SU(3) chiral linear σ model for positive and negative parity baryons in dense matter

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An SU(3) chirally symmetric linear sigma model for positive and negative parity spin-half baryons is developed. Ground state baryons are represented by a linear combination of two baryon fields, then we can construct two types of BBM couplings with mass dimension four. This model well reproduce baryon and meson masses at vacuum, but the EOS of symmetric matter is calculated to be too stiff as in the case of the SU(2) model.

1. Introduction

Strangeness is expected to play essential roles in dense matter. The possibility of hyperon admixture in neutron stars was pointed out from the beginning before the discovery of neutron stars, and the strange quarks change the order of the QCD phase transition at zero baryon density. Furthermore, we have had many interesting reports suggesting cold and dense matter formation with anti-kaons in this conference.

Although recent developments in hypernuclear physics have revealed various properties of bare and effective YN and YY interactions, we do not have enough information on nuclear matter EOS at high densities with strangeness admixture yet. Even at $\rho \sim \rho_0$, recently measured (π^-, K^+) spectra can be reproduced either with very repulsive or almost zero Σ^- potential. At higher densities, theoretical and experimental ambiguities are much larger. In order to reduce uncertainties in the EOS model including hyperons, it would be useful to invoke chiral SU(3) symmetry in addition to the flavor SU(3) symmetry.

Chiral SU(2) linear sigma model has been widely used as a basic model of pions and nucleons. In applying to nuclear matter, however, we have several problems. First, a naive model leads to a sudden change of chiral condensate from around $\sigma \sim f_\pi$ to 0 at a density below ρ_0. This abnormal vacuum problem was solved by Boguta [1] by introducing the $\sigma\omega$ coupling. Second, chiral linear sigma models generally give too stiff EOS. There are some prescriptions proposed so far, such as the loop effects [2] and higher order terms in σ [3], but all of these may not be conclusive. For example, since the double well structure of the effective potential is considered to be generated by the loop effects of quarks in the NJL model, we should take special care to include the loops of mesons in the model. If the model is extended to SU(3), we may have some hope to soften the EOS because of new degrees of freedom in meson and baryon sector.

In this work, we develop an SU(3) chirally symmetric linear sigma model for positive and negative parity spin-half baryons. By introducing two-types of chiral SU(3) symmetric MB couplings [4,5], we can construct a Lagrangian in the mass dimension four which reproduces the vacuum masses of positive and negative parity baryons as well as scalar and pseudoscalar mesons reasonably. Next we include vector mesons, and we discuss the properties of high density matter.

2. SU(3) Chiral Sigma Model with Spin 1/2 Baryons

Linear sigma model is one of the traditional and simple model which respects the chiral symmetry. Nucleon masses are generated by the spontaneous breaking of the chiral symmetry, and the chiral condensate evolves as the nucleon density increases. Thus it is expected to be an appropriate starting model to describe nuclear matter properties.

In SU(2), the mesons and baryons transform under the left (L) and right (R) chiral rotation as,

$$\mathrm{SU(2)}: \quad N_L \to LN_L, \quad N_R \to RN_R, \quad M \to LMR^\dagger, \tag{1}$$

then the chiral invariant meson-baryon coupling is given by $\overline{N}_L M N_R + (h.c.)$. In SU(3), while the meson field $M_{ij} = q_{Li}\overline{q}_{Rj}$ forms an adjoint representation $(3, 3^*)$ and transforms as in the SU(2) case, the baryon representation is not unique. When baryons are constructed with a flavor asymmetric diquark pair and a quark, there are two possibilities in the spin 1/2 baryon field operators without derivatives [4],

$$B^{(1)}_{dc} = N^{-3}(q^T_{i,a} C\gamma_5 q_{j,b}) q_{k,d}\epsilon_{ijk}\epsilon_{abc} \ , \qquad B^{(2)}_{dc} = N^{-3}(q^T_{i,a} C q_{j,b}) q_{k,d}\epsilon_{ijk}\epsilon_{abc} \ . \tag{2}$$

Combination of these two fields gives following two baryons which transform homogeneously under chiral transformation,

$$\Psi^{(1)} = (B^{(1)} + \gamma_5 B^{(2)})/\sqrt{2} \ , \quad \Psi^{(1)}_L \to L\Psi^{(1)}_L L^\dagger \quad (8,1), \quad \Psi^{(1)}_R \to R\Psi^{(1)}_R R^\dagger \quad (1,8), \tag{3}$$

$$\Psi^{(2)} = (B^{(1)} - \gamma_5 B^{(2)})/\sqrt{2} \ , \quad \Psi^{(2)}_L \to L\Psi^{(2)}_L R^\dagger \ \ (3,3^*), \quad \Psi^{(2)}_R \to R\Psi^{(2)}_R L^\dagger \ \ (3^*,3). \tag{4}$$

