Vorticity and polarization in heavy-ion collisions

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Office of Science

Hadron Interactions and Polarization from
Lattice QCD, Quark model, and Heavy-ion collisions
@YITP, Kyoto

Important features in non-central heavy-ion collisions

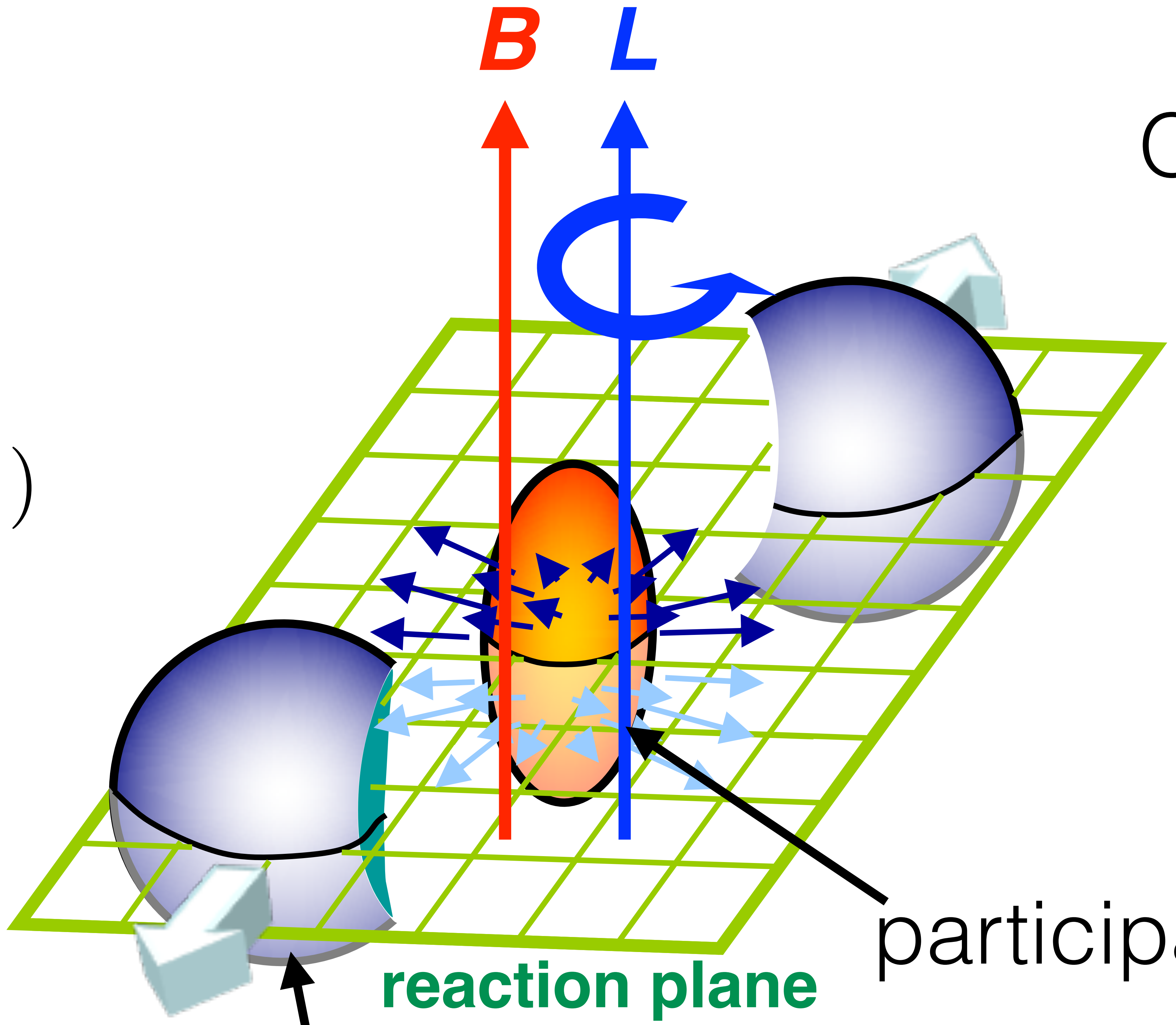
Strong magnetic field

$$B \sim 10^{13}\ \mathrm{T}$$

$$(eB \sim \mathrm{MeV}^2\ (\tau = 0.2\ \mathrm{fm}))$$

D. Kharzeev, L. McLerran, and H. Warringa,
Nucl.Phys.A803, 227 (2008)
McLerran and Skokov, Nucl. Phys. A929, 184 (2014)

→Chiral magnetic effect
Chiral magnetic wave



participants

spectators

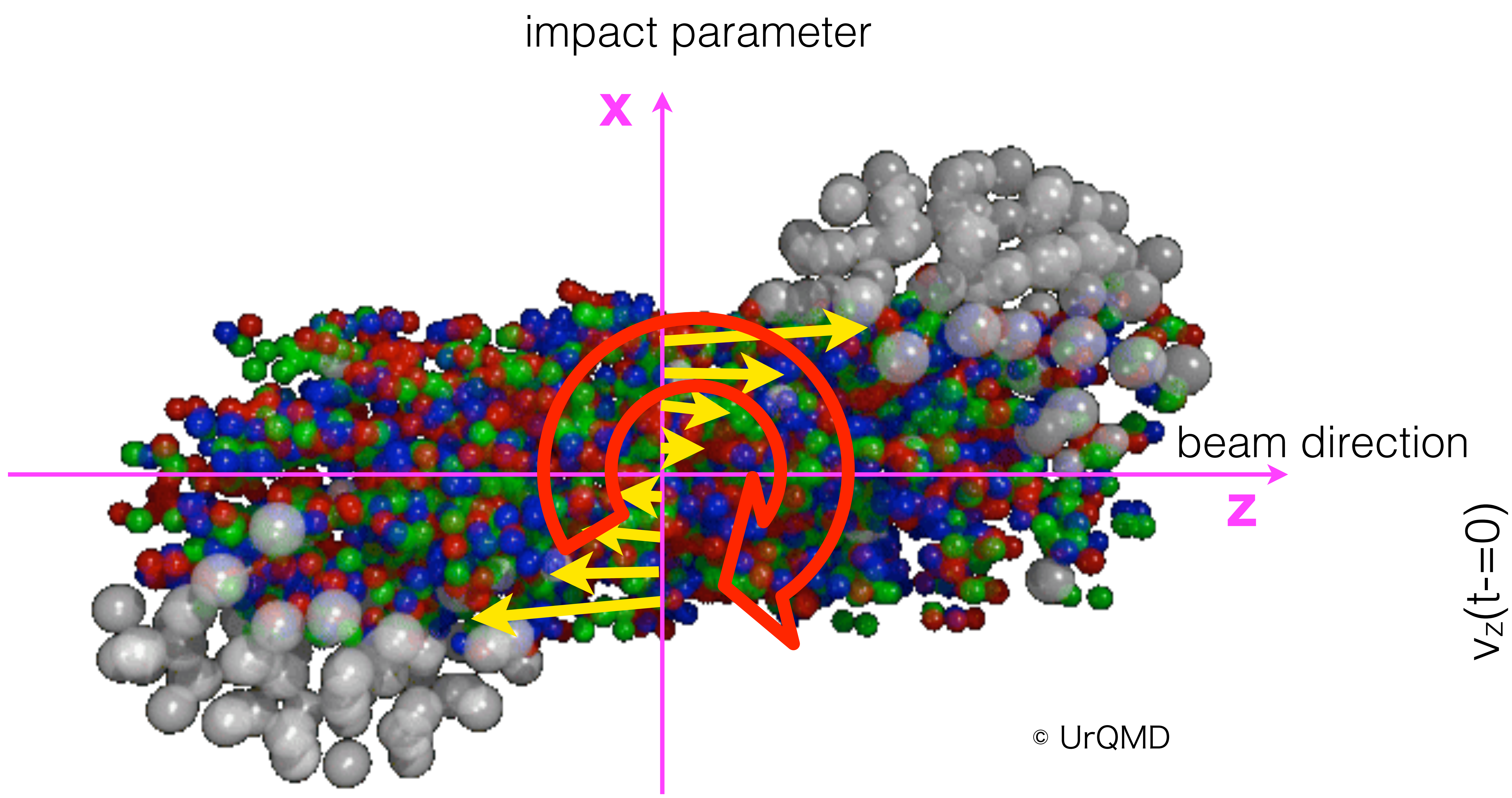
Orbital angular momentum

$$L \sim 10^5 \hbar$$

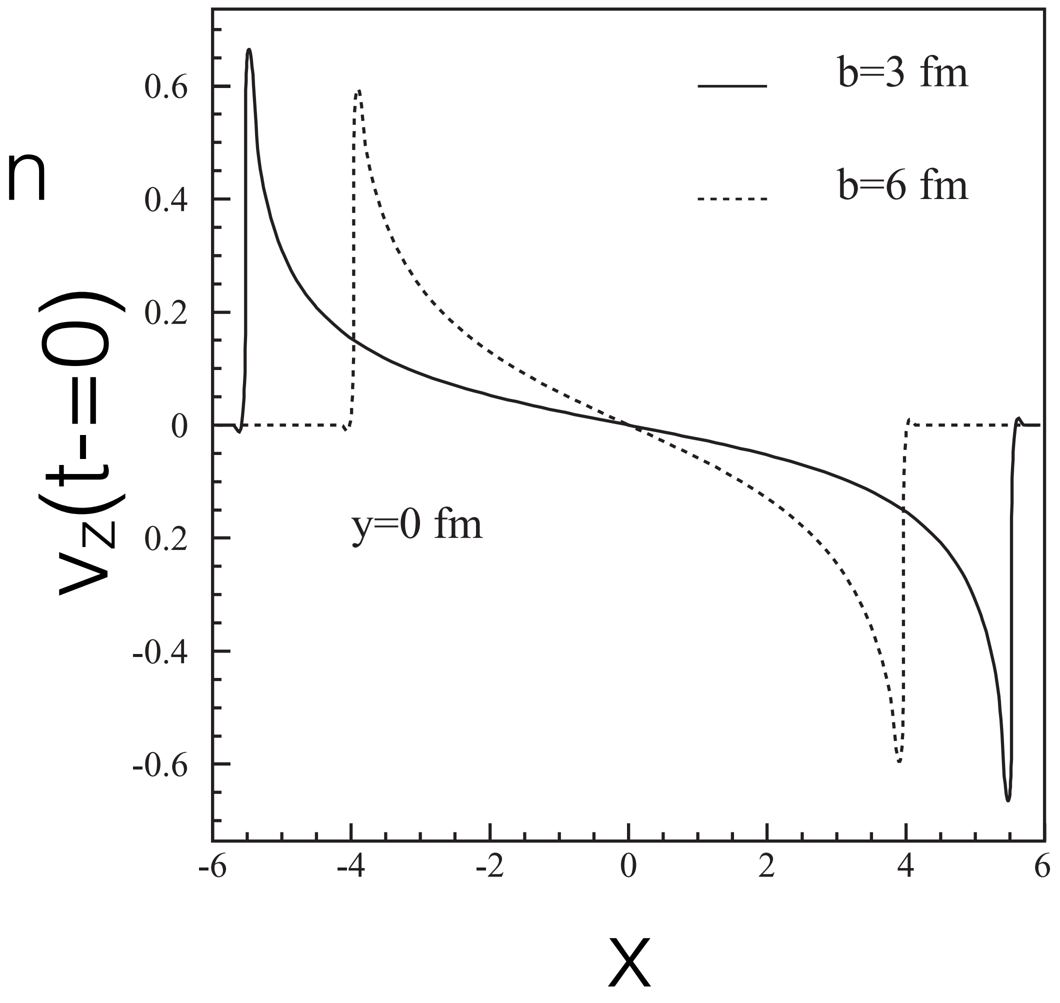
Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005)

→Vorticity and particle polarization
Chiral vortical effect

Vorticity in HIC



Becattini et al., PRC77, 024906 (2008)

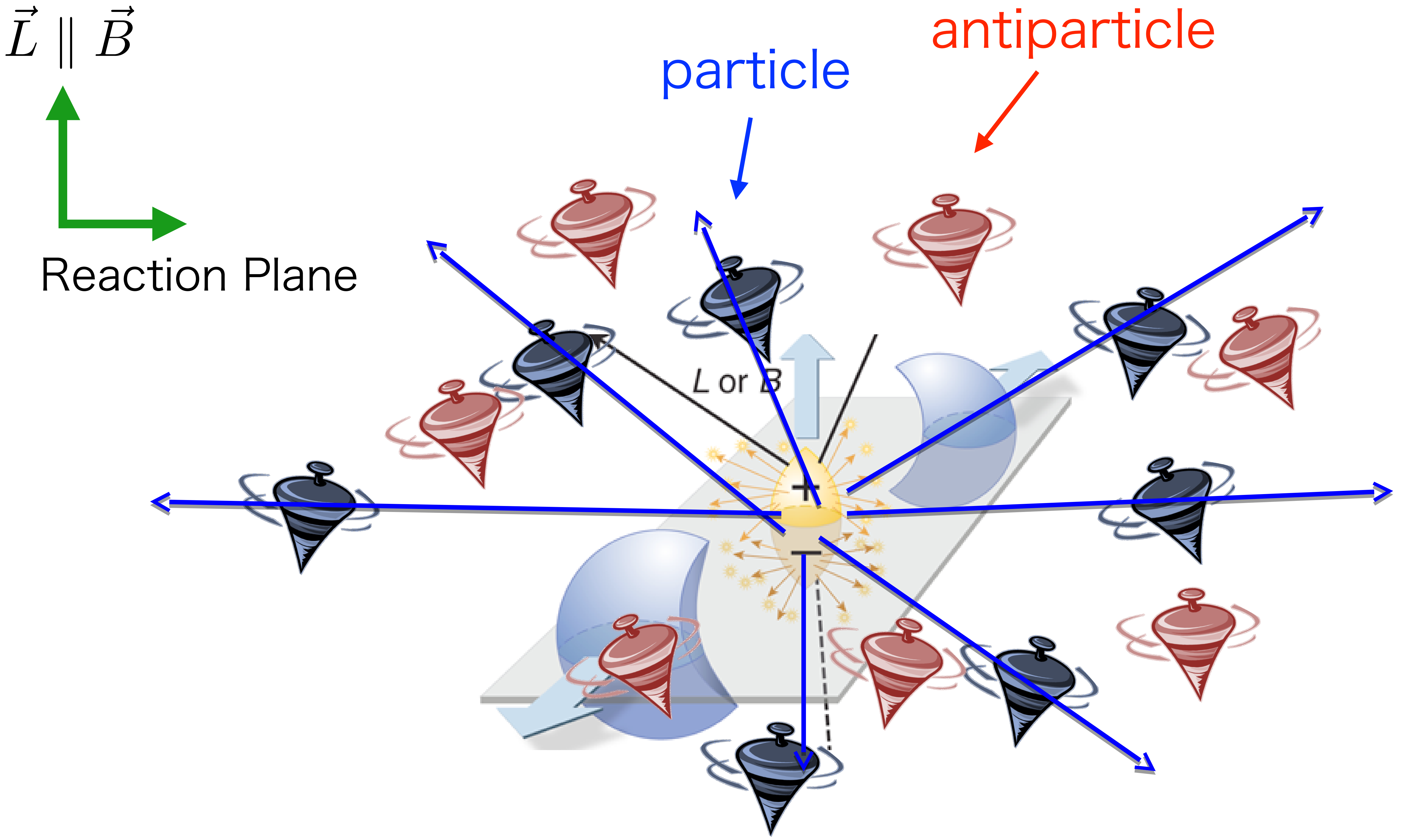


In non-central collisions,
the initial collective longitudinal flow velocity depends on x.

