SU(2) Chiral sigma model and the properties of neutron stars *)

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We discuss SU(2) chiral sigma model in the nuclear matter using mean field approach at high density. In this model we include a dynamically generated isoscalar vector field and higher order terms in scalar field. With inclusion of these, we reproduce the empirical values of the nuclear matter saturation density, binding energy and the nuclear incompressibility. The desirable value of incompressibility is chosen according to the recently estimated heavy-ion collision data. We then apply the same dynamical model to neutron-rich matter in beta equilibrium relating to neutron star structure. The maximum mass and corresponding radius of the stable non-rotating neutron stars are found to be in the observational limit.

§1. Introduction

It has been argued that the chiral symmetry is a good hadron symmetry[1] which ranks only below the isotopic spin symmetry. The spontaneous breaking of this symmetry generates the constituent quark masses and hence various hadron masses, including the nucleon mass. So, the theories should possess this symmetry in discussing the dense nuclear matter, where dynamical modifications of hadron masses are expected. In the last decades, the three-body forces in the equation of state at high density has been emphasized by several authors.[2),3)] This also supports the importance to study the chiral sigma model, because the non-linear terms in the chiral sigma Lagrangian can give rise to the three-body forces. The theories of dense nuclear matter should be capable of describing the bulk properties of nuclear matter, such as binding energy per nucleon, saturation density, compression modulus/incompressibility and symmetry energy as well. So far, the theory in dense nuclear matter was not available for long time which describes all the nuclear matter properties and possess chiral symmetry.

A chiral Lagrangian using scalar (sigma) field was originally introduced by Gell-Mann and Levy[4] and later the importance of chiral symmetry in the nuclear matter was emphasized by Lee and Wick.[5] The usual theory of pions does not possess the empirically desirable saturation properties in the nuclear matter. For this reason, the isoscalar vector field with the dynamically generated mass is introduced in the theory of nuclear matter and possible to have a saturating nuclear matter equation of state.[6] In the standard sigma model, the value of incompressibility parameter of nuclear matter turns out to be quite large, several times the desirable value and can be reduced only by introducing the scalar field self-interactions with adjustable coefficients. There have been several earlier papers[7),8)] made attempt to derive the chiral

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sigma model equation of state at high density matter with normal nuclear matter saturation as well as desirable incompressibility value. However, in these theories, the mass of isoscalar vector field is not generated dynamically. This treatment can be considered as a shortcoming in the chirally symmetry model.

Few years ago, we have proposed the SU(2) chiral sigma model[9)] to describe the properties of nuclear matter, where we have adopted an approach in which the mass of isoscalar vector field is generated dynamically. To ensure the saturation properties of nuclear matter, inclusion of such a vector field is necessary. The nucleon effective mass thus acquires a self-consistent density dependence both on the scalar and the vector meson fields, and these meson fields are treated in the mean-field theory. To describe the nuclear matter properties, we have two parameters in the theory: the ratio of the coupling constants to the scalar and to the isoscalar vector fields. In this procedure the value of incompressibility at saturation density gives relatively high, which is an undesirable feature as far as nuclear matter at saturation and higher densities are concerned. In the present calculation, we rectify the above shortcoming at nuclear matter saturation density by including the higher order terms of scalar field potential in our proposed chiral sigma Lagrangian.[9)] In this way, we get extra two parameters in the mean-field approach, these are fitted with the nuclear matter properties at the saturation density.

The paper is organized as follows: In section 2 we briefly describe the proposed SU(2) chiral sigma model and the derivation of equation of state in the nuclear matter density. Section 3 contains the neutron star matter equation of state in beta equilibrium with the neutron star results. The conclusion and summary are presented in section 4.

§2. The SU(2) chiral sigma model and equation of state

The Lagrangian for an SU(2) chiral sigma model can be written as follows (we choose $\hbar = c = 1$),

