



# Three-body couplings in RMF and its effects on hyperonic star equation of state<sup>☆</sup>

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## Abstract

We develop a new relativistic mean field (RMF) model including explicit three-body couplings and apply it to hyperonic systems and neutron star matter. The recent observation of the massive mass neutron star has given us a puzzle; when strange hadrons including hyperons are taken into account, the equation of state (EOS) becomes too soft to support the observed two-solar-mass neutron star. Three-baryon repulsion is a promising ingredient to answer this puzzle. We demonstrate that it is possible to consistently explain the massive neutron star and hypernuclear data when we include three-body couplings and modify the hyperon vector couplings from the flavor SU(3) values.

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## 1. Introduction

In constructing dense matter equations of state (EOSs), it is strongly desired to respect hypernuclear data; hyperons are expected to emerge in the neutron star core, and they drastically soften the dense matter EOS. With hyperons, the maximum mass of neutron stars is usually calculated to be  $(1.3 - 1.6)M_{\odot}$ , while nucleonic EOSs without hyperons predict  $(1.5 - 2.7)M_{\odot}$  [1]. Contrary to these understandings, a two-solar-mass neutron star ( $M = 1.97 \pm 0.04M_{\odot}$ ) is discovered recently by using the Shapiro delay, and most of the EOSs including strange hadrons are claimed to be ruled out [2]. The Shapiro delay is a consequence of the general relativity, and we need to respect this observation seriously. When hyperons are introduced into nucleonic EOSs compatible with the massive neutron star, it is also questionable even for them to explain the recent observation. Thus we need to find either the reason why hyperons do not appear in dense neutron star matter or the mechanism how EOSs can be stiff enough even with hyperons.

One of the possible mechanisms to make the EOS stiffer is the three-baryon repulsion. In microscopic g-matrix calculations, the three-baryon repulsion is found to be necessary to support the  $1.44 M_{\odot}$  neutron star when hyperons are included [3]. In relativistic mean field (RMF) models, attractive contribution from the scalar potential grows more slowly than the baryon density, and in terms of non-relativistic languages, this behavior can be interpreted as the *implicit* three-body repulsion caused by relativistic treatments. This relativistic effect had been considered to be enough to support neutron stars even if hyperons are taken into account, until the two-solar-mass neutron star was

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discovered. When hyperon-meson couplings are chosen away from the SU(6) values and the  $\omega$  meson self-energy is ignored, the calculated neutron star maximum mass can be compatible with the observed massive neutron star [4]. However, these hyperon-meson couplings have not been seriously verified in finite hypernuclear systems, and the density dependence of the vector potential in these treatments would not be compatible with the relativistic Brückner-Hartree-Fock (RBHF) calculation [5], whose results ensure that RMF models are reasonable. Thus the most natural way to make the EOS with hyperons stiff enough would be to introduce *explicit* three-body repulsion.

In this work, we examine how three-body couplings affect the neutron star matter EOS in the framework of RMF model. Specifically, we include the interaction terms of baryon-meson-meson (*BMM*) and three-meson (*MMM*) couplings. Two mesons in the *BMM* term couple with two other baryons, and three mesons in the *MMM* term couple with three baryons. Thus *BMM* and *MMM* couplings correspond to the explicit three-body forces. Each term in the RMF Lagrangian can be characterized by the number  $n = B/2 + M + D$ , where  $B$  is the number of baryon fields,  $M$  is the number of non-Goldstone boson fields, and  $D$  is the number of derivatives in each term. The baryon-meson coupling terms  $\bar{\psi}M\psi$  belong to  $n = 2$  hierarchy, and the present three-body coupling terms correspond to  $n = 3$  one. Thus, they are assigned to the next-to-leading order interactions in the expansion in  $n$  [6]. While some of the  $n = 3$  terms are considered to be absorbed by field redefinitions [6], we need to modify  $n = 4$  terms to compensate the redefinitions. We demonstrate the importance of  $n = 3$  terms, especially the *BMM* terms, on the dense matter EOS.

