Strong coupling limit/region of lattice QCD

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We study the phase diagram of quark matter and nuclear properties based on the strong coupling expansion of lattice QCD. Both of baryon and finite coupling correction are found to have effects to extend the hadron phase to a larger μ direction relative to T_c. In a chiral RMF model with logarithmic sigma potential derived in the strong coupling limit of lattice QCD, we can avoid the chiral collapse and normal and hypernuclei properties are well described.

§1. Introduction

Understanding the properties of nuclei and nuclear matter from QCD is one of the ultimate goals in nuclear physics. In a standard roadmap, it is necessary to describe hadrons in QCD, to derive the bear nucleon-nucleon interaction, to obtain the effective nuclear force, and to solve nuclear many-body problems. An alternative way would be to obtain the effective potential (free energy density) in QCD, to represent this effective potential as the density functional in hadronic degrees of freedom, and to apply this density functional to nuclear many-body problems. For this purpose, the most instructive approach may be to combine the strong coupling limit (SCL) of lattice QCD[1)–4)] and the relativistic mean field (RMF) models, since Monte-Carlo simulations of lattice QCD for dense matter[5)] are not yet easy at present.

In this proceedings, we discuss the baryon and finite coupling effects on the phase diagram in the strong coupling region of lattice QCD,[4),6)] and nuclear properties in a chiral RMF model based on the SCL effective potential.[7),8)]

§2. Effective potential at strong coupling of lattice QCD

The strong coupling limit of lattice QCD (SCL-LQCD) predicts the high T second order chiral phase transition at $\mu = 0$ and the high density first order transition at $T = 0$, and it well explains hadron masses. While these predictions explain the real world qualitatively, quantitative understanding of the phase diagram is not achieved yet. For example, the ratio $R_{\mu T} = \mu_c(T = 0)/T_c(\mu = 0)$ should be larger than two, but it is less than 1/3 in SCL-LQCD without baryon effects. In this work, we discuss the effects of baryons[4)] and finite coupling on the shape of the phase boundary.

At strong coupling ($g \gg 1$), the plaquett contribution ($\propto 1/g^2$) is perturbative and the effective action is obtained by integrating spatial links as,[2)]

$$S = S_{\rm SCL} + \mathit{\Delta} S_g + \mathcal{O}(1/d, 1/g^2\sqrt{d}, 1/g^4) \ , \tag{2·1}$$

$$S_{\rm SCL} = \frac{1}{2}\sum_x \left(e^{\mu}V_x - e^{-\mu}V_x^{\dagger}\right) - \frac{1}{2}(M, V_M M) - (\bar{B}, V_B B) + m_0 \sum_x M_x \ , \tag{2·2}$$

$$\varDelta S_g = \frac{\beta_t}{2d}\sum_{x,j>0}(V_x^\dagger V_{x+\hat{j}} + V_x^\dagger V_{x-\hat{j}}) - \frac{\beta_s}{d-1}\sum_{x,k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}} \ , \quad (2\cdot3)$$

where $(A,B) = \sum_x A_x B_x$, $M_x = \bar{\chi}_x^a \chi_x^a$, $B_x = \varepsilon_{abc}\chi_x^a\chi_x^b\chi_x^c/6$, $V_x = \bar{\chi}_x U_0(x)\chi_{x+\hat{0}}$, $\beta_t = d/2N_c^2 g^2$, $\beta_s = (d-1)/8N_c^4 g^2$, $d=3$ is the spatial dimension, and $V_M(x,y)$ and $V_B(x,y)$ represent mesonic and baryonic inverse propagators. In SCL, the effective potential for $N_c = 3$ without baryon effects has been known as,[2),3)]

$$\mathcal{F}_{\mathrm{eff}}^{(T)}(\sigma) = \frac{1}{2}a_\sigma\sigma^2 + \mathcal{F}_{\mathrm{eff}}^{(q)}(a_\sigma\sigma + m_0; T, \mu)\,, \quad (2\cdot4)$$

$$\mathcal{F}_{\mathrm{eff}}^{(q)}(m_q) = -T\log\left[C_\sigma^3 - C_\sigma/2 + \cosh(3\mu/T)/4\right]\,, \quad (2\cdot5)$$

where $C_\sigma = \cosh(\mathrm{arcsinh}\,(m_q)/T)$ and $a_\sigma = d/2N_c$, and the scalar field $\sigma = -\langle M\rangle$ is the chiral order parameter. This effective potential gives the ratio of $R_{\mu T} \sim 0.33$.

