Vorticity and spin polarization in heavy-ion collisions

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Motivation of the talk

Quark-gluon plasma: "The most vortical fluid"

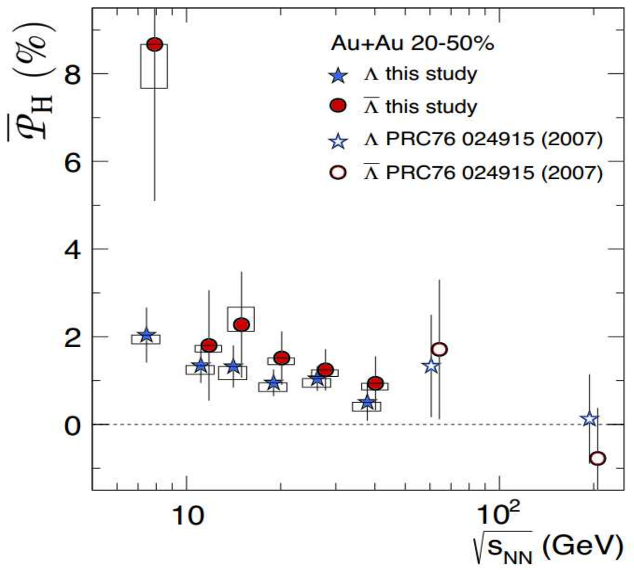
LETTER

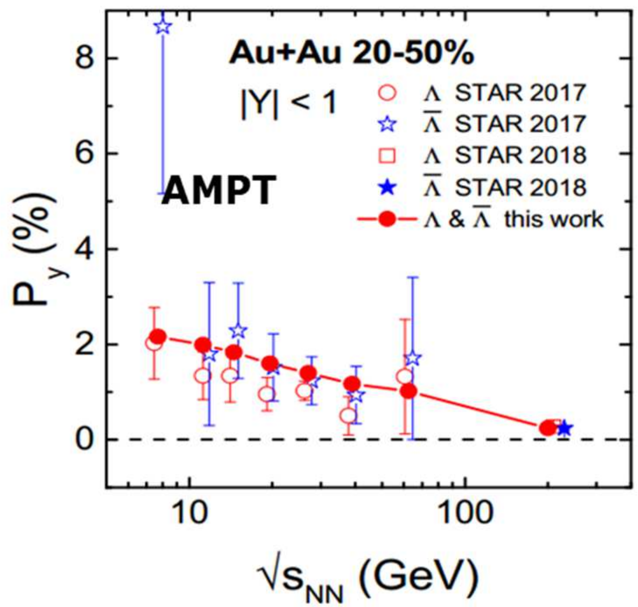
doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*

See talk by Niida



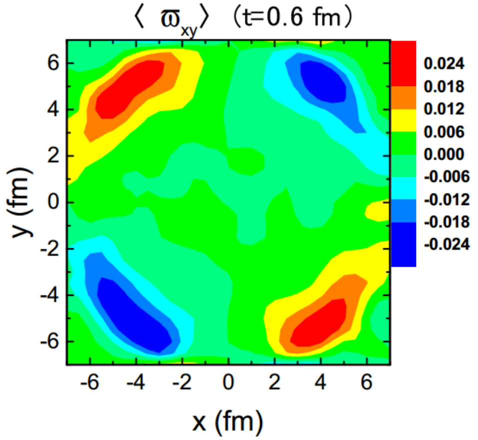


Experiment = Theory

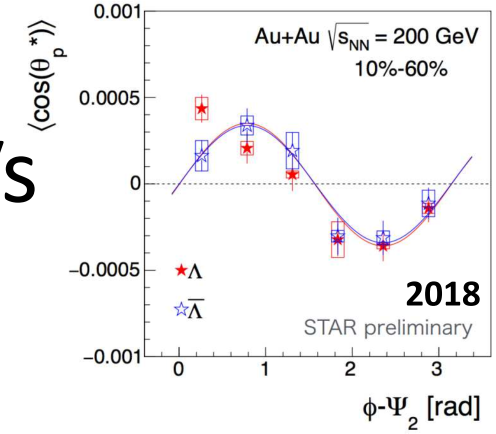
Motivation of the talk

- **But: discrepancies between theory and experiments**

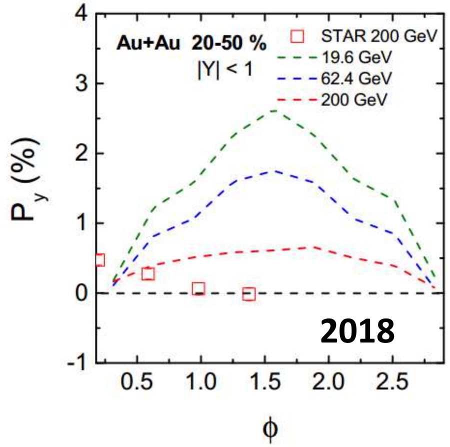
1) longitudinal polarization vs ϕ



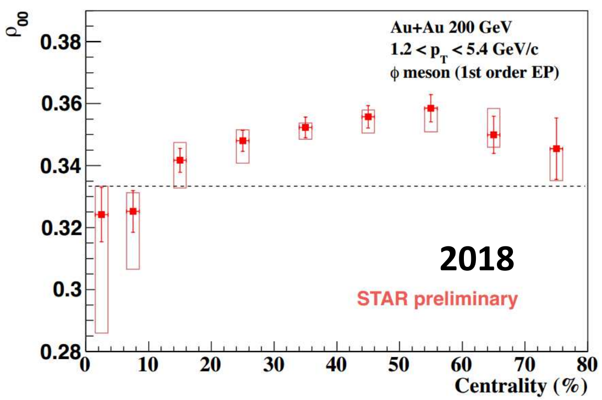
Vs



2) Transverse polarization vs ϕ



3) Vector meson spin alignment



Experiment Refs:

STAR Collaboration, arXiv:1805.04400

Niida, Quark matter 2018

C. Zhou, Quark matter 2018

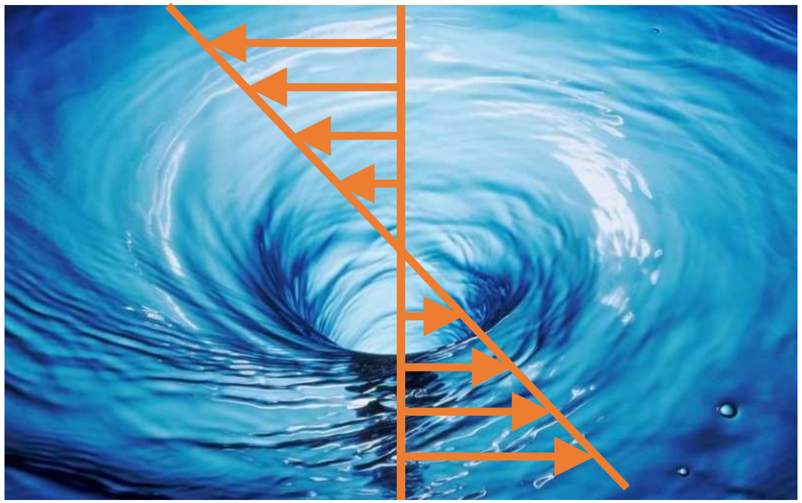
B. Tu, Quark matter 2018

Motivation of the talk

- **To resolve the discrepancies, from the theory side, we need to:**
  - Understand the properties of fluid vorticity itself
  - Understand the magnetic field contribution, the resonance decays contribution, … …
  - Find other observables which are always helpful: spin-alignment at central collisions, the chiral vorticity effects, … …
  - Understand how vorticity polarizes spin and how the spin polarization evolve: spin kinetic theory or spin hydrodynamics (**See talk by Taya**)

Vorticity in heavy-ion collisions

Fluid vorticity



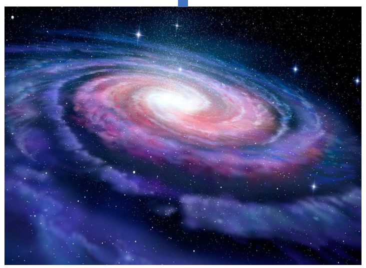
$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$$