Generally, the ground state baryon octet is assigned to $\Psi^{(1)}$ as in Ref. [6]. With this choice, the lowest dimension (dimension four) BBM coupling is not allowed, then the understanding of baryon mass becomes different from that in the SU(2) case. On the other hand, $\Psi^{(2)}$ can have the dimension four coupling with mesons [5], but only the D-type coupling appears.

In this work, we assume that the ground state baryon octet is realized as a linear combination of the two baryon fields following the work by Christos [4]. Under this assumption, we necessarily treat positive and negative parity spin 1/2 baryons simultaneously, and we have following two chiral invariant interactions with dimension four [4,5],

$$\mathcal{L}^{\mathrm{Tr}} = -\sqrt{2} g_{\mathrm{tr}} \, \mathrm{Tr}\left(\overline{\Psi}^{(1)}_L M \Psi^{(2)}_R + \overline{\Psi}^{(1)}_R M^\dagger \Psi^{(2)}_L\right) + h.c. \ , \tag{5}$$

$$\mathcal{L}^{\mathrm{Det}} = -\sqrt{2} g_{\mathrm{det}} \mathrm{Det}'\left(\overline{\Psi}^{(2)}_R, M, \Psi^{(2)}_L\right) + h.c. \ , \tag{6}$$

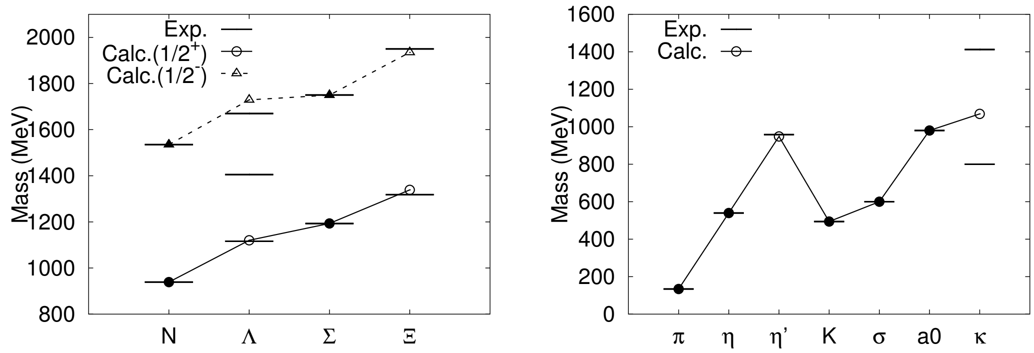

Figure 1. Baryon (left) and meson (right) mass spectrum. Bars show the experimental data, and points connected with lines show the calculated masses. Filled marks indicate fitted masses.

where $\mathrm{Det}'(A,B,C) \equiv \epsilon_{ijk}\epsilon_{lmn}A_{il}B_{jm}C_{kn}$. We have included the explicit strange quark mass term, which breaks both the chiral and flavor SU(3) symmetry, in $B^{(1,2)}$ fields. In the mean field approximation, the baryon Lagrangian is given by,

$$\mathcal{L}_B^{\mathrm{MF}} = \sum_i \left(\overline{\Psi}_i^{(1)}, \overline{\Psi}_i^{(2)}\right) \begin{pmatrix} i\partial_\mu\gamma^\mu & -g_{\mathrm{tr}}\sigma_i - n_i^s m_s \\ -g_{\mathrm{tr}}\sigma_i - n_i^s m_s & i\partial_\mu\gamma^\mu + g_{\mathrm{det}}\sigma_i' \end{pmatrix} \begin{pmatrix} \overline{\Psi}_i^{(1)} \\ \overline{\Psi}_i^{(2)} \end{pmatrix} , \tag{7}$$

where $i = 1 \sim 8$, σ_i and σ_i' are the combinations of chiral condensates corresponding to the i-th baryon. The meson matrix is replaced to its expectation value, $\langle M \rangle = \mathrm{diag}\left(\sigma/\sqrt{2}, \sigma/\sqrt{2}, \zeta\right)$. By diagonalizing the mass matrix, we can obtain the baryon mass spectrum. There are three independent parameters in the baryon sector, and we can reproduce the baryon mass spectrum reasonably well, as shown in the left panel of Fig. 1.

