Fundamental and phenomenological approaches to high density hadronic matter

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Abstract

We study the properties of kigh density hadronic and quark matter in fundamental and phenomenological approaches. First we discuss the phase diagram of quark matter at finite temperature (T) and chemical potential (μ) in the strong coupling limit of lattice QCD for color SU(3) with baryon effects. In an analytically derived expression of the effective free energy as a function of T and μ, baryon effects are shown to extend the hadron phase to a larger chemical potential direction at low temperatures. Next we apply the idea from the strong coupling limit of lattice QCD to the chiral potential in RMF. By introducing logarithmic sigma potential, we can well describe both of the nuclear matter and finite nuclei in a chiral RMF model.

1 Introduction

Exploring various phases of quark and nuclear matter has recently attracted much attention both theoretically and experimentally. While the phase transition from hadron phase to QGP at high temperatures and at zero baryon chemical potential is predicted from lattice QCD, and various experimental signals at RHIC suggest the formation of QGP. On the other hand, properties of highly compressed cold matter are difficult to study in Monte-Carlo simulation of lattice QCD. Then in order to attack the problem of compressed baryonic matter, it is necessary to invoke some approximations in QCD or to apply some effective models. One of the most instructive approximations to investigate the finite temperature T and chemical potential μ of QCD is to consider the strong coupling limit of lattice QCD. Another useful approach is to include essential features of QCD, such as the chiral symmetry, into relativistic mean field (RMF) models, which have been successfully applied to various nuclear many-body problems.

In this report, first we study the phase diagram of quark matter at finite temperature (T) and finite chemical potential (μ) in the strong coupling limit of lattice QCD for color SU(3) [1]. We derive an analytical expression of the effective free energy as a function of T and μ. We take account of both the mesonic and baryonic composite terms in the $1/d$ expansion of the lattice QCD action, and perform the temporal link variable (U_0) integral exactly in the Polyakov gauge with anti-periodic boundary condition for fermions, while we ignore the effects of finite diquark condensate. Next, we study nuclear matter and finite nuclei in a flavor SU(2) chirally symmetric relativistic mean field model containing a logarithmic potential term of σ, $-\log\sigma^2$ [2], derived from the strong coupling limit of the lattice QCD at vacuum. A phenomenological ω self-interaction term, $(\omega_\mu\omega^\mu)^2$, is also included in the effective Lagrangian. By requiring that nuclear matter saturation properties and finite

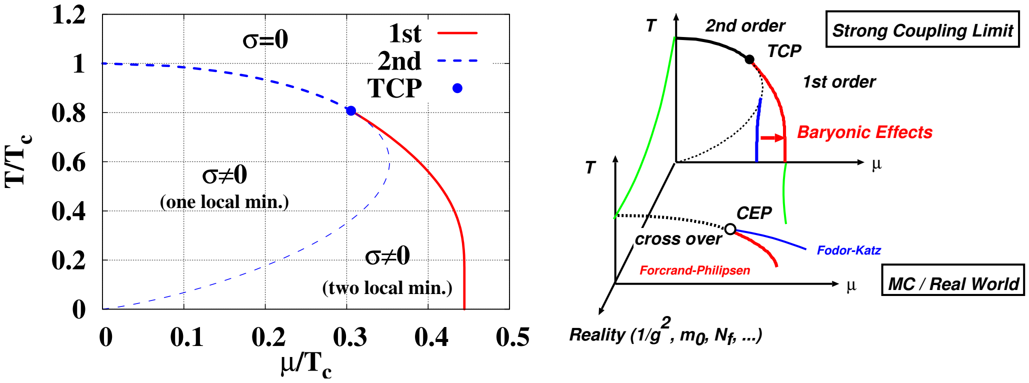

Figure 1: Left: phase diagram of quark matter in the strong coupling limit of lattice QCD with baryonic effects. Right: expected phase diagram evolution from the strong coupling limit towards the real world.

nuclear binding energies are reproduced, we have fixed the four free parameters in the model. The obtained EOS is as soft as those in phenomenologically successful RMF models such as NL and TM models [3]. Bulk properties of proton closed even-even nuclei are also well explained in a comparable precision to other sophisticated RMF models.

2 Phase diagram in the strong coupling limit of lattice QCD

One of the most instructive approximations to investigate the finite temperature T and chemical potential μ of QCD would be to consider the strong coupling limit of lattice QCD. In this limit, we can analytically derive the effective free energy (free energy density), then by using this effective free energy we can discuss the phase diagram of quark matter.