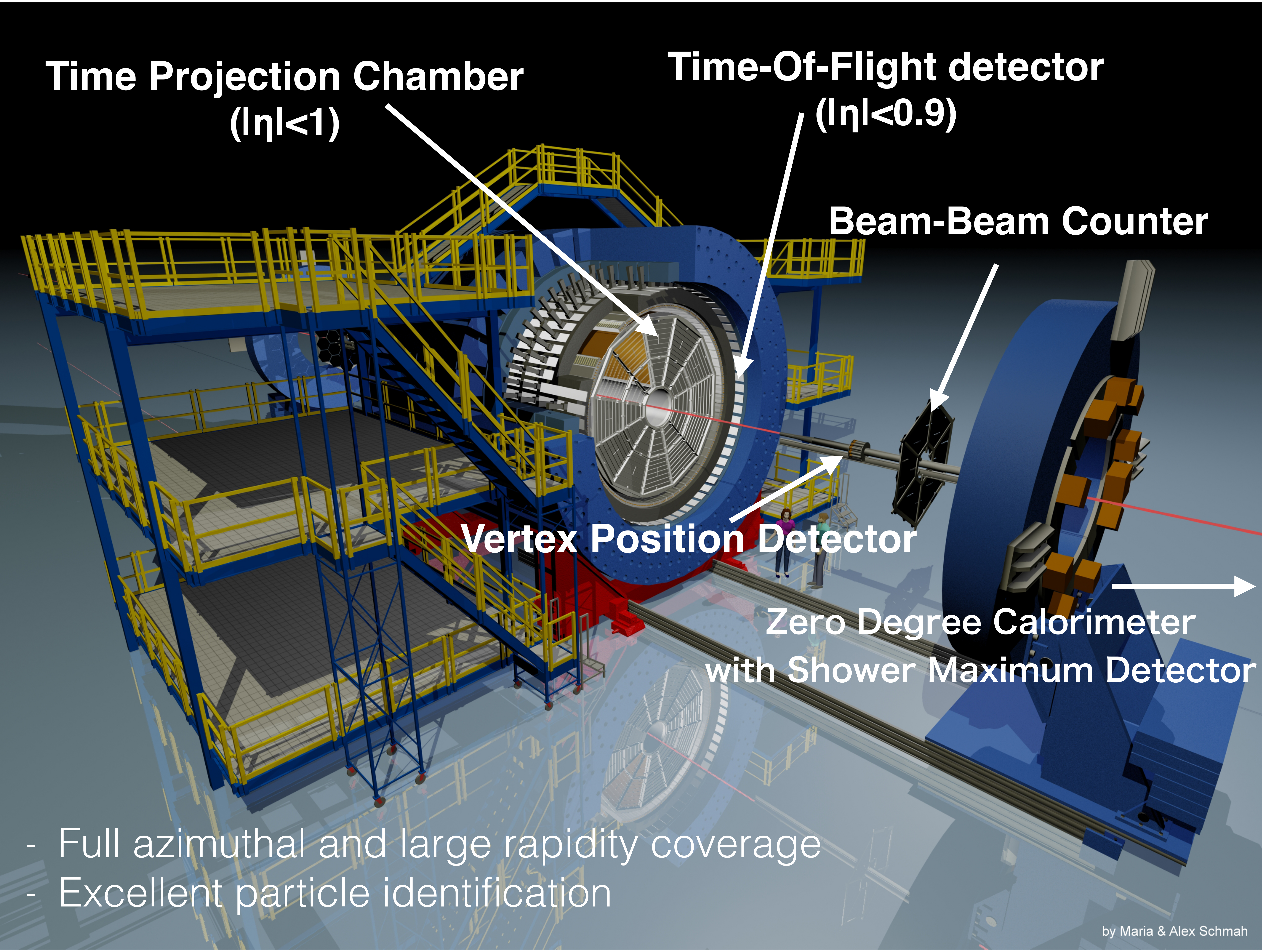
$$\omega_y = \frac{1}{2}(\nabla \times v)_y \approx -\frac{1}{2}\frac{\partial v_z}{\partial x}$$

Global polarization

- Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005)
- S. Voloshin, nucl-th/0410089 (2004)



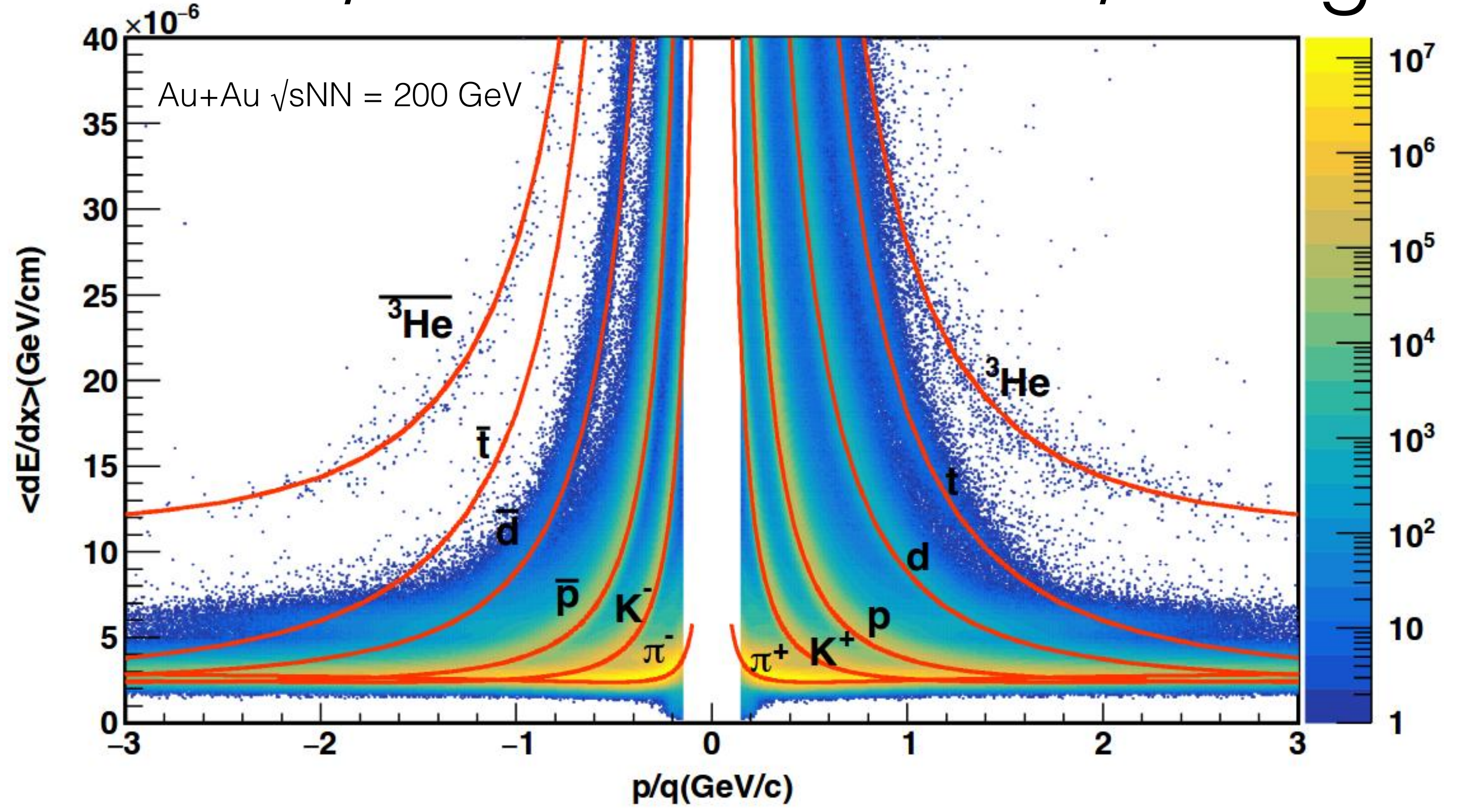
STAR Detectors



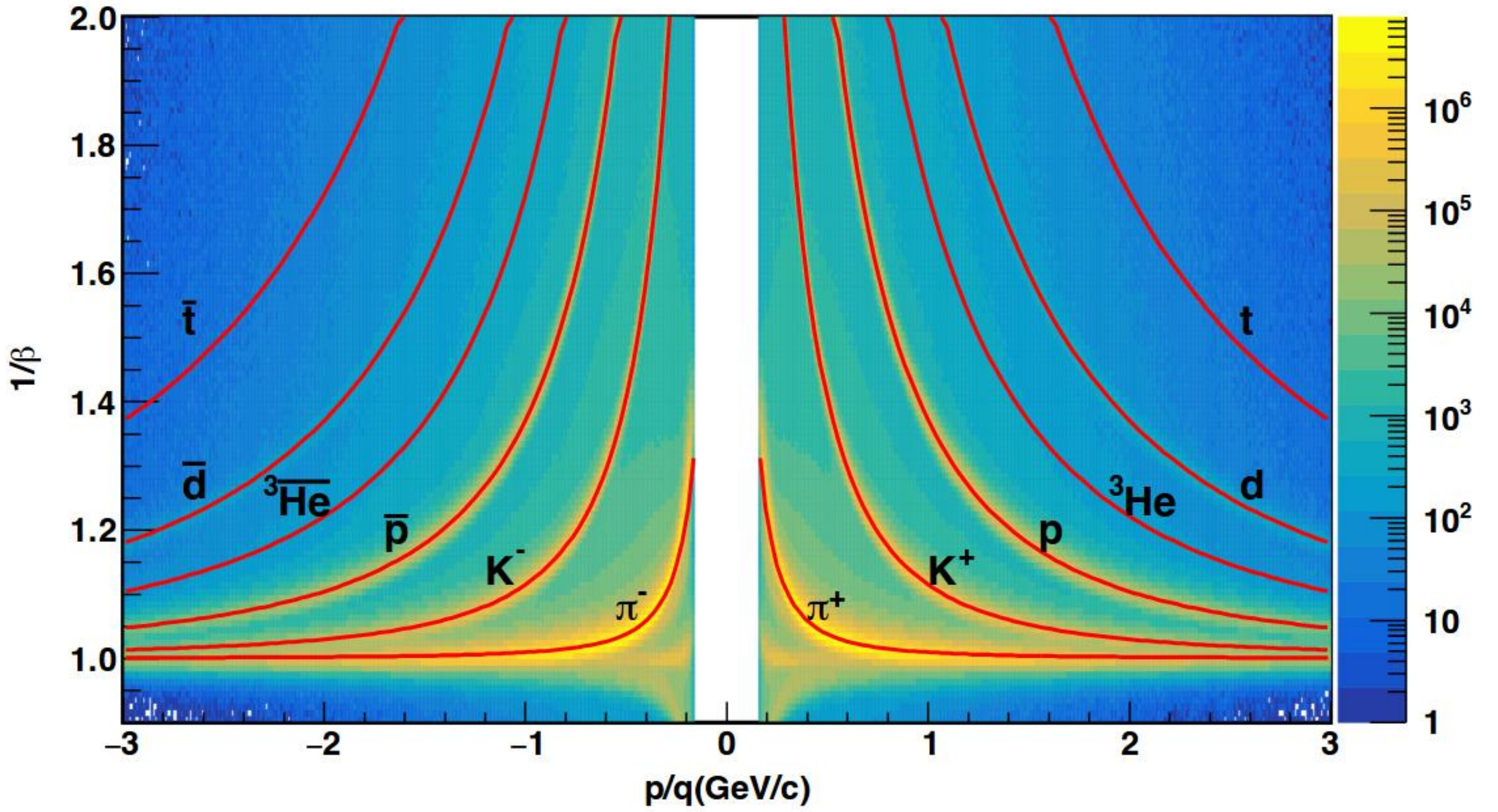
- Full azimuthal and large rapidity coverage
- Excellent particle identification

by Maria & Alex Schmah

TPC dE/dx vs momentum/charge



TOF 1/β vs momentum/charge



How to measure the polarization?

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

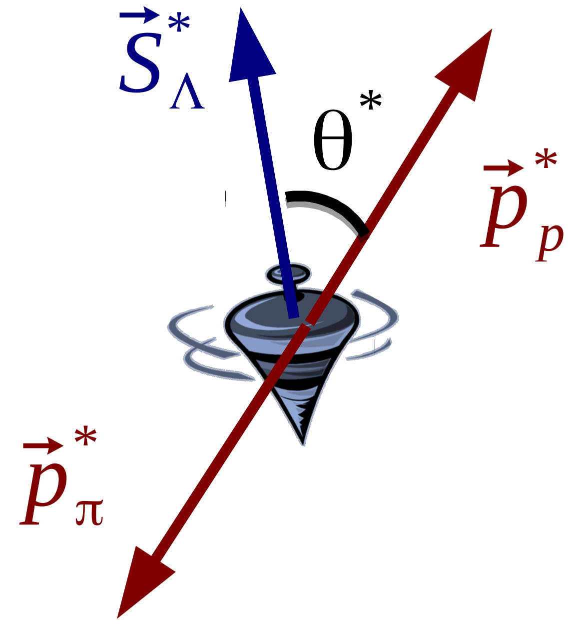
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}}\mathbf{P}_{\mathrm{H}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

P_H: Λ polarization

p_p^*: proton momentum in the Λ rest frame

α_H: Λ decay parameter

($\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)



$$\Lambda \rightarrow p + \pi^-$$

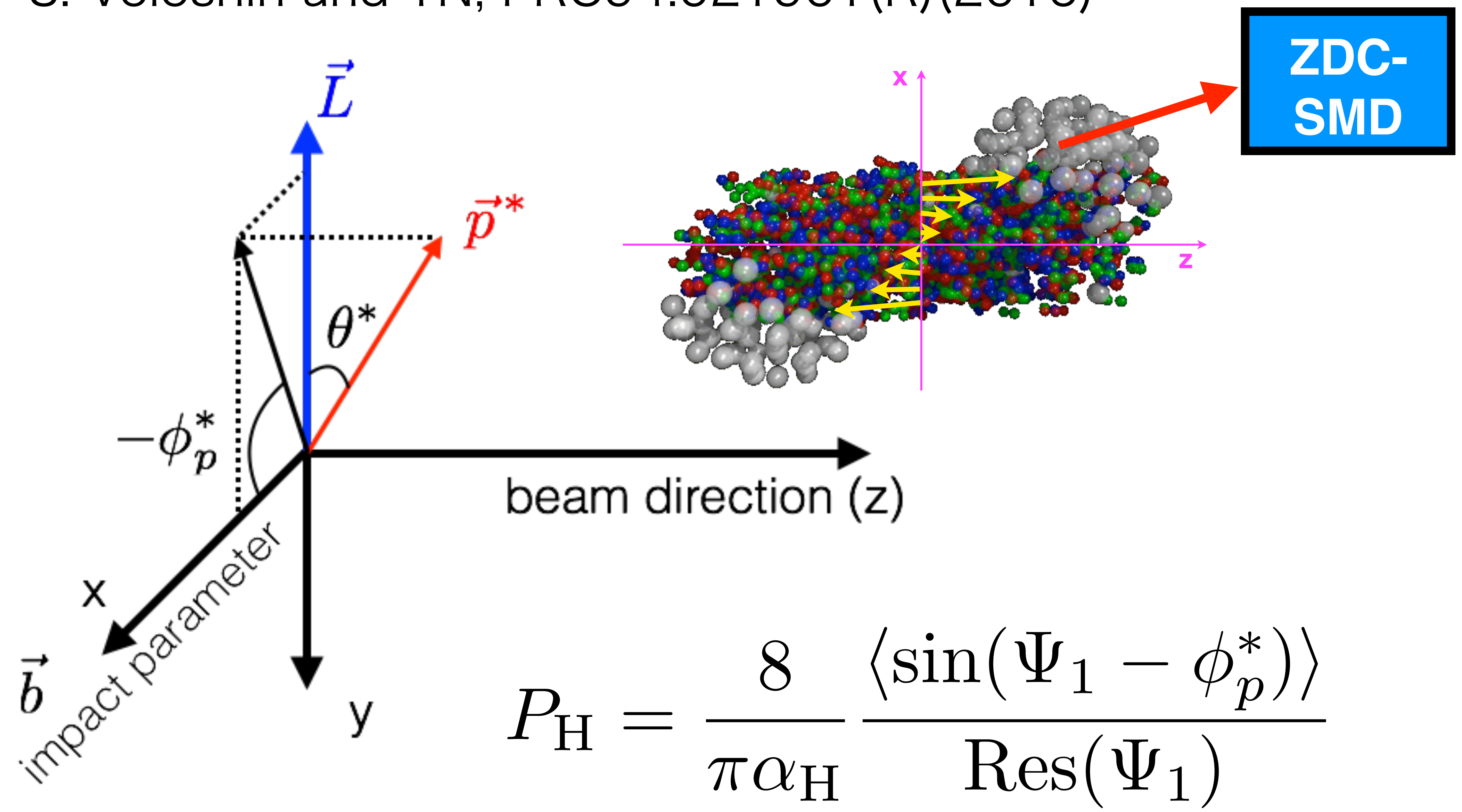
(BR: 63.9%, cτ ~7.9 cm)

C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)

- S. Voloshin and TN, PRC94.021901(R)(2016)



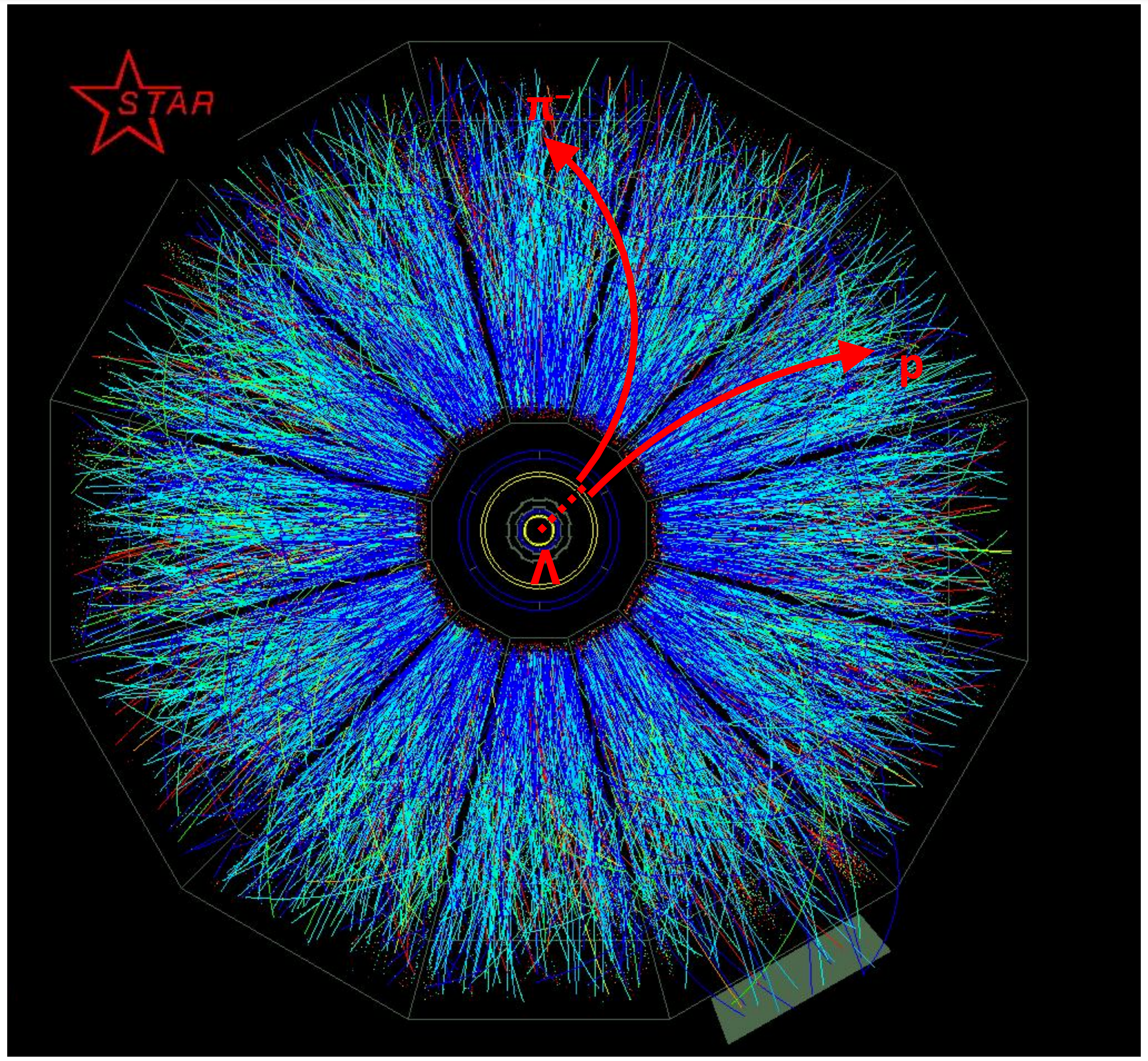
$$P_{\mathrm{H}} = \frac{8}{\pi\alpha_{\mathrm{H}}}\frac{\langle\sin(\Psi_1 - \phi_p^*)\rangle}{\mathrm{Res}(\Psi_1)}$$

