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Brown-Rho Scaling in the Strong Coupling Lattice QCD

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We examine the Brown-Rho scaling for meson masses in the strong coupling limit of lattice QCD with one species of staggered fermion. Analytical expression of meson masses is derived at finite temperature and chemical potential. We find that meson masses are approximately proportional to the equilibrium value of the chiral condensate, which evolves as a function of temperature and chemical potential.

Keywords: Meson mass; Brown-Rho scaling; Strong Coupling Limit of Lattice QCD.

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1. Introduction

The origin of masses has been one of the major driving forces in physics. For hadrons, a large part of their masses are generated by the chiral condensate. Since the chiral condensate may vary significantly in hot and/or dense matter, hadron masses would be also modified. Brown and Rho conjectured a scaling behavior of hadron masses in dense medium,[1]

$$m_\sigma^*/m_\sigma \approx m_N^*/m_N \approx m_\rho^*/m_\rho \approx m_\omega^*/m_\omega \approx f_\pi^*/f_\pi \ . \tag{1}$$

This scaling law (referred to as the *Brown-Rho scaling*) suggests that the partial restoration of the chiral symmetry can be experimentally accessible by measuring in-medium hadron masses, and triggered many later theoretical and experimental works. Theoretically, a similar behavior is also found in the NJL model[2] and in the QCD sum rule.[3] Experimentally, enhancement of dileptons is observed below the ρ and ω meson masses in heavy-ion collisions at SPS[4] and RHIC[5] and in pA reactions.[6] The interpretation of these enhancements is still under debate, then it is important to examine the Brown-Rho scaling in QCD. In the lattice Monte-Carlo (MC) simulations, it is possible to measure hadron masses quantitatively in vacuum and at finite temperature, but it is not easy to perform MC simulations at high densities because of the sign problem. Furthermore, MC simulations with small quark masses are expensive.

In this work, we investigate meson masses in the strong coupling limit of lattice QCD (SCL-LQCD) at finite temperature and density.[7] In SCL-LQCD combined with the mean field approximation, it is possible to obtain analytical expressions of the effective potential at finite temperature (T) and quark chemical potential (μ).[8,9] Hadron masses are studied in SCL-LQCD at zero temperature,[10,11] but not at finite temperatures. We find that the Brown-Rho scaling for meson masses approximately holds at finite T and μ in SCL-LQCD with one species of staggered Fermion in the leading order of the $1/d$ expansion.

2. Meson masses in the strong coupling limit lattice QCD

In SCL ($g \to \infty$), we can ignore pure gluonic action proportional to $1/g^2$, and obtain the following partition function after integrating spatial link variables in the leading order of the $1/d$ expansion,[8,9]

$$\mathcal{Z} = \int \mathcal{D}[\chi, \bar{\chi}, U_0]\, e^{\frac{1}{2}\sum_{x,y} M(x) V_M(x,y) M(y) - S_F^{(t)} - m_0 \sum_x \bar{\chi}(x)\chi(x)} \ , \tag{2}$$

$$\begin{aligned} S_F^{(t)} &= \frac{1}{2}\sum_{\mathbf{x},n} \left[e^{\mu}\bar{\chi}(\mathbf{x},n) U_0(x)\chi(\mathbf{x},n+1) - e^{-\mu}\bar{\chi}(\mathbf{x},n+1) U_0^\dagger(x)\chi(\mathbf{x},n) \right] \\ &\equiv \frac{1}{2}\sum_{\mathbf{x},n,m} \bar{\chi}_a(\mathbf{x},n) V^{(t)}_{na,mb}(\mathbf{x}) \chi^b(\mathbf{x},m) \ , \end{aligned} \tag{3}$$

where the mesonic composite and their propagators are defined as $M(x) = \bar{\chi}_a(x)\chi^a(x)$ and $V_M(x,y) = \sum_{j=1}^{d}(\delta_{y,x+\hat{j}} + \delta_{y,x-\hat{j}})/4N_c$, and d denotes the spatial dimension. We introduce a mesonic auxiliary field (σ) through the Hubbard-Stratonovich transformation, then the action is separated into terms containing quarks and time-like link variables on the same spatial points, and it becomes possible to perform the integral over quark and time-like link variables. The effective action for σ is obtained as,

