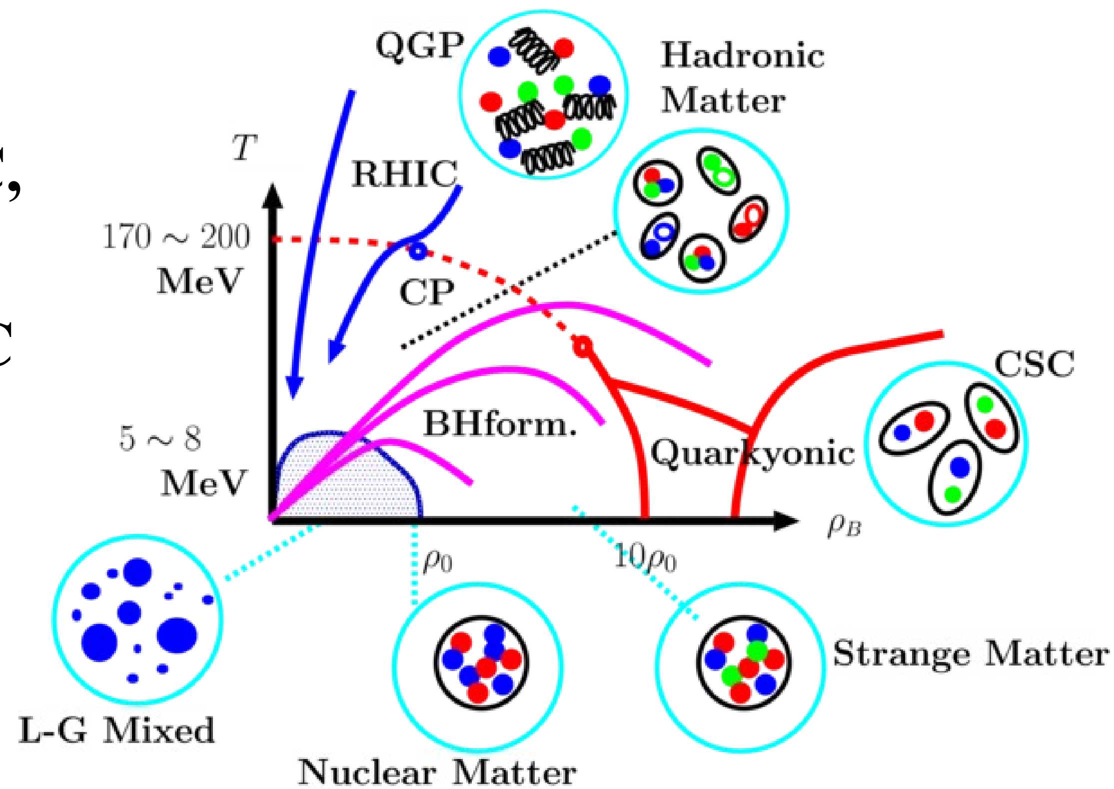




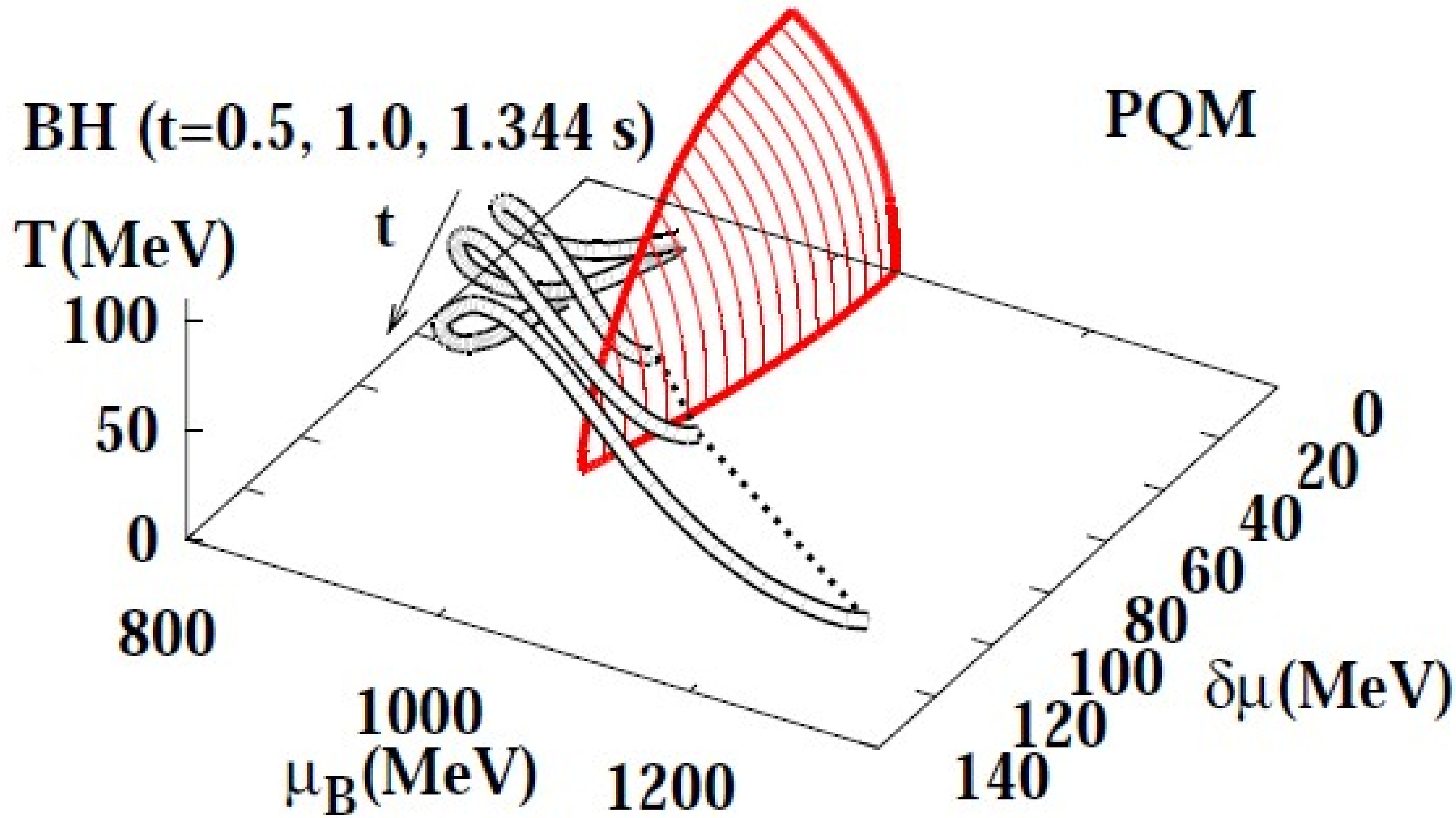
Munster, '80

QCD Phase diagram

- Phase transition at high *T*
  - Physics of early universe: Where do we come from ?
  - RHIC, LHC, Lattice MC, pQCD, ….
- High μ transition
  - Physics of neutron stars: Where do we go ?