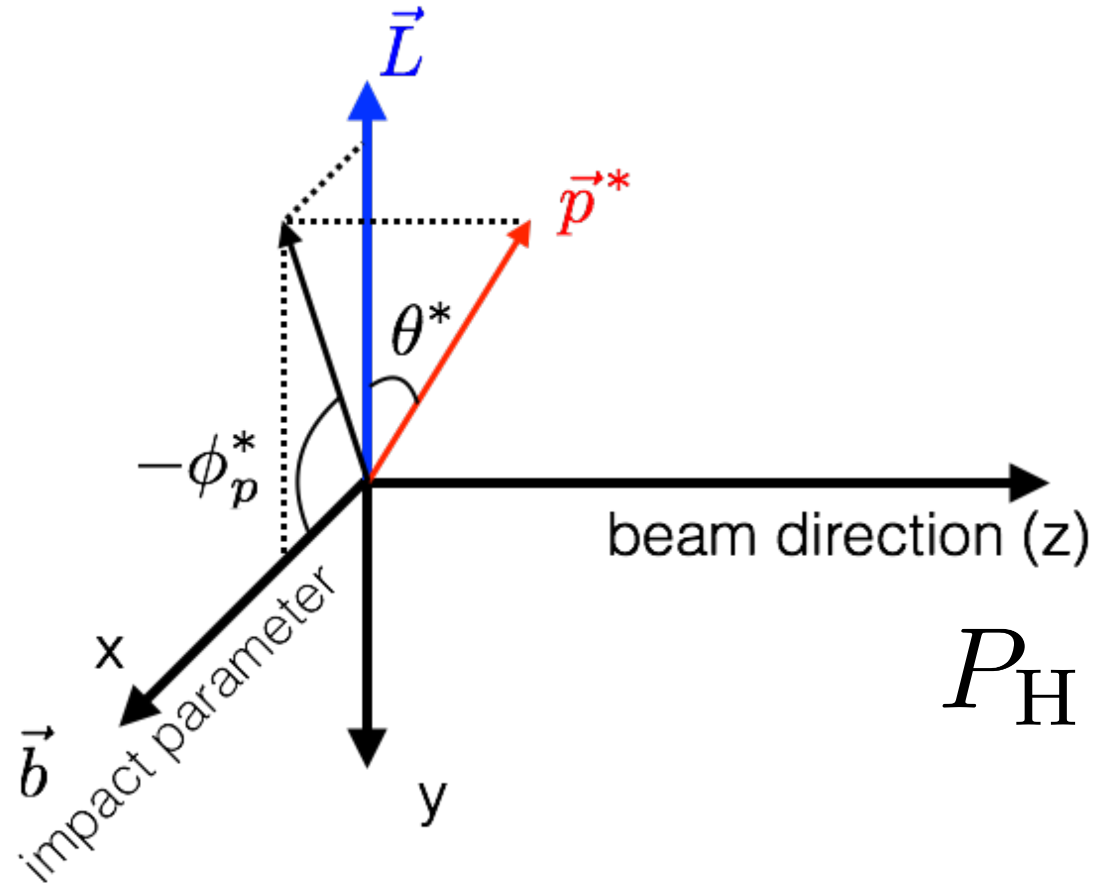
Ψ_1: azimuthal angle of the impact parameter

ϕ_p^*: ϕ of daughter proton in Λ rest frame

STAR, PRC76, 024915 (2007)

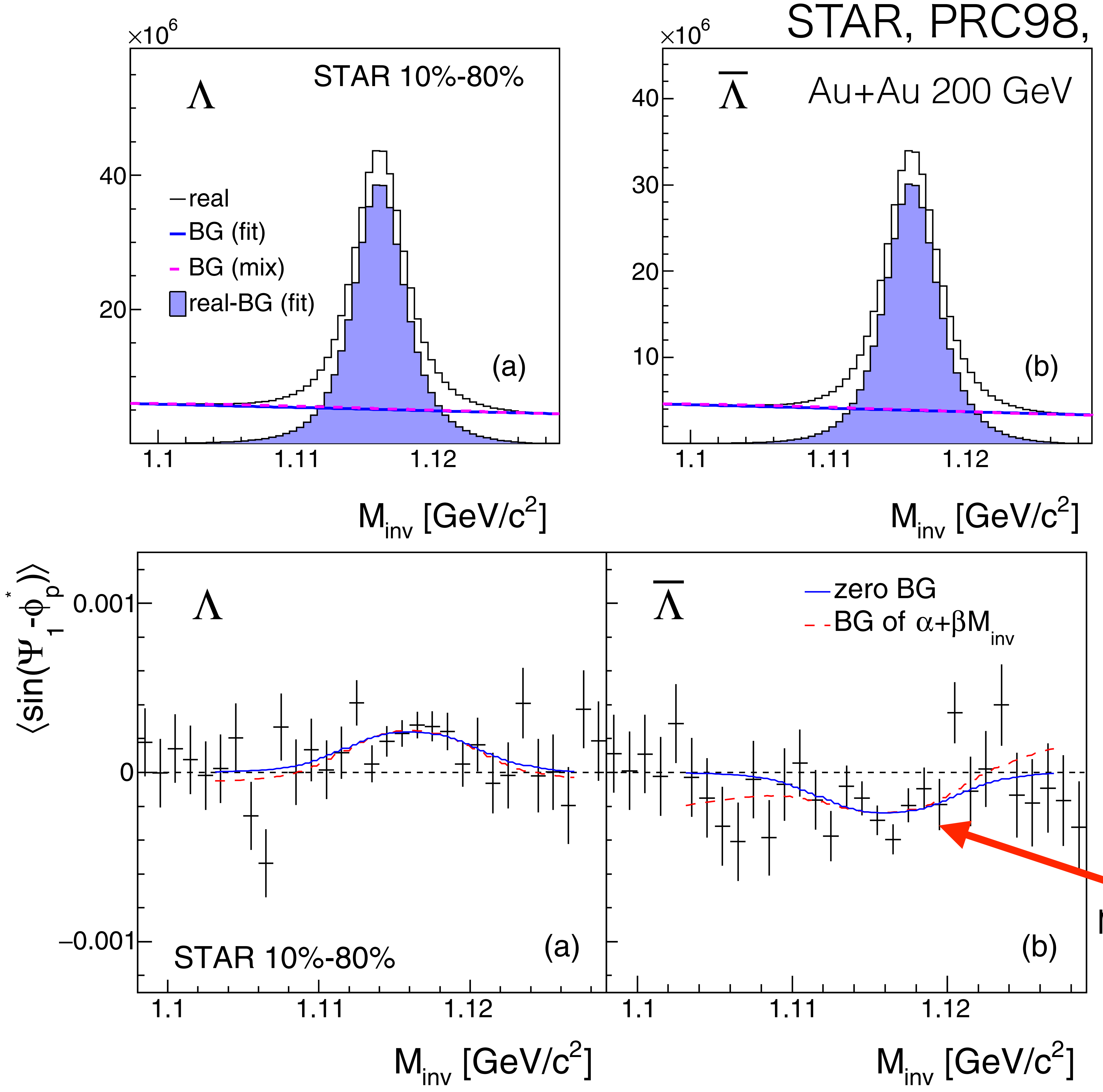
Signal extraction with Λ hyperons





$$P_{\rm H} = \frac{8}{\pi\alpha_{\rm H}} \frac{\langle \sin(\Psi_1 - \phi_p^*) \rangle}{{\rm Res}(\Psi_1)}$$

STAR, PRC98, 014910 (2018)



negative for anti-Λ

$$\alpha_H = -\alpha_{\bar{H}}$$

$$\langle \sin(\Psi_1 - \phi_p^*) \rangle^{\rm obs} = (1 - f^{\rm Bg}(M_{\rm inv})) \langle \sin(\Psi_1 - \phi_p^*) \rangle^{\rm Sg} + f^{\rm Bg}(M_{\rm inv}) \langle \sin(\Psi_1 - \phi_p^*) \rangle^{\rm Bg},$$

Feed-down effect

- Only ~25% of measured Λ and anti-Λ are primary, while ~60% are feed-down from $\Sigma^{*}\to\Lambda\pi$, $\Sigma^{0}\to\Lambda\gamma$, $\Xi\to\Lambda\pi$
- Polarization of parent particle R is transferred to its daughter Λ

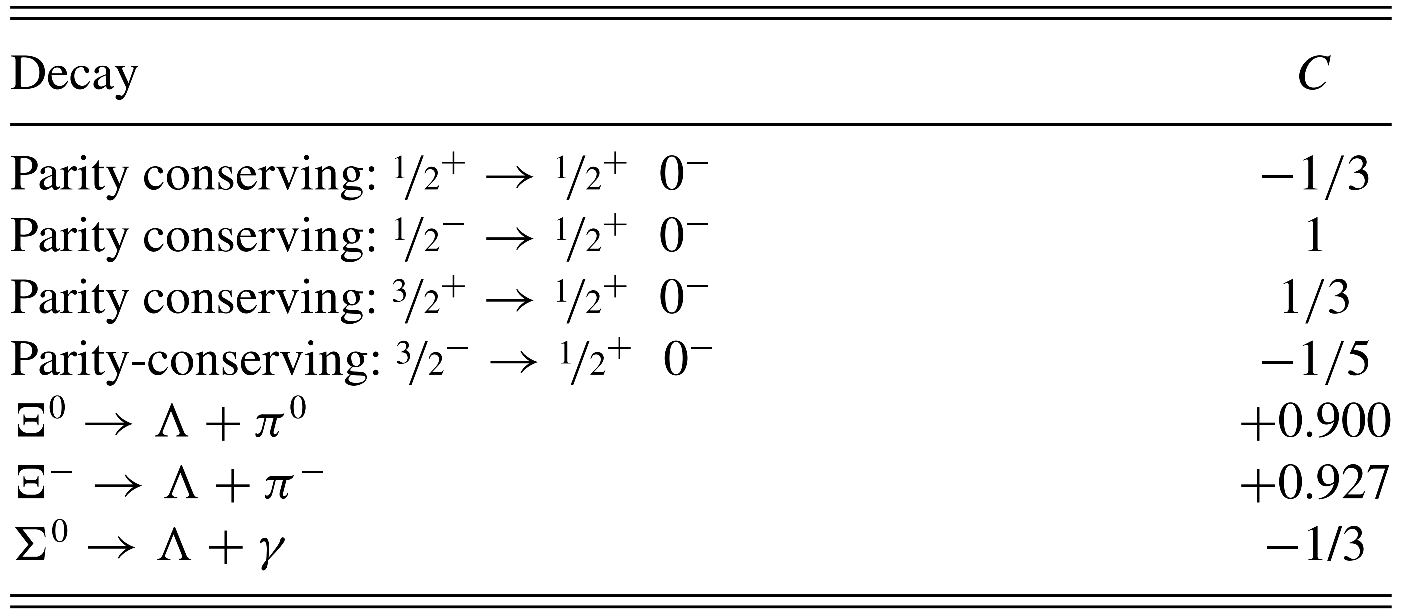
$$\mathbf{S}^{*}_{\Lambda} = C\mathbf{S}^{*}_{R}$$

$$\langle S_y \rangle \propto \frac{S(S+1)}{3}\left(\omega + \frac{\mu}{S}B\right)$$

Becattini, Karpenko, Lisa, Upsal, and Voloshin, PRC95.054902 (2017)

$C_{\Lambda R}$: coefficient of spin transfer from parent R to Λ
S_R : parent particle's spin
$f_{\Lambda R}$: fraction of Λ originating from parent R
μ_R : magnetic moment of particle R

$$\begin{pmatrix} \varpi_{\mathrm{c}} \\ \\ B_{\mathrm{c}}/T \end{pmatrix} = \begin{bmatrix} \frac{2}{3}\sum_{R}\left(f_{\Lambda R}\,C_{\Lambda R} - \frac{1}{3}f_{\Sigma^0 R}\,C_{\Sigma^0 R}\right)\,S_R(S_R+1) & \frac{2}{3}\sum_{R}\left(f_{\Lambda R}\,C_{\Lambda R} - \frac{1}{3}f_{\Sigma^0 R}\,C_{\Sigma^0 R}\right)\,(S_R+1)\,\mu_R \\ \\ \frac{2}{3}\sum_{\overline{R}}\left(f_{\overline{\Lambda}\overline{R}}\,C_{\overline{\Lambda}\overline{R}} - \frac{1}{3}f_{\overline{\Sigma}^0\overline{R}}\,C_{\overline{\Sigma}^0\overline{R}}\right)\,S_{\overline{R}}(S_{\overline{R}}+1) & \frac{2}{3}\sum_{\overline{R}}\left(f_{\overline{\Lambda}\overline{R}}\,C_{\overline{\Lambda}\overline{R}} - \frac{1}{3}f_{\overline{\Sigma}^0\overline{R}}\,C_{\overline{\Sigma}^0\overline{R}}\right)\,(S_{\overline{R}}+1)\,\mu_{\overline{R}} \end{bmatrix}^{-1} \begin{pmatrix} P^{\mathrm{meas}}_{\Lambda} \\ \\ P^{\mathrm{meas}}_{\overline{\Lambda}} \end{pmatrix}$$



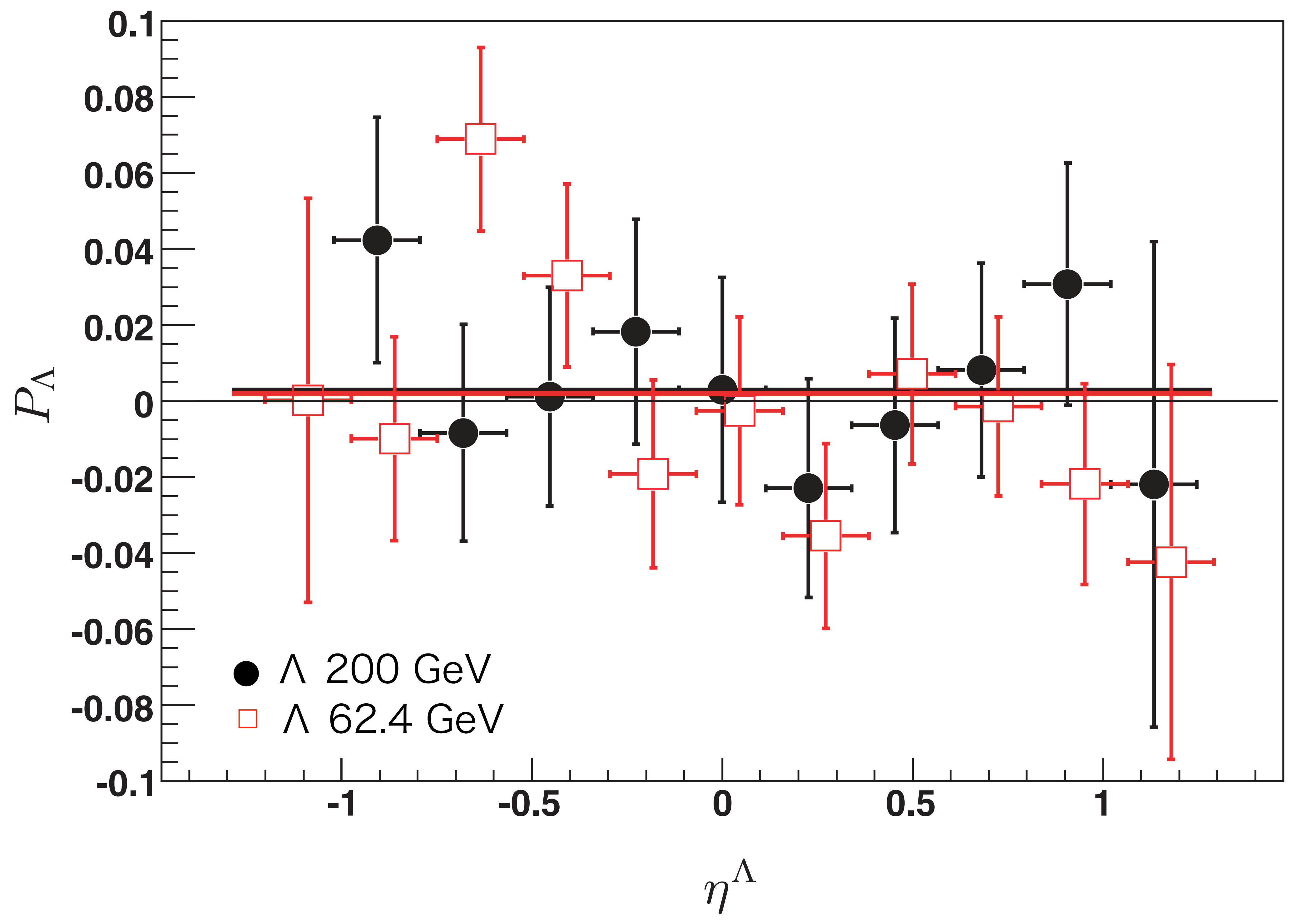
Decay	C
Parity conserving: $1/2^{+} \to 1/2^{+}\ 0^{-}$	$-1/3$
Parity conserving: $1/2^{-} \to 1/2^{+}\ 0^{-}$	1
Parity conserving: $3/2^{+} \to 1/2^{+}\ 0^{-}$	$1/3$
Parity-conserving: $3/2^{-} \to 1/2^{+}\ 0^{-}$	$-1/5$
$\Xi^{0} \to \Lambda + \pi^{0}$	+0.900
$\Xi^{-} \to \Lambda + \pi^{-}$	+0.927
$\Sigma^{0} \to \Lambda + \gamma$	$-1/3$