Local angular velocity

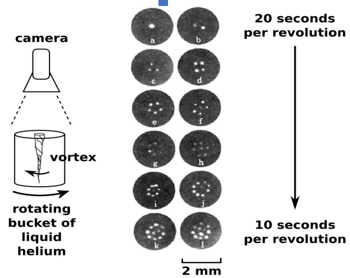
Fluid vorticity

- Vortices: common phenomena in fluids across a very broad hierarchy of scales

$10^{21}m$ | $10^{2}-10^{-2}m$ | $10^{-5}-10^{-8}m$ | $10^{-15}m$

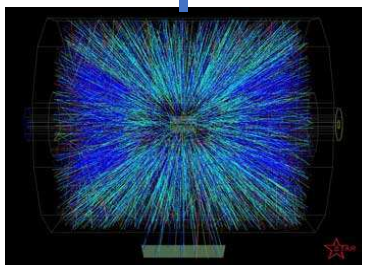


Rotating galaxies



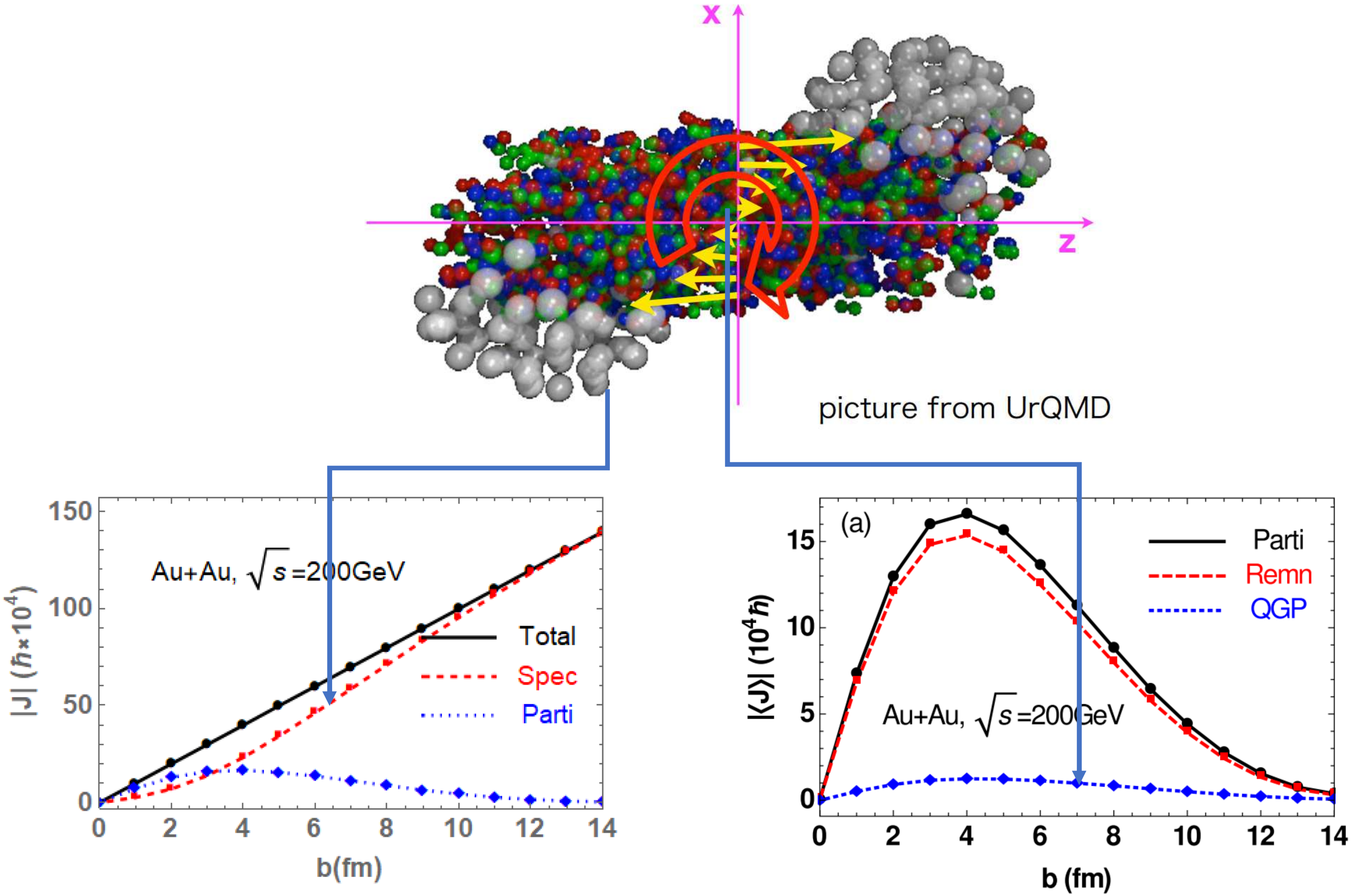
Superfluid helium

Tornados, ocean vortices, …

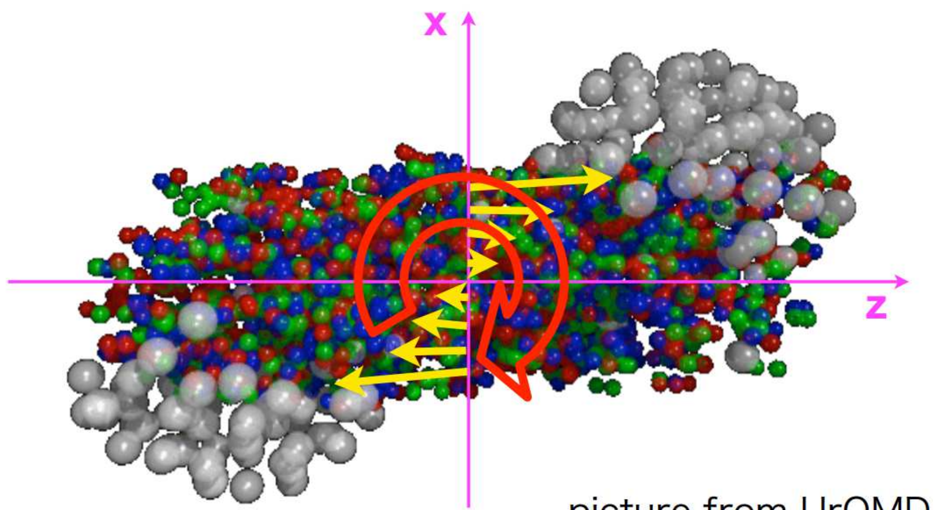


Quark gluon matter

Angular momentum in HIC



Angular momentum in HIC



picture from UrQMD

Global angular momentum

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)

Local vorticity

$$\omega \sim ?$$

Velocity field in partonic model

- To calculate the vorticity, we need to know the velocity

Definition of velocity field in HIJING or AMPT model

$$v_1^a(x) = \frac{1}{\sum_i \Phi(x,x_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(x,x_i) = \frac{J^a}{J^0}$$ **~ Particle flow velocity**

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x,x_i)}{\sum_i [p_i^0 + (p_i^a)^2/p_i^0] \Phi(x,x_i)} = \frac{T^{0a}}{T^{00} + T^{aa}}$$ **~ Energy flow velocity**

Smearing function Phi

$$\Phi_G(x,x_i) = \frac{K}{\tau_0 \sqrt{2\pi\sigma_\eta^2} 2\pi\sigma_r^2} \exp\left[-\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2} \right]$$

Parameters are so chosen that with hydro, it is consistent with elliptic data (Pang-Wang-Wang 2012)

Velocity field in partonic model

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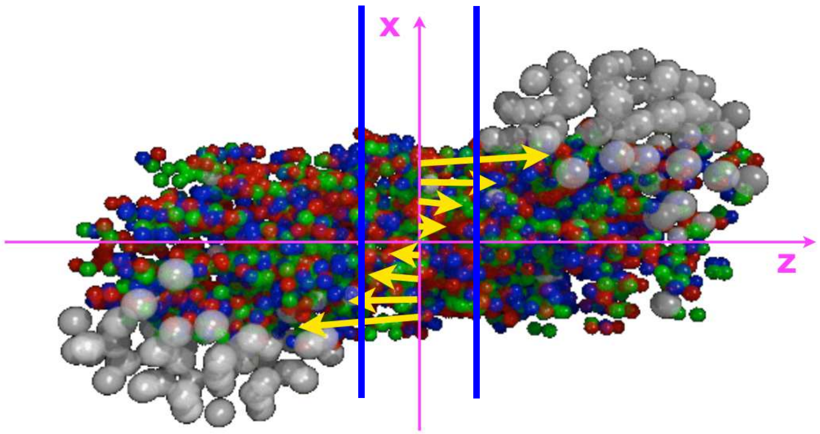
Definition of vorticity field (for each definition of v)

$$\omega_1 = \nabla \times v,$$ **~ nonrelativistic definition**

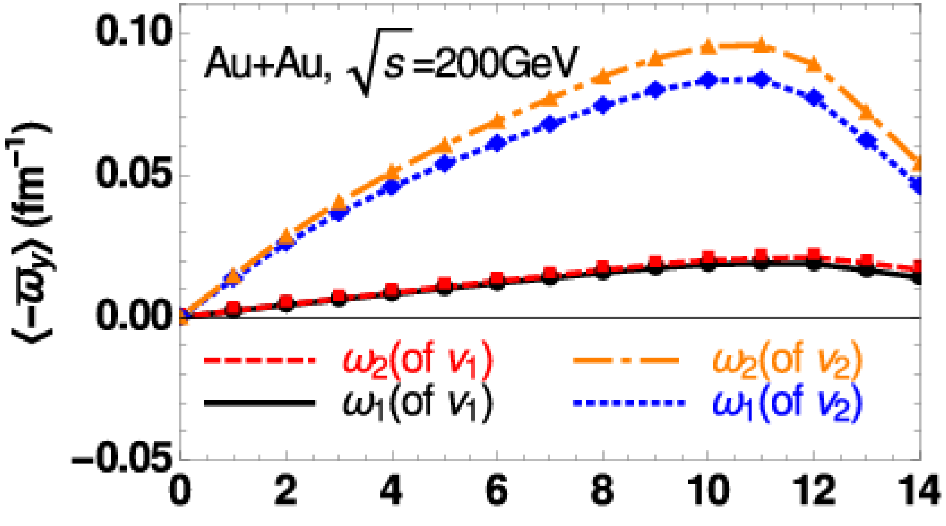
$$\omega_2^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$ **~ relativistic definition**

$$\varpi_{\mu\nu} = \frac{1}{2}[\partial_\nu(u_\mu/T) - \partial_\mu(u_\nu/T)]$$ **~ relativistic thermal vorticity**

Vorticity by global AM

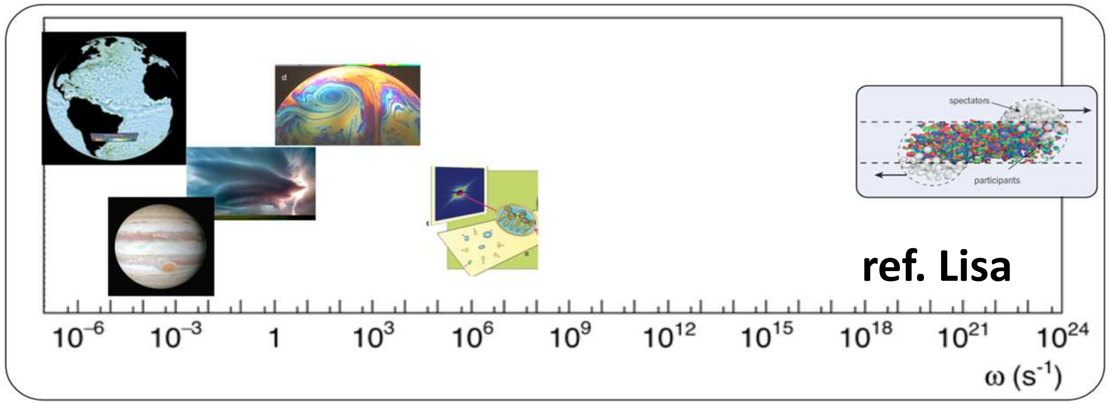


Mid rapidity



Deng-XGH 2016 b(fm)