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\big(\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu \boldsymbol{\pi} + \partial_\mu\sigma\partial^\mu\sigma\big) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \\ &- \frac{\lambda}{4}\big(x^2 - x_o^2\big)^2 - \frac{\lambda B}{6m^2}\big(x^2 - x_o^2\big)^3 - \frac{\lambda C}{8m^4}\big(x^2 - x_o^2\big)^4 \\ &- g_\sigma\bar{\psi}\big(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\big)\psi + \bar{\psi}\big(i\gamma_\mu\partial^\mu - g_\omega\gamma_\mu\omega^\mu\big)\psi \\ &+ \frac{1}{2}{g_\omega}^2 x^2\omega_\mu\omega^\mu + \frac{1}{24}\xi {g_w}^4(\omega_\mu\omega^\mu)^2 \end{aligned} \tag{1}$$

where $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $x^2 = \boldsymbol{\pi}^2 + \sigma^2$, ψ is the nucleon isospin doublet, $\boldsymbol{\pi}$ is the pseudoscalar-isovector pion field and σ is the scalar field. The Lagrangian includes dynamically an isoscalar vector field (ω_μ) that couples to the conserved baryonic current $j_\mu = \bar{\psi}\gamma_\mu\psi$. B and C are the constant parameters included the higher order self-interaction of the scalar field in the potential. The last term in the Lagrangian is the fourth order of omega fields and ξ is the constant parameter. For simplicity, we put ξ to be zero in our calculation.

The interactions of the scalar and the pseudoscalar mesons with vector generate a mass for the latter through the spontaneous breaking of the chiral symmetry . The masses for the nucleon, the scalar meson and the vector meson are respectively represented by

$$M = g_\sigma x_o, \quad m_\sigma = \sqrt{2\lambda} x_o, \quad m_\omega = g_\omega x_o \ , \tag{2}$$

where x_o is the vacuum expectation value of the σ field, $\lambda = ({m_\sigma}^2 - {m_\pi}^2)/(2{f_\pi}^2)$, here m_π is pion mass and f_π is the pion decay coupling constant and g_ω and g_σ are the coupling constants for the vector and scalar fields respectively. Throughout this paper, we limit ourselves in the mean field treatment and ignore the explicit role of π mesons. Therefore, the explicit breaking term of the chiral symmetry is omitted in the Lagrangian.

The equation of motion of fields are obtained by adopting the mean-field approximation. This approach has been extensively used to obtain field theoretical equation of state models for high density matter. Using mean-field ansatz, the equation of motion for isoscalar vector field is:

$$\omega_0 = \frac{n_B}{g_\omega x^2} \tag{3}$$

and the equation of motion for scalar field in terms of $y \equiv x/x_o$ is:

$$\begin{aligned} (1-y^2) - \frac{B}{m^2 c_\omega}(1-y^2)^2 + \frac{C}{m^4 c_\omega^2}(1-y^2)^3 \\ + \frac{2 c_\sigma c_\omega n_B^2}{M^2 y^4} - \frac{c_\sigma \gamma}{\pi^2} \int_o^{k_F} \frac{k^2 dk}{\sqrt{k^2 + M^{\star 2}}} = 0 \end{aligned} \tag{4}$$

where $M^\star \equiv yM$ is the effective mass of the nucleon and

$$c_\sigma \equiv g_\sigma^2/m_\sigma^2, \qquad c_\omega \equiv g_\omega^2/m_\omega^2. \tag{5}$$

The symbol n_B is the nucleon number density or equivalently the Fermi momentum $k_F = (6\pi^2 n_B/\gamma)^{1/3}$ (γ = nucleon spin degeneracy factor).

The equation of state is calculated from the diagonal components of the conserved total stress tensor corresponding to the Lagrangian together with the mean-field equation of motions for the fermion field and a mean-field approximation for the meson fields. These are the total energy density (ε) and pressure (P) of the many-nucleon system:

$$\begin{aligned} \varepsilon = & \frac{M^2(1-y^2)^2}{8c_\sigma} - \frac{B}{12 c_\omega c_\sigma}(1-y^2)^3 + \frac{C}{16 m^2 c_\omega^2 c_\sigma}(1-y^2)^4 \\ & + \frac{c_\omega n_B^2}{2y^2} + \frac{\gamma}{2\pi^2} \int_o^{k_F} k^2 dk \sqrt{k^2 + M^{\star 2}} \\ P = & -\frac{M^2(1-y^2)^2}{8c_\sigma} + \frac{B}{12 c_\omega c_\sigma}(1-y^2)^3 - \frac{C}{16 m^2 c_\omega^2 c_\sigma}(1-y^2)^4 \\ & + \frac{c_\omega n_B^2}{2y^2} + \frac{\gamma}{6\pi^2} \int_o^{k_F} \frac{k^4 dk}{\sqrt{k^2 + M^{\star 2}}}. \end{aligned} \tag{6}$$

The energy per nucleon is $E/A = \varepsilon/n_B$, where $\gamma = 4$ for symmetric nuclear matter.