15%-20% dilution of primary Λ polarization
(model-dependent)

First paper on Λ polarization from STAR

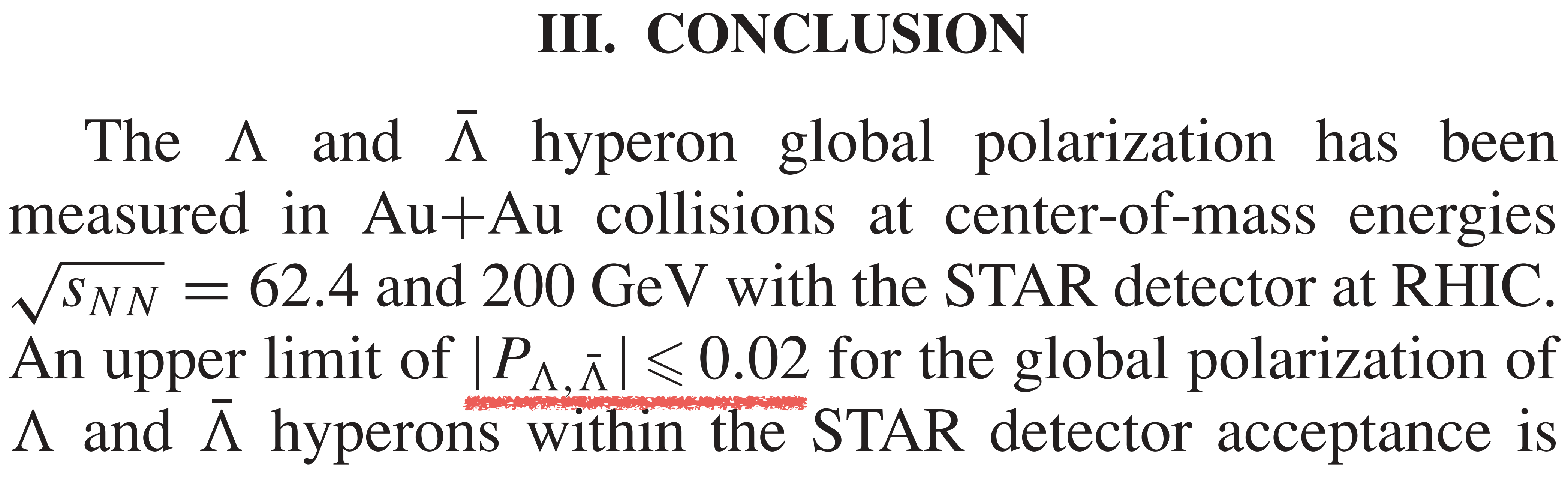
PHYSICAL REVIEW C **76**, 024915 (2007)

Global polarization measurement in Au+Au collisions



Au+Au collisions at $\sqrt{s_{NN}}$ = 62.4 and 200 GeV in 2004 with very limited statistics (~9M events)

Results are consistent with zero…
giving an upper limit of P_H<2%

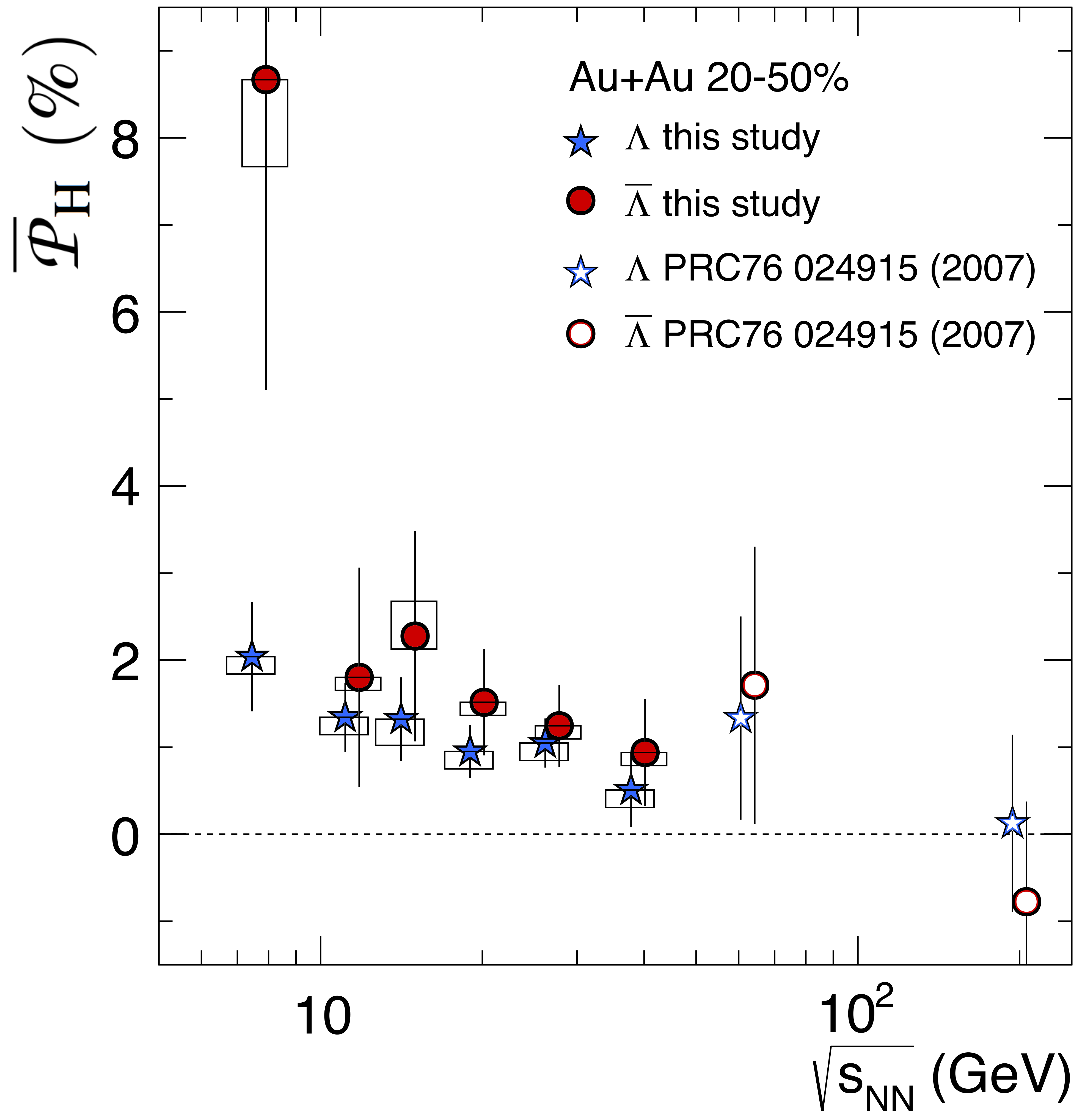


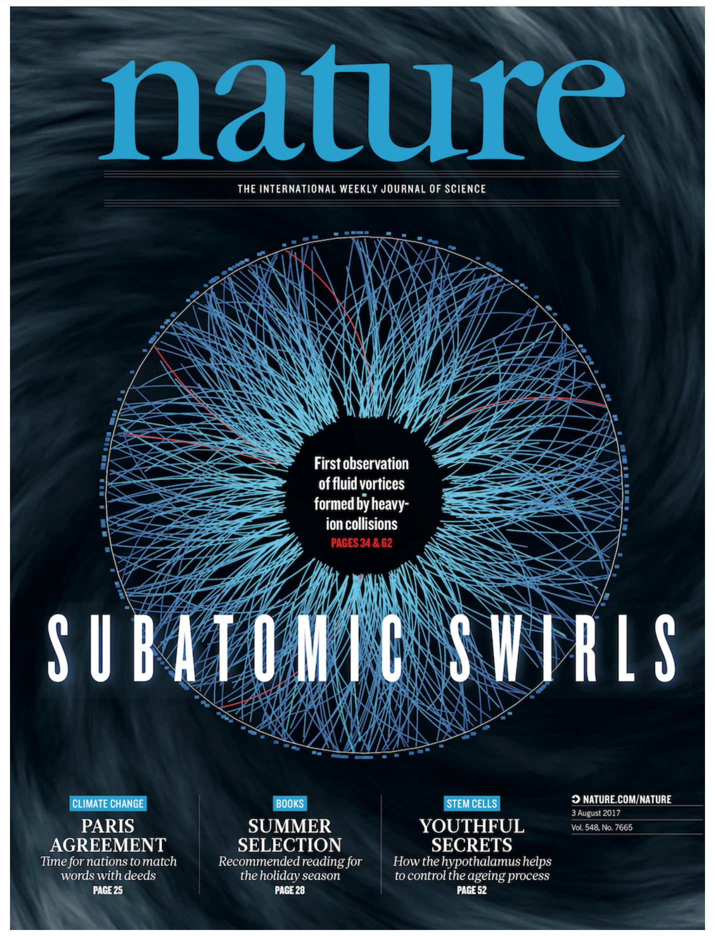
III. CONCLUSION

The Λ and $\bar{\Lambda}$ hyperon global polarization has been measured in Au+Au collisions at center-of-mass energies $\sqrt{s_{NN}} = 62.4$ and 200 GeV with the STAR detector at RHIC. An upper limit of $|P_{\Lambda,\bar{\Lambda}}| \leqslant 0.02$ for the global polarization of Λ and $\bar{\Lambda}$ hyperons within the STAR detector acceptance is

First observation of fluid vortices in HIC

STAR, Nature 548, 62 (2017)





#38

The Fastest Fluid

by Sylvia Morrow

Superhot material spins at an incredible rate.

Positive polarization signal at lower energies!

- polarization looks to increase in lower energies
- anti-Λ is systematically larger than Λ

Becattini, Karpenko, Lisa, Upsal, and Voloshin, PRC95.054902 (2017)

μΛ: Λ magnetic moment
T: temperature at thermal equilibrium

$$P_\Lambda \simeq \boxed{\frac{1}{2}\frac{\omega}{T}} + \frac{\mu_\Lambda B}{T}$$

$$P_{\bar{\Lambda}} \simeq \boxed{\frac{1}{2}\frac{\omega}{T}} - \frac{\mu_\Lambda B}{T}$$

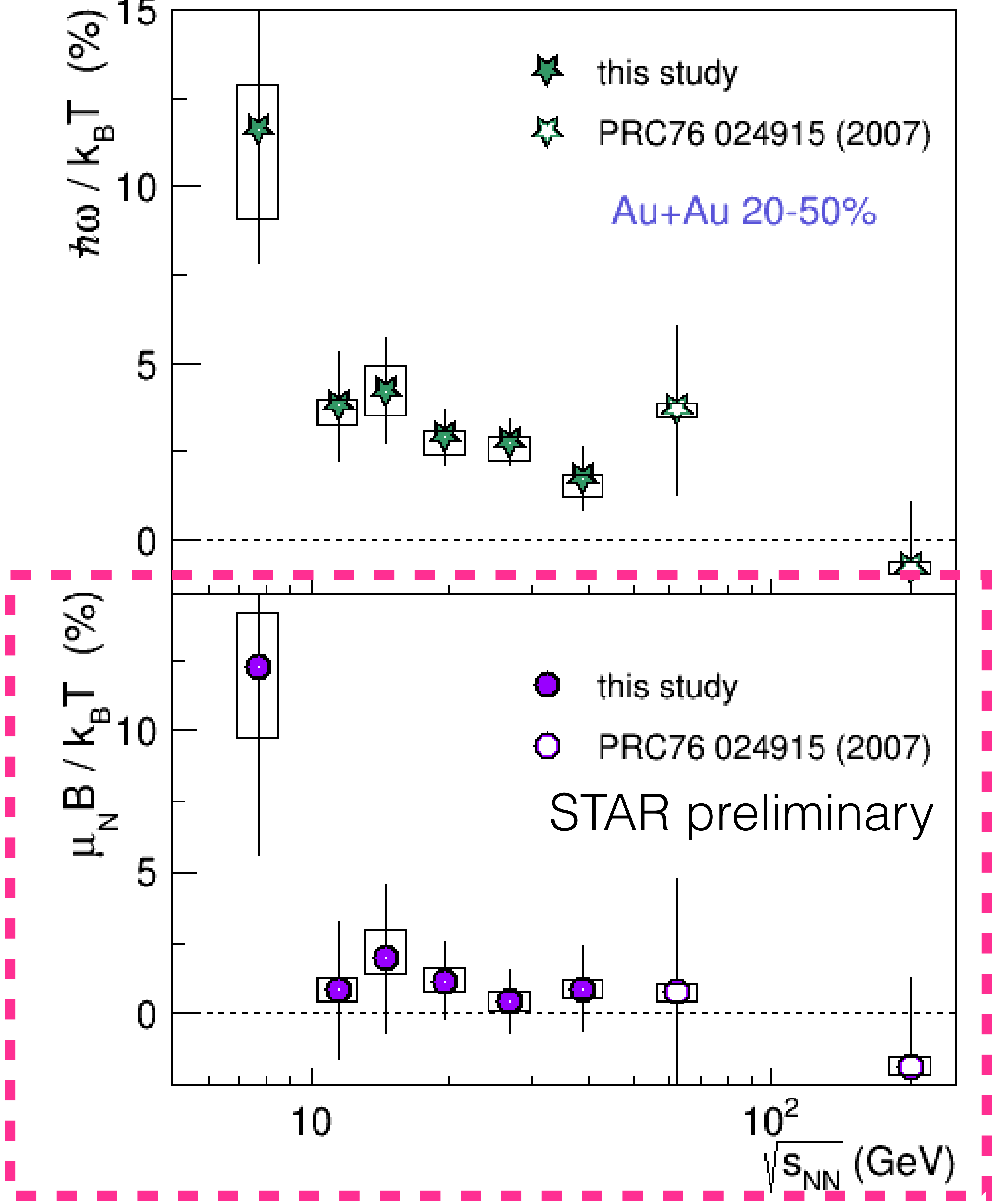
$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar$$
$$\sim 0.02\text{-}0.09\ \mathrm{fm}^{-1}$$
$$\sim 0.6\text{-}2.7 \times 10^{22}\mathrm{s}^{-1}$$

(T=160 MeV)

The most vortical fluid ever observed!

Possible probe of magnetic field

QM17, I. Upsal (STAR)



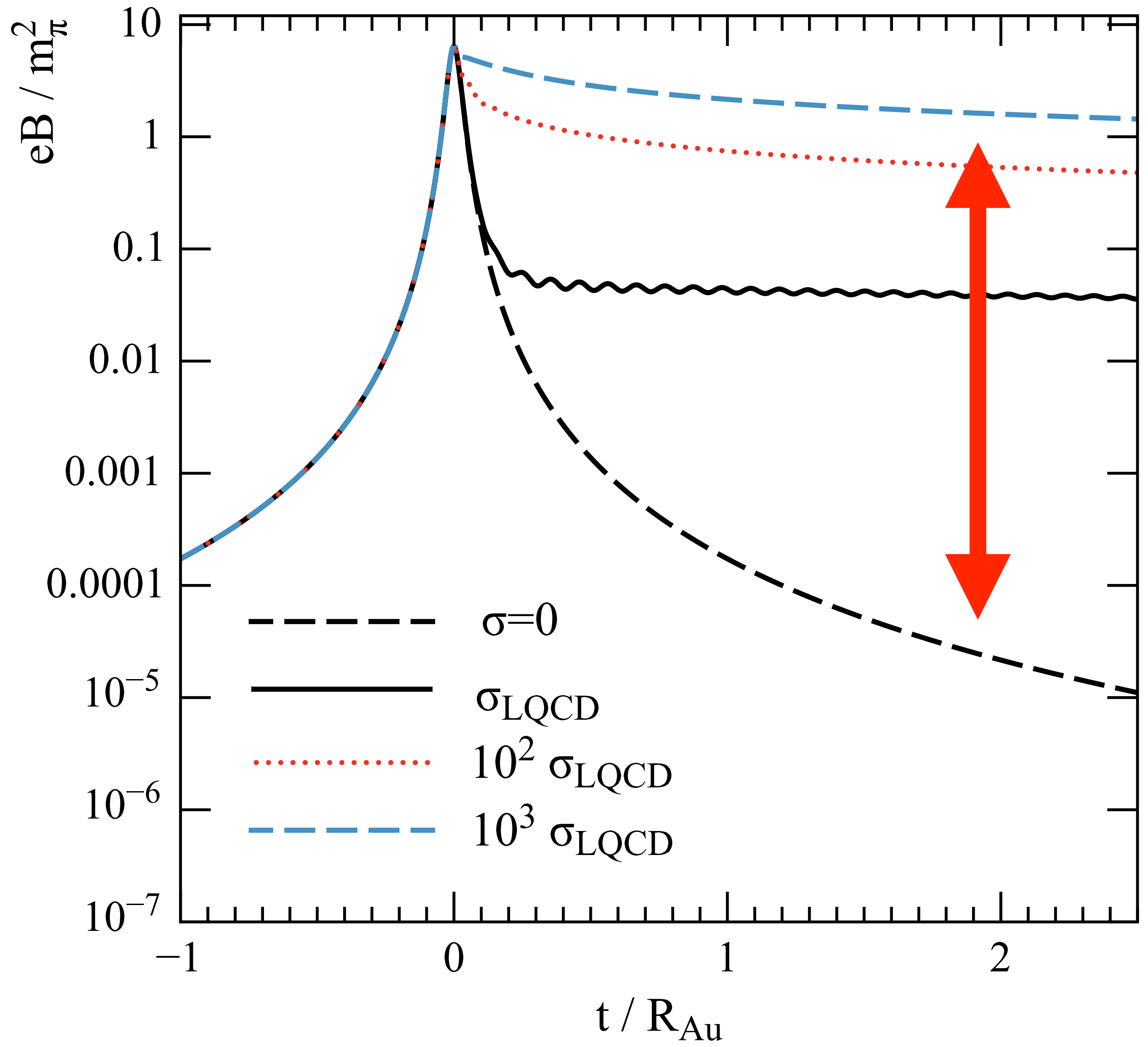
Becattini, Karpenko, Lisa, Upsal, and Voloshin, PRC95.054902 (2017)

$$P_\Lambda \simeq \frac{1}{2}\frac{\omega}{T} \boxed{+ \frac{\mu_\Lambda B}{T}}$$

$$P_{\bar{\Lambda}} \simeq \frac{1}{2}\frac{\omega}{T} \boxed{- \frac{\mu_\Lambda B}{T}}$$

