Hadron mass spectrum in strong coupling limit of lattice QCD at finite temperature and density for color SU(3)

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We investigate the thermodynamical evolution of hadron masses in the strong coupling limit of lattice QCD (SCL-LQCD) with one species of staggered fermion for color SU(N_c), including $N_c = 3$. We directly derive Brown-Rho scaling for the meson masses in SCL-LQCD. We propose a mechanism of the "discrepancy" between the baryon mass and baryon critical chemical potential in the strong coupling.

§1. Introduction

Exploring the chiral phase transitions in Quantum Chromo-dynamics (QCD) at finite temperature (T) and finite quark chemical potential (μ) is one of the most interesting problems in high energy physics. The *Strong Coupling Limit of Lattice QCD (SCL-LQCD)* provides an analytic and instructive framework for the chiral phase transition, and has a long history of study.

One of the interesting problems at finite T and μ is the meson mass modification in medium. A significant enhancement in the low-mass side of the vector meson peaks over the known hadronic sources has been observed.[1)] Effective model studies[2)–4)] indicate that such a mass shift may be a signal of the partial chiral symmetry restoration, while the interpretation is still under debate. Here an interesting question is raised: Can we describe meson mass modification in SCL-LQCD ?

Another interesting problem, *Baryon Mass Puzzle*, is raised by the Monte-Carlo (MC) group.[5)] Naively, the chiral phase transition is expected to occur when a baryon chemical potential $N_c\mu$ approaches to the baryon mass M_B, which indicates a relation $N_c\mu_c \simeq M_B$.[6)] On the other hand, previous SCL-LQCD studies show the discrepancy, $N_c\mu_c \neq M_B$. This discrepancy has been recently confirmed in the MC study. The MC result for M_B at strong coupling[5)] is clearly different from $N_c\mu_c$ obtained in SCL-LQCD. Since SCL-LQCD should be consistent with MC in the strong coupling region, the discrepancy is a crucial problem and should be seriously investigated.

In this proceedings, we investigate meson[8)] and baryon masses in SCL-LQCD, and attack the problems mentioned above

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§2. Meson mass spectrum

The starting point is the lattice QCD action with one species of staggered fermion (χ). In the strong coupling limit ($g \to \infty$), the plaquette terms ($\propto 1/g^2$) becomes negligible, and then the action is,

$$S_{LQCD} \to \frac{1}{2}\sum_x \Big[\big(\eta_{\nu,x} \bar{\chi}_x U_{\nu,x} \chi_{x+\hat{j}} - \eta^{-1}_{\nu,x}(h.c) \big) + 2m_0 (\bar{\chi}\chi)_x \Big] \ . \tag{2·1}$$

where U_ν represents the link variable, and $\eta_{\nu,x}$ denotes the lattice chemical potential μ[9] and the Kogut-Susskind sign factor, $(\eta_{0,x}, \eta_{j,x}) = (e^\mu, (-1)^{x_0+\cdots+x_{j-1}})$. In the chiral limit ($m_0 \to 0$), the action has the chiral symmetry. In the meson mass derivation, we employ the finite temperature treatment,[10] where the periodic or anti-periodic nature in the temporal direction is respected and the temporal link $U_{\nu=0}$ is fixed to the specified gauge (called temporal gauge), $U_0 = \{e^{i\theta^1(\mathbf{x})}, \cdots, e^{i\theta^{N_c}(\mathbf{x})}\}$ at $\tau = N$ and $U_0 = \mathbf{1}_c$ at $\tau \neq N$. In the following, we consider the color SU($N_c = 3$) case in $3+1$ dimension ($d = 3$). Taking account of the leading order of the $1/d$ expansion[11] and utilizing the mean field approximation, an analytic expression for the effective action $S_{\mathrm{eff}}[\sigma]$ has been known.[6),7),10),12),13] The equilibrium value of chiral condensate $\bar{\sigma}$ is derived from the minimum of the effective potential ($\propto S_{\mathrm{eff}}(\bar{\sigma})$).