Vorticity in Au+Au@RHIC at $b = 10$ fm is $10^{20} - 10^{21} s^{-1}$

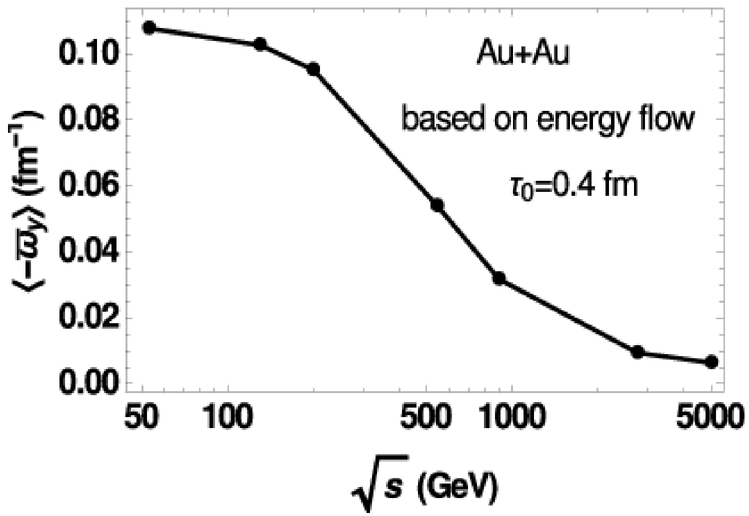


See also: Becattini etal 2015,2016; Jiang-Lin-Liao 2016;Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; … …

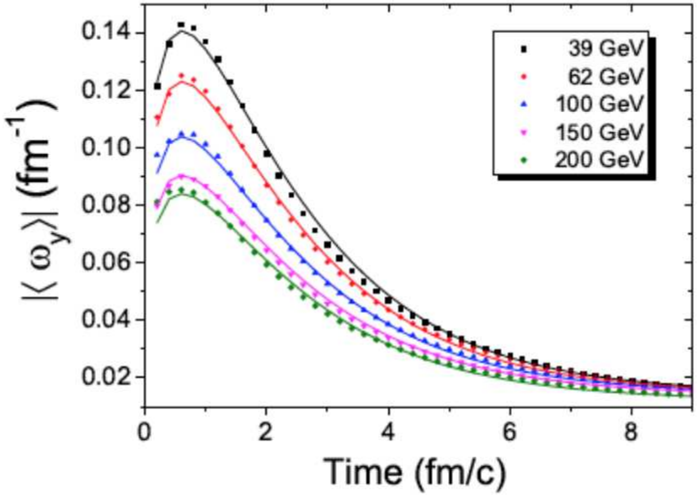
Vorticity by global AM

- Collision energy dependence

Deng-XGH 2016



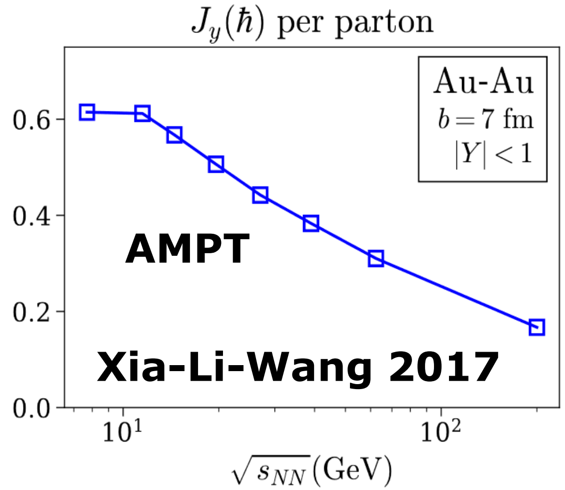
Jiang-Liao 2016



•Consistent with the Lambda polarization result of STAR

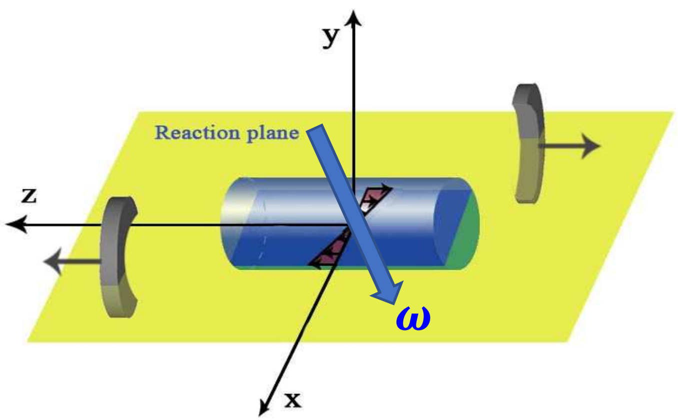
•With increasing energy, more AM carried by high-rapidity particles, midrapidity closer to Bjorken expansion

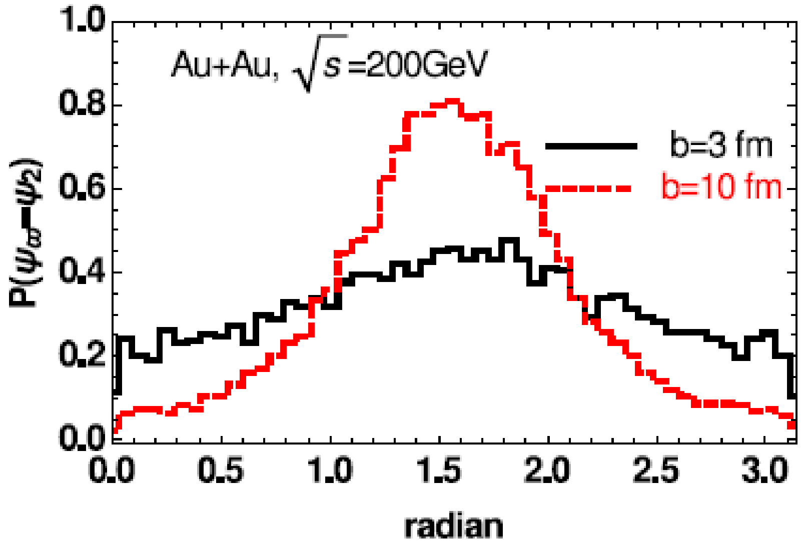
•Indicates stronger vortical effect at lower energy (beam energy scan, NICA, FAIR,JPARC, HIAF)



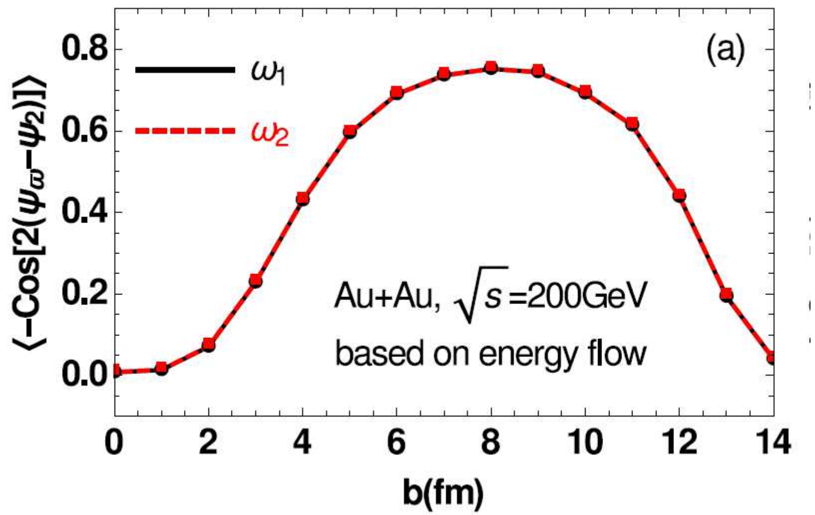
Vorticity by global AM

- Event-by-event azimuthal fluctuation

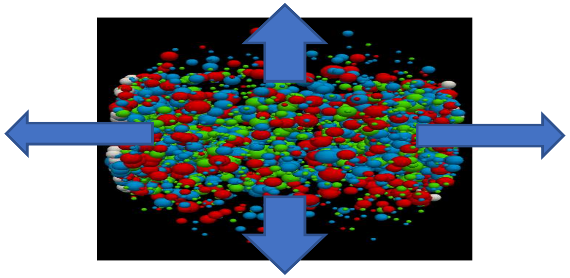


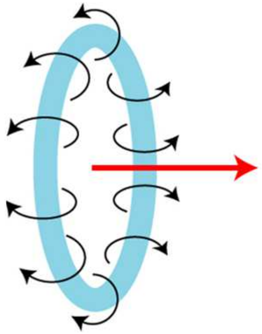


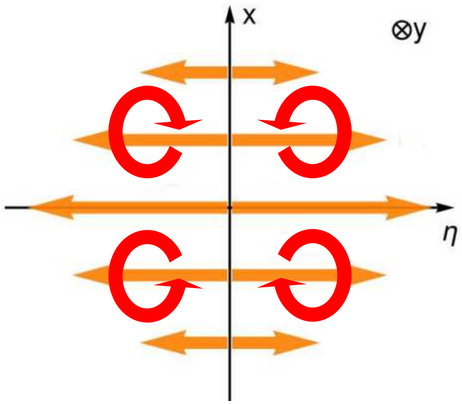
- For small and very large b, fluctuation so strong that correlation with PP is lost
- Moderate b, Gaussian around pi/2
- Suppress the correlation with the matter geometric plane