μ_Λ: Λ magnetic moment

McLerran and Skokov, Nucl. Phys. A929, 184 (2014)



conductivity increases B-lifetime

$$B = (P_\Lambda - P_{\bar{\Lambda}}) k_B T / \mu_N$$

$$\sim 5.0 \times 10^{13}\ \text{[Tesla]}$$

nuclear magneton $\mu_N = -0.613\mu_\Lambda$

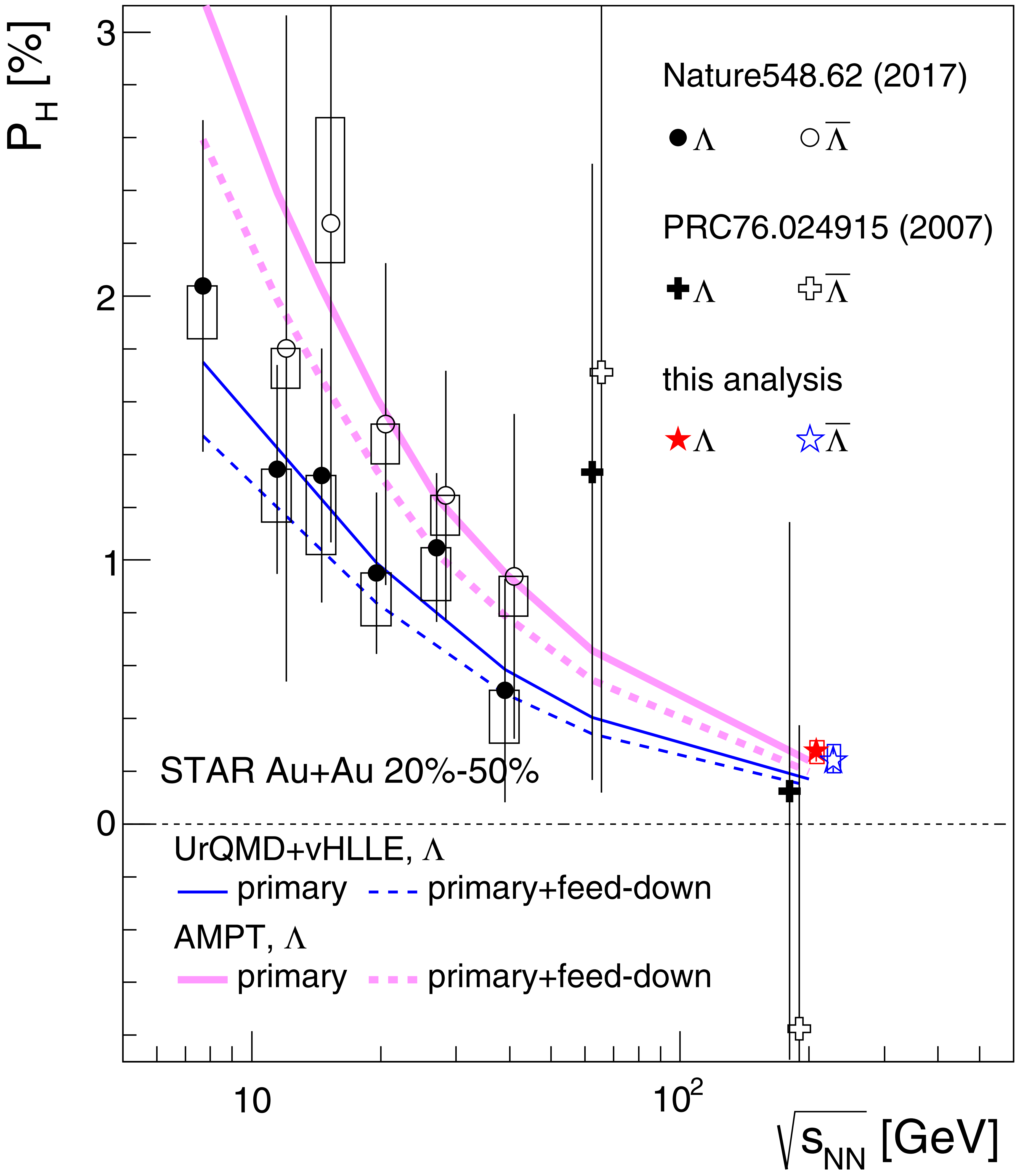
$$B \sim 10^{13}\ \text{T}$$

$$(eB \sim \text{MeV}^2\ (\tau = 0.2\ \text{fm}))$$

Extracted B-field at freeze-out assuming local thermal equilibrium, although it's consistent with zero.
Need more data! →BES-II and Isobaric collisions

Positive signal at $\sqrt{s_{NN}}$ = 200 GeV

STAR, PRC98, 014910 (2018)



$$P_H(\Lambda)\ [\%] = 0.277 \pm 0.040(\text{stat}) \pm^{0.039}_{0.049}\ (\text{sys})$$

$$P_H(\bar{\Lambda})\ [\%] = 0.240 \pm 0.045(\text{stat}) \pm^{0.061}_{0.045}\ (\text{sys})$$

- 5-7σ significance, comparable to the combined result of 7.7-39 GeV

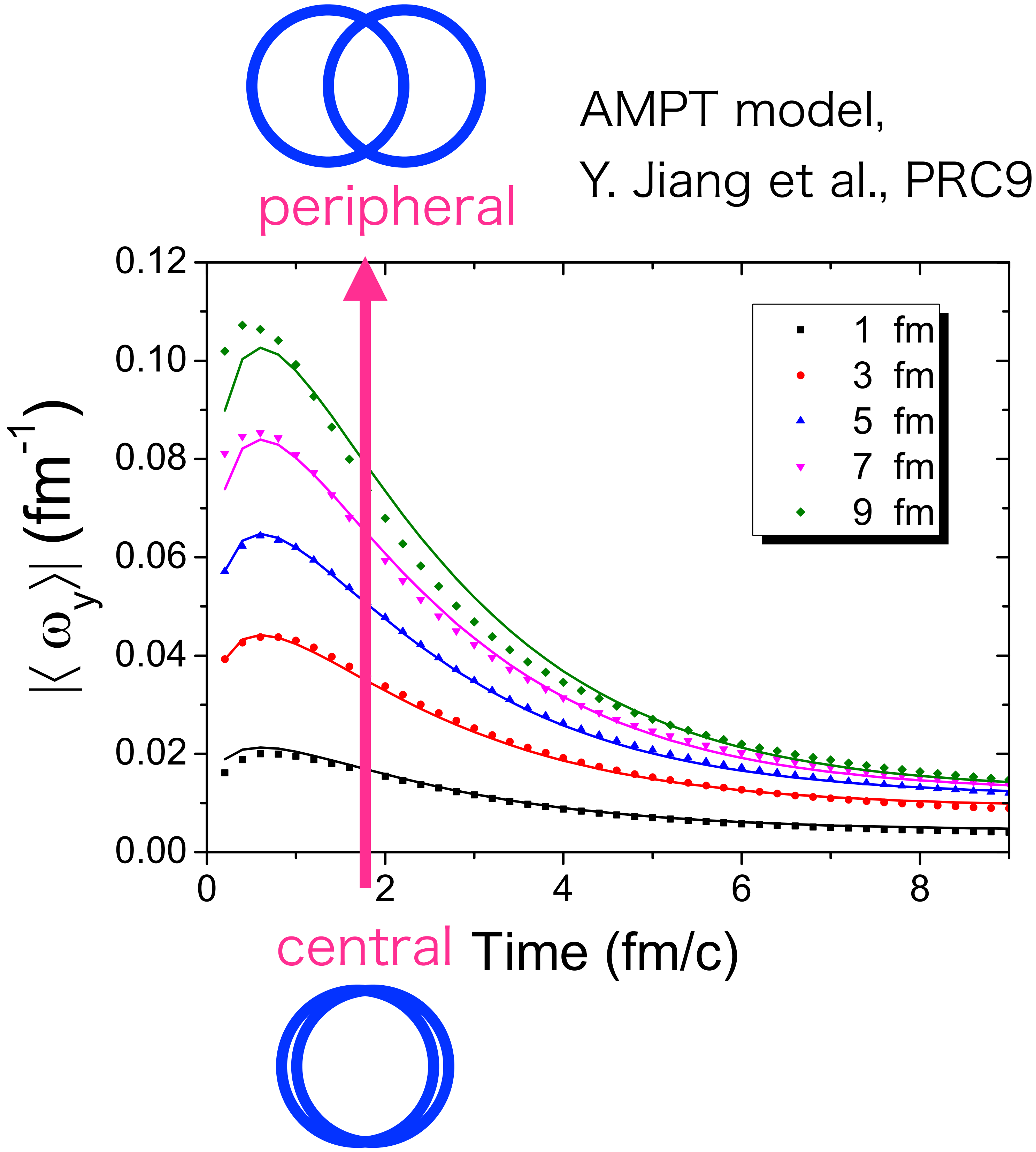
- Feed-down ~15%-20% reduction of P_H (model-dependent)

  Becattini, Karpenko, Lisa, Upsal, and Voloshin, PRC95.054902 (2017)

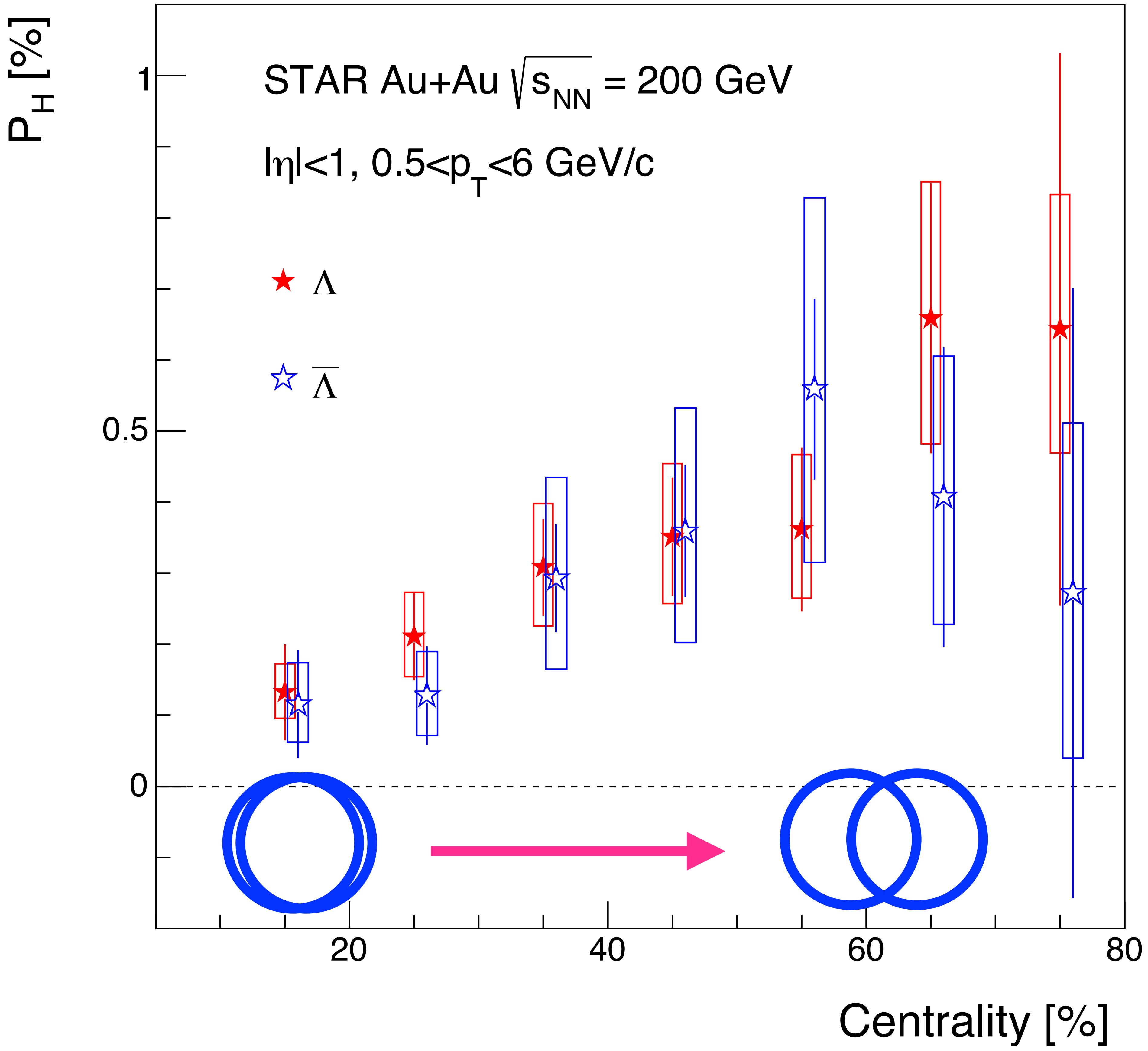
UrQMD+vHLLE: I. Karpenko and F. Becattini, EPJC(2017)77:213
AMPT: H. Li et al., Phys. Rev. C 96, 054908 (2017)

Centrality dependence of P_H

AMPT model,
Y. Jiang et al., PRC94, 044910 (2016)