$$S[\sigma] = \frac{1}{2}\sum_{x,y}\sigma(x) V_M^{-1}(x,y)\sigma(y) + \frac{1}{T}\sum_{\mathbf{x}} V_{\text{eff}}(\mathbf{x}) \tag{4}$$

$$= \frac{L^d}{T}\mathcal{F}_{\text{eff}}(\bar{\sigma}) + \frac{1}{2}\sum_{x,y}\delta\sigma(x) G_\sigma^{-1}(x,y)\delta\sigma(y) \ , \tag{5}$$

where $N = 1/T$ and L stand for the temporal and spatial lattice sizes.

The equilibrium value $\bar{\sigma}$ is determined by the effective potential minimum, $\partial\mathcal{F}_{\text{eff}}/\partial\bar{\sigma} = 0$. In order to obtain the inverse propagator G_σ^{-1}, we need to know the interaction term, $V_{\text{eff}}(\mathbf{x})$, as a functional of $\sigma(\mathbf{x},n)$. Fäldt and Petersson showed that V_{eff} is obtained as a function of $X_N[\sigma]$, which is a functional of $\sigma_n = \sigma(\mathbf{x},n)$ [9],

$$\begin{aligned} e^{-V_{\text{eff}}/T} &= \int \mathcal{D}U_0\, \mathrm{Det}^{(NN_c)} \left[V^{(t)}_{na,mb} + 2(m_0+\sigma_n)\delta_{nm} \right] \\ &= \int dU_0\, \mathrm{Det}^{(N_c)} \left[X_N[\sigma]\otimes \mathbf{1}_c + e^{-\mu/T}U_0^\dagger + (-1)^N e^{\mu/T} U_0 \right] \ , \end{aligned} \tag{6}$$

where $\mathrm{Det}^{(n)}$ denotes the $n \times n$ determinant, and the temporal gauge $U_0 = \mathrm{diag}(e^{i\theta_1}, \cdots, e^{i\theta_{N_c}})$ is adopted in the second line. $X_N[\sigma]$ is given as,

$$X_N = \begin{vmatrix} I_1 & e^{\mu} & 0 & \cdots & & e^{-\mu} \\ -e^{-\mu} & I_2 & e^{\mu} & & & 0 \\ 0 & -e^{-\mu} & I_3 & & & 0 \\ \vdots & & & \ddots & & \\ & & & & I_{N-1} & e^{\mu} \\ -e^{\mu} & 0 & 0 & \cdots & -e^{-\mu} & I_N \end{vmatrix} - \left[e^{-\mu/T} + (-1)^N e^{\mu/T} \right] . \tag{7}$$

where $I_k = 2(\sigma_k + m_0)$. Since X_N is expressed in an explicit determinant form, its derivatives by σ_n are also given in the determinant of smaller matrices. In obtaining the meson propagator, it is enough to evaluate the U_0 integral and determinants in equilibrium,[8,9] and these are given as follows,

$$X_N = e^{E/T} + (-1)^N e^{-E/T} , \tag{8}$$

$$V_{\rm eff} = -T \log \left[2\cosh(N_c\mu/T) + \frac{\sinh[(N_c+1)E/T]}{\sinh[E/T]} \right] , \tag{9}$$

