

## Elliptic Flow in a Hadron String Cascade Model at 130 GeV Energy

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**Abstract.** We present the analysis of elliptic flow at  $\sqrt{s} = 130A\text{GeV}$  energy in a hadron-string cascade model. We find that the final hadronic yields are qualitatively described. The elliptic flow  $v_2$  is reasonably well described at low transverse momentum ( $p_T < 1\text{ GeV}/c$ ) in mid-central collisions. However, this model is found to underestimate  $v_2$  at high  $p_T$  or in peripheral collisions.

**Keywords.** heavy-ion collisions, elliptic flow

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### 1. Introduction

In high-energy heavy-ion collisions, strongly interacting matter at high temperature and/or density is created. When two nuclei collide, nucleon-nucleon (NN) interactions produce huge number of particles — hadrons, strings and partons —, and these particles will develop collectivity and eventually thermalize. The prime aim of high-energy heavy-ion collisions is to explore the properties of this extreme state of matter far from stable nuclei. Especially the confirmation of the deconfined phase of quarks and gluons, e.g., quark-gluon plasma (QGP) [1] is of current interest. Although it is not straightforward to connect the observable in heavy-ion collision experiments with the properties of matter under equilibrium, several observable such as collective flows have been proposed to be sensitive to these properties [2,3].

Recent RHIC experiments [4–13] have shown that radial and elliptic flows are strongly generated. Both these flows are studied using hydrodynamic model [14–17] for different hadron species produced in Au + Au collisions at  $\sqrt{s} = 19.6 - 200A\text{GeV}$ . Among these, the elliptic flow ( $v_2$ ) has attracted much attention at high incident energies, and it is measured in a wide energy range from GSI-SIS ( $\sim 1A\text{ GeV}$ ) [18], BNL-AGS ( $2 - 10.6A\text{ GeV}$ ) [19], to CERN-SPS ( $40 - 158A\text{ GeV}$ ) [20], in addition to BNL-RHIC. In non-central collisions, collective flow is characterised by the azimuthal correlations between particle momenta and the reaction plane, which is defined by the incident momentum and the impact parameter vectors  $\mathbf{b}$ . The elliptic flow  $v_2$  is the anisotropy of particle emission in- and

out-of reaction plane. In non-central collisions of heavy nuclei, the nucleus-nucleus overlap initially has a lens or almond shape. This spatial anisotropy creates stronger pressure gradient in-plane direction than out of plane by rescatterings, provided the squeezing from spectators are negligible. Since the almond shape is obscured in the later stage, strong  $v_2$  is generally believed to be an evidence of early thermalization. For example, to explain the strong elliptic flow at RHIC, it is required to set the thermalization time of the order of 1 fm/c in hydrodynamical [14] studies.

The above considerations imply the importance of partonic interactions in the early stage at RHIC. Hadrons are mainly produced through the fragmentation of jets or strings after the formation time  $\tau_f \sim 1$  fm/c. Strings are often considered to interact before hadronization only when they contain constituent quarks of the original hadron [21,22]. Thus most of hadrons, which are made of created  $\bar{q}q$ , have to wait for  $\tau_f$  before rescattering. On the other hand, partons can make rescatterings in the early stages within the hadron formation time. Since the number of freed partons in the primary nucleon-nucleon interaction is expected to be much larger at RHIC than at SPS, we expect significant difference in the elliptic flow as a function of the transverse momentum, because some of particles are present at high transverse momenta at RHIC energy due to hard gluon-gluon scattering, and the high  $p_T$  particles goes off the reaction zone quickly, and thus their elliptic flows reflect early stage thermalization. However, recent observation at SPS shows similar behaviour of the elliptic flow like RHIC as a function of the particle transverse momentum [20]. In addition, there are some reports in which the elliptic flow at RHIC is underestimated in hadron-string cascade models [24–26]. In order to reproduce the flow, the strings had to be melted to partons again and the partons had to scatter with cross section of 6-8 mb [26]. Therefore, there is a possibility that the strong elliptic flow at RHIC may or may not be a consequence of partonic interactions in the early stage. In order to clarify this point, it is necessary to understand the mechanism of generating strong elliptic flow at RHIC, and examine the elliptic flow at SPS energies in comparison with that at RHIC in hadron-string cascade models.

Another interesting point is the method of extracting  $v_2$ . In a standard method in experimental data analyses [27], the reaction plane is determined from particle momentum distribution, and  $v_2$  is calculated using the azimuthal angle from this reaction plane. Since in this standard treatment there are several sources of non-flow contributions such as the resonance decays, jets, mini-jets and Coulomb interaction,  $v_2$  values are extracted also from particle azimuthal angle correlations and they are compared with the reaction plane method [28]. While the results from the reaction plane and particle correlation methods seems to be in agreement within the error bars, the four particle correlations significantly reduces the non-flow contribution compared to the two particle correlations. For example, in the two particle correlation method, it would be affected in the presence of mini-jet production, when a major part of particles are produced from a few mini-jets, azimuthal angle correlation would be determined by the cone-angle of jets rather than the collective flows [29] in a model calculation. In the reaction plane method, particles from mini-jets are also used to estimate the reaction plane, and thus the reaction plane is correlated with mini-jets. Thus it is also interesting to compare  $v_2$  values using the two methods in the model calculations for SPS and RHIC energies.