The meson mass is obtained as the pole energy of the mesonic propagator in equilibrium, $\sigma = \bar{\sigma}$. In the present work, we have utilized the formulation developed by Faldt *et al.*,[12] and have obtained the explicit expression of inverse propagator in the momentum space,

$$\left.\frac{S_{\mathrm{eff}}[\sigma]}{\delta\sigma_{\omega\mathbf{k}}\delta\sigma_{\omega'\mathbf{k}'}}\right|_{\sigma\to\bar{\sigma}} = \delta_{\omega\omega'}\delta_{\mathbf{k}\mathbf{k}'}\left[\frac{2N_c}{\sum_j \cos k_j} + \frac{2N_c\bar{\sigma}}{d}\frac{2\sinh E(\bar{\sigma})}{\cos\omega + \cosh 2E(\bar{\sigma})}\right] , \tag{2·2}$$

where $E(\bar{\sigma}) = \sinh^{-1}(m_0 + \bar{\sigma})$ corresponds to the quark excitation energy.

To obtain the meson mass spectrum from Eq. (2·2), we employ the Susskind interpretation[11] for the staggered flavors:[14] $(\omega, \mathbf{k}) \to (iM_m, \mathbf{0}) + \pi\delta_\nu,\ \delta_\nu = 0$ or 1. In this scheme, we find four different real valued meson masses,

$$M_m^{(\kappa)}(\bar{\sigma}(T,\mu)) = 2\ \sinh^{-1}\sqrt{(\bar{\sigma}+m_0)\left(\frac{\kappa+d}{d}\bar{\sigma}+m_0\right)} \ , \tag{2·3}$$

where $\kappa = -d, -d+2, \ldots, d,\ (d = 3)$ represents the staggered flavor.

The mode $\kappa = -d = -3$ corresponds to the chiral partner of the chiral condensate σ, and we find the PCAC relation, $M_m^{(\kappa=-3)} \approx 2\sqrt{\bar{\sigma}m_0}$ for a small current quark mass. Therefore we regard the mode $\kappa = -d$ as the pion. The identification of other modes are discussed in the previous work.[8),11] Here we concentrate on the scaling law instead of discussing details of the mode identification. In the left panel of Fig. 1, we show μ dependence of the meson masses. In this figure, all meson masses except for π are found to follow a similar scaling property. In fact, we find that these masses are approximately proportional to $\bar{\sigma}$

$$M_m^{(\kappa)}(\bar{\sigma}(T,\mu)) \approx 2\sqrt{\frac{\kappa+d}{d}}\,\bar{\sigma} \quad (\kappa = -d+2, -d+4, \ldots, d) \ , \tag{2·4}$$

in the chiral limit ($m_0 = 0$) and for small values of $\bar{\sigma}$. Namely, the Brown-Rho (BR) scaling[2] is found to hold approximately. This is originated to the fact the system is essentially governed by the quark excitation $E(\bar{\sigma}(T, \mu))$ (See, Eq.(2·2)) and does not contain $T = N^{-1}$ and μ explicitly. As a result, the meson masses depend on T and μ only through the equilibrium value of the chiral condensate $\bar{\sigma}(T, \mu)$, and BR scaling holes.

§3. Baryon mass spectrum

The baryon mass has been derived by Damgaard *et al.*[6] and by Kluberg-Stern *et al.*[11] in the zero temperature treatment, where the one link integral is performed also for the temporal links (U_0). We consider the leading and the next to leading order of the $1/d$ expansion. The former (latter) corresponds to the mesonic (baryonic) effects, and we introduce auxiliary fields $\sigma \propto \langle \chi\bar{\chi} \rangle$ and $(b, \bar{b})$, $b \sim \varepsilon\chi\chi\chi$, respectively. The baryon mass M_B can be obtained as a function of $\bar{\sigma}$ (and m_0) from the pole of the propagator of $(b, \bar{b})$ as,

$$M_B(\bar{\sigma}, m_0) = E_B(\mathbf{0}) \ , \quad E_B(\mathbf{p}) = \sinh^{-1}\left[\sqrt{\mathbf{p}^2 + \left[2(\bar{\sigma} + m_0)\right]^{2N_c}/4}\right] \ . \quad (3\cdot1)$$