We have recently demonstrated that the baryonic composite action can be represented in the mean field approximation at zero diquark condensate as,[4)]

$$e^{(\bar{B},V_B,B)} \simeq e^{-N_s^3 N_\tau\left[a_\omega\omega^2/2 + \varDelta\mathcal{F}_{\mathrm{eff}}^{(b)}(g_\omega\omega)\right] - \sum_x\left[(\alpha^2+\gamma^2)M^2/2 + \alpha\omega M\right]}\ , \quad (2\cdot6)$$

where auxiliary baryon determinant is represented in $\varDelta\mathcal{F}_{\mathrm{eff}}^{(b)}$.[4)] At equilibrium, the baryon potential field ω is approximately proportional to σ, and the effective potential is found to be

$$\mathcal{F}_{\mathrm{eff}}^{(Tb)}(\sigma) = \frac{1}{2}b_\sigma\sigma^2 + \mathcal{F}_{\mathrm{eff}}^{(q)}(b_\sigma\sigma) + \varDelta\mathcal{F}_{\mathrm{eff}}^{(b)}(g_\sigma\sigma)\,. \quad (2\cdot7)$$

Two parameters in this effective potential, b_σ and g_σ, are related to the decomposition parameters, α and γ. introduced in baryonic composite decomposition.

The effective potential with $1/g^2$ correction was derived by Bilić *et al.*,[3)] where $V^\dagger V$ term was generated by the derivative of $\mathcal{F}_{\mathrm{eff}}^{(q)}$. Since $\varDelta S_g$ is not in the bilinear form in χ and $\bar{\chi}$, we here bosonize the plaquett contributions and apply the mean

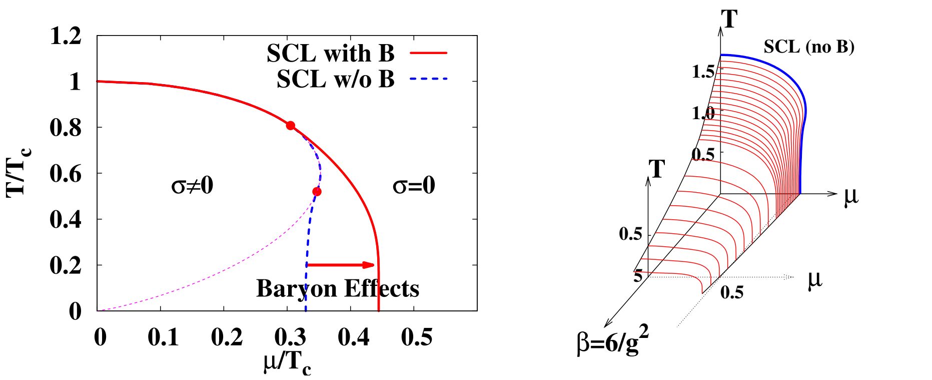

Fig. 1. The phase diagram in the strong coupling limit (left panel), and its evolution with $\beta = 6/g^2$ (right panel).

field approximation,[6)]

$$\Delta S_F \simeq N_s^3 N_\tau \left[\frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2\right] + \frac{\beta_t\varphi_t}{4}\sum_x (V_x - V_x^\dagger) - \beta_s\varphi_s \sum_{x,j>0} M_x M_{x+\hat{j}} \,. \quad (2\cdot 8)$$

The auxiliary fields have expectation values of $\langle\varphi_t\rangle = \langle V^\dagger - V\rangle$ and $\langle\varphi_s\rangle = 2\langle M_x M_{x+\hat{j}}\rangle$. These correction terms have a similar structure to the SCL effective action (2·2), and they lead to the modifications of the quark mass and effective chemical potential as $\widetilde{m}_q = \sigma d(1 + 4N_c\beta_s\varphi_s - \beta_t\varphi_t\cosh\mu)/2N_c$ and $\widetilde{\mu} = \mu - \beta_t\varphi_t\sinh\mu$. At equilibrium, we can put $\varphi_s = 2\sigma^2 + \mathcal{O}(1/g^2)$, and the effective free energy up to $\mathcal{O}(1/g^2)$ without baryon effects is obtained as,

$$\mathcal{F}_{\rm eff}^{(1/g^2)} = \frac{d}{4N_c}\sigma^2 + 3d\beta_s\sigma^4 + \frac{\beta_t}{4}\varphi_t^2 - N_c\beta_t\varphi_t\cosh\mu + \mathcal{F}_{\rm eff}^{(q)}(\widetilde{m}_q; T, \widetilde{\mu}) \,. \quad (2\cdot 9)$$

As shown in Fig. 1, both of baryons and finite coupling corrections have effects to extend the hadronic phase in the larger μ direction relative to T_c, while each of these is not enough to explain the empirical ratio of $R_{\mu T}$. It would be interesting to evaluate both of these effects simultaneously.