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial\sigma_n \partial\sigma_{n+k}}\right|_{\sigma=\bar\sigma} = \left[\frac{dV_{\rm eff}}{dX_N} \frac{\partial^2 X_N}{\partial\sigma_n\partial\sigma_{n+k}} + \frac{d^2 V_{\rm eff}}{dX_N^2} \frac{1}{N^2} \left[\frac{dX_N^{(0)}}{d\bar\sigma} \right]^2 \right]_{\sigma=\bar\sigma} , \tag{10}$$

$$\left.\frac{\partial^2 X_N}{\partial\sigma_n \partial\sigma_{n+k}}\right|_{\sigma=\bar\sigma} = \frac{2}{\cosh^2 E} \left[\cosh NE - e^{i\pi k} \cosh[(N-2k)E] \right] , \tag{11}$$

where $E = \mathrm{arcsinh}\,[\bar\sigma + m_0]$ denotes the one-dimensional quark energy. By requiring null average fluctuation, $\sum_k \delta\sigma_k = 0$, we ignore those terms independent from k in the derivative of $V_{\rm eff}$. The Fourier transform of the inverse propagator, $\widetilde{G}_\sigma^{-1} \equiv \mathrm{F.T.}(G_\sigma^{-1})$, is then given as,

$$\widetilde{G}_\sigma^{-1}(\mathbf{k},\omega) = \frac{2N_c}{\kappa(\mathbf{k})} - \frac{\partial V_{\rm eff}(\bar\sigma, T, \mu)}{\partial\bar\sigma} \frac{2\sinh E(\bar\sigma)}{\cos\omega + \cosh 2E(\bar\sigma)} , \tag{12}$$

where $\kappa(\mathbf{k}) = \sum_j \cos k_j$.

The meson propagator $\widetilde{G}_\sigma$ depends on T and μ as well as on $\bar\sigma$ via the interaction term $V_{\rm eff}(\bar\sigma, T, \mu)$. However, the equilibrium condition $\partial\mathcal{F}_{\rm eff}/\partial\bar\sigma = 0$ with $\mathcal{F}_{\rm eff} = N_c\bar\sigma^2/d + V_{\rm eff}$ reads $\partial V_{\rm eff}/\partial\bar\sigma = -2N_c\bar\sigma/d$, and removes the explicit T and μ dependence,

$$\widetilde{G}_\sigma^{-1}(\mathbf{k},\omega) = \frac{2N_c}{\kappa(\mathbf{k})} + \frac{2N_c\bar\sigma}{d} \frac{2\sinh E(\bar\sigma)}{\cos\omega + \cosh 2E(\bar\sigma)} . \tag{13}$$

As a result, the meson propagator depends on T and μ only through the equilibrium value of the chiral condensate, $\bar\sigma(T,\mu)$.

Meson masses are obtained as the pole energy $\omega = iM + \delta_\pi$ of the propagator at *zero* momentum, $\mathrm{k}_j = \delta_\pi$, where $\delta_\pi = 0$ or π.[11] This appears from the taste degrees

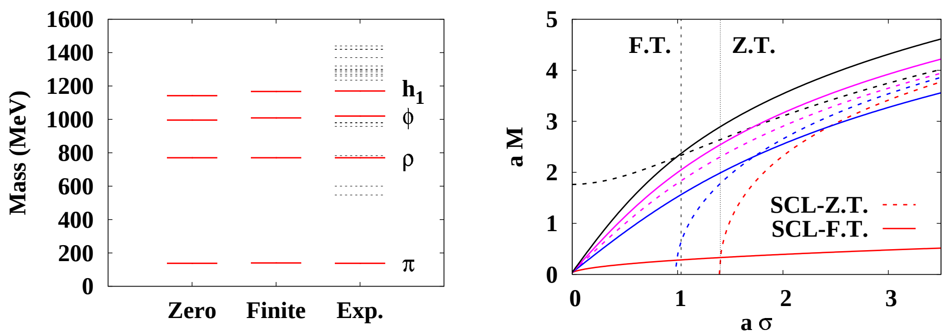

Fig. 1. Meson mass spectrum in vacuum (left) and meson masses as functions of σ (right) in the zero temperature (Z.T.) and finite temperature (F.T.) treatment in the strong coupling limit of lattice QCD.

of freedom. With the choice of $\omega = iM + \pi$, the masses are found to be

$$M = 2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_0)\left(\frac{\kappa+d}{d}\bar{\sigma}+m_0\right)}\ , \tag{14}$$

where $\kappa = -d, -d+2, \ldots, d$.

