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Femtoscopic study of $N\Xi$ interaction and search for the H dibaryon state around the $N\Xi$ threshold

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Abstract We study the momentum correlation functions of $p\Xi^-$ and $\Lambda\Lambda$ pairs produced in high-energy nuclear collisions by using the coupled-channel framework and the $N\Xi$-$\Lambda\Lambda$ coupled-channel baryon-baryon potentials recently obtained from lattice QCD calculations at almost physical quark masses. The calculated results are found to well describe the correlation function data from pp and pPb collisions. This agreement confirms the $S = -2$ baryon-baryon potentials from lattice QCD and supports the existence of the H dibaryon as a virtual state around the $N\Xi$ threshold.

Keywords Correlation function · Lattice baryon-baryon interaction · Dibaryon

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1 Introduction

The H dibaryon state has been discussed for the last several decades. The H particle consisting of $uuddss$ quarks with $I = 0$, $S = -2$ and $J = 0^+$ quantum numbers was predicted to be deeply bound from the $\Lambda\Lambda$ threshold [1], but it has not been discovered despite serious experimental searches. Eventually the discovery of the double Λ hypernucleus [2] ruled out the deeply bound H. Yet people are still interested in the fate of H, since it is one of the leading $6q$ $L = 0$ dibaryon candidates [3]; the Pauli exclusion principle of quarks does not operate and the color-spin interaction acts attractively in H. At present, the main question is whether the H dibaryon state is bound or not with respect to the $N\Xi$ threshold.

In order to obtain information on the pairwise interaction as well as to deduce the existence of a bound state around the threshold, femtoscopy is helpful. The momentum correlation function of two particles is enhanced at low momenta, when the interaction is attractive and the source is small. When the source size is comparable with the absolute value of the scattering length, by comparison, the correlation function is suppressed if there is a bound state [4], while it is enhanced if there is no bound state. This idea was utilized to give a conjecture that a bound state would exist in the $N\Omega$ channel [4]; the $p\Omega$ correlation function is strongly enhanced in high-multiplicity pp collision events [5] but is suppressed at low momenta in Au+Au collisions [6]. This source size dependence suggests the existence of an $N\Omega$ bound state as predicted by the lattice QCD calculation [7].

Recently, the $N\Xi$-$\Lambda\Lambda$ coupled-channel potentials have been obtained from the lattice QCD calculation [8], where the H dibaryon state is found as a virtual state. A virtual state has an eigenenergy whose real part is below the threshold but has an eigenmomentum whose imaginary part is negative, as shown in Fig. 1. It is not regarded as a resonance from the former. From the latter, the wave function diverges as $\exp(ikr)/r = \exp(-\mathrm{Im}(k)r + i\mathrm{Re}(k)r)/r$ with k being the eigenmomentum, so it is not a bound state. In the s-wave, a bound state evolves to a virtual state with decreasing attraction, and sometimes further evolves to a resonance [9]. Thus the results in [8] tells us that the H is barely unbound. For example, the pole position was found to depend on the analysis details [10], and lattice QCD calculations predict a bound H dibaryon below the $\Lambda\Lambda$ threshold with heavier quark masses or in the flavor SU(3) limit [11–13]. Experimental data are needed to decide *to be bound or not to be bound.*

In this work, we investigate the $p\Xi^-$ and $\Lambda\Lambda$ correlation functions by using the $N\Xi$-$\Lambda\Lambda$ coupled-channel baryon-baryon potentials obtained from the lattice

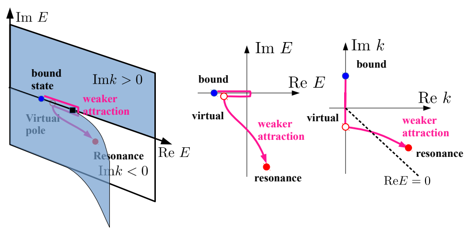

Fig. 1 Schematic picture of the s-wave pole position in the complex energy and momentum space.