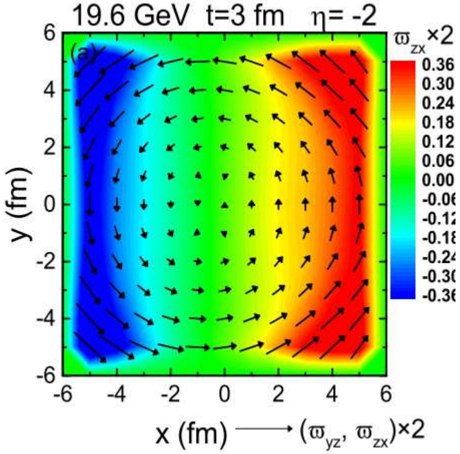
Vorticity due to expansion

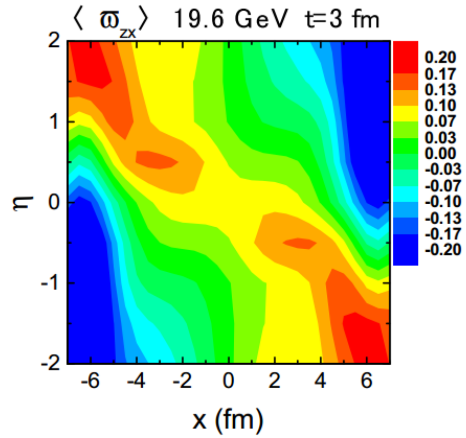




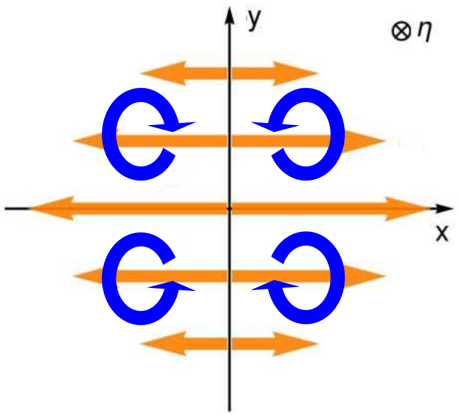


Transverse

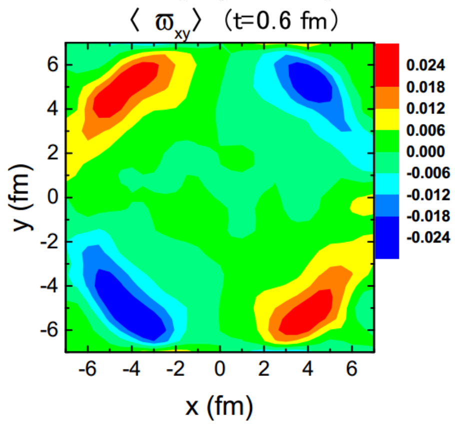




Thermal vorticity



Longitudinal

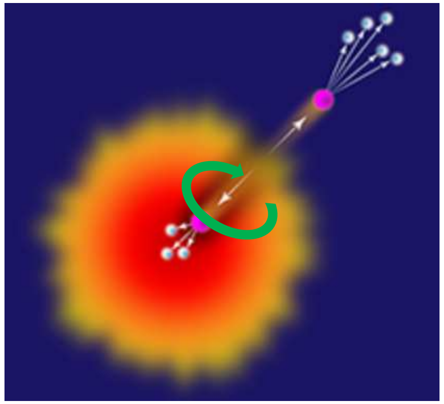


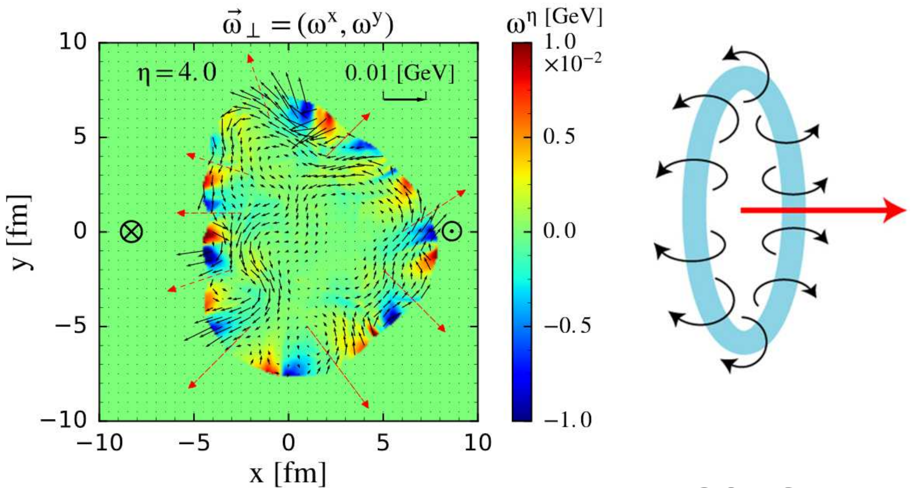
D.X.Wei-W.T.Deng-XGH, 1810.00151

(see also: Becattini etal 2017; Jiang-Lin-Liao 2016; Xia-Li-Wang 2017; Teryaev-Usubov 2015, …)

Other sources of vorticity

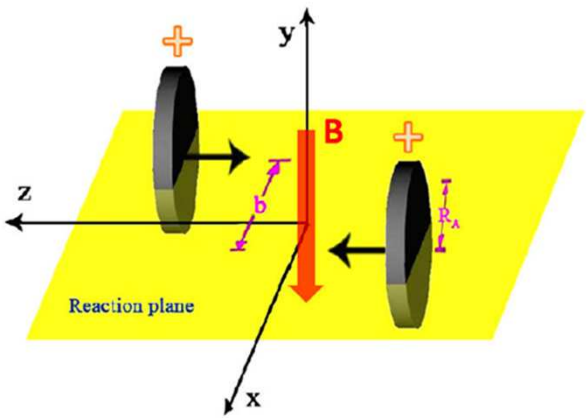
1) Jet

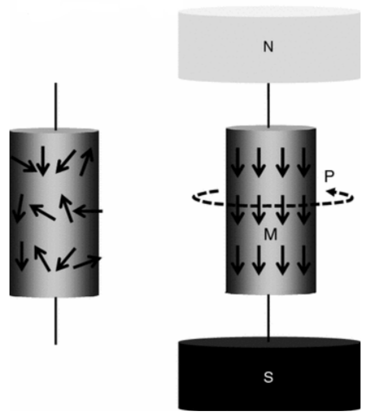




Pang-Peterson-Wang-Wang 2016

2) Magnetic field

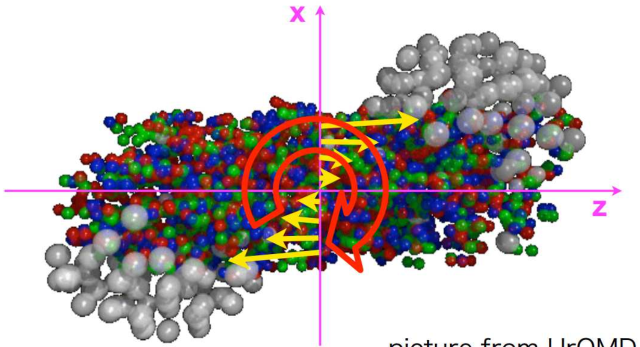




Einstein-de-Haas effect

Main message:

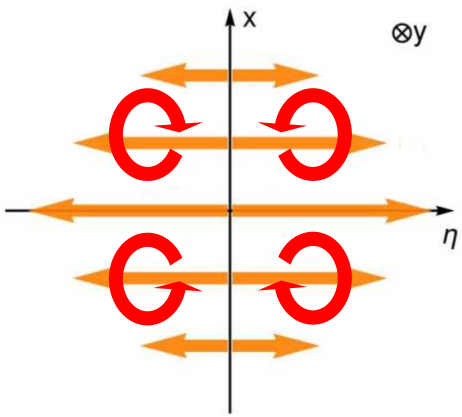
1. Global AM induces strong vorticity in HICs

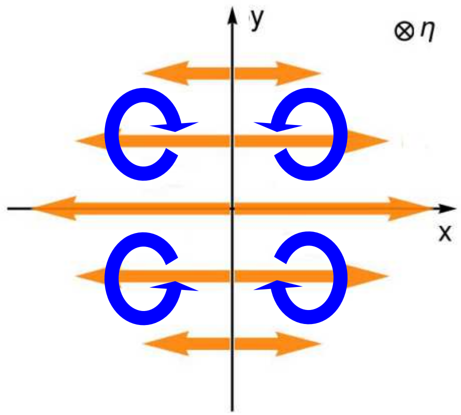


picture from UrQMD

$: \omega \approx 10^{21} - 10^{22}\ s^{-1}$

2. Inhomogeneous expansion: quadrupoles in both xy and xz planes



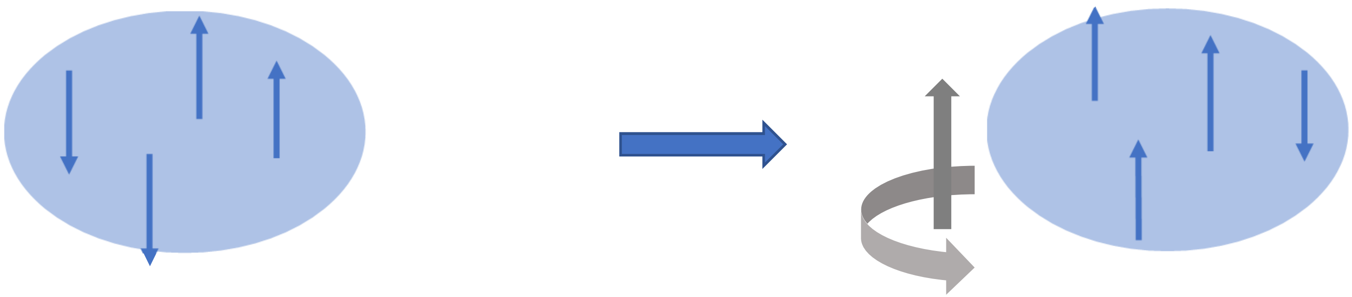


How the vorticity polarize spin?

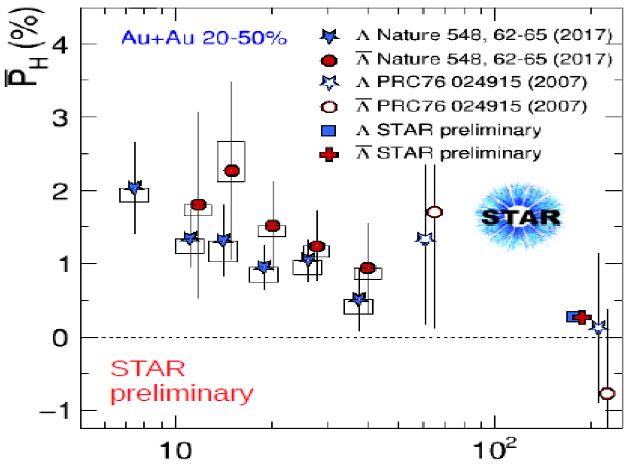
Spin-vorticity coupling

Early consideration: Liang-Wang 2004; Voloshin 2004

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{J} \quad \Rightarrow \quad \frac{dN}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{J})/T}$$



$$\text{P} = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \sim \frac{\langle \omega \rangle}{T}$$



STAR, Nature **548**, 62–65 (2017)

Possible magnetic-field contribution. A way to measure B?