In the above equations, we have four parameters: the nucleon coupling to scalar c_σ, vector c_ω and the coefficients in scalar potential terms i.e. B and C. These are obtained by fitting the saturation values of binding energy/nucleon (−16.3 MeV), baryon density (0.153 fm^{-3}), effective (Landau) mass ($0.85M$).[10] The nuclear incompressibility is somewhat uncertain at saturation and therefore we choose it between 250 MeV −350 MeV, i.e. ∼ 300 MeV according to recent inference from the heavy-ion collision data.[11),12)] These values are $c_\omega = 1.9989\text{fm}^2$, $c_\sigma = 6.8158\text{fm}^2$, $B = -100.$ and $C = -133.6$.

In Fig. 1, we present the energy/nucleon vs the baryon density. The solid line (MCH) correspond to modified chiral sigma model presented above, where the nuclear incompressibility is around 300 MeV. This equation of state is much softer than the original chiral sigma model (CH) which is represented by the dotted line. This is due to additional terms B and C in the modified chiral sigma model in the potential term in equation (1). For comparison purpose, we display the dashed line (NL3), which has been derived from the recent heavy-ion collision data[13] with incompressibility around 340 MeV . We notice from Fig. 1 that MCH model is softer than NL3 model at high density, this is due to the slight difference in the incompressibility.

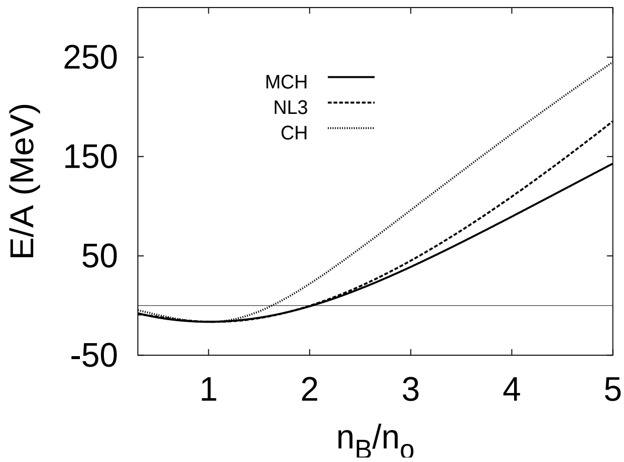

Fig. 1. Energy per nucleon vs baryon density in units of n_0. The solid line (MCH) correspond to modified chiral sigma model, where the nuclear incompressibility is around 300 MeV, the dashed line (NL3) is for the equation of state that derived from the recent heavy-ion collision data[13] with incompressibility around 340 MeV and the dotted line is for the original chiral sigma model[9] with very high incompressibility.

§3. Equation of state in neutron star matter

In the interior of neutron stars i.e. at high density, the neutron chemical potential exceeds the combined mass of proton and electron. Therefore, the asymmetric matter with an admixture of electron rather than pure neutron matter is more likely composition of matter in neutron star interiors. The concentrations of neutron, proton and electron can be determined from the condition of beta equilibrium ($n \leftrightarrow p + e + \bar{\nu}$)

and from charge neutrality assuming that neutrinos are not degenerate. These are:

$$\mu_n = \mu_p + \mu_e, \quad n_p = n_e, \tag{7}$$

where μ_i is the chemical potential of particle species i. For the purpose of describing the neutron rich matter, we include the interaction due to isospin triplet ρ meson in Lagrangian (1). The following terms are taken in the Lagrangian:

$$-\frac{1}{4}\boldsymbol{G}_{\mu\nu}\cdot\boldsymbol{G}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{2}g_\rho\bar{\psi}(\boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau}\gamma^\mu)\psi\ , \tag{8}$$

where $\boldsymbol{G}_{\mu\nu} \equiv \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$. Using mean-field approximation in the equation of motion for ρ, the following density dependence equation is obtained:

$$\rho_o^3 = \frac{g_\rho}{2m_\rho^2}(n_p - n_n). \tag{9}$$

From the semi-empirical nuclear mass formula, the symmetric energy coefficient is:

$$a_{sym} = \frac{c_\rho k_F^3}{12\pi^2} + \frac{k_F^2}{6\sqrt{(k_F^2 + M^{\star 2})}} \tag{10}$$