In this paper, we analyse the elliptic flow in heavy-ion collisions at RHIC energy of  $\sqrt{s_{NN}} = 130$  GeV in a hadron-string cascade model with jet production (JAM) [22]. We will show that final hadronic yield ( $dN/d\eta$  and  $p_T$  distribution) at RHIC are roughly ex-

plained in JAM.

In addition, the elliptic flow ( $v_2$ ) at low  $p_T$  ( $p_T < 1$  GeV/c) in mid-central collisions (Centrality  $< 20\%$ ) is reasonably well described. However,  $v_2$  is underestimated at high transverse momentum or in peripheral collisions. As a result, this model underestimates  $v_2$  as a function of pseudo-rapidity by around 30% and  $v_2$  as a function of transverse momentum for  $p_T > 0.5$  GeV/c, for minimum bias events. Next we compare the elliptic flows at SPS and RHIC energies. Calculated results as well as the experimental data show that the transverse momentum dependence of the elliptic flow at SPS is very similar to that at RHIC. The similarity in the present hadron-string cascade model comes from the late growth of the elliptic flow,  $t \sim 10$  fm/c. Since the transverse momentum is generally much smaller than the longitudinal momentum in string fragmentation, the spatial ellipticity does not become very small when hadrons are formed. If this late growth is the true mechanism to enhance the elliptic flow, it becomes almost a flat function of the (pseudo)rapidity. The data from the PHOBOS collaboration [12] does not necessarily support this behaviour. We also compare the  $v_2$  values calculated in the reaction plane method and two particle correlation method. It is found that two methods give similar values in JAM, while they give significantly different results in a model without hadronic rescatterings.

## **2. Transport Model and Elliptic Flow Calculation**

In order to analyse high-energy heavy-ion collisions, many cascade-type models have been developed upto SPS energies and have met with some successes in explaining the data [40,23]. The most basic feature of hadronic cascade models is the implementation of hadron-hadron elementary cross sections. At low energies, pions are mainly produced through resonance formation and decay. At medium energies ( $\sqrt{s} = 3 - 10$  GeV), string formation and decay becomes dominant [21,22,30-32]. There are some models in which multi-pions are directly produced according to the parameterisation of experimental data [33], or by using multi-particle production model such as the multi-chain model [34]. At high energies ( $\sqrt{s} \geq 20$  GeV), partonic interactions becomes important especially in describing high  $p_T$  hadrons. Jet production is estimated by folding the partonic pQCD cross sections with parton distribution in hadrons [30,35]. In describing the elliptic flow at RHIC, we believe that all the above processes have to be included. In the nucleon-nucleon ( $NN$ ) collisions at  $\sqrt{s_{NN}} = 130$  GeV, jet production (hard process) cross section amounts to around 15 % of the total cross section, and the rest of the  $NN$  cross section is dominated by the string formation, then both of these processes are required at the first chance  $NN$  collisions during the nucleus-nucleus ( $AA$ ) collisions. Low energy hadron-hadron collisions are also necessary, since they are responsible to later thermalizations in  $AA$  collisions.

The next important feature in cascade models is the way to treat multiple scatterings. In principle, each particle-particle collision is regarded as an incoherent process in cascade models, but several coherent effects have been found to be important. One of them is the rescattering of strings containing constituent quarks in the original hadron. This effect is pointed out by Sorge [21] in the analysis of proton rapidity distribution at SPS; if a string produced in the primary  $NN$  collision does not interact before hadronization, the di-quark in the baryonic string would have almost the same velocity as the incident nucleon, then the stopping power at SPS cannot be explained.

In this work, we analyse the elliptic flow using a hadron-string cascade simulation model, JAM [22]. The main features of this model are as follows. (1) At low energies ( $\sqrt{s} < 4 - 5$  GeV), inelastic hadron-hadron collisions are modeled by the resonance productions based on the idea from RQMD [21,31] and UrQMD [32]. (2) Above the resonance region, soft string excitation is implemented along the lines of the HIJING model [35]. String fragmentation to hadrons is calculated in the Lund fragmentation model, using PYTHIA program [30]. (3) String interaction before hadronization is simulated by the rescattering of hadrons having original constituent quarks within a formation time, assuming the additive quark model cross section. (4) Multiple mini-jet production is included in the same way as the HIJING model. Jet production cross section and the number of jets are calculated using an eikonal formalism for perturbative QCD (pQCD) and hard parton-parton scatterings with initial and final state radiation are simulated using PYTHIA [30] program. Produced mini-jets later fragment to hadrons through string configurations. (5) Mini-jet production in  $NN$  collision is regarded as independent, and the coherence between different nucleons is not taken into account. (6) Parton rescattering among different mini-jets are not included. The detail description of this model is given in Ref. [22,23]

The first three points are the most dominating part in the present calculation at RHIC. However, the effect to the present calculation by the later three points is insignificant. These are evident in the results, these are discussed in the next section.