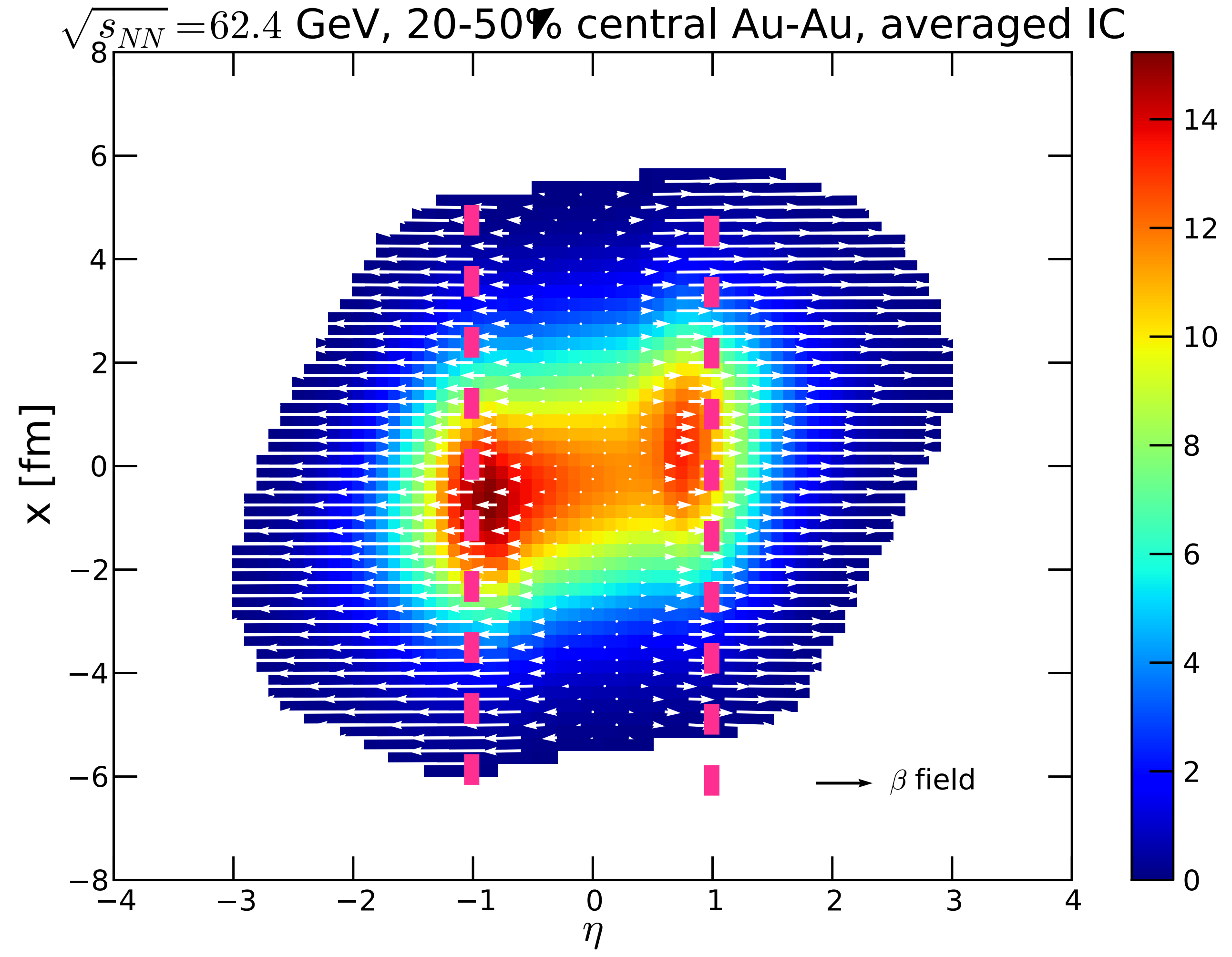


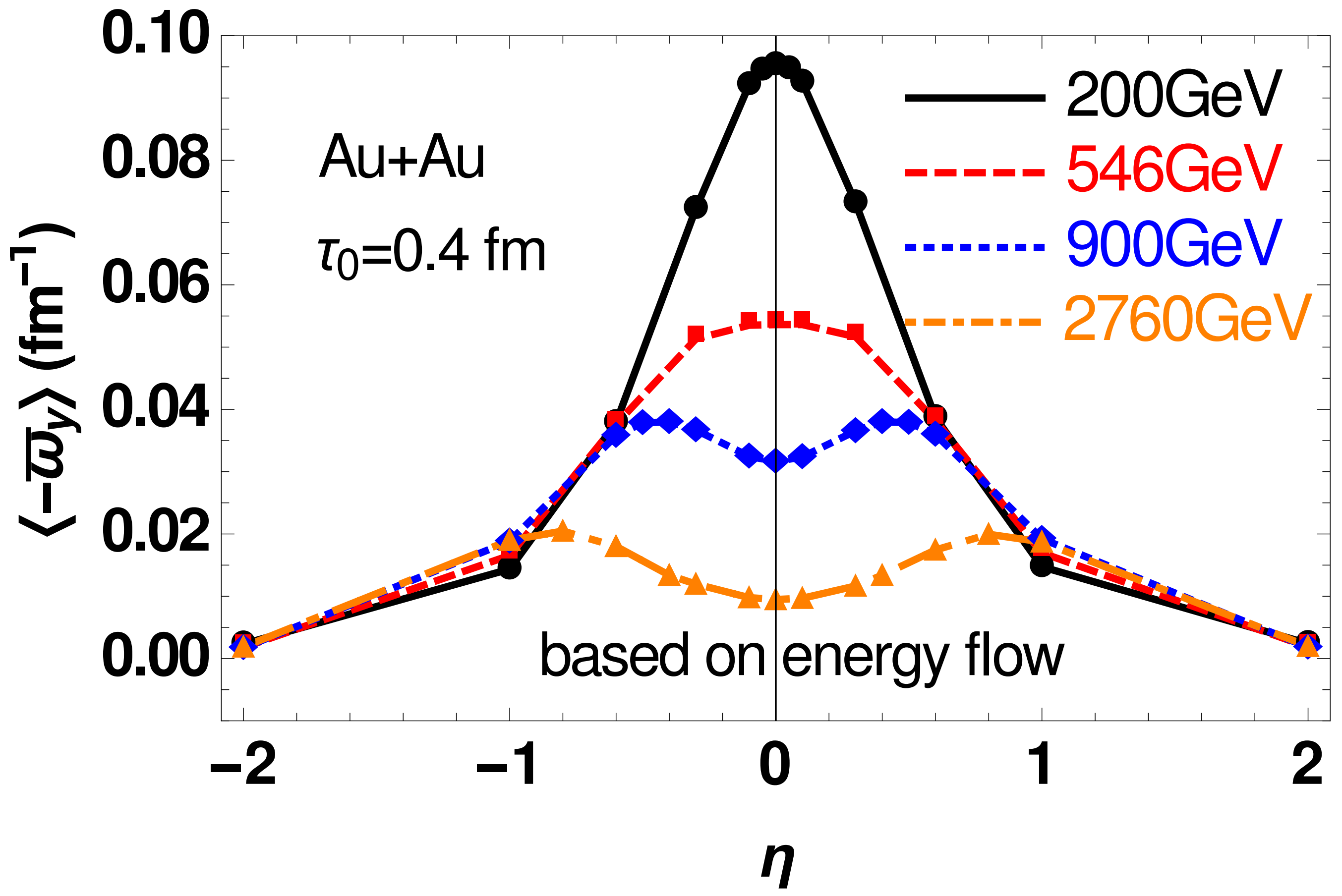
STAR, PRC98, 014910 (2018)



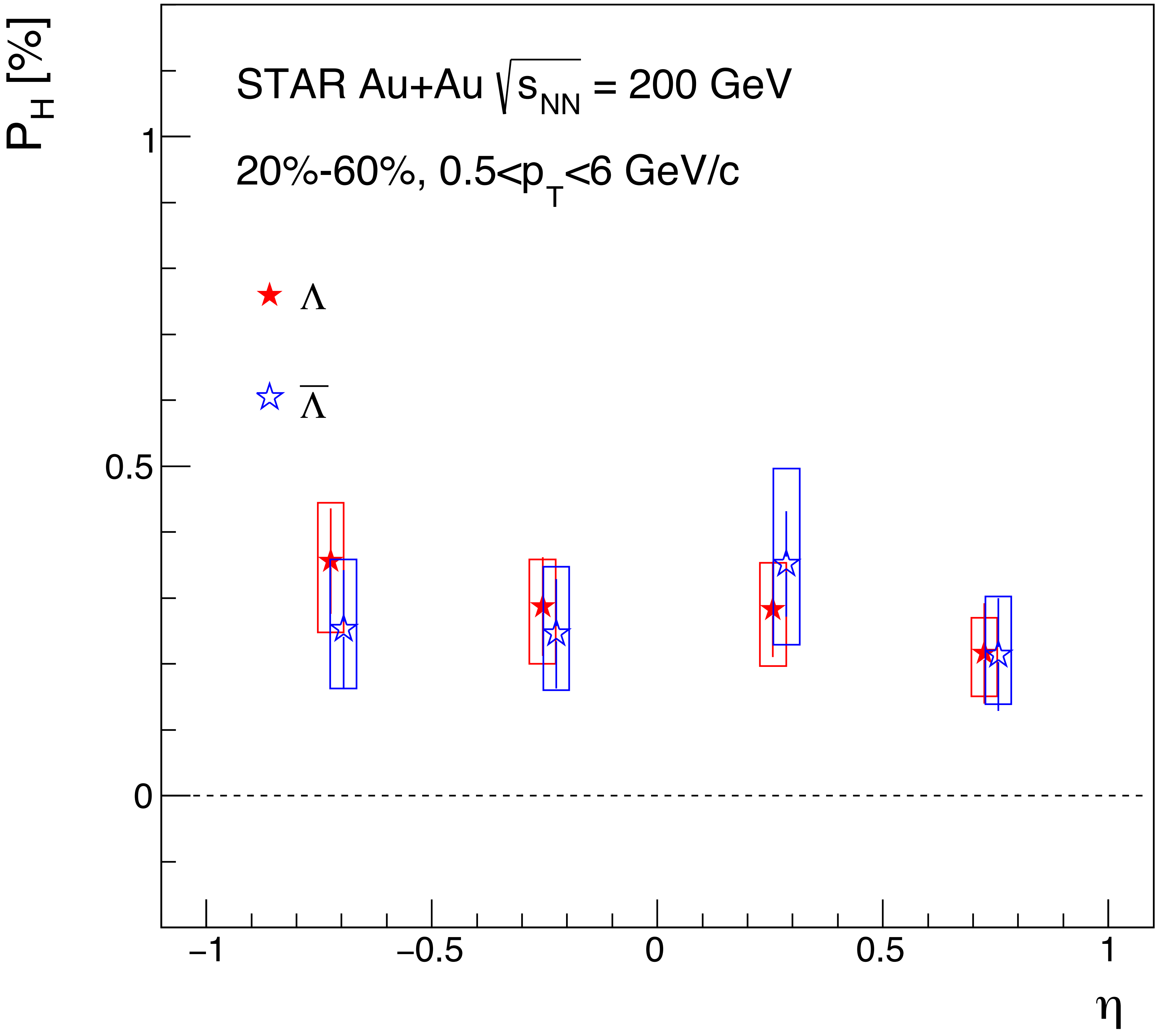
In most central collision → no initial angular momentum
As expected, the polarization decreases in more central collisions

η dependence of P_H





STAR, PRC98, 014910 (2018)



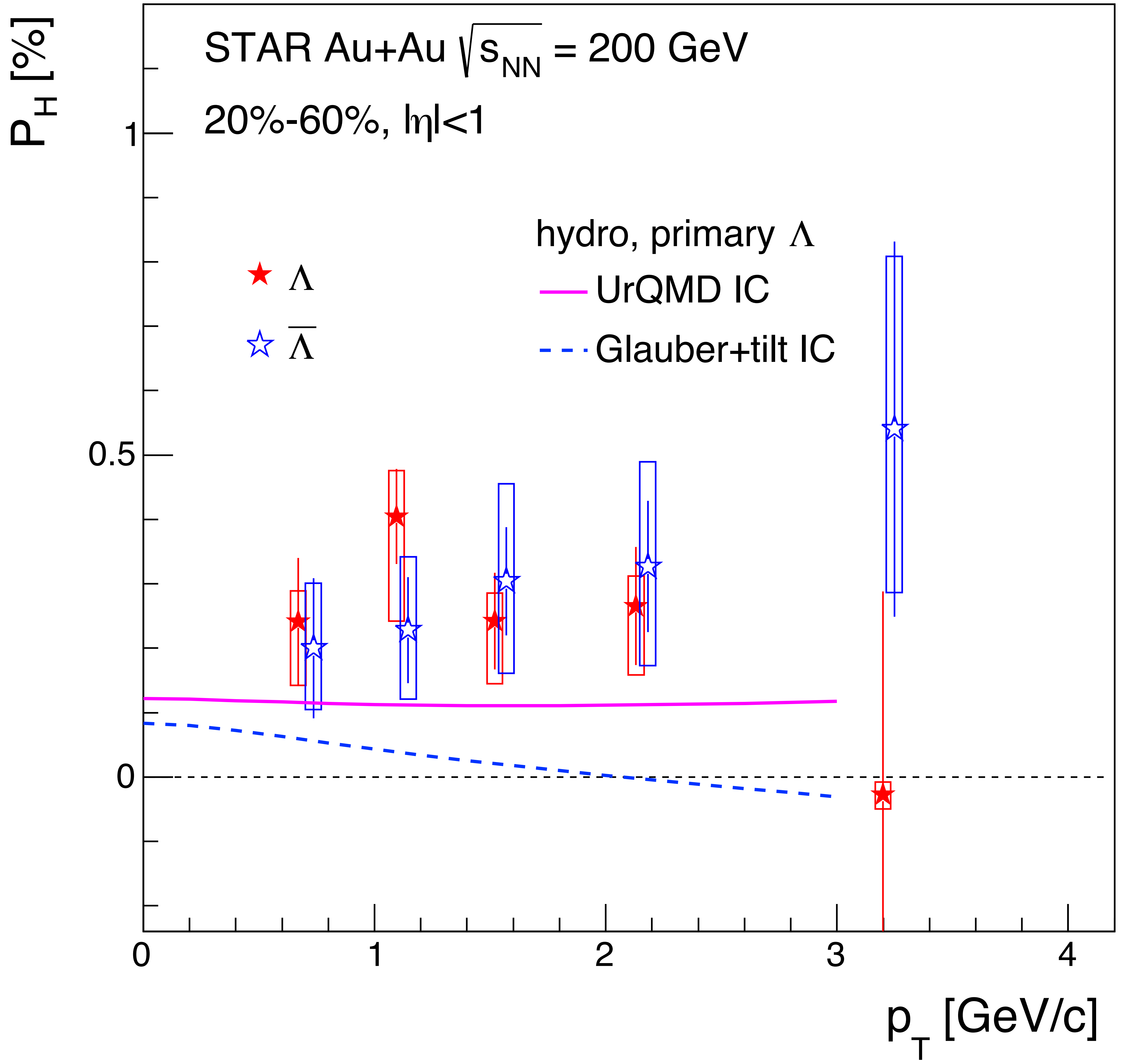
- Shear flow structure/initial flow velocity would be stronger in forward/backward region
- Expect rapidity dependence of the polarization

I. Karpenko and F. Becattini, EPJC(2017)77:213
W.-T. Deng and X.-G. Huang, arXiv:1609.01801

- The data do not show significant η dependence
  - Maybe due to baryon transparency at higher energy
  - Also due to event-by-event C.M. fluctuations

p_T dependence of P_H

STAR, PRC98, 014910 (2018)



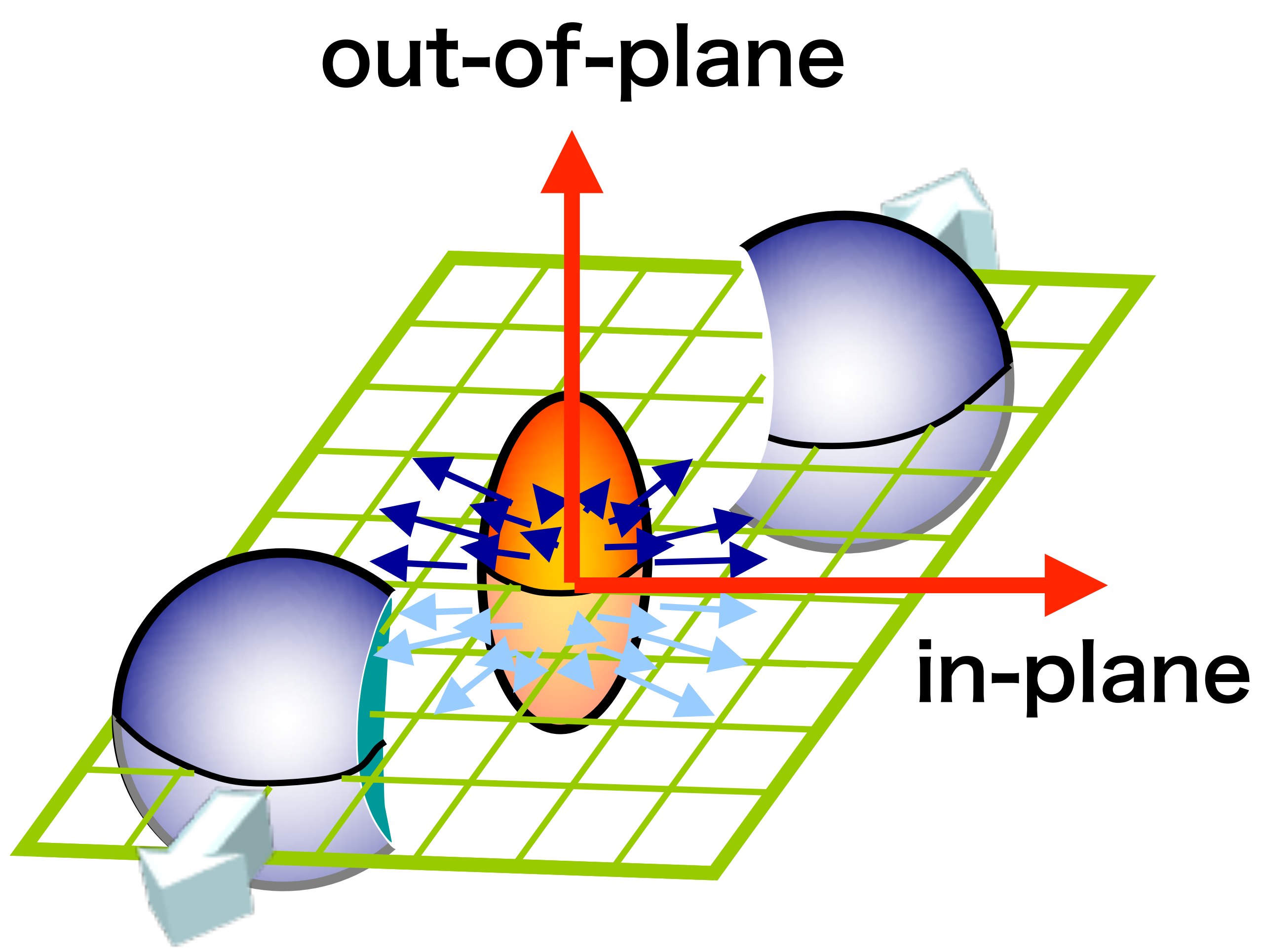
- ☐ No significant p_T dependence, as expected from the initial angular momentum of the system
- ☐ Hydrodynamic model underestimates the data. Initial conditions affect the magnitude and dependence on p_T

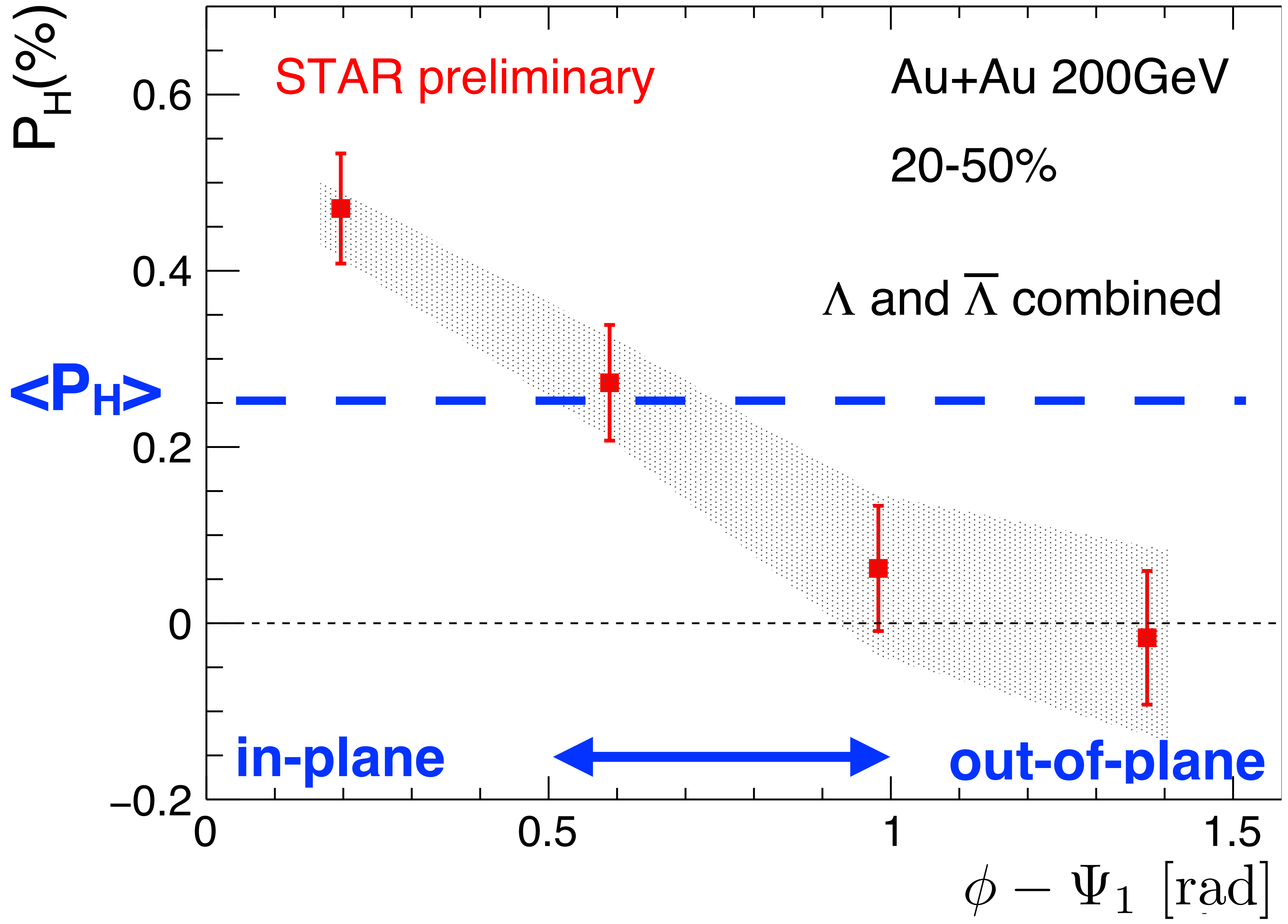
3D viscous hydrodynamic model with 2 initial conditions (ICs)