where $c_\rho \equiv g_\rho^2/m_\rho^2$ and $k_F = (6\pi^2 n_B/\gamma)^{1/3}(n_B = n_p + n_n)$. We fix the coupling constant c_ρ by requiring that a_{sym} correspond to the empirical value 32 MeV.[10] This gives $c_\rho = 4.66\mathrm{fm}^2$. The inclusion of ρ meson in the Lagrangian will contribute a term $= m_\rho^2(\rho_o^3)^2/2$ to the energy density and pressure. Then the equation of state for neutron rich nuclear matter in beta equilibrium is calculated using the expression for the energy density ε and the pressure P as given as follows:

$$\begin{aligned}
\varepsilon = {} & \frac{M^2(1-y^2)^2}{8c_\sigma} - \frac{B}{12c_\omega c_\sigma}(1-y^2)^3 + \frac{c_\omega n_B^2}{2y^2} \\
& + \frac{C}{16m^2c_\omega^2c_\sigma}(1-y^2)^4 + \sum_i \varepsilon_{FG} + \frac{1}{2}m_\rho^2{\rho_0}^2\ , \\
P = {} & -\frac{M^2(1-y^2)^2}{8c_\sigma} + \frac{B}{12c_\omega c_\sigma}(1-y^2)^3 + \frac{c_\omega n_B^2}{2y^2} \\
& - \frac{C}{16m^2c_\omega^2c_\sigma}(1-y^2)^4 + \sum_i P_{FG} + \frac{1}{2}m_\rho^2{\rho_0}^2\ .
\end{aligned} \tag{11}$$

In these equations ε_{FG} and P_{FG} are the relativistic non- interacting energy density and pressure of the baryons (with effective masses) and electrons (i), respectively.

Fig. 2, displays the pressure versus the total mass-energy density for the neutron rich matter in beta equilibrium. The solid line is for the modified chiral sigma model MCH considered in the present calculation. In this model, the two extra terms are included with two parameters B and C in the Lagrangian.

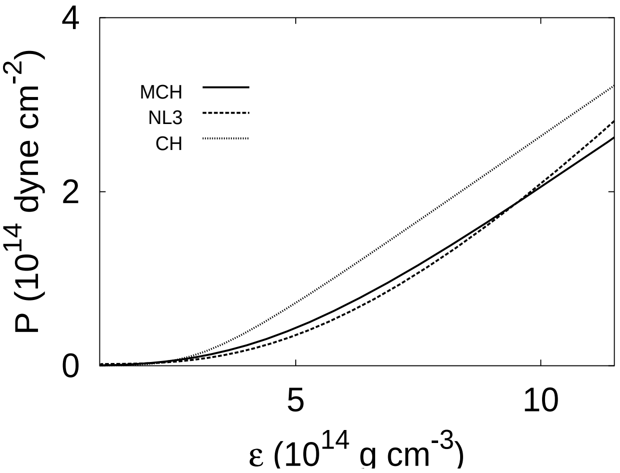

Fig. 2. The neutron star matter pressure vs the energy density. The models are same as Fig. 1.

Inclusion of these two parameters in the potential term give the reasonable value of incompressibility at the saturation density of nuclear matter. Hence, MCH model is much softer than the original CH model in the neutron star matter equation of state. The physics behind this is that the pressure generated due to self-interaction of scalar fields at high density is less i.e., the pressure gradient with density decreases. The dashed line (NL3) represents the neutron rich equation of state, which has been derived from the heavy-ion collision data.[13)] From Fig. 2, we notice in comparison to NL3 model that MCH model shows stiffer at low density range and becomes softer after three times the normal nuclear matter density.

The mass and radius of the neutron star are characterized by the structure of a neutron star. These are determined from the equations that describe the hydrostatic equilibrium of degenerate stars without rotation in general relativity, called Tolman-Oppenheimer-Volkoff (TOV) equations:[14),15)]

$$\frac{dp}{dr} = -\frac{G(\epsilon + p/c^2)(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)} \ , \quad \frac{dm}{dr} = 4\pi r^2 \epsilon \ , \tag{12}$$

where p and ϵ are the pressure and total mass energy density and $m(r)$ is the mass contained in a volume of radius r. The symbol G is the gravitational constant and c is the velocity of light. To integrate the TOV equations, one need to know the equation of state for the entire expected density range of neutron star, starting from the high density at center to the surface densities. Therefore, we construct the composite equation of state for the entire neutron star density span by joining our equation of state of high density neutron rich matter to that given by (i) 10^{14} to 5×10^{10} g cm^{-3},[16)] (ii) 5×10^{10} g cm^{-3} to 10^3 g cm^{-3}[17)] and (iii) less than 10^3 g cm^{-3}.[18)] Thus we integrate the TOV equations for newly constructed equation of state and given central density $\epsilon(r = 0) = \epsilon_c$ with boundary condition $m(r = 0) = 0$ to give R and M. The radius R is defined by the point where $P \sim 0$, or, equivalently, $\epsilon = \epsilon_s$, where ϵ_s (7.8g cm^{-3}) is the density expected at the star surface. The total mass is then given by $M = m(R)$.