The elliptic flow  $v_2$  is defined as the second Fourier component in the azimuthal angle distribution relative to the reaction plane,

$$\frac{dN(\eta, p_T)}{d\phi} = v_0 (1 + 2v_1 \cos \phi + 2v_2 \cos 2\phi + \dots) . \quad (1)$$

If we know the true reaction plane, we can easily calculate  $v_2$  using the final state hadron momenta as,

$$v_2(\eta, p_T) = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle . \quad (2)$$

While in a model calculation we know the true reaction plane, it is not obvious in experiments. One of the way to estimate  $v_2$  without the knowledge of the true reaction plane is to use the two particle correlation [29].

$$\begin{aligned} \langle \cos 2(\phi_i - \phi_j) \rangle &= \langle \cos 2\phi_i \rangle \langle \cos 2\phi_j \rangle + \langle \sin 2\phi_i \rangle \langle \sin 2\phi_j \rangle \\ &= v_2(\eta_1, p_{T1}) v_2(\eta_2, p_{T2}) , \end{aligned} \quad (3)$$

where the average is taken over particles with the condition  $(\eta_i, p_{Ti}) = (\eta_1, p_{T1})$  and  $(\eta_j, p_{Tj}) = (\eta_2, p_{T2})$ .

### 3. Results

We have generated around 10k minimum bias ( $0 < b < 13$  fm) simulation events at RHIC energy ( $\sqrt{s} = 130A$  GeV), and 5k events at SPS energy ( $\sqrt{s} = 17A$  GeV ( $E_{inc} = 158A$  GeV)). Default parameters in JAM are adopted except for a little wider  $p_T$  width in the

string decay and a larger partonic minimum  $p_T$  ( $p_0 = 2.7\text{GeV}/c$ ) to get a stiffer  $p_T$  distribution of charged particles in  $pp$  collisions at RHIC. Phase space coordinates of the final collision/decay points are recorded, and  $v_2$  values are calculated later using the phase space data. In central or mid-central collisions, we choose impact parameter range corresponding to experimental centrality (i.e., Centrality =  $\pi b^2/\sigma_{\text{geo}}$ , where  $\sigma_{\text{geo}} = 720\text{ fm}^2$ ) in Au+Au collisions [9].

First we show the pseudo-rapidity  $\eta$  distribution of charged particles at RHIC energy ( $\sqrt{s_{NN}} = 130\text{ GeV}$ ) in the upper panel of Fig. 1. Calculated charged hadron distribution explains well the PHOBOS data at large  $\eta$  for central collisions [11], although we find a slight overestimate at mid-rapidities compared to the PHOBOS [11], BRAHMS [13], and PHENIX [9] data. This trend applies also to mid-central collisions as well.

In the lower panel of Fig. 1, we show the calculated elliptic flow of charged particles for 5 – 53% and minimum bias events as a function of  $\eta$  in comparison with data from PHOBOS [12] and STAR [4,5,8] collaboration at RHIC energy ( $\sqrt{s_{NN}} = 130\text{ GeV}$ ). The data from STAR [8] is not from the minimum bias but 5 – 53% of the total hadronic cross section and from the four particle correlations. At mid-rapidity, the calculated values for  $v_2$  underestimate the data by around 30%. In addition, the present model shows flat  $v_2$  behaviour at  $|\eta| < 2$ , which contradicts to the enhancement at small  $\eta$  observed by PHOBOS collaboration [12]. This is not a specific problem in the present model. For example, a full 3D hydrodynamical model explains the strong elliptic flow at mid-rapidities, while it gives very flat elliptic flows and consequently overestimates the data at large rapidities for all reasonable initial conditions [17].