## 2. Model description

We adopt here an RMF Lagrangian,  $\mathcal{L}_{\text{RMF}} = \mathcal{L}_{n=2} + V_{\text{eff}} + \mathcal{L}_{n=3}$ , where  $\mathcal{L}_{n=2}$  corresponds to the ordinary RMF Lagrangian including  $n = 2$  two-body couplings, and  $V_{\text{eff}}$  represents the meson self-energies, which include a logarithmic potential of scalar-isoscalar mesons and  $\omega^4$  potential [7]. For the three-body coupling terms, we first consider symmetric nuclear matter, where  $\sigma$  and  $\omega$  mesons are involved. Here we assume that nucleons do not couple with  $\bar{s}s$  mesons ( $\zeta$  and  $\phi$  for scalar and vector mesons, respectively), as usually assumed. Three-body interaction terms,  $\mathcal{L}_{n=3}$ , for symmetric nuclear matter are taken to be

$$\mathcal{L}_{n=3}^{\sigma\omega} = -\frac{1}{f_\pi} \sum_B \bar{\psi}_B \left[ g_{\sigma\sigma B} \sigma^2 + g_{\omega\omega B} \omega_\mu \omega^\mu - g_{\sigma\omega B} \sigma \omega_\mu \gamma^\mu \right] \psi_B - c_{\sigma\omega} f_\pi \sigma \omega_\mu \omega^\mu. \quad (1)$$

The first and second terms modify the effective mass of nucleons,  $M_N^* = M_N - (g_{\sigma N} \sigma - g_{\sigma\sigma N} \sigma^2 / f_\pi - g_{\omega\omega N} \omega^2 / f_\pi)$ , where  $\omega$  represents the temporal component. These  $n = 3$  terms help to weaken the effective mass reduction from the  $n = 2$  coupling with  $\sigma$ . The third term modifies the vector potential of nucleons at high density,  $U_v = (g_{\omega N} - g_{\sigma\omega N} \sigma / f_\pi) \omega$ , and the fourth term represents the  $\omega$  meson mass shift at finite density. In RBHF calculations, the vector potential is almost proportional to the baryon density at low densities, but the vector potential to baryon density ratio  $U_v / \rho_B$  is calculated to be suppressed around  $\rho_0$  or at higher densities. This suppression in RBHF is sometimes simulated by the  $\omega^4$  term [8] or by the density dependent coupling [9]. When we simulate the suppression of  $U_v / \rho_B$  only with the  $\omega^4$  term, it is calculated to be monotonically decreasing with increasing  $\rho_B$ . With a large coefficient of  $\omega^4$ , EOS at high density is thus too softened to support the massive neutron star [7]. The density dependent vector coupling is usually chosen to decrease and be saturated with increasing  $\rho_B$ . With the present  $n = 3$  coupling terms in Eq. (1) in addition to the  $\omega^4$  term, we try to simulate the decreasing and saturating behavior of the  $U_v / \rho_B$  ratio found in the RBHF results.

For the massive neutron star puzzle, hyperon-meson coupling is another essential ingredient. In many of the RMF parameter sets, the hyperon- and nucleon-vector meson coupling ratio is chosen to be  $R = g_{\omega Y} / g_{\omega N} \simeq 2/3$  ( $Y = \Lambda, \Sigma$ ) based on the spin-flavor SU(6) symmetry or the quark counting arguments. This choice is the main reason why we cannot support the massive neutron star with hyperons. Mesons in RMF models describe scalar and vector potentials coming from various origins; pion exchanges, correlation from two-baryon short range repulsion, meson pair exchanges, and so on, in addition to the meson fields consisting of  $\bar{q}q$ . Thus it is not mandatory to impose the flavor SU(3) relations among the coupling constants. In our previous work [7], we have adopted a more phenomenological prescription; while the hyperon-isoscalar vector couplings ( $g_{\omega Y}, g_{\phi Y}$ ) have been chosen to be the flavor SU(3) values, other couplings in the  $S = -1$  hyperon sector ( $g_{\sigma Y}, g_{\zeta Y}, g_{\rho\Sigma}$ ) have been fitted to the hypernuclear data, including the potential depth in symmetric matter around  $\rho_0$ , the bond energy of the double  $\Lambda$  hypernuclei, and the  $\Sigma^-$  atomic shift data. It should be noted that we need to adopt the  $\Sigma$ - $\rho$  coupling constant much smaller than the flavor SU(3) value, in order to fit the  $\Sigma^-$  atomic shift data [7, 10]. We here explore the results using the hyperon- $\omega$  coupling value other than the SU(3) values. Specifically, we examine the results with  $R = 0.8$  in the later discussion.