Once we obtain the pole location, the baryon loop contribution for the effective potential $\mathcal{F}_{\text{eff}}$ can be evaluated by the Matsubara formulation, and we obtain,

$$\mathcal{F}_{\text{eff}}(\bar{\sigma}; m_0, \mu) = \frac{N_c\bar{\sigma}^2}{d+1} + \frac{1}{4\pi\Lambda^3/3}\int_0^{\Lambda\simeq\pi/2} dp\ 4\pi p^2 \begin{cases} -N_c\mu & (E_B \leq N_c\mu) \\ -E_B & (E_B \geq N_c\mu) \end{cases} . \quad (3\cdot2)$$

The second term represents the baryon loop effects and this indicates that $E_B(\mathbf{p})$ corresponds to the baryon excitation energy.

We have now clarified the explicit relations among E_B, $N_c\mu$, M_B and $\mathcal{F}_{\text{eff}}$, which are crucially important to investigate the *Baryon Mass Puzzle*, "Why $N_c\mu_c \neq M_B$?". The effective potential Eq. (3·2) indicates that the chiral phase transition is caused by the competition between the energy gain from the chemical potential (*i.e.* $-N_c\mu$) in the chiral restored matter ($\sigma \sim 0$) and the effective potential in the chiral broken vacuum (*i.e.* $\mathcal{F}_{\text{eff}}(\bar{\sigma}; m_0, \mu = 0)$). Hence the critical chemical potential μ_c is expected to satisfy,

$$N_c\mu_c = -\mathcal{F}_{\text{eff}}(\bar{\sigma}; m_0, \mu = 0) \simeq -N_c\bar{\sigma}^2_{\mu=0}/(d+1) + M_B \ , \quad (3\cdot3)$$

rather than $N_c\mu_c = M_B$. In the second equality, we tentatively ignore the momentum effects in Eq. (3·2), which corresponds to consider only the temporal hopping for baryons. The first term appears through bosonization, and would be interpreted as the term to cancel the double counting of the interaction. Thus we have reached the conclusion that the phase transition at finite μ can occur at $N_c\mu < M_B$, because the baryon mass is generated by the interaction and we have to take account of the counter term $(-N_c\bar{\sigma}^2/(d+1))$ in the effective potential.

Finally we compare the explicit value of the baryon mass M_B and the critical chemical potential $N_c\mu_c$ with the previous results (Fig. 1). It is found that the current result is consistent with the previous ones.

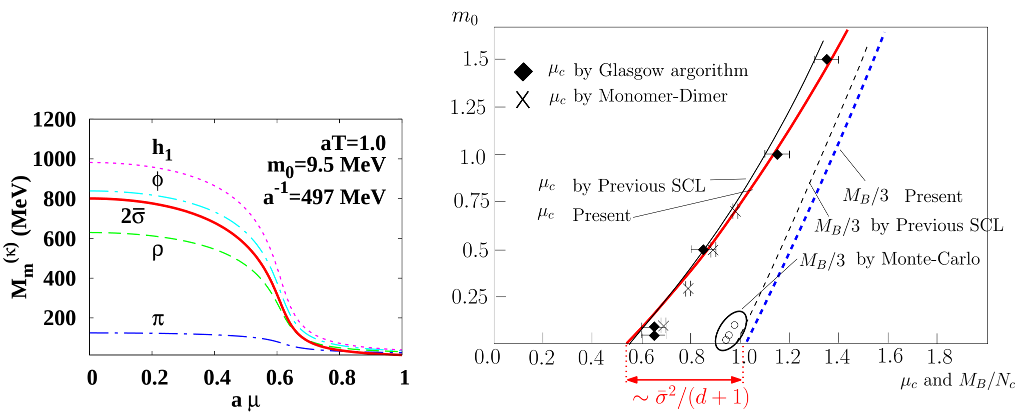

Fig. 1. Left: Quark chemical potential dependence of meson mass and chiral condensate at $m_0 = 9.5$ (MeV) and $T = 497$ (MeV). Right: μ_c and M_B/N_c for each current quark mass (m_0). Thick solid and thick dashed lines are the current results, which represents μ_c and M_B/N_c respectively and compared with some previous results: Small circle[5), thin dashed line[11),16)], ♦[16)], ×[15)], thin solid line[6)].

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