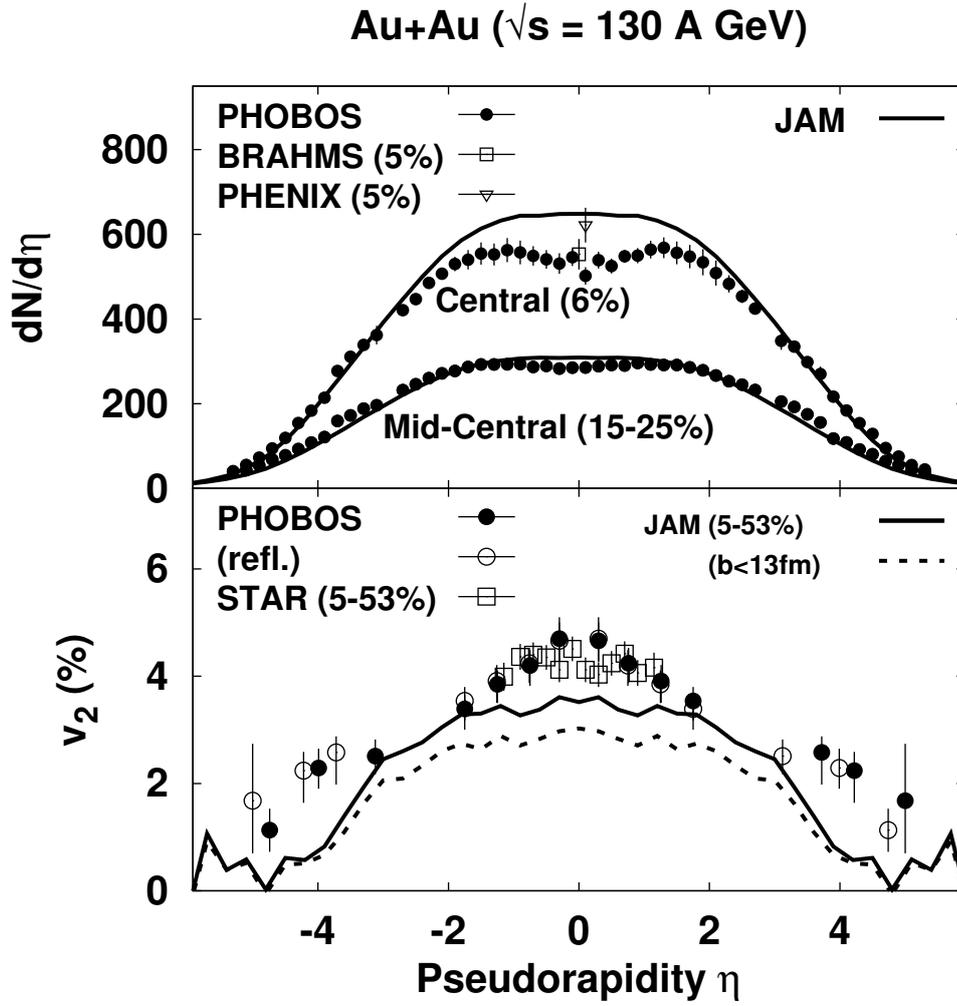
In the upper panel of Fig. 2, we show the calculated  $p_T$  spectra of pions and protons at mid-rapidities for minimum bias events in comparison with data from PHENIX collaboration at RHIC energy ( $\sqrt{s_{NN}} = 130\text{ GeV}$ ) [10]. Pion and proton  $p_T$  spectra are reproduced qualitatively. We need to take account of weak decays of strange baryons to explain the proton yield.

In the lower panel of Fig. 2, we display JAM results on elliptic flow of charged particles as functions of transverse momenta for minimum bias events at RHIC energy. In this figure, we notice that our model gives a reasonable description of the data in the low  $p_T$  region ( $p_T < 0.5\text{ GeV}/c$ ). For larger  $p_T > 0.5\text{ GeV}/c$  values, however, we underestimate  $v_2$  by round 10-40 % for charged particles.

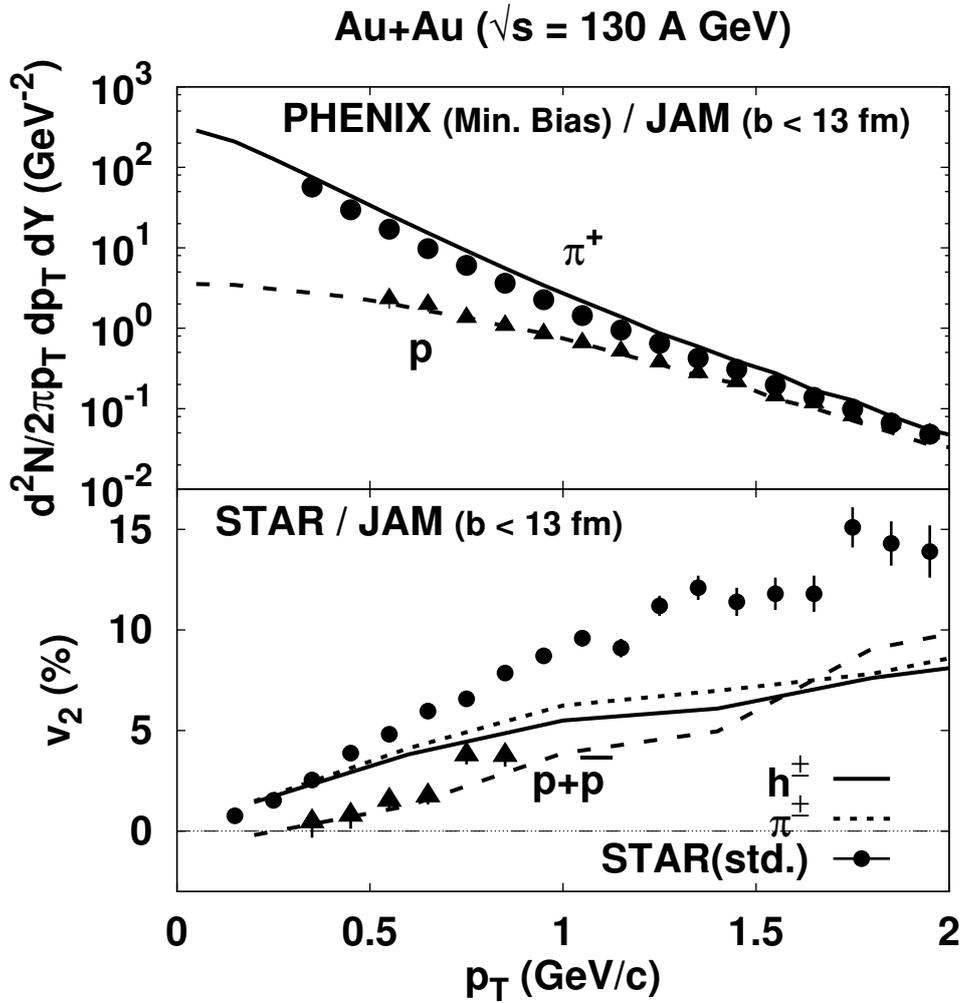
Another interesting point to be noticed here is that the calculated elliptic flow is sensitive to the particle masses as a function of  $p_T$ . The particles which are having smaller masses have higher values of elliptic flow at small  $p_T$ , such as pions and kaons and these are linear functions of  $p_T$ . For higher mass particles such as protons,  $v_2$  behaves non-linearly with  $p_T$ . Similar characteristics are observed in the STAR data [5] as well.