The results for star structure parameters are listed in Table I and are shown in Fig. 3. Fig. 3 shows the plot of mass vs central density. The models are same as in Fig. 2.

Table I. Table I: Neutron star structure parameters

ε_c (g cm^{-3})	R (km)	$M/M_\odot$	z	I (g cm^2)	
4.0×10^{14}	13.74	0.79	0.10	0.82×10^{45}	
6.0×10^{14}	13.76	1.38	0.19	1.77×10^{45}	
8.0×10^{14}	13.57	1.74	0.27	2.38×10^{45}	
1.0×10^{14}	13.28	1.93	0.32	2.66×10^{45}	
1.5×10^{15}	12.62	2.10	0.40	2.73×10^{45}	MCH
1.8×10^{15}	12.30	2.12	0.43	2.64×10^{45}	
2.0×10^{15}	12.11	2.12	0.44	2.57×10^{45}	
2.5×10^{15}	11.74	2.10	0.46	2.39×10^{45}	
3.0×10^{15}	11.44	2.07	0.47	2.24×10^{45}	
4.0×10^{14}	09.36	0.48	0.08	0.30×10^{45}	
6.0×10^{14}	11.14	1.06	0.18	1.06×10^{45}	
8.0×10^{14}	11.78	1.52	0.27	1.78×10^{45}	
1.0×10^{14}	11.82	1.80	0.35	2.22×10^{45}	
1.5×10^{15}	11.78	2.09	0.45	2.56×10^{45}	NL3
1.8×10^{15}	11.22	2.14	0.51	2.55×10^{45}	
2.0×10^{15}	11.06	2.16	0.54	2.51×10^{45}	
2.5×10^{15}	10.70	2.16	0.57	2.37×10^{45}	
3.0×10^{15}	10.43	2.14	0.59	2.23×10^{45}	
3.0×10^{14}	13.50	0.57	0.07	0.52×10^{45}	
4.0×10^{14}	14.39	1.26	0.16	1.78×10^{45}	
6.0×10^{14}	14.93	2.07	0.30	3.73×10^{45}	
8.0×10^{14}	14.73	2.36	0.38	4.36×10^{45}	
1.0×10^{14}	14.43	2.47	0.42	4.48×10^{45}	CH
1.5×10^{15}	13.75	2.51	0.47	4.19×10^{45}	
1.8×10^{15}	13.44	2.49	0.49	3.96×10^{45}	
2.0×10^{15}	13.25	2.48	0.49	3.81×10^{45}	
2.5×10^{15}	12.88	2.42	0.50	3.49×10^{45}	

In Table I, we listed the additional parameters of interest, these are the moment of inertia I, the surface redshift $z = \frac{1}{\sqrt{1-2GM/Rc^2}} - 1$ as a function of the central density of the star (for details see in Ref.[9]). These are important for the dynamics and transport properties of pulsars. From Fig. 3 and Table I, we notice that the maximum masses of the stable neutron stars are $2.1M_\odot$, $2.2M_\odot$ and $2.5M_\odot$ and corresponding radii are 12.1 km, 10.7 km and 13.6 km for MCH, NL3 and CH equation of states respectively. The solid line (MCH) represents the modified chiral sigma model considered in the present calculation. The dashed line (NL3) is the equation of state derived from the recent heavy-ion collision data, where the dotted line (CH) is for the original chiral sigma model. The corresponding central densities are 2.0×10^{15} g cm^{-3} ($>$ 7 times nuclear matter density), 2.5×10^{15} g cm^{-3} ($>$ 8 times nuclear matter density) and 1.5×10^{15} g cm^{-3} ($>$ 5 times nuclear matter density) for MCH, NL3 and CH respectively at the maximum neutron star masses. From the neutron star structure point of view, our present model MCH is comparable to that of NL3 model, which has been compiled from the recent heavy-ion collision data.[13] These maximum masses calculate in our models are in the range of recent observations,[19–22] where the observational consequence are given below.