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{J} - \boldsymbol{m} \cdot \boldsymbol{B}$$

Spin-vorticity coupling

More careful examination: Becattini-Chandra-Grossi 2013; Fang-Pang-Wang-Wang 2016

$$S^{\mu}(x,p) = -\frac{s(s+1)}{6m}(1-n_F)\epsilon^{\mu\nu\rho\sigma}p_{\nu}\varpi_{\rho\sigma}(x) + O(\varpi)^2$$

where $n_F(p_0)$ is the Fermi-Dirac distribution function and $p_0 = \sqrt{\boldsymbol{p}^2 + m^2}$

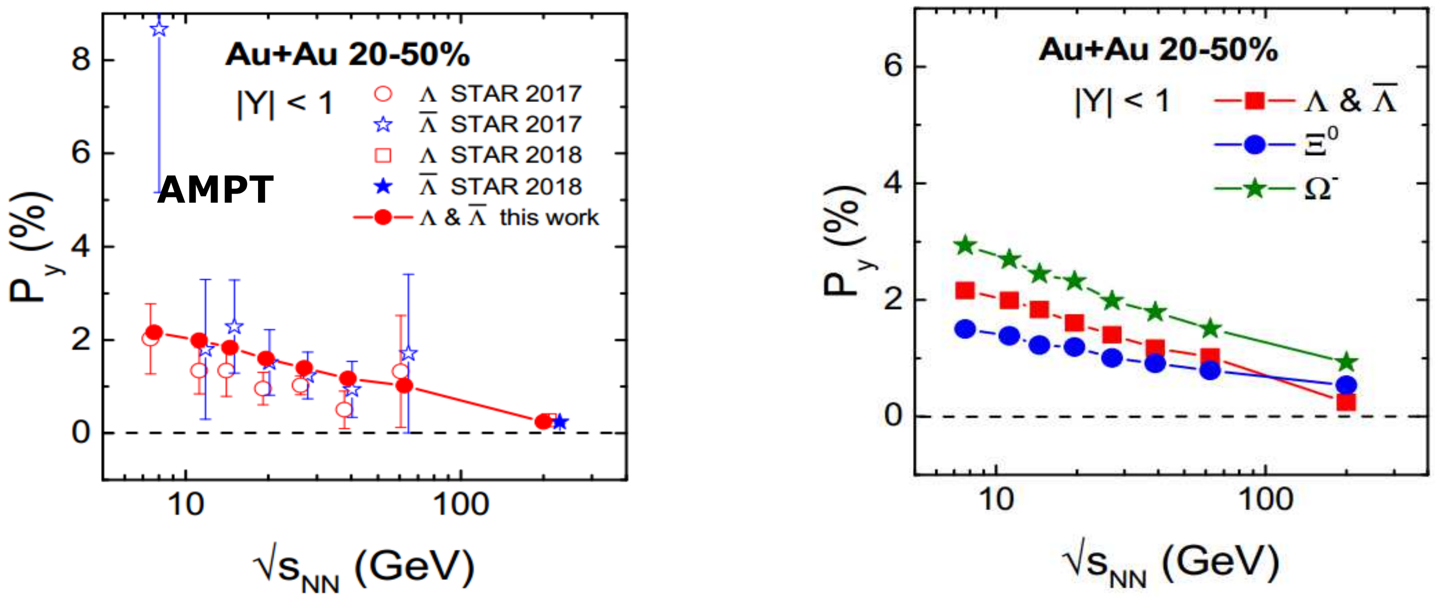
Rest frame of particle: $$\boldsymbol{S}^* = \boldsymbol{S} - \frac{\boldsymbol{p}\cdot\boldsymbol{S}}{p_0(p_0+m)}\boldsymbol{p}$$

Polarization in direction n: $$P_n = \frac{1}{s}\boldsymbol{S}^*\cdot\boldsymbol{n}$$

Assumption used: thermal equilibrium. Is spin degree of freedom thermalized in HICs? Open question.

Hyperon polarization

- Global spin polarization



- Mass ordering among $\Omega^-(sss)$, $\Xi^0(uss)$, and $\Lambda(uds)$.
- Magnetic moments $\mu_\Omega : \mu_\Xi : \mu_\Lambda = 3:2:1$. Test magnetic contribution.

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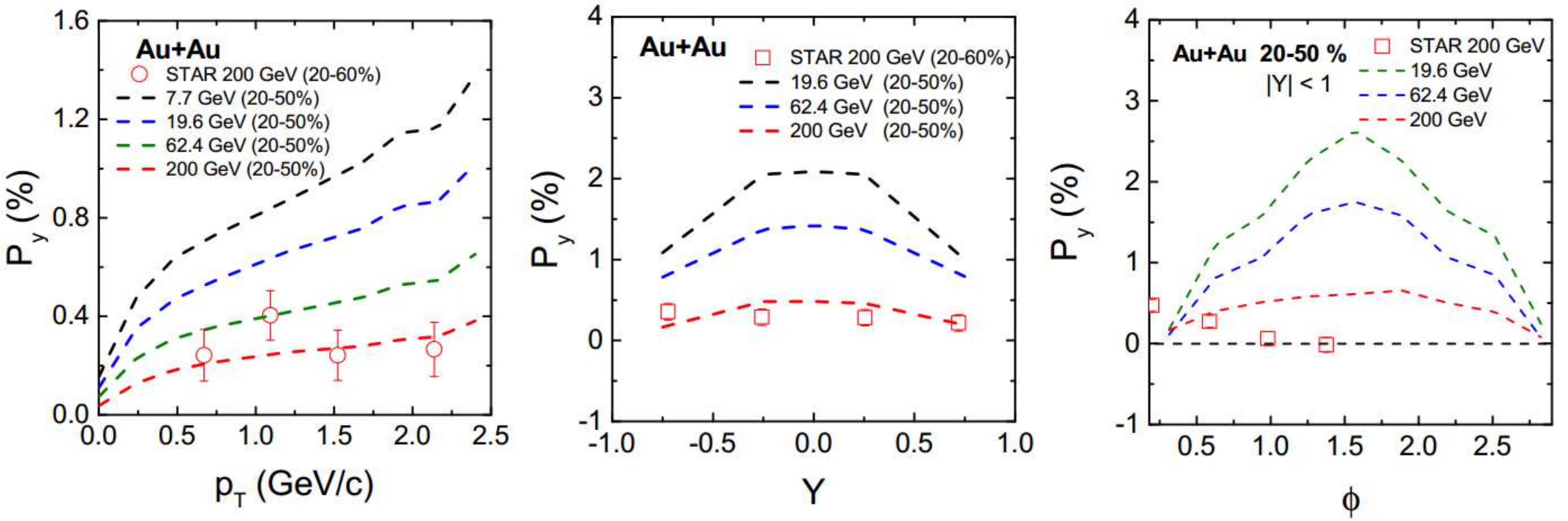
Hyperon polarization

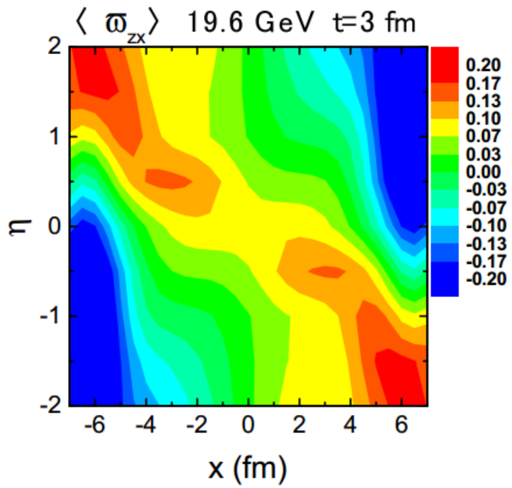
- p_T, rapidity, and azimuthal dependence, theory vs expts.



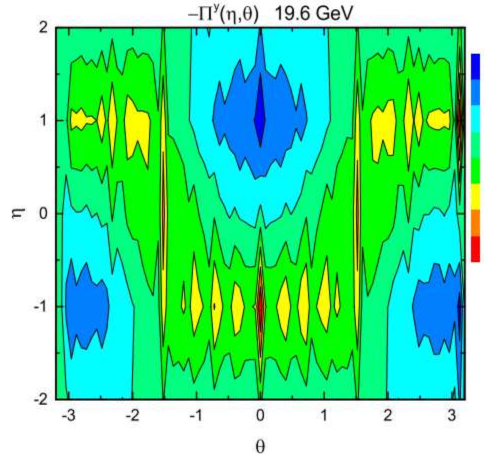
- Theory consistent with experiments in p_T and rapidity dependence.
- Puzzle: opposite ϕ dependence in theory and experiment.