QCD calculation at almost physical quark masses [8]. Because of the $p\Xi^-$-$n\Xi^0$-$\Lambda\Lambda$ coupling, we need to use the correlation function formula with the coupled-channel effects, which may be called the Koonin-Pratt-Lednicky-Lyuboshits-Lyuboshits (KPLLL) formula [14–17]. We also need to take care of the Coulomb potential and the threshold difference. In addition to the mass difference between $N\Xi$ and $\Lambda\Lambda$, the isospin breaking of the $N\Xi$ thresholds ($p\Xi^-$ and $n\Xi^0$) is also relevant to the pole position of the virtual state. The calculated correlation functions are compared with the recently obtained correlation function data of $p\Xi^-$ [18] and $\Lambda\Lambda$ [19] pairs. In the comparison, we assume that the source function in the relative coordinate is a static Gaussian and the source size R is regarded as a variable parameter. We find that the calculated results agree with the data well when the source size is fitted to the data. Thus the $S = -2$ baryon-baryon potentials from the lattice QCD are examined by the data. In addition, the data support the existence of H as a virtual state around the $N\Xi$ threshold. Details of the present work are given in Ref. [20].

2 $p\Xi^-$ and $\Lambda\Lambda$ correlation functions

The correlation function $C(\boldsymbol{q})$ with coupled-channel effects from a chaotic static spherical source for a non-identical particle pair is given as [15–17]

$$C(q) = 1 + \sum_j \int_0^\infty 4\pi r^2 dr\, \omega_j S_j(r) \left[|\psi_j^{(-)}(q_j; r)|^2 - \delta_{j1} |j_0(qr)|^2 \right], \quad (1)$$

where $S_j(r)$, ω_j, and q_j are the normalized source function, its weight, and the eigenmomentum in the jth channel, respectively. We here ignore the Coulomb potential tentatively, assume that only the s-wave component is modified by the strong interaction, and impose the $\Psi^{(-)}$ boundary condition, where the outgoing wave exists only in the measured channel ($j = 1$) in the correlation function; $\psi_{j=1}^{(-)}(q; r) \to [\exp(iqr) + A_1(q)\exp(-iqr)]/2iqr$ ($r \to \infty$) is the regular solution in the s-wave having the same outgoing wave component as that in the plain wave, and the wave functions in other channels contain only the incoming waves as $\psi_{j\neq1}^{(-)}(q_j; r) \to A_j(q)\exp(-iq_j r)/2iq_j r$. From Eq. (1), the correlation function is found to represent the average enhancement of the squared wave function within the source in the single channel case. By comparison, $C(q)$ is enhanced also by the wave functions in other channels in the coupled-channel case.

Equation (1) can be used also for the bound state diagnosis. If there is a bound state in a single channel problem, the wave function at low energies has a node at $r \simeq a_0$ with $a_0 (> 0)$ being the scattering length defined by the phase shift as $q \cot\delta = -1/a_0 + r_{\mathrm{eff}} k^2/2 + \mathcal{O}(k^4)$. Then for the source having the size comparable to a_0, the squared wave function is suppressed on average and then the correlation function is suppressed from unity. For more quantitative discussions, the Lednicky-Lyuboshits model [21] is useful. By substituting the asymptotic wave function in Eq. (1), an analytic expression of the correlation function is obtained. In the zero effective range case, $q \cot\delta = -1/a_0$, we further obtain a simple expression

$$C(x, y) = 1 + \frac{1}{x^2 + y^2} \left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_1(2x) - x F_2(2x) \right] \quad (2)$$

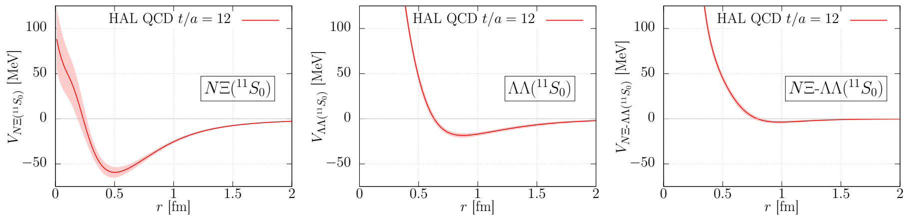

Fig. 2 The s-wave coupled-channel $S = -2$ HAL QCD baryon-baryon potentials. The colored shadow denotes the statistical error of each potential.