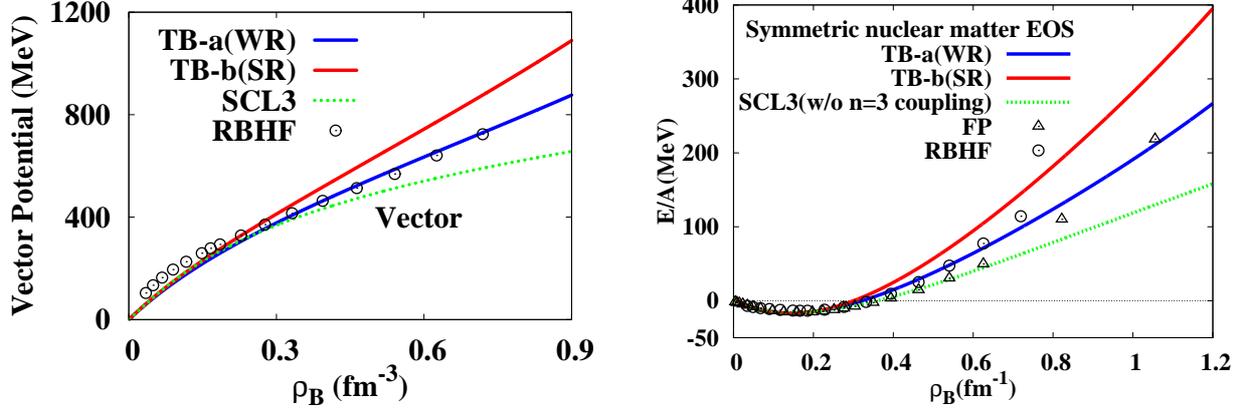
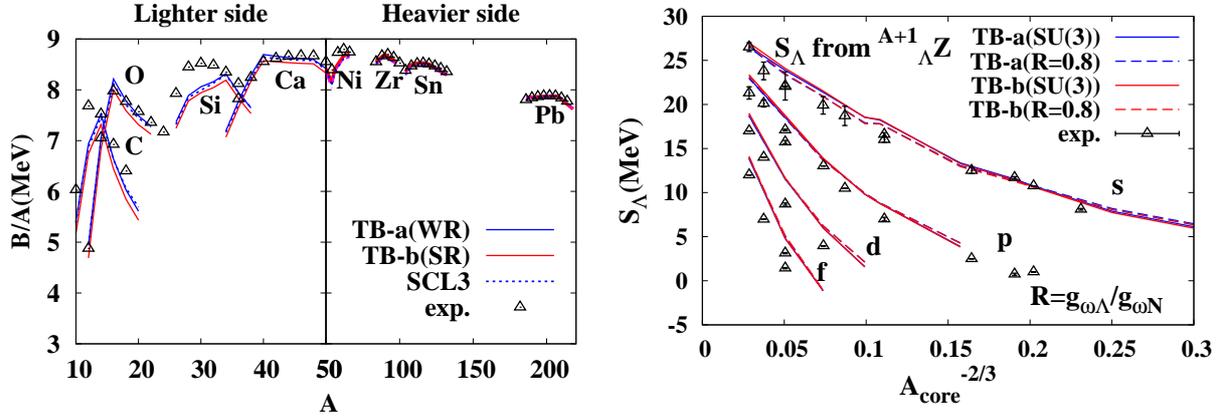


Figure 1. Calculated vector potential and EOS in symmetric nuclear matter with TB-a and TB-b parameter set in comparison with SCL3 results.

Figure 2. Calculated binding energies and  $\Lambda$  separation energies ( $S_{\Lambda}$ s) based on TB-a and TB-b parameter sets.

### 3. Results

We prepare two different three-body coupling parameter sets, TB-a and TB-b, based on our previous RMF model (SCL3 RMF model), with which we can describe known properties of finite and infinite nuclear systems. Three-body couplings in TB-a are determined so as to reproduce the density dependence of the vector potential in RBHF at high densities, and the ones in TB-b are chosen to give more repulsive potential as shown in the left panel of Fig. 1.  $n = 2$  couplings are modified to reproduce the saturation point. Compared to symmetric nuclear matter EOS in SCL3, EOSs in TB-a and TB-b are obviously stiffened especially at high  $\rho_B$  as shown in the right panel of Fig. 1. We find that TB-a EOS is in good agreement with RBHF EOS and stiffer than FP EOS [11].