In contrast, hydrodynamic models show excellent agreement upto  $p_T \sim 1.5\text{ GeV}$  in central and semi-central collisions [15], which is not shown in this figure. It fails at high  $p_T$ , due to saturation and onset of hard processes and fails at peripheral collisions due to incomplete early-time thermalization.

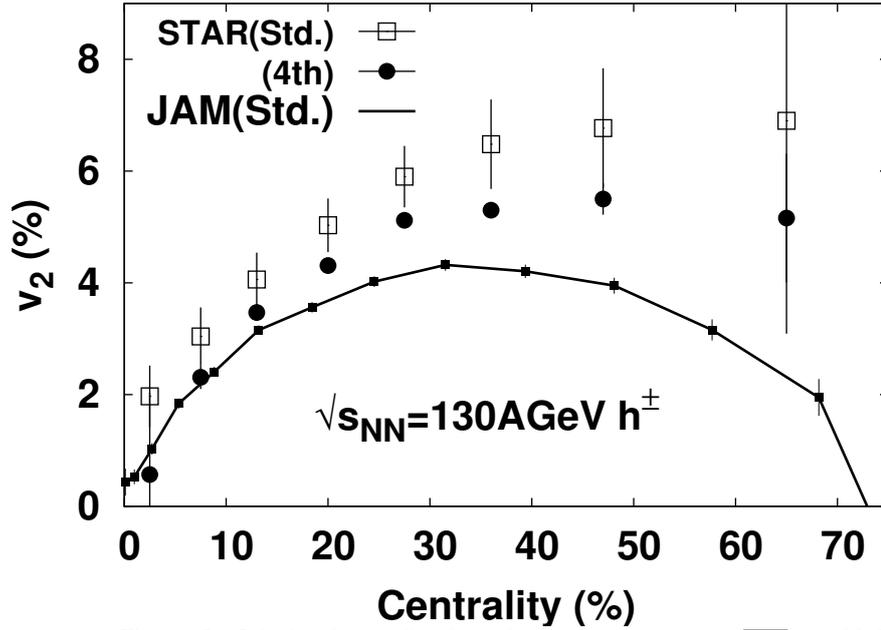
Figure 3 shows the centrality dependence of the elliptic flow. The data are taken from STAR collaboration, where reanalysed data by using cumulant methods for 4 particles show smaller values of  $v_2$  than in the reaction plane method [8]. Our model (solid line) explains the data in mid-central region (Centrality < 20%), but it underestimates the data in peripheral collisions. This underestimate may be due to the lack of rescattering of partons and interaction between hadrons and partons. In comparison to hydrodynamic models,



**Figure 1.** Pseudo-rapidity dependence of charged particle  $dN/d\eta$  (upper panel) and  $v_2$  (lower panel) for minimum bias events at RHIC ( $\sqrt{s_{NN}} = 130$  GeV) in comparison with PHOBOS [11,12] BRAHMS [13], PHENIX [9], and STAR [4,5] data.



**Figure 2.** Transverse momentum  $p_T$  distribution (upper panel) and  $p_T$  dependence of  $v_2$  (lower panel) for minimum bias events at RHIC ( $\sqrt{s_{NN}} = 130$  GeV) in comparison with PHENIX [10] and STAR [5] data.