  - RHIC-BES, FAIR, J-PARC, Astro-H, Grav. Wave, …
  - Sign problem in Lattice MC → Model studies, Approximations, Functional RG, ...



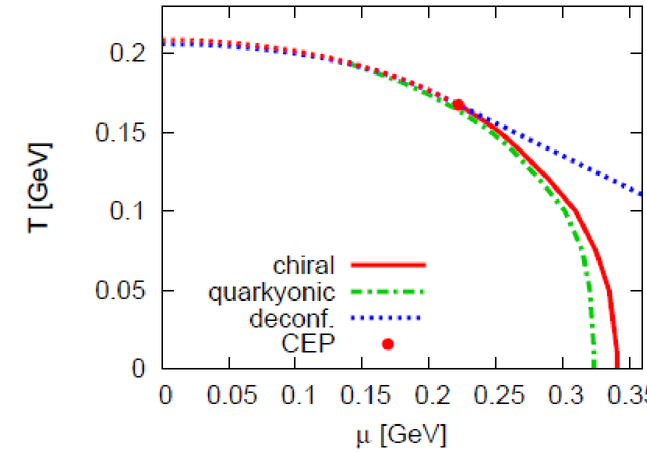
CP sweep during BH formation



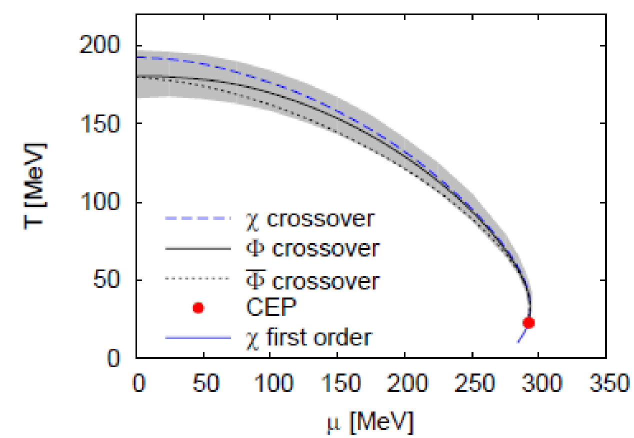
AO, H. Ueda, T. Z. Nakano, M. Ruggieri, K. Sumiyoshi, PLB, to appear [arXiv:1102.3753 [nucl-th]]