In Ref. [1], we have derived the following effective free energy:

$$\mathcal{F}_{\text{eff}}(\sigma; T, \mu) = \tfrac{1}{2} b_\sigma \sigma^2 + F_{\text{eff}}^{(q)}(b_\sigma \sigma; T, \mu) + \Delta F_{\text{eff}}^{(b)}(g_\sigma \sigma) \;, \tag{1}$$

$$F_{\text{eff}}^{(q)}(\sigma; T, \mu) = -T \log \left[\tfrac{4}{3} \left(C_\sigma^3 - \tfrac{1}{2} C_\sigma + \tfrac{1}{4} C_{N_c\mu} \right) \right] \;, \tag{2}$$

$$C_\sigma = \cosh \left[\operatorname{arcsinh} (b_\sigma \sigma)/T \right] \;, \quad C_{N_c\mu} = \cosh N_c \mu / T \;, \tag{3}$$

where $F_{\text{eff}}^{(q)}$ and $\Delta F_{\text{eff}}^{(b)}$ are the potential energies from quark and baryon integrals. Compared to that in models without baryon effects [4], the polarizability of the chiral condensate b_σ is modified from baryon effects, and we explicitly have the potential term from baryon integral. Compared to that in models with diquark condensate but at zero temperature [5], we find that it is important to take account of finite temperature effects, i.e. anti-periodic boundary condition in the temporal lattice.

One of the problems in the strong coupling limit approaches is that the ratio of the critical baryon chemical potential at zero temperature with respect to the

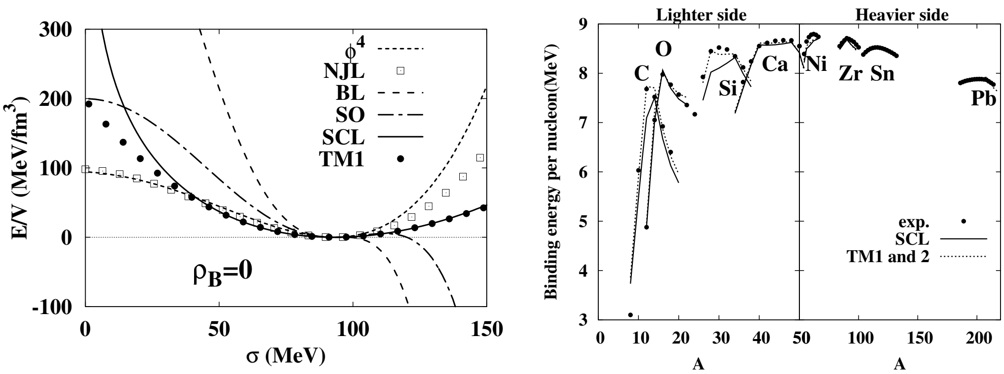

Figure 2: Left: vacuum energy density as a function of σ in various models. Right: binding energies per nucleon in the present chiral RMF model in comparison with TM models.

critical temperature at zero chemical potential, $R_{\mu T} \equiv 3\mu_c(T=0)/T_c(\mu=0)$, is found to be much smaller than the empirical value or that suggested in Monte-Carlo simulations. Thus for a quantitative discussion, the strong coupling limit in the chiral limit with one species of staggered fermion is not enough, and it is necessary to take care of finite quark mass m_0, multi-staggered fermions, finite $1/g^2$, other order parameters than the chiral condensate, and/or other mechanisms towards the real world in order to explain large $\mu_c(T=0)$ relative to T_c, as illustrated in the right panel of Fig. 1.

3 Chiral RMF model with logarithmic sigma potential

Relativistic mean field (RMF) models have been successfully applied to various nuclear many-body problems, and it is based on the idea of σ meson, which would be the chiral partner of pions. Thus it would be natural to expect that RMF can be a starting point of finite baryon density hadronic models provided that the chiral symmetry is included. However, this has not been achieved in a satisfactory way, mainly because of the instability of the normal vacuum generated with the restriction of the chiral symmetry.

In Ref. [2], we have developed an SU(2) chiral symmetric RMF model containing a logarithmic potential term of σ, $-\log\sigma^2$, derived from the strong coupling limit of the lattice QCD at vacuum.

$$V(\sigma) = b_\sigma \sigma^2 - a_\sigma \log \sigma^2 - c_\sigma \sigma \ . \qquad (4)$$

With this logarithmic chiral potential, the normal vacuum is found to be the state of absolute energy minimum as shown in the left panel of Fig. 2, and this normal vacuum is stable even above the normal nuclear density. In addition, we can well describe the nuclear matter saturation properties and binding energies of finite nuclei. In the right panel of Fig. 2, we show the calculated binding energies of proton (sub-)closed even-even nuclei in comparison with data and those calculated in TM models [3].

Since the chiral potential adopted in this work is based on the strong coupling limit of lattice QCD, we can improve it in various ways. First, it is very important to extend the present model to flavor SU(3) chiral RMF model in order to discuss the high density matter, where strangeness is expected to emerge. Preliminary work in this direction tells us that the EOS is softened due to the hidden strange mesons (ζ or f_0), and it is possible to describe single and double Λ hypernuclei. Secondly, the above chiral potential (4) is the one which is derived at zero-temperature, and it cannot describe the transition to chirally restored phase. As shown in the previous section, we already have the finite temperature chiral potential, it is an interesting direction to include the finite T effects. Then it may become possible to describe finite nuclei and quark matter in one theoretical framework.

4 Summary

We have discussed the properties of hadronic and quark matter in a view of the strong coupling limit of lattice QCD from QCD side and nuclear side. It would be valuable to investigate from these two aspects of nuclear physics: The former approach is a fundamental one, and we have a lot of theoretical works to be compared with. The latter is a phenomenological one, and we have a lot of observables to be compared with.

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