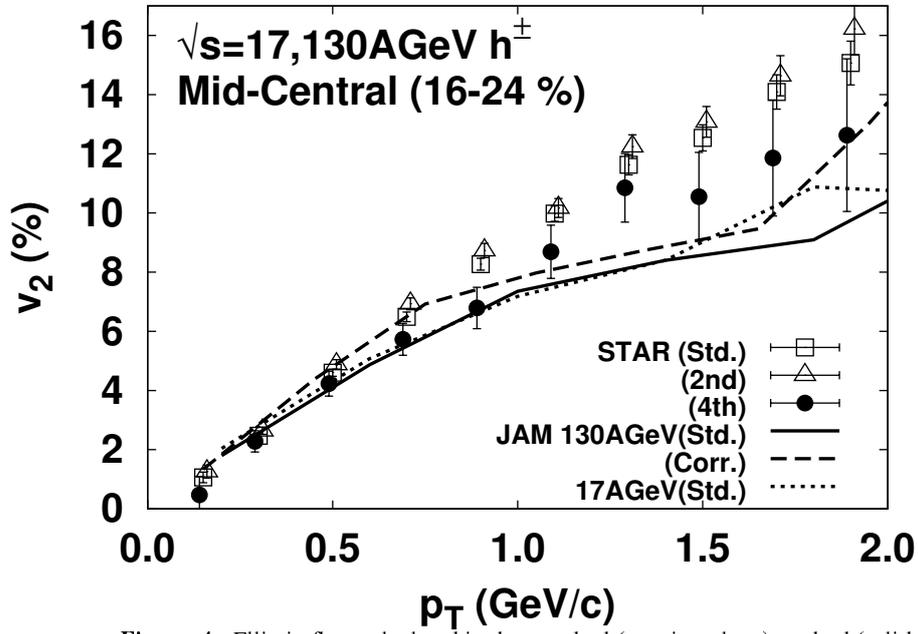
**Figure 3.** Calculated centrality dependence of  $v_2$  at RHIC ( $\sqrt{s_{NN}} = 130$  GeV) in comparison with STAR [8] data.

the hydrodynamic prediction overestimates the elliptic flow data in the most peripheral region [14], which is not shown in the figure. Hence, a large degree of thermalization are favoured in central collisions and the non-equilibrium effects are required to reduce  $v_2$  in peripheral collisions.

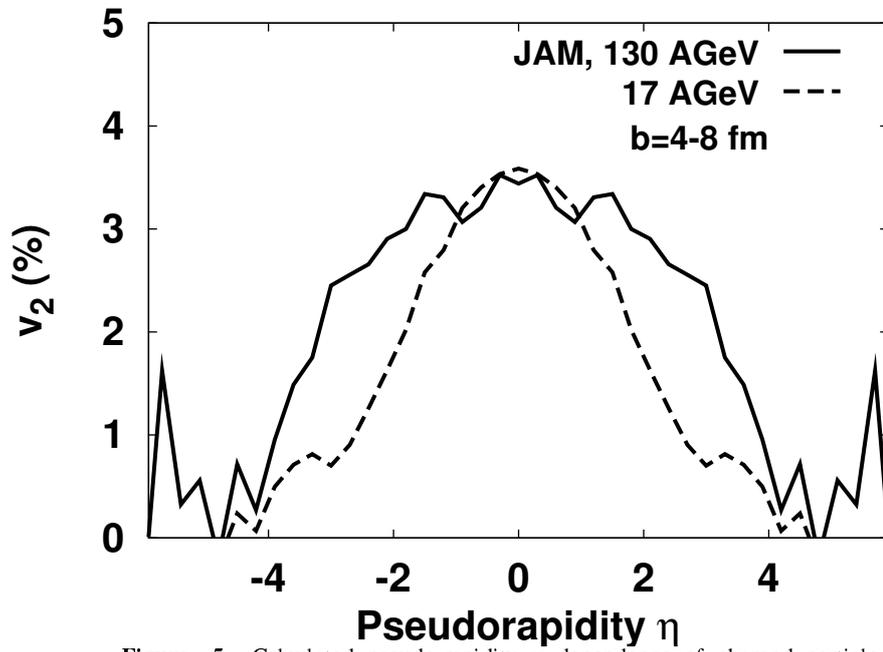
Dependence of  $v_2$  on the analysis method can appear in theoretical model results, too. While the reaction plane is given a priori by the incident momentum and the impact parameter directions, correlation coming from jets may modify the two particle azimuthal angle correlation [29]. In Fig. 4, we show  $v_2$  results at RHIC (Au+Au,  $\sqrt{s_{NN}} = 130$  GeV) and SPS (Pb+Pb,  $\sqrt{s_{NN}} = 17$  GeV) for mid-central collisions calculated by using the standard reaction plane method ( $v_2 = \langle \cos 2\phi \rangle$ , solid line for RHIC, and dotted line for SPS) and the two particle correlation method ( $v_2^2 = \langle \cos 2(\phi_i - \phi_j) \rangle$ ), dashed line for RHIC). Two results at RHIC are in agreement with each other within the statistical error bar. And we find that the elliptic flows are similar at SPS and RHIC energies when we see them as a function of  $p_T$ . If the hadronic final state interaction is the main source of  $v_2$ , we expect a similar trend also at lower energies as SPS. Compare to data the calculated results are underestimated at about 10-20% from  $p_T > 1$  to 2 GeV/c.