Transverse spin harmonic flow

- **How to test the local structure of vorticity?**



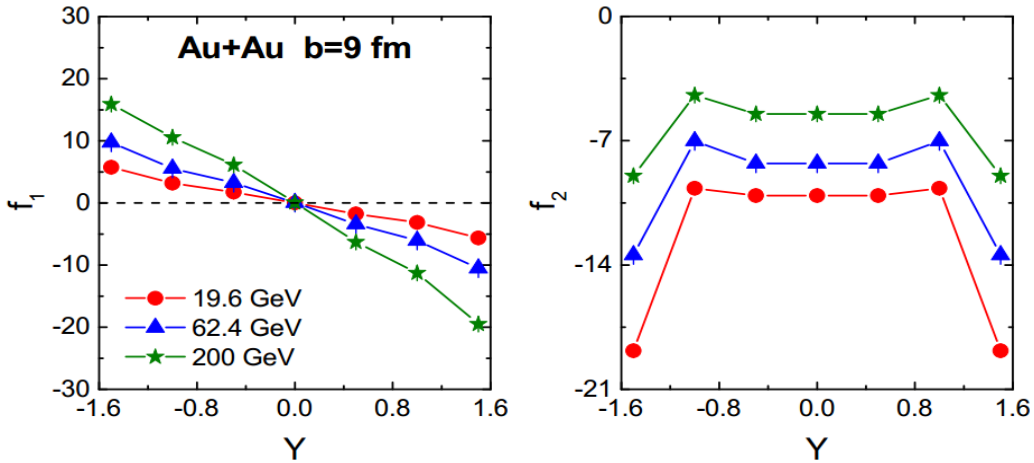
Initial spatial anisotropy



Final momentum anisotropy

- **Harmonic decomposition of the transverse polarization**

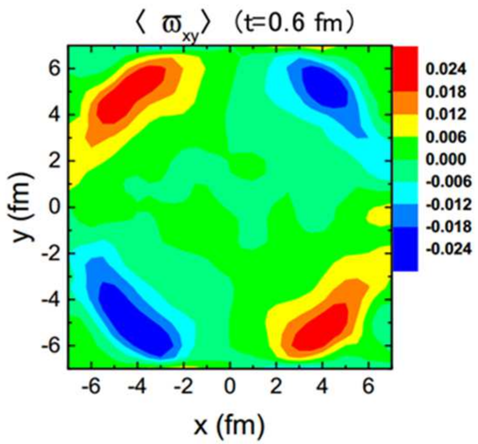
$$P_y(Y,\phi) = \frac{1}{2\pi} P_y(Y)\{1 + 2\sum_{n=1}^{\infty} f_n \cos[n(\phi - \Phi_n)]\}$$

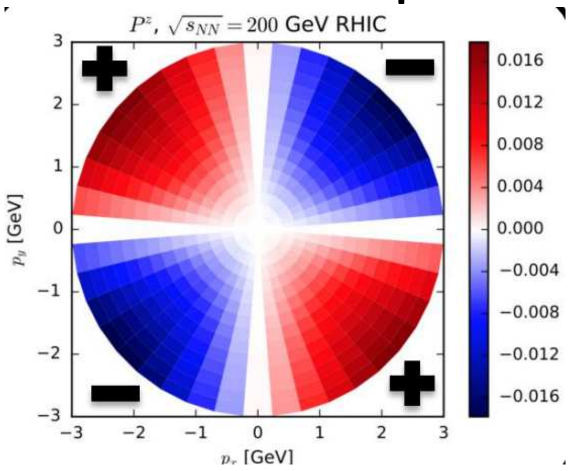


Longitudinal spin harmonic flow

- **Longitudinal vortical quadrupole.**

Becattini-Karpenko 2017

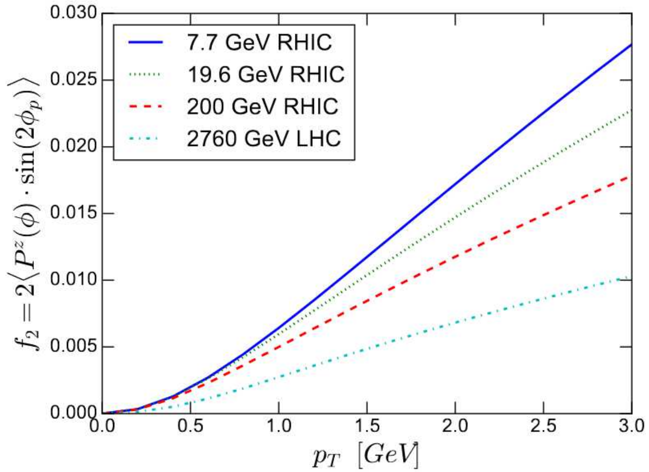




Initial spatial anisotropy **Final momentum anisotropy**

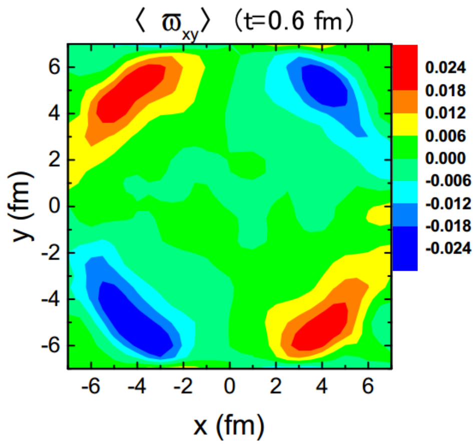
- **Harmonic decomposition of the longitudinal polarization**

$$P_z(Y,\phi) = \frac{P_z(Y)}{2\pi}\{1 + 2\sum_{n=1}^{\infty} f_n \cos[n(\phi - \Psi_n)]\}$$

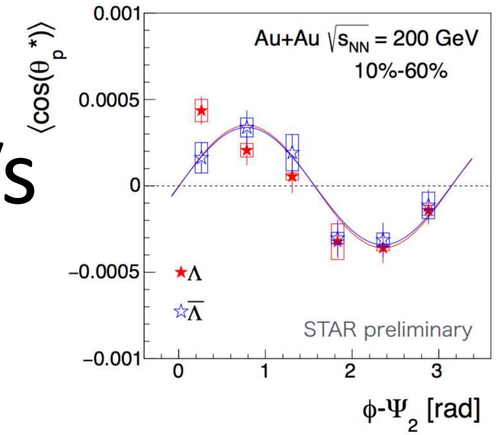


The sign problem

- **Longitudinal sign problem:**

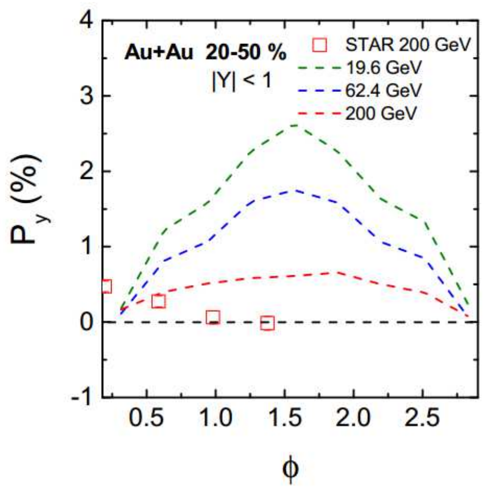


Vs



- **Transverse sign problem:**

Data: STAR Collaboration
Calculation: Wei-Deng-XGH 2018



Spin alignment

ϕ-spin alignment

- Vorticity can also polarize spin of vector mesons, e.g. ϕ
- Consider recombination $q + \overline{q} \rightarrow \phi$, the density matrix of q:

$$\rho^q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}$$

- The density matrix of ϕ is obtained from $\rho^q \otimes \rho^{\overline{q}}$ in basis of |↑↑), |↑↓)- |↓↑), and (↓↓ |

$$\rho^V = \begin{pmatrix} \frac{(1+P_q)(1+P_{\bar{q}})}{3+P_q P_{\bar{q}}} & 0 & 0 \\ 0 & \frac{1-P_q P_{\bar{q}}}{3+P_q P_{\bar{q}}} & 0 \\ 0 & 0 & \frac{(1-P_q)(1-P_{\bar{q}})}{3+P_q P_{\bar{q}}}) \end{pmatrix}$$

- Suppose $P_q = P_{\overline{q}}$,

$$\rho_{00}^{\rho(\mathrm{rec})} = \frac{1-P_q^2}{3+P_q^2}$$

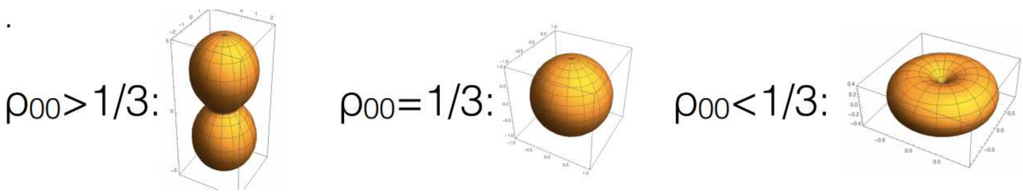
Liang-Wang 2005

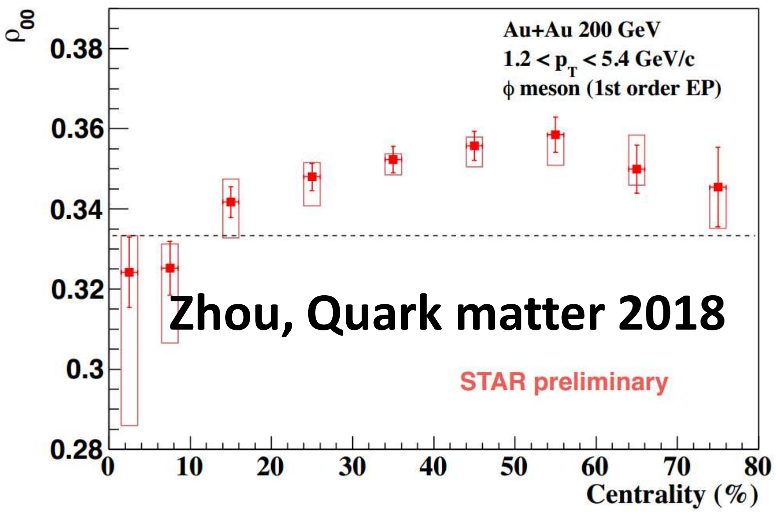
Smaller than 1/3

ϕ-spin alignment

- Φ decay via strong process, no parity violation, it is not easy to determine its spin polarization states, but

$$\frac{dN}{d(\cos\theta^*)} = N_0 \times \left[\left(1-\rho_{00}\right) + (3\rho_{00}-1)\cos^2\theta^* \right]$$





Puzzle: for most centrality, $\rho_{00} > \frac{1}{3}$

Magnetic field contribution?