QCD based approaches to Cold Dense Matter

- **Effective Models**
  (P)NJL, (P)QM, Random Matrix, …
  E.g.: K.Fukushima, PLB 695('11)387 (PNJL+Stat.).
- **Functional (Exact, Wilsonian) RG**
  E.g.: T. K. Herbst, J. M. Pawlowski, B. J. Schaefer, PLB 696 ('11)58 (PQM-FRG).
- **Expansion / Extrapolation from μ=0**
  - **AC, Taylor expansion, … → μ/T<1**
  - **Cumulant expansion of θ dist. (S. Ejiri, ...)**
- **Strong Coupling Lattice QCD**
  - **Mean field approaches**
  - **Monomer-Dimer-Polymer (MDP) simulation**



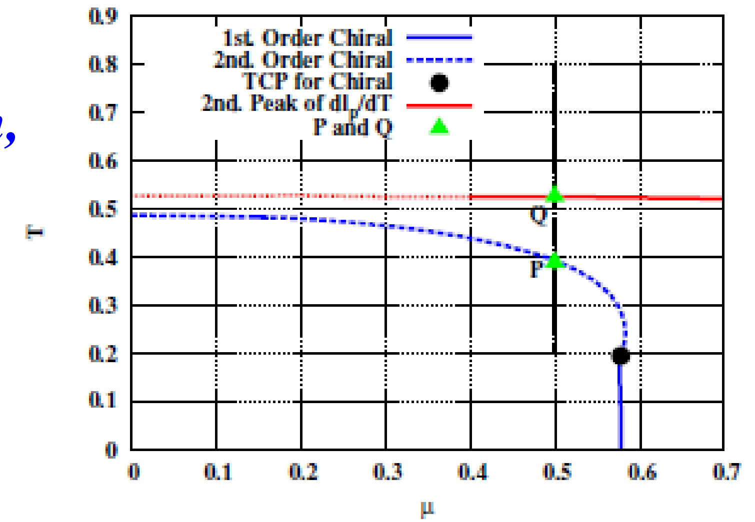
McLerran, Redlich, Sasaki ('09)



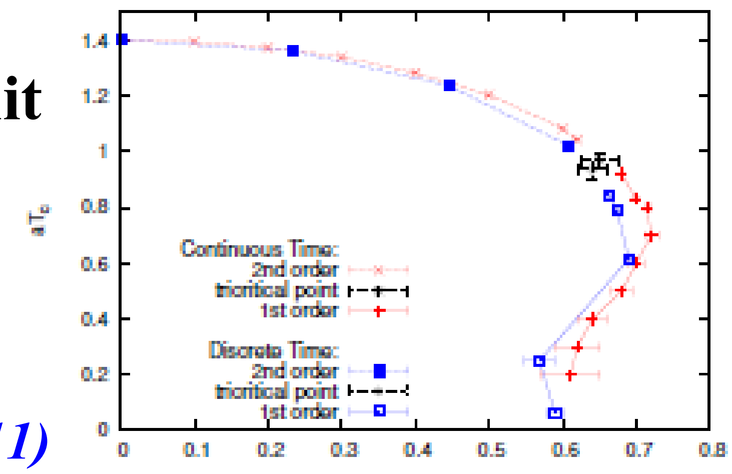
Herbst, Pawlowski, Schafer, ('11)

Strong Coupling Lattice QCD for finite μ

- **Mean Field approaches**
  Damagaard, Hochberg, Kawamoto ('85); Bilic, Karsch, Redlich ('92); Fukushima ('03); Nishida ('03); Kawamoto, Miura, AO, Ohnuma ('07).
- **MDP simulation**
  Karsch, Mutter('89); de Forcrand, Fromm ('10); de Forcrand, Unger ('11)
  - Partition function = sum of config. weights of various loops.
  - Applicable only to Strong Coupling Limit ($1/g^2=0$) at present



Miura, Nakano, AO, Kawamoto, arXiv:1106.1219



de Forcrand, Unger ('11)

Can we include both fluctuation and finite coupling effects ?
→ One of the candidates = Auxiliary field MC