We show pseudo-rapidity  $\eta$  dependence of charged hadron elliptic flow  $v_2$  in mid-central ( $4 < b < 8$  fm; not same as Fig. 1) collisions at RHIC (Au+Au,  $\sqrt{s_{NN}} = 130$  GeV, solid line), SPS (Pb+Pb,  $\sqrt{s_{NN}} = 17$  GeV, dashed line) [23] in Fig. 5. It is noteworthy that the present model explains the elliptic flow data at SPS energies [23] We find that elliptic flows at mid-rapidity have similar values for RHIC and SPS.

From the above analysis, one question arises: Why can the elliptic flow take similar



**Figure 4.** Elliptic flow calculated in the standard (reaction plane) method (solid line for RHIC, and dotted line for SPS), two particle azimuthal angle correlation (dashed line for RHIC) in comparison with RHIC-STAR data in several analysis methods [8].



**Figure 5.** Calculated pseudo-rapidity  $\eta$  dependence of charged particle  $v_2$  for mid-central ( $b = 4 - 8$  fm) events at RHIC ( $\sqrt{s_{NN}} = 130$  GeV; solid line) and SPS ( $\sqrt{s_{NN}} = 17$  GeV; dashed line) energies.

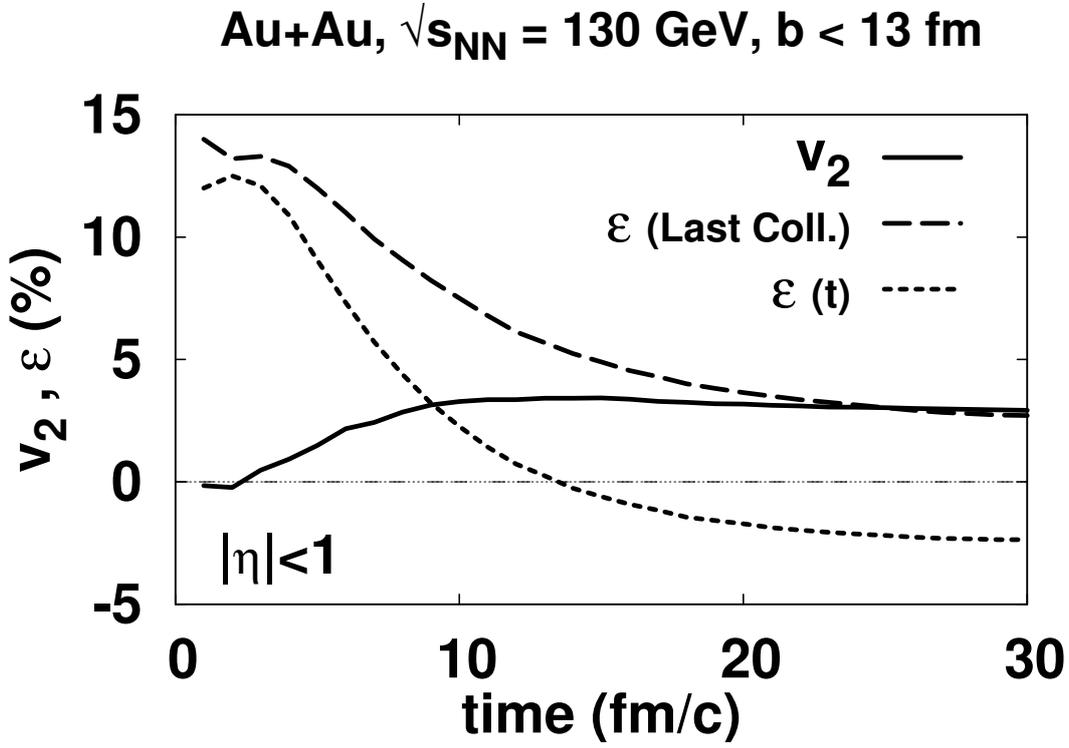


Figure 6. Time dependence of  $v_2$  and  $\epsilon$  for formed hadrons.

values at RHIC and SPS energies in a hadronic cascade model ? In order to give a hint to this question, we have made a simple time-dependent analysis. Elliptic flow is calculated as a function of time  $t$  for formed hadrons (after their formation time) in the pseudo-rapidity range  $|\eta| < 1$ , and shown by the solid line in Fig. 6. It is found that  $v_2$  grows slowly in the time  $t < t_{v_2} \simeq 10$  fm/c. For  $v_2$  to grow, the spatial eccentricity,  $\varepsilon = \langle (y^2 - x^2)/(y^2 + x^2) \rangle$ , has to be finite. We show in Fig. 6 the eccentricity  $\varepsilon$  of formed hadrons at time  $t$  ( $\varepsilon(t)$ , short dotted line) and  $\varepsilon$  at last collision point before  $t$  ( $\varepsilon(\text{Last Coll.})$ , dashed line). The eccentricity is calculated by using the last interaction points before  $t$ . During the time of  $v_2$  growth ( $t < 10$  fm/c),  $\varepsilon$  is large enough to drive  $v_2$  to increase, but this increase is not significant to explain the data at RHIC, while the present model works very well upto SPS energies [23]. The present time scale of  $v_2$  growth is longer compared to hydrodynamical calculations. For example, momentum anisotropy is shown to grow until  $\tau \simeq 2$  fm/c in 2D hydrodynamical calculation with equation of state (EOS) having the phase transition from QGP to hadron gas [16].