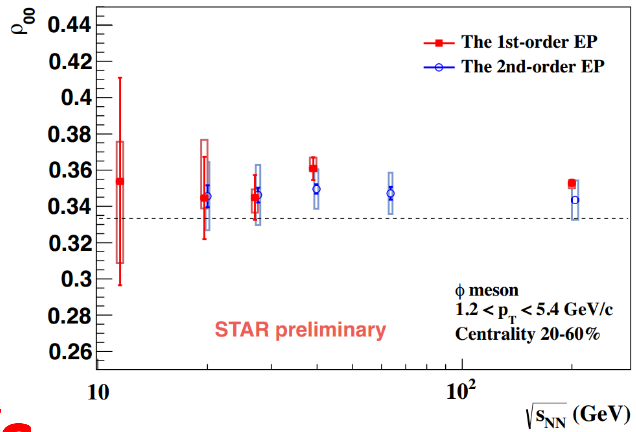
Fragmentation?

Gluon contribution?

… …

ϕ-spin alignment

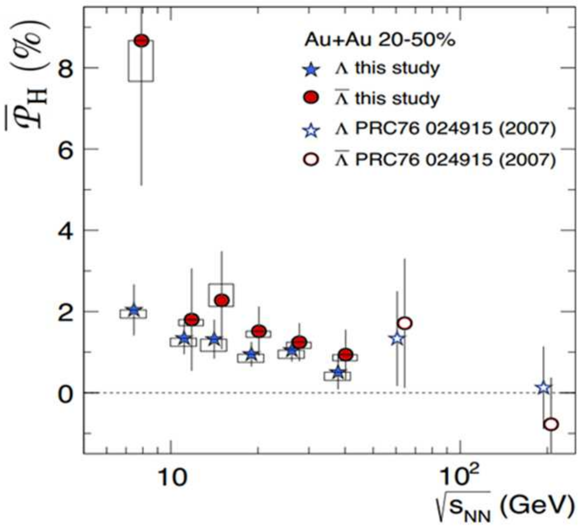
- Φ decay via strong process, no parity violation, it is not easy to determine its spin polarization states, but

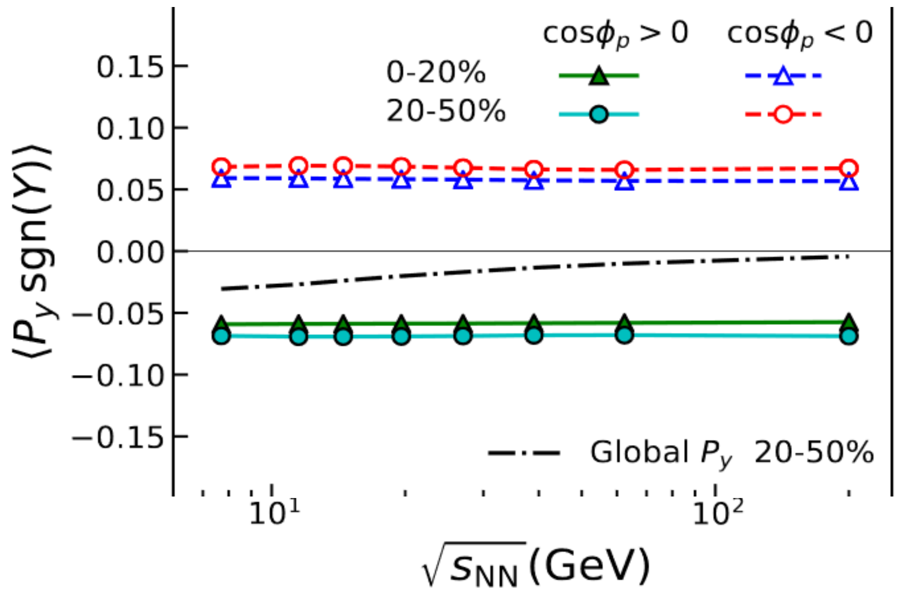


No significant energy dependence

Can be understood. As ρ_{00} depends on P_q^2

Vs

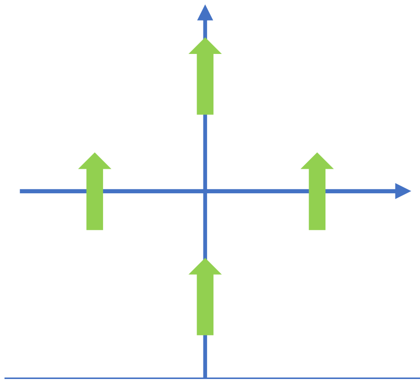




Xia-Li-Wang 2018

ϕ-spin alignment

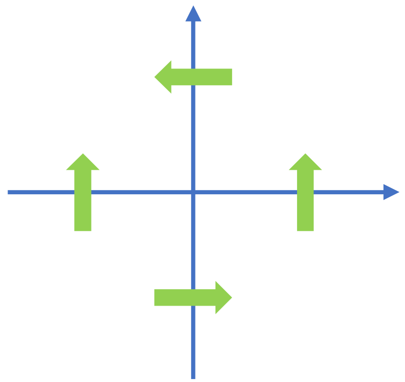
- **Spin configuration for vector mesons:**
  ρ_{11}~|↑↑) (↑↑|, ρ_{-1-1}~|↓↓) (↓↓ |, ρ_{00}~ [|↑↓)- |↓↑)][(↑↓|- (↓↑|]



$$\rho_{00} = \frac{1 - P_y^2}{3 + P_y^2}$$

Liang-Wang 2005

$$\rho_{00}^{V(rec)} < 1/3 \quad \text{for} \quad q^\uparrow + \bar{q}^\uparrow \to V$$

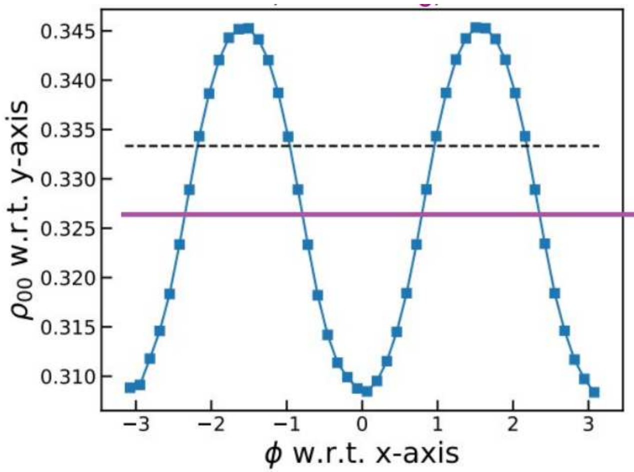


$$\rho_{00} = \frac{1 - P_y^2 + P_x^2 + P_z^2}{3 + P^2}$$

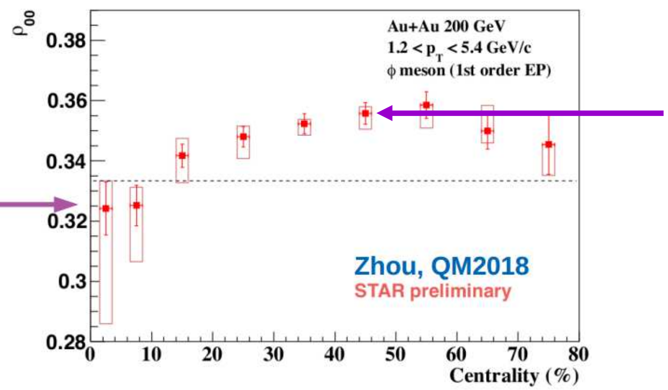
X.L.Xia-XGH, in preparation

ϕ-spin alignment

- **Predictions for central collisions:**



(Au+Au 200 GeV, b=0 fm, 1.2<pT<5.4 GeV)

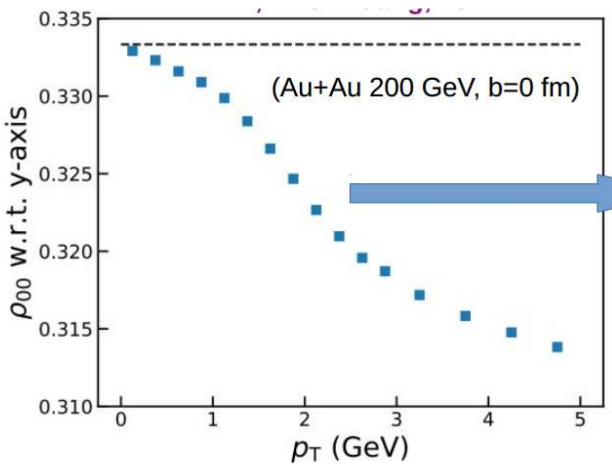


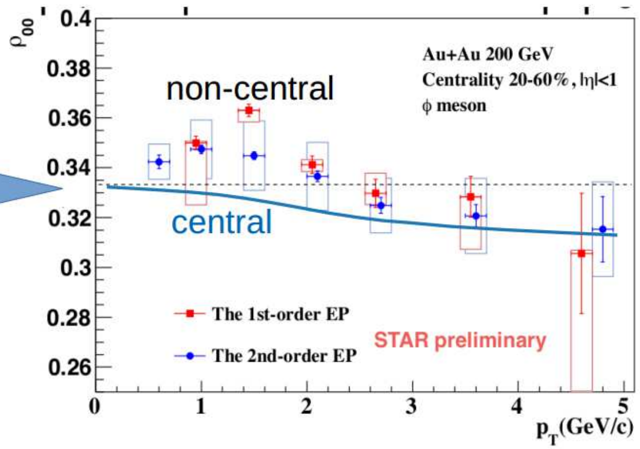
- EP is not needed in central case.
- Remove uncertainty caused by EP resolution.