3. Application to Symmetric Nuclear Matter

Now we apply the meson-baryon coupling described in the previous section to the study of EOS of symmetric nuclear matter within the relativistic mean field (RMF) approximation. The mean field Lagrangian in symmetric nuclear matter is given by,

$$\mathcal{L}^{\mathrm{MF}} = \mathcal{L}_B^{\mathrm{MF}} + \mathcal{L}_M^{\mathrm{MF}} + \mathcal{L}_V^{\mathrm{MF}} , \tag{8}$$

$$\mathcal{L}_M^{\mathrm{MF}} = \frac{\mu^2}{2}(\sigma^2+\zeta^2) - \frac{\lambda}{4}(\sigma^2+\zeta^2)^2 - \frac{\lambda'}{4}(\sigma^4+2\zeta^4) + c\sigma^2\zeta - \nu\zeta^2 + H_\sigma\sigma + H_\zeta\zeta , \tag{9}$$

$$\mathcal{L}_V^{\mathrm{MF}} = g_{\mathrm{BV}}\omega\rho_B + \frac{\lambda_{\mathrm{VS}}}{2}\sigma^2\omega^2 . \tag{10}$$

We have seven free parameters in the pseudoscalar-scalar meson Lagrangian $\mathcal{L}_M^{\mathrm{MF}}$. All of these parameters except for one (sigma meson mass) are fixed by fitting the pion and kaon decay constants, and meson masses as shown in the right panel of Fig. 1. For vector mesons, we employ the Boguta's scenario [1], i.e. the dynamical mass generation of isoscalar vector meson ω. There are two types of vector meson coupling with baryons, but only the sum of those coupling constants is meaningful in symmetric nuclear matter. We denote this sum as a free parameter g_{BV}.

We have carried out the self-consistent calculation of σ, ζ, ω and the nucleon scalar density for given baryon densities. These values evolves smoothly due to the $\sigma\omega$ coupling term. We have found that, with an empirical value $m_\sigma = 600$ MeV, we can reproduce neither the incompressibility nor the binding energy. Increasing the sigma mass, the incompressibility and the E/A changes to the right direction. We can reproduce the empirical value of $E/A \sim -16$ MeV with $m_\sigma \sim 800$ MeV as shown in Fig. 2, but the incompressibility is calculated to be too large, $K > 600$ MeV.

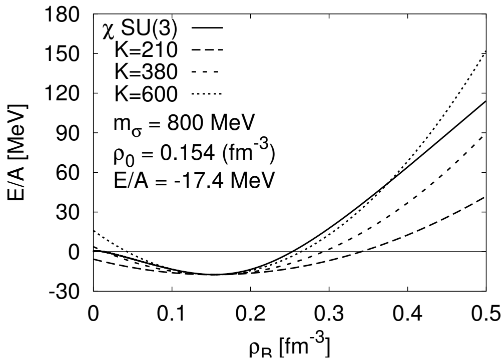

Figure 2: EOS of symmetric nuclear matter.

In the present model, the difference from the SU(2) case lies in the modification of ζ, representing the strange quark condensate, $\langle \bar{s}s \rangle$. However, since ζ does not couple to nucleons directly, the deviation from the vacuum value is not large. As a result, we have obtained similar results to those in the SU(2) model.

4. Summary

In this work, we have developed an SU(3) chirally symmetric linear sigma model with positive and negative parity spin 1/2 baryons, and applied this model to the study of symmetric nuclear matter. In order to construct the lowest dimension (dimension four) BBM coupling having both of F and D type, we need to incorporate two types of baryons, $\Psi^{(1)}$ and $\Psi^{(2)}$. With the explicit strange quark mass term, we can reproduce the baryon and meson spectrum reasonably well, except for $\Lambda(1405)$.

In the application to symmetric nuclear matter, vector meson mass is assumed to be generated by the sigma meson, and the ω-nucleon coupling constant g_{BV} and the sigma meson mass m_σ are taken as free parameters. We have found that the EOS becomes too stiff as in the case of the SU(2) model. Inclusion of other terms such as full octet vector mesons which couple to scalar mesons, higher dimension coupling terms $BMBM$ or M^6, M^8, or loop effects may help this problem. Works in these directions are in progress.

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