Possibility of s-wave pion condensates in neutron stars revisited

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(Dated: September 7, 2009)

We examine possibilities of pion condensation with zero momentum (s-wave condensation) in neutron stars by using the pion-nucleus optical potential U and the relativistic mean field (RMF) models. We use low-density phenomenological optical potentials parameterized to fit deeply bound pionic atoms or pion-nucleus elastic scatterings. Proton fraction (Y_p) and electron chemical potential (μ_e) in neutron star matter are evaluated in RMF models. We find that the s-wave pion condensation hardly takes place in neutron stars and especially has no chance if hyperons appear in neutron star matter and/or b_1 parameter in U has density dependence.

PACS numbers: 21.65.Jk, 26.60.-c, 36.10.Gv, 25.80.Dj,

Pion condensation in neutron stars has a long history of study from the first suggestion in early 1970s by Migdal [1] and Sawyer [2]. The pion-nucleon interaction is attractive in p-wave, then the main interest along this line was to explore possible appearance of pionic excitations with zero energy and *finite* momentum, *i.e.* p-wave pion condensation [3] in nuclear matter. Possibilities of p-wave pion condensation in finite nuclei had been investigated extensively in 1970's and 1980's. Those possibilities were denied by the non-observation of anomalous angular momentum distribution in the inelastic excitation of the pionic quantum numbers [4]. Possibilities of p-wave pion condensation at high densities were also considered to be improbable based on the universal repulsion assumption, $g'_{N\Delta} \sim g'_{NN} \sim 0.6$ - 0.8 [5]. In 1990's, new experiments on the Gamow-Teller giant resonances were performed, and the sum rule value including the $2p-2h$ states was found to be around 90 % [6], suggesting that the transition to the Δ region is weak and $g'_{N\Delta} \sim 0.2$ would be smaller than g'_{NN} [7]. In addition, microscopic variational calculation [8] suggests π^0 condensation in symmetric nuclear matter at high densities ($\rho_B > 0.2\ \mathrm{fm}^{-3}$) generated from the Δ mediated three nucleon force. Thus at present, we cannot completely deny the possibility of pion condensation in dense matter, and it is necessary to examine all the ingredients of πN and π-nucleus interactions with updated experimental and theoretical knowledge.

The study of the in-medium pion properties, especially the s-wave pion-nucleus interaction, has been recently developed in experiment. Precise observations of deeply bound atomic states of π^- in Pb and Sn isotopes [9–13] and low energy pion-nucleus elastic scattering [14] provide us with detailed information of density dependent optical potentials at low densities. Theoretical calculations of in-medium pion self-energy have also experienced much progress based on chiral dynamics [15–18].

Together with these developments, it may be interesting to revisit pion condensation at high densities, such as in neutron stars. Motivated by the recent progress in the s-wave pion-nucleus interaction, we concentrate on the study of pion condensation with zero momentum (s-wave condensation), for simplicity. Even such a limited investigation would make progress of our understanding of dense matter. The s-wave pion condensation takes place in neutron stars with nuclear matter instability where the transition $n \to p\pi^-$ becomes energetically possible [19]. This happens with $\mu_n - \mu_p \geq E_\pi$, where μ_n and μ_p are the neutron and proton chemical potentials, respectively, and E_π is the π^- energy at rest. Due to β equilibrium under the charge neutral and neutrino-less conditions, $\mu_n - \mu_p$ can be written as the electron chemical potential $\mu_e = \mu_n - \mu_p$. Since nn interaction is more repulsive than pn, μ_n is pushed up then μ_e increases compared to Fermi gas value in neutron rich matter. For example, relativistic mean field (RMF) models suggest [19, 20] that μ_e largely exceeds the in-vacuum π^- mass at nuclear density $\rho_B = (1 \sim 5)\rho_0$ in neutron stars, where ρ_0 is the saturation density.