Isovector  $n = 2$  coupling and  $\Lambda$ -meson couplings are obtained by fitting finite nuclear binding energies and  $S_{\Lambda}$ s. As shown in Fig. 2, we can describe these observables reasonably well, except for the binding energies of light  $jj$  closed nuclei. We may introduce the tensor coupling of vector mesons to obtain a larger  $ls$  potential, but the tensor coupling does not modify the uniform matter EOS in the mean field treatment. When we modify the vector coupling ratio  $R$  from the flavor SU(3) value to 0.8, both scalar and vector potentials become stronger, and  $S_{\Lambda}$ s show slightly different trends as shown in the right panel of Fig. 2. Thus, the modification of  $R$  may be examined by measuring  $\Lambda$  hypernuclear systems more precisely.

In Fig. 3, we show the neutron star matter EOS (NS EOS) obtained by using the parameter sets determined as described above. By including  $n = 3$  three-body couplings, we can obtain stiffer NS EOSs without spoiling the

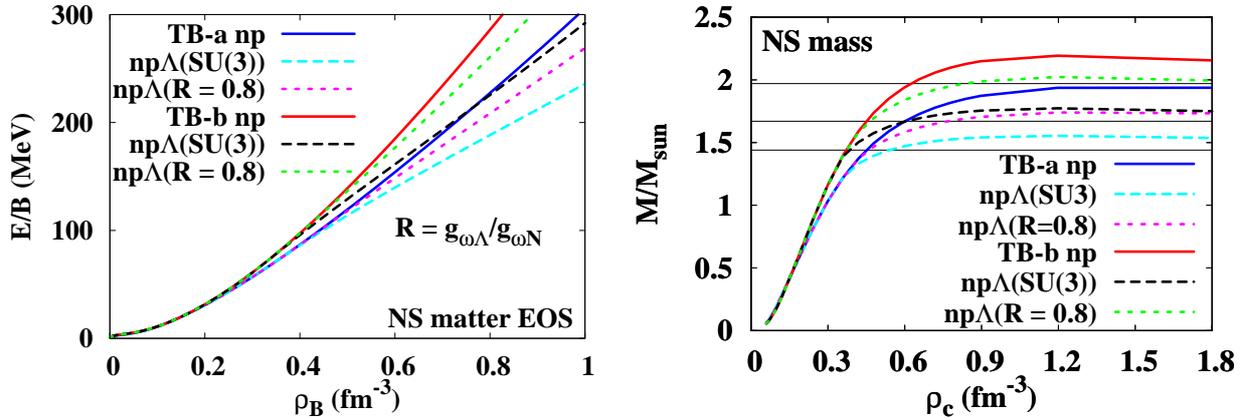


Figure 3. Neutron star matter EOSs as functions of  $\rho_B$  and neutron star maximum masses as functions of the central density.

nuclear matter saturation and finite nuclear properties. We also plot the mass-central density curve in the right panel of Fig. 3. We find that the calculated maximum mass can exceed  $1.97M_{\odot}$  by adopting repulsive three-body and hyperon-vector couplings (TB-b and  $R = 0.8$ ), while TB-a seems to be difficult to support the massive neutron star.

#### 4. Summary and conclusion

We have examined three-body couplings ( $n = 3$ ) in the framework of the relativistic mean field (RMF) model. We can obtain stiffer EOSs at high densities while keeping the nuclear matter properties around  $\rho_0$  and finite nuclei. We have also demonstrated that finite hypernuclear properties are reasonably well described with a modified  $g_{\omega\Lambda}$  from the flavor SU(3) value. By these two modifications in the RMF Lagrangian, we can obtain the neutron star matter EOSs, which support the recently observed two-solar-mass neutron star. These findings indicate that it would be possible to answer the massive neutron star puzzle; we can explain recently observed massive neutron star even if we respect hypernuclear data. The two ingredients discussed here — three-body coupling and modification of the hyperon-vector meson coupling — may afford a key to understand the maximum mass of neutron stars. More careful tuning of parameters and other interaction terms may be necessary for a more satisfactory description of finite normal nuclei and hypernuclei. Isovector part of  $n = 3$  couplings should be also investigated, and will be reported elsewhere.

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