§3. Chiral RMF with SCL Effective Potential

RMF models are powerful tools in describing nuclear matter and finite nuclei, but we have a so-called *chiral collapse* problem[9)] in a naive chiral RMF model based on the ϕ^4 theory; the normal vacuum jumps to an abnormal one below the normal nuclear density. The effective potential in SCL-LQCD gives us a hint to solve this problem. In a zero temperature treatment of SCL-LQCD,[1),4)] the effective potential is found to have the form, $\mathcal{F}_{\rm eff} = b_\sigma\sigma^2/2 - N_c\log\sigma$, and the divergent behavior at $\sigma \to 0$ helps to avoid the chiral collapse.

We have recently developed a chiral SU(2) RMF model[7)] with logarithmic sigma potential,

$$U_\sigma = \frac{b_\sigma}{2}{\rm tr}(\mathcal{M}\mathcal{M}^\dagger) - a_\sigma \log\det(\mathcal{M}\mathcal{M}^\dagger) - c_\sigma\sigma$$
$$\sim \frac{b_\sigma}{2}\sigma^2 - 2a_\sigma\log\sigma - c_\sigma\sigma \,, \quad (3\cdot 1)$$

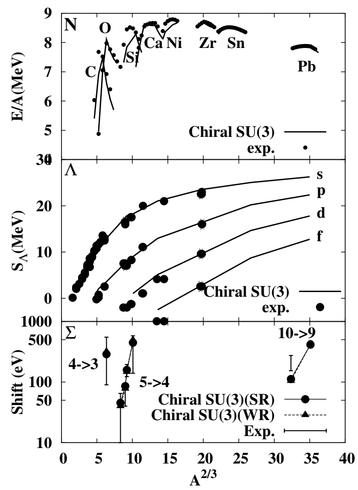

Fig. 2. Nuclear binding energies (top), single Λ hypernuclear separation energies (middle), and atomic shift of Σ^- atom (bottom) in the present chiral SU(3) RMF.

where $\mathcal{M}$ denotes the meson matrix. In this RMF, we can well describe symmetric nuclear matter equation of state (EOS) and bulk properties of finite nuclei.[7)] In a chiral SU(3) RMF model,[8)] we include the determinant interaction ($\det\mathcal{M} + \det\mathcal{M}^\dagger$)

simulating $U_A(1)$ anomaly. After fitting binding energies and charge rms radii of normal nuclei, we find that the symmetric matter EOS becomes softer than that in the chiral SU(2) RMF due to the scalar meson with hidden strangeness, $\zeta = \bar{s}s$, which couples with σ through the determinant interaction.

In this chiral SU(3) RMF, we can find hyperon-meson coupling constants which fits existing data of Λ separation energies in single hypernuclei, $\Lambda\Lambda$ bond energy ($\Delta B_{\Lambda\Lambda}$) in ${}^{6}_{\Lambda\Lambda}$He, and atomic shifts of Σ^- atoms, as shown in Fig. 2.

While the above sigma potential has divergence at $\sigma \to 0$, the effective potential derived in the finite T treatments have a linear term for small values of σ at $T \to 0$,

$$\mathcal{F}_{\rm eff}^{(T)} \to \frac{a_\sigma}{2}\sigma^2 - N_c \,\mathrm{arcsinh}\left(a_\sigma \sigma\right) , \tag{3·2}$$

which is enough to stabilize the normal vacuum. We may conclude that gluons play decisive roles to avoid the chiral collapse as pointed out in Ref. 10).

§4. Summary

In this paper, we have investigated the phase diagram of quark matter and nuclear properties based on the strong coupling expansion of lattice QCD. In the first part, we find that baryons and finite coupling corrections have favorable effects to extend the hadron phase to a larger μ direction with respect to T_c. In the second part, we have shown that we can well describe normal and hypernuclear properties with a logarithmic potential. For the understanding of dense matter, it is important to respect both of chiral symmetry and strangeness degrees of freedom. In addition, the present work may be suggesting the importance of implicit role of gluons, which generate the effective potential of hadrons.

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