In this paper, we examine whether the condition for the s-wave π^- condensation, $E_\pi = \mu_e$, is satisfied in neutron stars for E_π and μ_e obtained in our present knowledge of low density pion optical potentials and equation of state (EOS) of neutron star matter. For E_π, we use various pion optical potentials fitted so as to reproduce the pionic atom [12, 21–25] and π-nucleus elastic scattering data [14]. Although the fitted optical potentials in normal nuclei may not be extrapolated to higher density and/or highly asymmetric nuclear matter, it is interesting to examine the present status and to think about next steps. For μ_e, we adopt the results calculated in RMF with several parameter sets [26–29], which explain the bulk properties of nuclei such as the binding energy and the charge radius in a wide mass range. We also use proton fractions (Y_p) evaluated with RMF for calculating pion optical potentials[1]. There are several theoretical works on the s-wave pion condensation at higher densities [30], while the connection to low density phenomena observed in experiments is not clear yet. Since we concentrate on the possibility of the s-wave π^- condensate, we do not consider the double pole condition for $\pi^-\pi_s^+$ pair creation, which takes place with finite momentum.

The π^- energy in uniform matter may be evaluated by $E_\pi = \sqrt{m_\pi^2 + 2m_\pi U}$ ($\mathbf{p} = 0$) with the real part of the *energy-independent* potential U based on Ericson-Ericson pa-

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[1] A preliminary study has been done along the same line for limited combinations of pion potentials and an RMF parameter set [20].

TABLE I: Pion potential parameters. The upper four sets by the pionic atom are taken from Ref. [10], in which b_0 and $B_0^{\rm Im}$ were readjusted to reproduce the recent data of the deeply bound π^- states in Pb with fixing the other parameters as the original values given in Ref. [21] for T, [22] for BFG, [23] for SM and [24] for ET.

system	parameter set	$\tilde{b}_0$ (m_π^{-1})	$\tilde{b}_1$ (m_π^{-1})	$B_0^{\rm Re}$ (m_π^{-4})	α
	T	-0.034	-0.078	0	0
pionic	BFG	-0.025	-0.085	-0.021	0
atom	SM	-0.027	-0.12	0	0
	ET	-0.020	-0.0873	-0.049	0
	NOG [25]	-0.013	-0.105	0	0
	KY[12]	-0.0233	-0.1473	-0.019	0.367
pion-nucleus	F-C[14]	-0.009	-0.114	-0.040	0
scattering	F-W[14]	-0.009	-0.081	-0.040	0.391

rameterization [31]:

$$U = -\frac{2\pi}{m_\pi}\left[(1+\epsilon)\left(b_0\rho_B + b_1\delta\rho\right) + \left(1+\frac{\epsilon}{2}\right)B_0^{\rm Re}\,\rho^{(2)}\right]$$

with $\epsilon = m_\pi/M_N$, $\rho_B = \rho_n + \rho_p$, $\delta\rho = \rho_n - \rho_p = \rho_B(1-2Y_p)$ and a squared density $\rho^{(2)}$ defined below. This potential is related to the pion self-energy via $\Sigma_\pi = 2m_\pi(U + iW)$ with an imaginary potential W. The s-wave πN potential parameters $(b_0, b_1, B_0^{\rm Re})$ was determined with special care from precise measurements of pionic atom and pion-nucleus scattering data.

In Table I, we summarize the parameter sets adopted here. The upper 6 sets were determined from the pionic atom data and the lower two from pion-nucleus scattering. For the former parameter sets except NOG, $b_0 = \tilde{b}_0$ and $\rho^{(2)} = \rho_B^2$ are used, whereas, for the latter, double scattering modifications were explicitly included by $b_0 = \tilde{b}_0 - 3(\tilde{b}_0^2 + 2\tilde{b}_1^2)k_F/(2\pi)$ with $k_F = (3\pi^2\rho_B/2)^{1/3}$, and $\rho^{(2)} = \rho_B^2 - \delta\rho^2$ is used. For NOG, $b_0 = \tilde{b}_0 + \delta b_0 - 3(1+\epsilon)(\tilde{b}_0^2 + 2\tilde{b}_1^2)k_F/(2\pi)$ with $\delta b_0 = -0.0053 m_\pi^{-1}$. In the parameter sets of KY and F-W, b_1 is assumed to have density dependence through $b_1 = \tilde{b}_1/(1 - \alpha\rho_B/\rho_0)$ [17, 32, 33] with finite α and $\rho_0 = 0.17~\mathrm{fm}^{-3}$. This is a consequence of the pion wave function renormalization associated with energy dependence of the optical potential [17, 33] and the renormalization was performed at $E_\pi = m_\pi$. For more realistic calculations in dense nuclear matter, the wave function renormalization should be done at the in-medium pion mass and other parameters should be also renormalized.