where $x = qR$, $y = R/a_0$, and F_1 and F_2 are known functions. By using the low energy limit, $F_1(x) \to 1$ and $F_2(x) \to x$ at $x \to 0$, we find that the correlation function at zero momentum takes large values ($C(0) \geq 2$) for small sources $-1.5 \leq R/a_0 \leq 0.34$ and takes a minimum value when the source size is comparable with a_0, $C(0) = 1 - 2/\pi \simeq 0.36$ at $R/a_0 = \sqrt{\pi}/2 \simeq 0.89$. In more realistic cases with a finite effective range, the Coulomb potential, and coupled-channel effects, the above discussion does not apply as it is, but the qualitative feature survives. For example, existence of an $N\Omega$ bound state is suggested as discussed in the Introduction. We also expect that $\Lambda(1405)$ having a bound state nature of $N\bar{K}$ would cause a dip in the pK^- correlation function from a large source [17]. How about the $N\Xi$-$\Lambda\Lambda$ system?

Recently, the HAL QCD collaboration reported the $N\Xi$-$\Lambda\Lambda$ coupled-channel potentials obtained from lattice QCD calculations at almost physical quark masses [8]. Potentials in the isospin-singlet–spin-singlet ($^{11}S_0$) channel are shown in Fig. 2. The $N\Xi$ diagonal potential has a depth around -50 MeV, which is much more attractive than the $\Lambda\Lambda$ diagonal potential. With these potentials, the scattering lengths in all the s-wave channels have negative real parts, and there is no bound state caused by the strong interaction in the $N\Xi$-$\Lambda\Lambda$ system. While there is no bound state, virtual states are found at $E_{\text{pole}} = 2250.5 \pm i0.3$ MeV in the $(+,-,+)$ sheet, where the sign of the imaginary part of the eigenmomenta of $\Lambda\Lambda$, $n\Xi^0$, and $p\Xi^-$, are $+$, $-$, and $+$, respectively. The real part of the energy of this pole is below the $n\Xi^0$ threshold by -3.93 MeV, but the wave function in the $n\Xi^0$ channel diverges at $r \to \infty$ and these are not bound states.

By using the $N\Xi$-$\Lambda\Lambda$ coupled-channel potentials, we have calculated the $p\Xi^-$ and $\Lambda\Lambda$ correlation functions [20]. We take account of effects of the Coulomb potential, the coupled-channels, and the threshold differences. Quantum statistics is also taken care of in the $\Lambda\Lambda$ correlation function. In Fig. 3, we compare the calculated results with the correlation function data of $p\Xi^-$ from 5.02 TeV pPb collisions [18] and $\Lambda\Lambda$ from 13 TeV pp collisions [19]. We regard the source size is a parameter, varied it in the range of 0.6 fm $\leq R \leq$ 1.6 fm, and show the results with the best fit source size, $R = 0.85$ fm for $p\Xi^-$ and $R = 1.11$ fm for $\Lambda\Lambda$. These values are smaller than that determined from the pp correlation function, but smaller source sizes for $S = -2$ pairs may be acceptable.