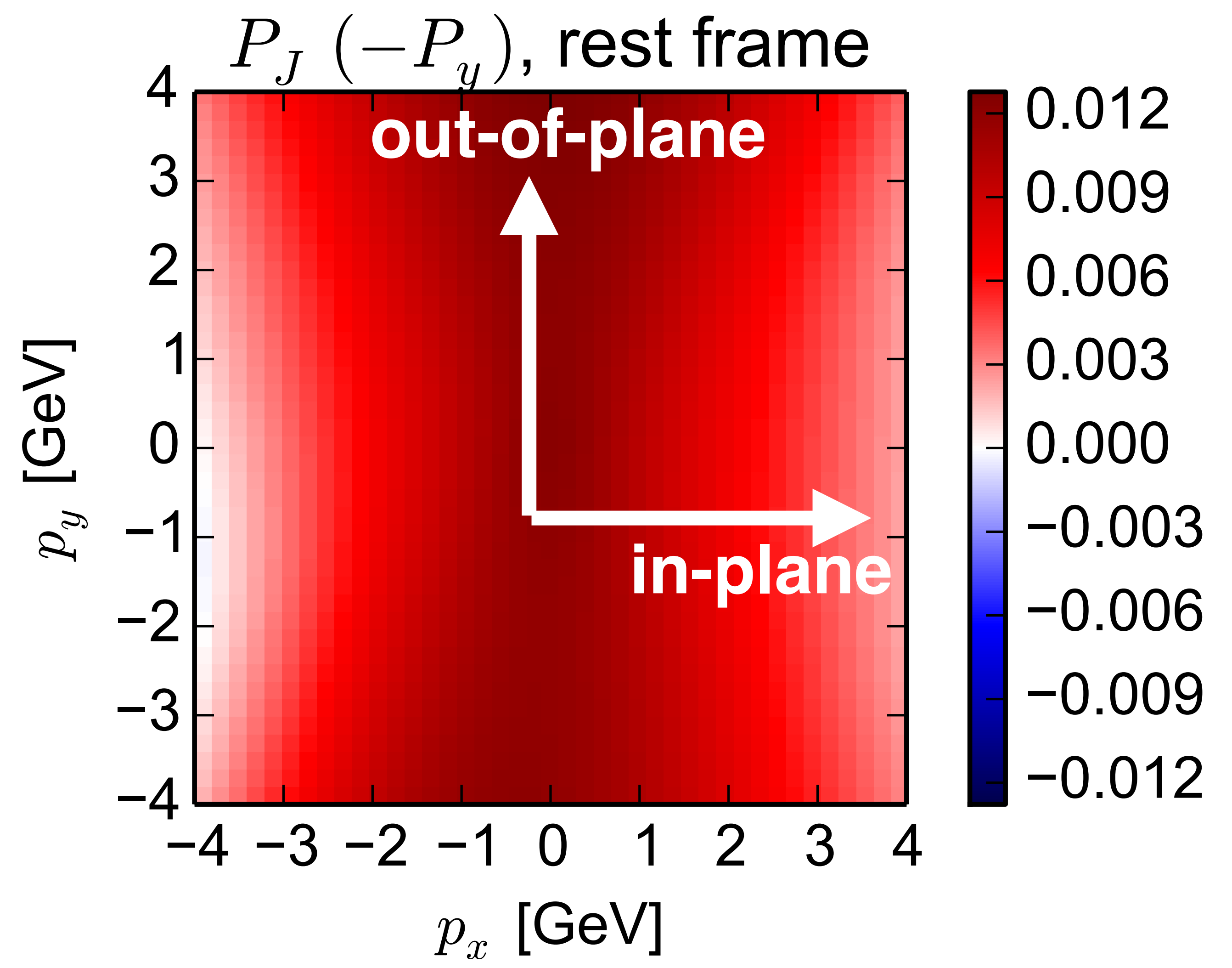
- UrQMD IC
- Glauber with source tilt IC

F. Becattini and I. Karpenko, PRL120.012302, 2018

Azimuthal angle dependence of P_H





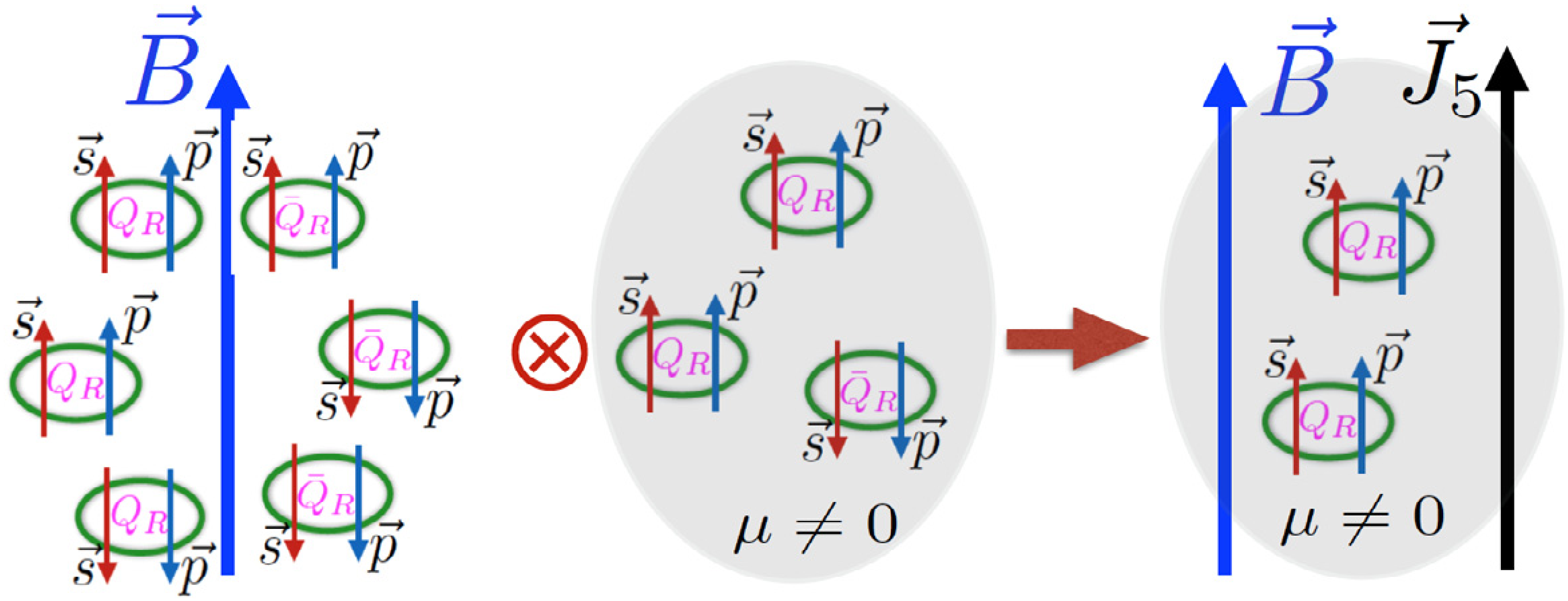


I. Karpenko and F. Becattini, EPJC(2017)77:213

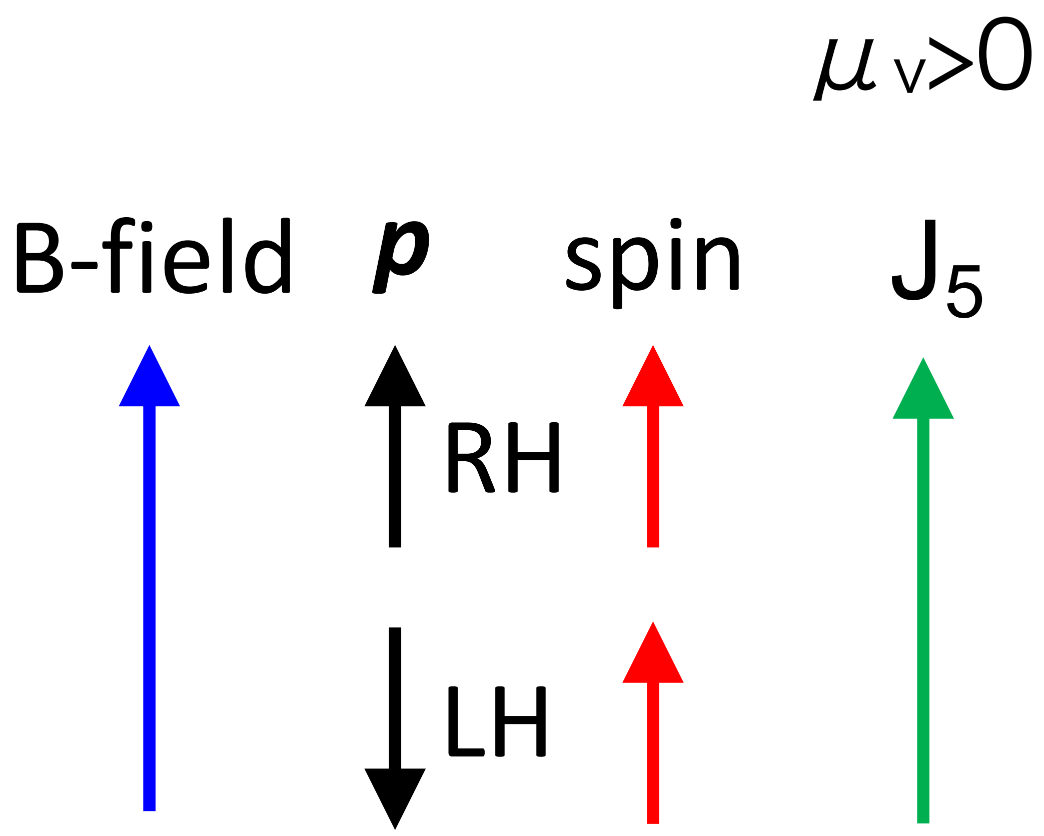
- ✦ Larger polarization in in-plane than in out-of-plane
- ✦ Opposite to hydrodynamic model! (larger in out-of-plane)

Λ polarization vs. charge asymmetry?

Chiral Separation Effect



$$\mathbf{J}_5 \propto \mu_\mathrm{v} \mathbf{B}$$

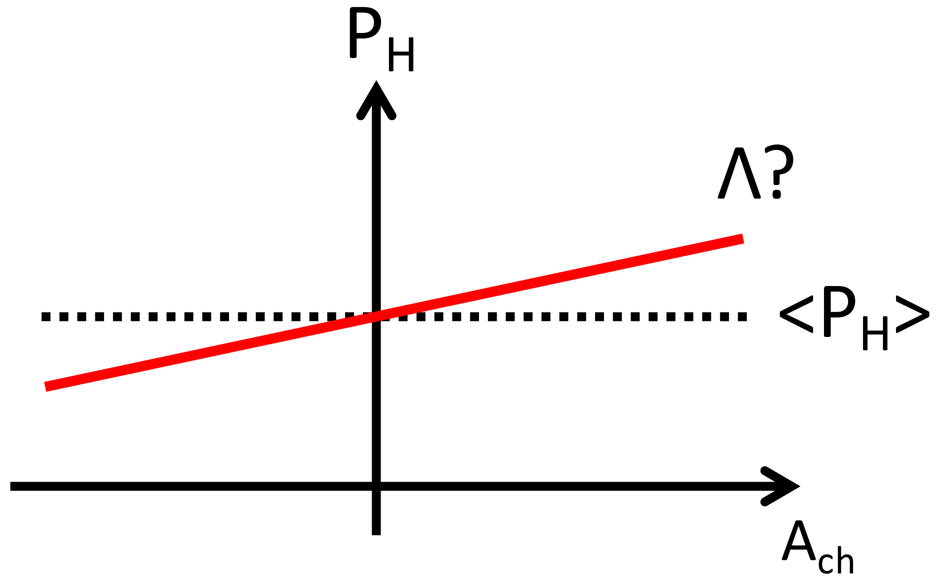


B-field + massless quarks + non-zero μ_v → axial current J_5
(spin alignment + spin and momentum in (anti)parallel for RH(LH) quarks)

- Λ polarization may have a contribution from the axial current J_5 induced by B-field (Chiral Separation Effect), S. Shlichting and S. Voloshin
- Use charge asymmetry A_{ch} instead of μ_v

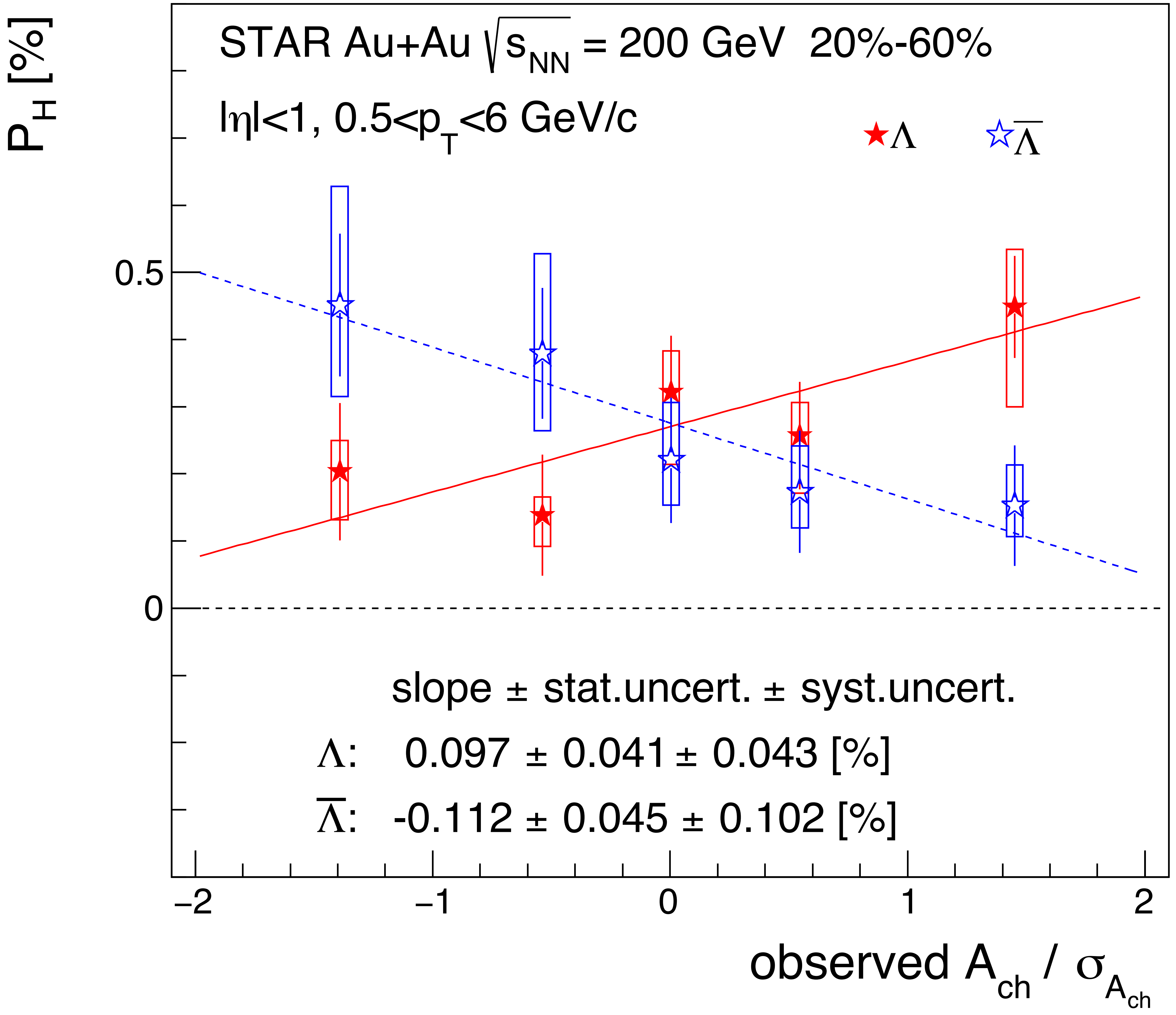
$$\mu_\mathrm{v}/T \propto \frac{\langle N_+ - N_- \rangle}{\langle N_+ + N_- \rangle} = A_\mathrm{ch}$$

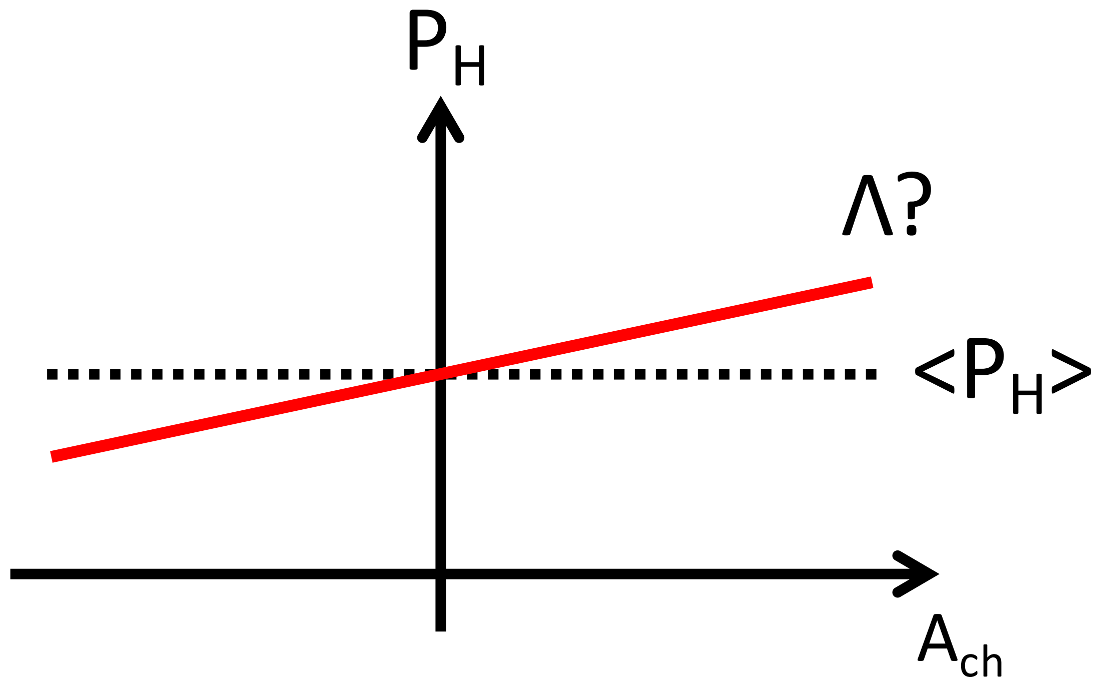
what's the expectation?
true for u-quark but also for Λ?