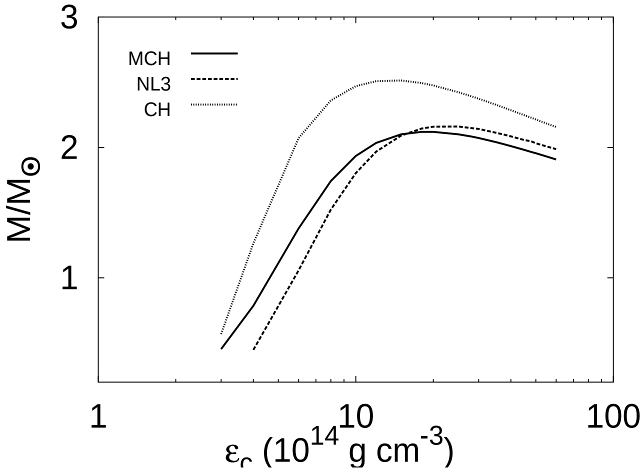

Fig. 3. Fig.3 The neutron star mass vs radius. The models are same as Fig. 1.

Very recently, it has been observed that the best determined neutron star masses[23] are found in binary pulsars and are all lie in the range $1.35{\pm}0.04 M_\odot$ except for the non-relativistic pulsars PSR J1012+5307 of mass $M = (2.1 \pm 0.8)M_\odot$.[19] There have been measured several X-ray binary masses, and the heaviest among them are Vela X-1 with $M = (1.9{\pm}0.2)M_\odot$[20] and Cygnus X-2 with $M = (1.8 \pm 0.4)M_\odot$.[21] From recent discovery of high-frequency brightness oscillations in low-mass X- ray binaries, the large masses of the neutron star in QPO 4U 1820-30 ($M = 2.3)M_\odot$[22] is confirmed and this provides a new method to determine the masses and radii of the neutron stars. Our results lie in the range of quasi-periodic oscillations method[22] as well as above non-relativistic pulsars limit of determining masses and radii of neutron stars.

Recently, it gives theoretical impetus to study the neutron star physics due to recent discovery[24] of afterglow in Gamma Ray Bursters (GRB), which allows determination of the very high redshifts (≥ 1) and thus the enormous distance and energy release of $E \sim 10^{53} - 10^{54}$ ergs in the γ ray alone, assuming isotropic emission. Candidates for such violent events include the merging of two neutron stars (or a neutron star and a black hole) in a binary system.[25] Also recently,[26] the conversion of a neutron star to a strange star as a possible central engine for GRB is considered. So far, the physics producing these GRB is not understood, though many cosmological models for GRB are proposed. Therefore, one of the probable model to explain such origin of GRB and the huge energy output are neutron stars or black hole objects.

§4. Summary and Conclusion

We have discussed the SU(2) chiral sigma model, where the isoscalar vector field is generated dynamically. In the present calculation, we have modified the SU(2) chiral sigma model by including the two extra terms in the Lagrangian to ensure the desirable value of incompressibility at the saturation density of symmetric nuclear matter. By employing these and using mean field approximation, we have obtained the nuclear equation of state at high densities. This is compatible to the equation of state that has been derived from the recent heavy-ion collision flow data.[11–13]

The nuclear equation of state is reasonably softer than the original SU(2) chiral sigma model considered earlier by us.[9] We then have implemented the same nuclear equation of state to neutron star matter calculation, where the compositions of star matter are neutron, proton and electron. The mass and radius of the neutron star are calculated from the TOV equations and these are $2.1M_{\odot}$ and 12.1 km, respectively. The corresponding central density is 2.0×10^{15}(>7 times the nuclear matter density) for the modified chiral sigma model. These values are comparable to the values calculated from the recent model (Nl3), which has been recently compiled from the heavy-ion collision data. The gross structural values are good agreement with the quasi-periodic oscillation method,[22] a new method to determine the masses and radii of the neutron stars.

In future we plan to investigate more systematic properties of neutron star in the presence of other baryons i.e., hyperons and mesons with (hidden-)strangeness in the SU(3) chiral sigma model with proper interaction between them derived from recent experiments. And moreover we are interested to look for the phase transition to quark matter (the new state of matter) by implementing this equation of state in to the heavy-ion collision simulation calculations.

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