The $p\Xi^-$ correlation function shows a strong enhancement $C(q) \geq 2$ at small q. When we switch off the Coulomb potential, the calculated correlation function at low momentum reaches $C(0) \simeq 1.9$ with $R = 1.2$ fm [20]. Then the data implies

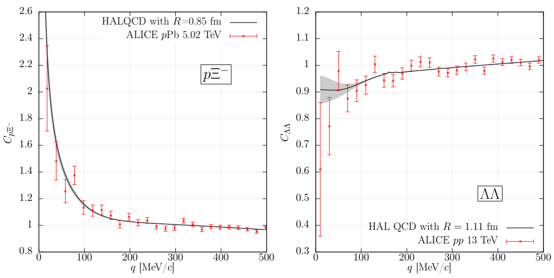

Fig. 3 Comparison of calculated $p\Xi^-$ and $\Lambda\Lambda$ correlation function and experimental data. The left panel shows the $p\Xi^-$ correlation function fitted to 5.02 TeV pPb collision data. The right panel shows the $\Lambda\Lambda$ correlation function fitted to 13 TeV pp collision data. The shaded area denotes the theoretical uncertainty of the correlation function. The ALICE data are taken from Refs. [18,19].

that the $N\Xi$ potential is attractive, and the ratio of the source size with the scattering length should not be large, say $|R/a_0| \leq 2$.

The $\Lambda\Lambda$ correlation function shows an small enhancement compared with the pure quantum statistical result, $C_{\mathrm{QS}}(q) = 1 - \lambda \exp(-4q^2R^2)/2 \simeq 0.78$ $(q \to 0)$, where λ is a pair purity parameter and we assume that $\lambda = (0.67)^2$ [22]. The small enhancement implies that the $\Lambda\Lambda$ potential is weakly attractive, as expected from the $\Lambda\Lambda$ bond energy in the double Λ hypernucleus [2] as well as the previous correlation function analyses [22,23]. We also note that there is a cusp at the $N\Xi$ thresholds, $q \simeq 180$ MeV$/c$, and there is a long tail at larger momenta. In previous analyses of the $\Lambda\Lambda$ correlation function [22–24], so-called the *residual source* was introduced to explain the long tail. In the present analysis, the long tail is found to appear naturally as a result of the coupled-channel effects.

3 Summary

We have investigated the $p\Xi^-$ and $\Lambda\Lambda$ correlation functions by using the $N\Xi$-$\Lambda\Lambda$ coupled-channel potentials from lattice QCD calculation at almost physical quark masses [8]. The calculated correlation functions are found to describe the data well. The agreement of the calculated results and the data implies that the $N\Xi$-$\Lambda\Lambda$ coupled-channel potentials from lattice QCD are reasonable. This is not trivial. For example, the boson exchange model is found to fail in describing the enhancement in the $p\Xi^-$ correlation function [25], while the baryon-baryon potentials from the chiral effective field theory seem to explain the data [16].

The strong enhancement of $p\Xi^-$ correlation function data at low momenta suggests that the $N\Xi$ potential is attractive and the absolute value of the $p\Xi^-$ scattering length would be comparable with the source size or larger, as long as the single channel Lednicky-Lyuboshits model applies. In order to deduce the sign of the scattering length, it is desired to have the correlation function data from a larger source. Recently, the RHIC-STAR collaboration measured the $p\Xi^-$ correlation function from Au+Au collisions [26]. The preliminary data seem to show enhanced correlation function both in central and mid-central collisions. Together with the ALICE data, it is plausible that there is no bound state in $N\Xi$

(except for the Ξ^- atom). These features are consistent with the lattice QCD baryon-baryon potentials, which predict the existence of the H dibaryon state as a virtual state around the $N\Xi$ thresholds.

In the $\Lambda\Lambda$ correlation function, we find that the coupled-channel calculation modifies the shape of the correlation function; the threshold cusps at $N\Xi$ thresholds and a long tail at higher momenta.

There are several comments in order. First, we have discussed only the s-wave potentials. Since the interaction range is of the order of 1 fm, effects of p-wave and higher partial waves appear in the high-momentum region, $q \geq 200$ MeV/c, where the correlation function is close to unity. Second, the $p\Xi^-$ correlation function represents the sum of squared wave functions in channels of $(I, J) = (0, 0), (0, 1), (1, 0)$ and $(1, 1)$. In order to discriminate the interactions in these channels, for example, hadron-deuteron correlation functions may be helpful [27].

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