The phenomenological potentials are determined with a fixed pion energy. The energy dependence of the optical potential can be estimated by theoretical calculations. The s-wave in-medium pion self-energy $\Sigma_\pi(E_\pi)$, equivalent to the optical potential, was derived based on the chiral perturbation theory in Ref. [15] within a linear density approximation. The in-medium pion mass is obtained by solving $m_\pi^{*2} - m_\pi^2 - \Sigma(m_\pi^*) = 0$, which automatically takes account of the wave function renormalization. Here we use the following self-energy, abbreviated as MOW [16],

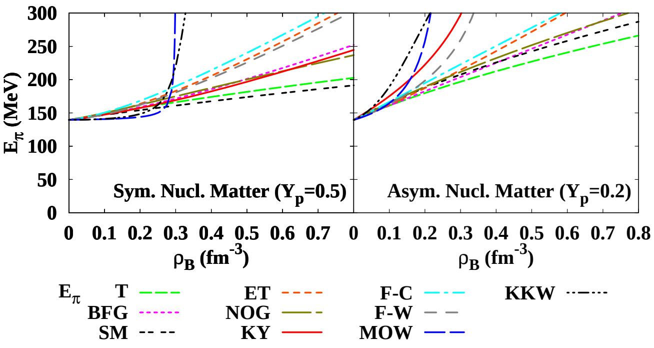

FIG. 1: (Color online) E_π in symmetric ($Y_p = 0.5$, upper) and asymmetric ($Y_p = 0.2$, lower) nuclear matter.

$$\Sigma(m_\pi^*) = c_1\frac{4\rho_B}{f^2}m_\pi^2 - \frac{2\rho_B}{f^2}m_\pi^{*2}\left(c_2 + c_3 - \frac{g_A^2}{8m_N}\right) + \frac{m_\pi^*\delta\rho}{2f^2}\,,$$

with $c_1 = -0.81$, $c_2 = 3.20$, $c_3 = -4.66$ in units of GeV^{-1} and $f = 88$ MeV. We also consider the π^- self-energy calculated by an in-medium chiral perturbation theory in $\mathcal{O}(p^5)$ discussed in Ref. [17] (KKW), which reproduces well the energies and widths of deeply bound π^- atomic states in Pb.

Let us see the model dependence of the pion energy E_π. In Fig. 1, we show E_π in the cases of symmetric ($Y_p = 0.5$) and asymmetric ($Y_p = 0.2$) nuclear matter. The proton fraction $Y_p = 0.2$ is a typical value in neutron star matter obtained in RMF, as shown later in Fig. 2. The negative signs of the coefficients $(b_0, b_1, B_0^{\rm Re})$ imply that π^- feels repulsive potential in nuclear matter. The phenomenological pion potentials in symmetric nuclear matter agree well with each other at low densities below ρ_0, whereas, in asymmetric nuclear matter with $Y_p = 0.2$, we have 50-100 MeV ambiguities at $\rho_B \sim \rho_0$. In order to fix the large ambiguity of the potentials in asymmetric nuclear matter, it is very interesting to obtain pionic atom and scattering data in neutron rich nuclei [34].