Large spatial eccentricity  $\varepsilon$  is considered to be a consequence of small  $p_T$  in the string fragmentation. At the hadronization time, the shift of particle transverse coordinate can be estimated as

$$\Delta r_T \simeq v_T \gamma \tau_f = p_T \tau_f / m. \quad (4)$$

Average  $p_T \sim 0.3 - 0.4$  GeV/c reads  $r_T \sim 2 - 3$  fm for pions, which is not very large compared to the nuclear radius. Thus the elliptic flow can grow slowly also in the hadronic cascade stage. The delay of  $v_2$  growth in hadron-string cascade models would be due to the *dead* time of interaction. Hadrons are formed gradually after  $t = \tau_f$ , and the  $\gamma$  factor and secondary string-hadron interaction make the delay time longer. Formed hadrons inherit the eccentricity of the original nucleon positions, then  $v_2$  can grow slowly after hadronization.

All the results shown in this paper suggest that in the present hadronic cascade model the energy used to increase number of particles rather than to increase pressure in the early stage compared to the real data. There are several probable reasons for this shortcoming. First one is the change of initial state parton momentum distribution, which comes from the saturation of gluons at low momentum. It is discussed that at very high energies, we can see huge number of gluons at small  $x$ , which saturate at some momentum  $Q_s$  due to gluon fusion, and it may be treated in a classical Yang-Mills field [36]. This saturation reduces the number of gluons at small momenta, and increases the higher momentum gluons. It would be possible to increase the initial pressure in exchange for reducing final hadron multiplicities [37,38]. The next possibility is the partonic rescatterings just after the initial mini-jet production. Generally, partons produced in mini-jets have large transverse momenta, which may be transferred to longitudinal momenta if they collide with other hadrons. The third candidate is the coherence in the initial mini-jet production. In the present model, we calculate mini-jet production at each  $NN$  collision rather than estimating the number of collisions through the thickness of nuclei in the eikonal approximation as in HIJING [35].

#### 4. Summary and Discussion

In this work, we have studied the elliptic flow at RHIC by using a jet implemented hadron-string cascade model (JAM). We have compared the calculated results of  $v_2$  with data as

a function of pseudo-rapidity, transverse momentum, and centrality. We find that the final hadronic yields are qualitatively described. The elliptic flow  $v_2$  is reasonably well described at low transverse momentum ( $p_T < 1$  GeV/c) in mid-central collisions. However, this model is found to underestimate  $v_2$  at high  $p_T$  or in peripheral collisions. As a result, this model underestimates  $v_2$  as a function of pseudo-rapidity by around 30% and  $v_2$  as a function of transverse momentum for  $p_T > 0.5$  GeV/c by about 10-40 %, for minimum bias events. The calculated elliptic flows at SPS and RHIC energies are found to be similar, and the reason of this similarity is discussed in terms of spatial eccentricity.

In the analysis of pseudo-rapidity dependence for minimum bias events, the elliptic flow is calculated to be almost constant in the pseudo-rapidity region  $|\eta| < 2$  in the present model calculation. Since similar behaviour is found also in the 3D hydrodynamical calculation [17], quick decrease of  $v_2$  at larger  $\eta$  may be due to non-equilibrium dynamics.

Hadronic description seems to fail at high transverse momentum ( $p_T > 0.5$  GeV/c for minimum bias events, and  $p_T > 1$  GeV/c for mid-central collisions). Since the spatial eccentricity quickly goes down for high momentum particles, especially in peripheral collisions where the participant size is small, we cannot give strong pressure effects on those particles. The similar feature was observed from the analysis of radial flow in the central collisions [7,39]. Thus the elliptic flow at high momenta seems to be actually sensitive to the thermalization in the early stage, and the above observations indicate that it is necessary to include additional processes which are effective in early thermalization than hadron-string cascade processes.

We have also shown the comparison of the calculated elliptic flow results at SPS and RHIC energies. At SPS energy, the elliptic flow as a function of  $p_T$  is calculated to be very similar to that at RHIC in the present model. Up to SPS energies, we already know that hadron-string description works quite nicely for directed and elliptic flows [40,23]. While the cascade model generally overestimate the elliptic flows, repulsive mean field effects suppress the elliptic flow values and we can explain the data quantitatively at AGS energies [40]. At SPS energies of 40-158 A GeV, mean field effects are still important to reduce the elliptic flows [23].

In summary, we find that a hadron-string cascade model generally underestimates the elliptic flow at RHIC energy. It may be stated that the natural explanation of this underestimates is to assume that partons produced in mini-jets interact frequently in the early stage at mid-rapidities and thermalized QGP is formed through these rescatterings, which is not included in the present model.

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