In the chiral limit ($m_0 = 0$), we always have a massless boson for $\kappa = -d$, as a consequence of the chiral symmetry in the present effective potential. For a small current quark mass, we find that $M(\kappa = -d) \approx 2\sqrt{\bar{\sigma} m_0}$, which may be regarded as the PCAC relation. Thus we regard the mode $\kappa = -d$ as the pion. We tentatively assign $\kappa = -1$ corresponds to ρ meson,[11] then we can fix the physical scale, $a^{-1} = 497$ MeV and $m_0 = 9.5$ MeV by fitting π and ρ meson masses in vacuum. With these parameters and the present assignment, $M(\kappa = 1, 3)$ seems to correspond to ϕ and h_1 mesons as shown in the left panel of Fig. 1. Contribution from the chiral condensate $\bar{\sigma}$ to meson masses (except for π) is found to be much larger than that from the current quark mass m_0 in vacuum, where $\bar{\sigma}_{\mathrm{vac}} \sim a^{-1}$.

In the right panel of Fig. 1, we show meson masses as functions of $\bar{\sigma}$. In the range $\bar{\sigma} \leq \bar{\sigma}_{\mathrm{vac}}$, meson masses with $\kappa = -1, 1, 3$ are approximately proportional to $\bar{\sigma}$. In the present finite temperature (F.T.) treatment, $\bar{\sigma}$ evolves as a function of temperature and density. As a result, meson masses are also modified in hot and/or dense matter, as shown in Fig. 2. These results should be compared with those in the zero temperature (Z.T.) treatment,[11]

$$\cosh M = 2(\bar{\sigma}+m_0)^2 + \kappa\ . \tag{15}$$

In vacuum, these meson masses explain observed the observed mass spectrum, but there is no (T, μ) dependence.

3. Summary and Discussion

In this work, we have examined the Brown-Rho scaling for meson masses in the strong coupling limit of lattice QCD with one species of staggered Fermion at finite

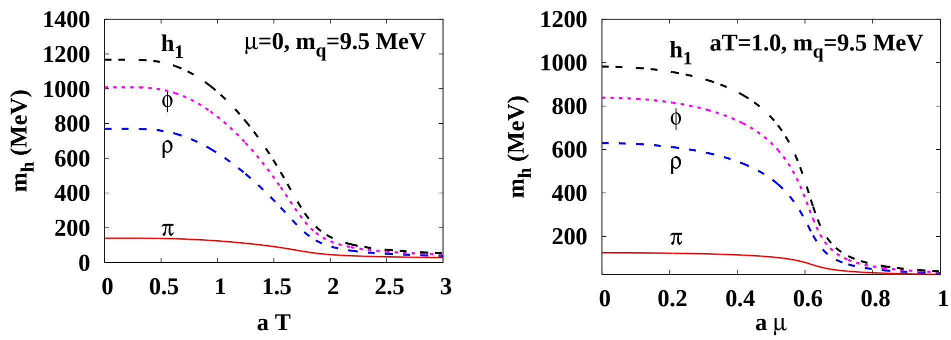

Fig. 2. Temperature (left) and chemical potential (right) dependence of meson masses.

temperature and chemical potential. Meson masses except for π are found to be approximately proportional to the equilibrium value of the chiral condensate, $\bar{\sigma}$. Since the condensate mode $(\omega, \mathbf{k}) = (0, \mathbf{0})$ corresponds to the chiral partner of π, we may assume that $\bar{\sigma}$ is proportional to the pion decay constant in medium, f_π^*. Under this assumption, we may conclude that the Brown-Rho scaling would hold in the strong coupling limit of QCD.

There are many more things to be clarified including the meson assignment as pointed out in the symposium, pion mass behavior around the chiral transition which is considered to grow in medium,[2] meson masses with the choice of $\omega = iM$, too high T_c in SCL,[12] and negative eigen values in V_M.[13]

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