RMF models have been developed to describe bulk properties of nuclei and nuclear matter with the mean field via the meson fields. We here adopt the RMF models, NL1 [26], NL3 [27], TM1 [28], SCL [29], having the Lagrangian in the following form,

$$\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\rm free} + \bar{\psi}\left[g_\sigma\sigma - g_\omega\not{\omega} - g_\rho\tau_z\not{\rho}\right]\psi + \frac{c_\omega}{4}\omega^4 - V_\sigma(\sigma),\\
V_\sigma =& \begin{cases} \frac{1}{3}g_3\sigma^3 + \frac{1}{4}g_4\sigma^4 & \text{(NL1, NL3, TM1)} \\ -a_\sigma f_{\rm SCL}(\sigma/f_\pi) & \text{(SCL)} \end{cases}\,,
\end{aligned}$$

where ψ, σ, ω, ρ represent nucleon and σ-, ω- and ρ-meson fields, respectively, and $f_{\rm SCL}(x) = \log(1-x) + x + x^2/2$. The model parameters are summarized in Table II. These RMF models describe the binding energies of heavy semi-double magic nuclei well, and are expected to give reasonable EOS of nuclear matter. We have solved the β equilibrium condition

TABLE II: RMF parameters. In SCL, g_3 and g_4 are from the expansion of $f_{\rm SCL}$.

	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[26]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[27]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[28]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[29]	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

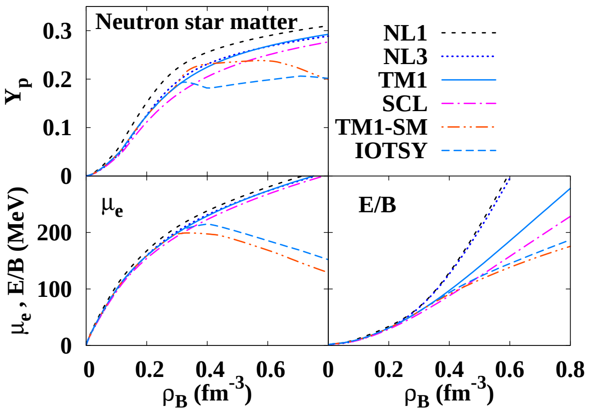

FIG. 2: (Color online) RMF results of proton fraction ($Y_p = \rho_p/\rho_B$, upper panel), electron chemical potential (μ_e, lower left panel) and energy per baryon (E/B, lower right panel).

in cold neutron star matter,

$$\mu_e = \mu_n - \mu_p \ , \quad \rho_e = \rho_p \ , \tag{1}$$

$$\mu_{n,p} = \sqrt{{M_N^*}^2 + k_F^2} + g_\omega \omega \mp g_\rho \rho \ , \tag{2}$$

where $M_N^* = M_N - g_\sigma \sigma$ represents the effective mass of nucleon. As shown in Fig. 2, calculated values of E/B, Y_p and μ_e in neutron star matter are consistent at low densities ($\rho_B < \rho_0$), since meson-baryon coupling constants are well determined by the binding energies of heavy-nuclei. Significant differences are found in E/B at higher densities, where the mesons have large expectation values and the self-interaction terms ($\sigma^3, \sigma^4, \omega^4$) contribute to E/B considerably. While we have small differences in Y_p and μ_e, the model dependence is smaller compared with those in E/B and E_π. As we can see from Eq.(2), $\mu_n - \mu_p$ is modified from the Fermi gas value with M^* by the ρ meson, whose coupling with nucleons is well constrained by nuclear binding energies, and higher order terms of the ρ meson are not included in the RMF models under consideration. As a result, model dependence of the isospin dependent potential, $g_\rho \rho$, is only around 10 MeV at $\rho_B = 0.8$ fm^{-3}. In Fig. 2 we also show the results of some RMF models including hyperons (TM1-SM [35] and IOTSY [20]). With hyperons, the proton fraction and electron chemical potential significantly decrease.

Now let us compare the electron chemical potential μ_e and the pion energy E_π as functions of ρ_B (Fig. 3). The RMF results of TM1 are adopted for the proton fraction (Y_p) to evaluate E_π. Results with hyperons denoted by IOTSY are also shown. The left panel of Fig. 3 shows the comparison of μ_e and E_π obtained from the pionic atom data, namely, Tauscher (T), Batty-Friedman-Gal (BFG), Seki-Masutani (SM), Ericson-Tauscher (ET) and Kienle-Yamazaki (KY). In these potentials, density dependence of the potential parameters is not taken into account except for KY. The right panel of Fig. 3 compares μ_e with E_π obtained from the pion-nucleus scattering data and the theoretical calculations.