Noncentral collisions:
Magnetic field ?

$$\rho_{00}^{mag} = \frac{1+P_y^2}{3-P_y^2} > \frac{1}{3}$$

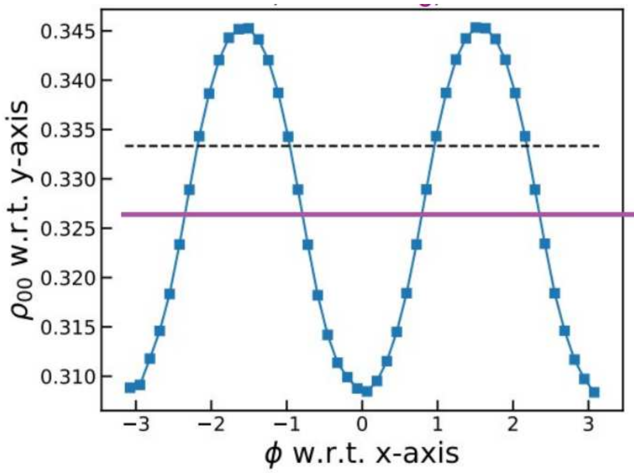
$$\rho_{00}^{vor} = \frac{1-P_y^2+P_x^2+P_z^2}{3+P^2}$$



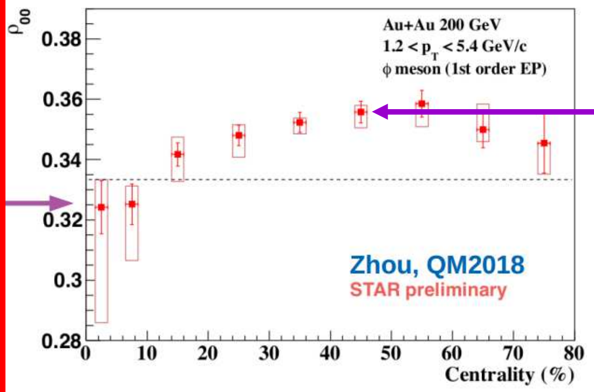


ϕ-spin alignment

- **Predictions for central collisions:**



(Au+Au 200 GeV, b=0 fm, 1.2<pT<5.4 GeV)



Noncentral collisions: Magnetic field ?

$$\rho_{00}^{mag} = \frac{1+P_y^2}{3-P_y^2} > \frac{1}{3}$$

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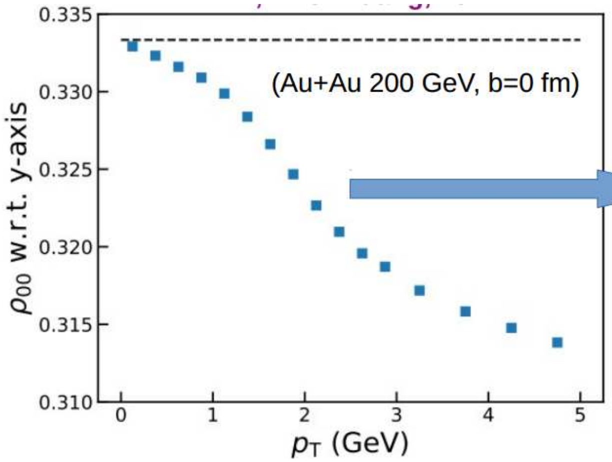
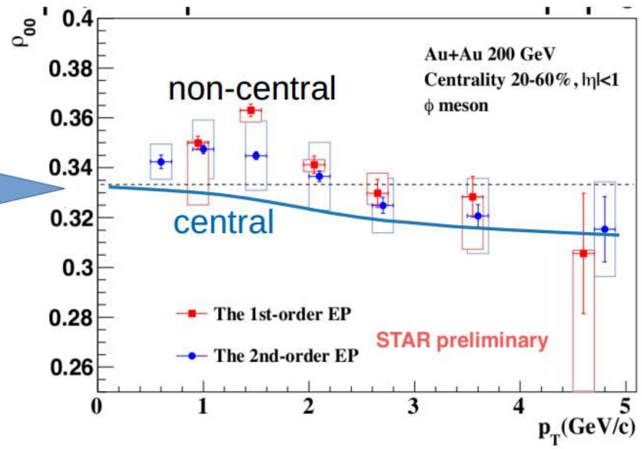
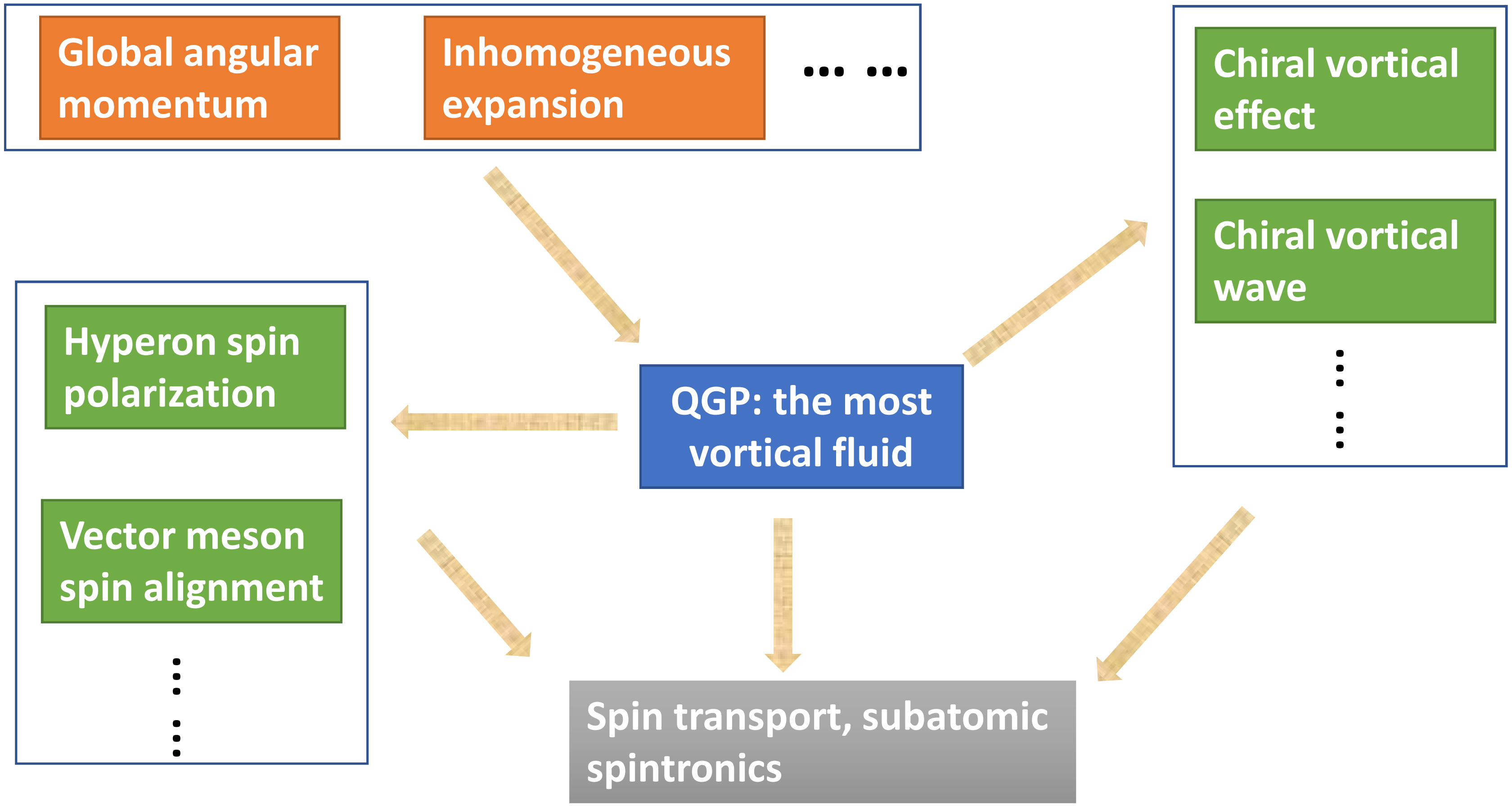




Well testable!
Evidence of circular vorticity

Summary

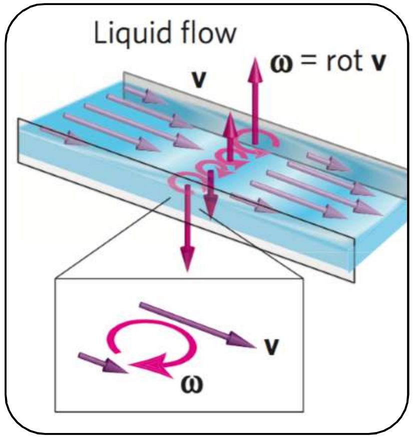
- Most vortical fluid created in HICs.
- Global polarization can be understood: vorticity induced by global AM
- Inhomogeneous expansion leads to quadrupolar vortical structure in transverse plane and reaction plane
- Sign problem in the azimuthal-angle dependence of both transverse and longitudinal polarizations
- Resonance decays don’t solve sign problem
- New observables: rapidity dependent spin harmonic flows, spin alignment in central collisions

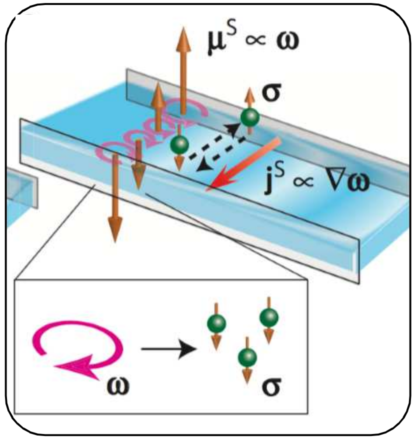

Global angular momentum
Inhomogeneous expansion
… …
Chiral vortical effect
Chiral vortical wave
Hyperon spin polarization
Vector meson spin alignment
QGP: the most vortical fluid
Spin transport, subatomic spintronics

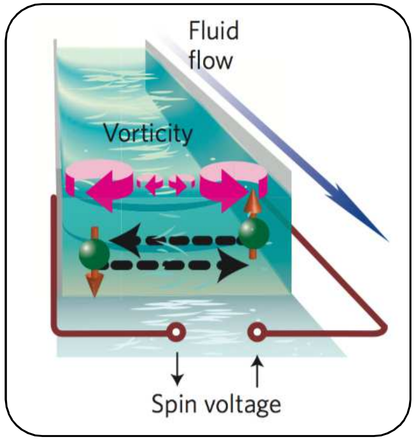
Thank you!

Subatomic spintronics

- **Spin hydrodynamic generation in Hg** (Takahashi, et al. Nat. Phys. (2016))







- **Subatomic spintronics in HIC: a new probe for QGP**