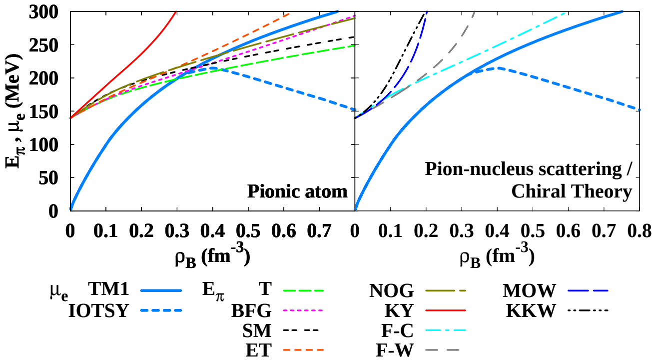

FIG. 3: (Color online) Electron chemical potentials in RMF models and pion energy in neutron stars.

We find that E_π obtained from the potentials with density-independent b_1 are very close to μ_e at high densities $\rho_B > 0.3$ fm^{-3}, and in further dense nuclear matter, μ_e exceeds E_π obtained with some of the parameter sets. We could have possibility for the s-wave pion condensation to take place in dense neutron star matter. However, since inclusion of hyperons makes μ_e suppressed, E_π is found to be larger than μ_e (IOTSY) in most cases. Thus the s-wave pion condensation would not take place, if hyperons could participate in neutron star matter.

We note that even more attractive hyperon potentials make μ_e smaller (TM1-SM). Also in non-relativistic variational treatments [8, 36], symmetry energy and μ_e are generally smaller at $\rho_B > \rho_0$ than in relativistic models. In the case of the density-dependent b_1, which is a consequence of the renormalization of the pion wave function and is required to explain the pionic atom data of Sn isotopes [12], the pion self-energies are more repulsive. Nevertheless, it is important to note that, as already mentioned, the renormalization of the wave function has been done only for the b_1 parameter in linear $\delta\rho$. Thus, for more quantitative discussion, it is necessary to improve the phenomenological and theoretical pion optical potentials in a consistent way, for instance as done in

Ref. [18]. It is also desired to include the effects of short-range and tensor correlations on μ_e under β-equilibrium [8] in relativistic frameworks [37].

In summary, we have discussed the in-medium pion energy in the context of possibility for the s-wave pion condensation to take place in neutron stars. We have compared the in-medium pion energies determined from pionic atom or pion-nucleus scattering data with the electron chemical potential evaluated in relativistic mean field (RMF) models, using the RMF result of the proton fraction. With our present limited knowledge of the in-medium pion properties obtained in experiments, we could conclude that the s-wave pion condensation would not take place in neutron stars with hyperons. It is certainly necessary to investigate in-medium pion self-energy theoretically in more elaborated prescription to go beyond nuclear density. Especially energy dependence of the pion self-energy should be treated in more proper ways for higher densities. At the same time, experimental observations of pionic atoms and scattering in neutron rich nuclei are essential to fix ambiguities in the pion optical potentials in asymmetric nuclear matter. Precise knowledge of pion self-energies at high density is also important to study finite temperature process such as black hole formations, where the temperature can be as large as $T = 70\,\mathrm{MeV}$ [38]. At such high temperatures, pion contribution could be significant depending on the in-medium pion mass.

We acknowledge S. Hirenzaki, T. Kunihiro, K. Sumiyoshi and K. Yazaki for useful discussions. This work was supported in part by the Grant-in-Aid for Scientific Research from MEXT and JSPS under the grant numbers, 17070002, 19540252, 20028004 and 20540273, and the Yukawa International Program for Quark-hadron Sciences (YIPQS). KT also thanks JSPS for the fellowship (20·4326).

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