Λ polarization vs charge asymmetry?

STAR, PRC98, 014910 (2018)

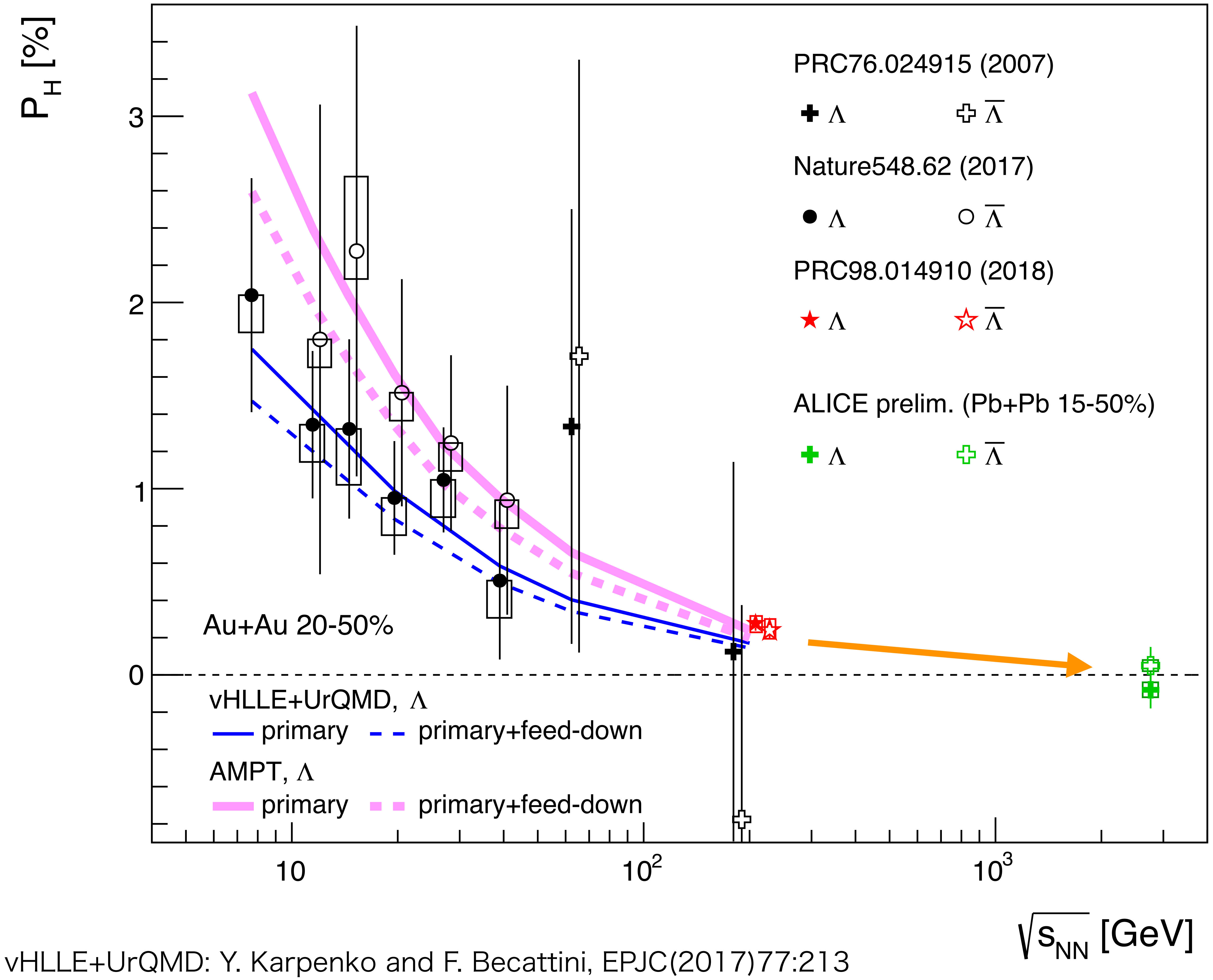




Slopes of Λ and anti-Λ seem to be different.
(statistical significance is ~2σ level)

Possibly a contribution from the axial current?

Go to the LHC energy

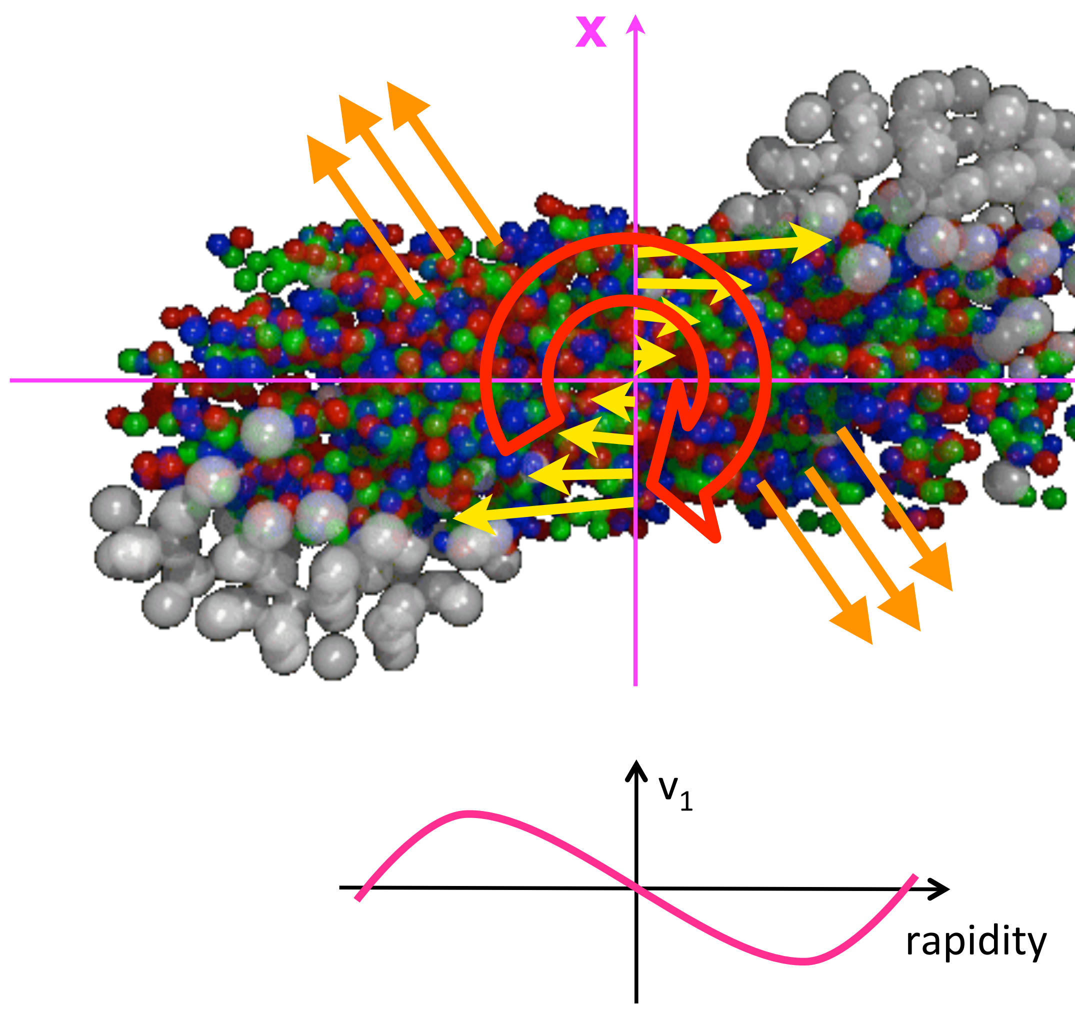


$$P_H(\Lambda)[\%] = -0.08 \pm 0.10\ (\mathrm{stat}) \pm 0.04\ (\mathrm{syst})$$
$$P_H(\bar{\Lambda})[\%] = \quad 0.05 \pm 0.10\ (\mathrm{stat}) \pm 0.03\ (\mathrm{syst})$$

M. Konyushikhin (ALICE), QCD Chirality Workshop 2017

vHLLE+UrQMD: Y. Karpenko and F. Becattini, EPJC(2017)77:213
AMPT: H. Li et al., Phys. Rev. C 96, 054908 (2017)

dv₁/dy vs polarization in data



F. Becattini et al., Eur. Phys. J. C (2015)75:406

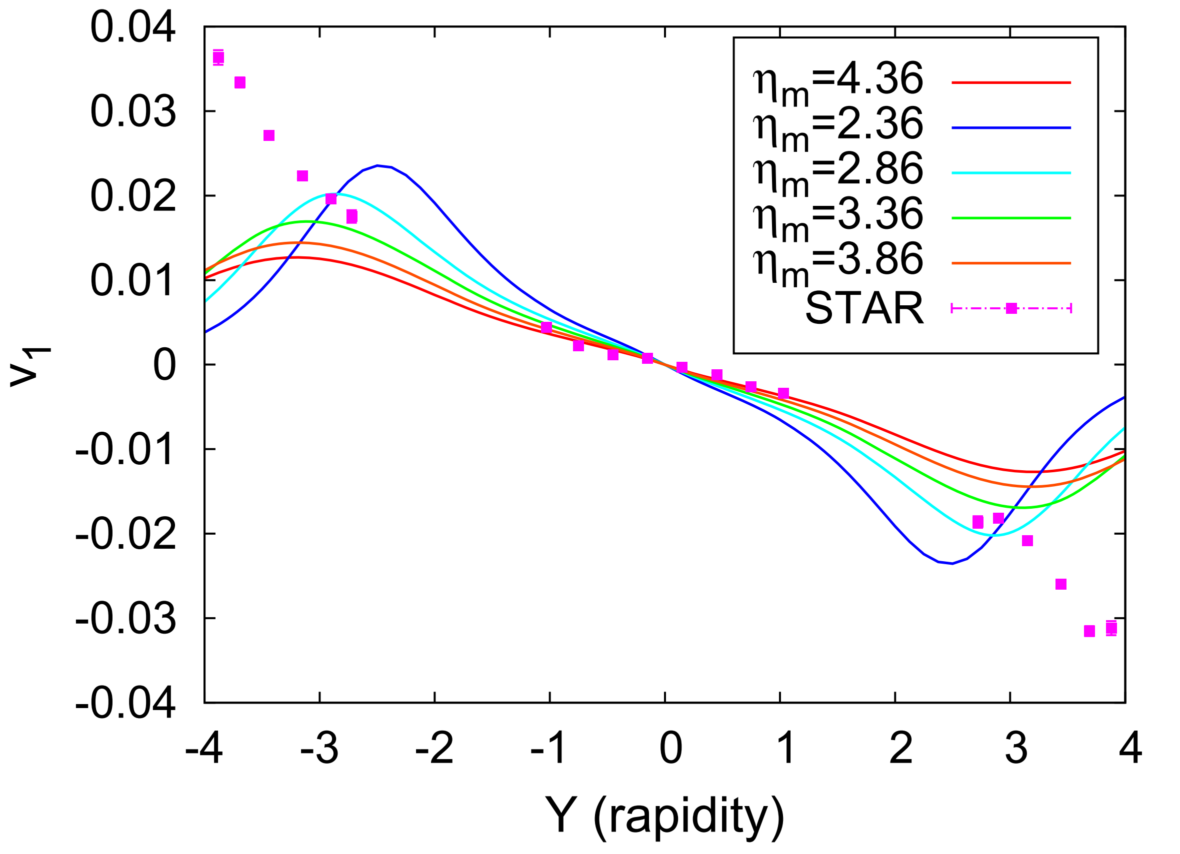
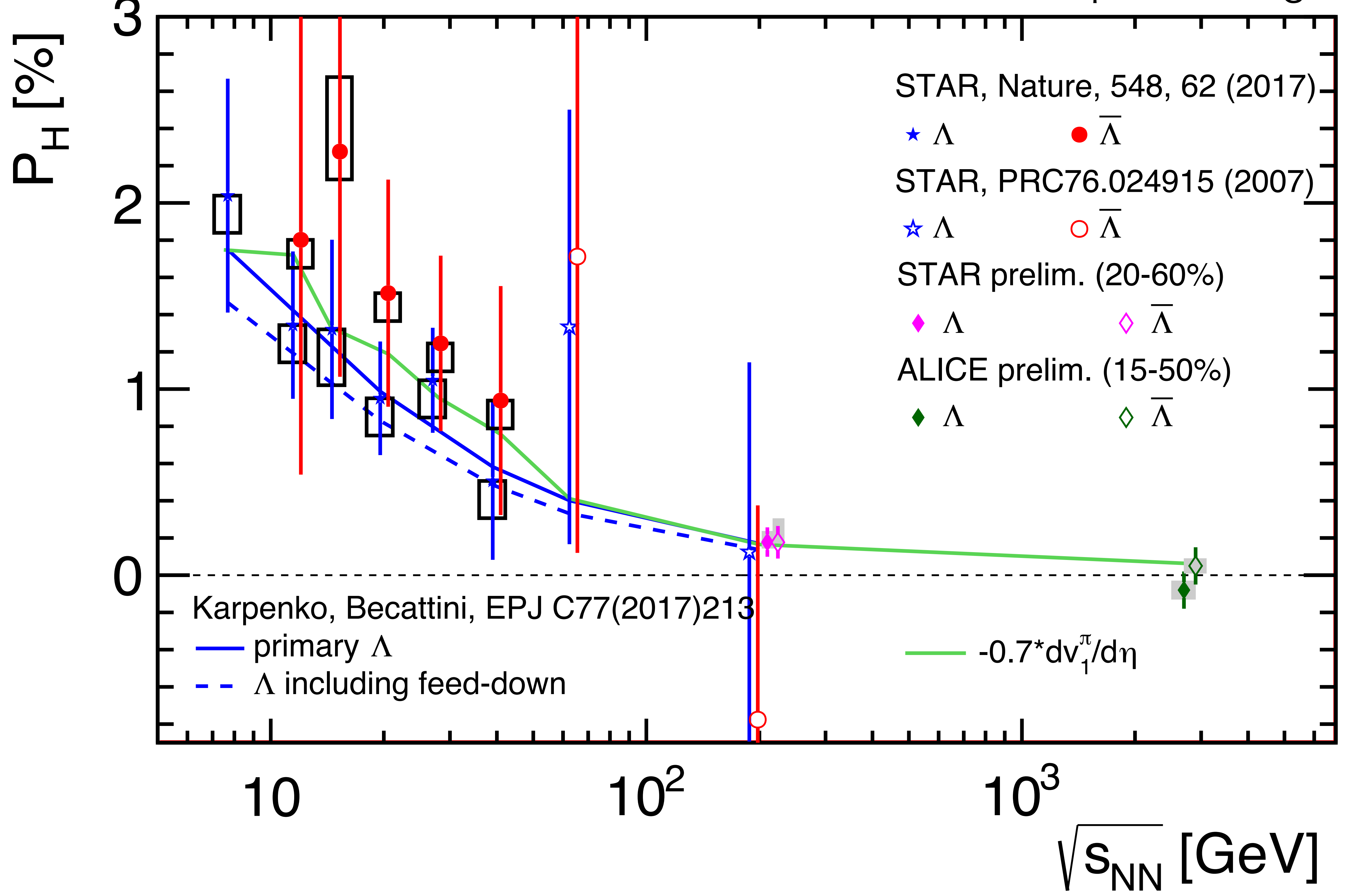


Fig. 6 Directed flow of pions for different values of η_m parameter with $\eta/s = 0.1$ compared with STAR data [22]

$$\frac{dN}{d\phi} \propto (1 + 2v_1\cos(\phi - \Psi) + 2v_2\cos(2\phi - 2\Psi) + \cdots)$$

- Vorticity is likely related to the directed flow.
- The tilted source accounting for vorticity provides a better description of v_1!

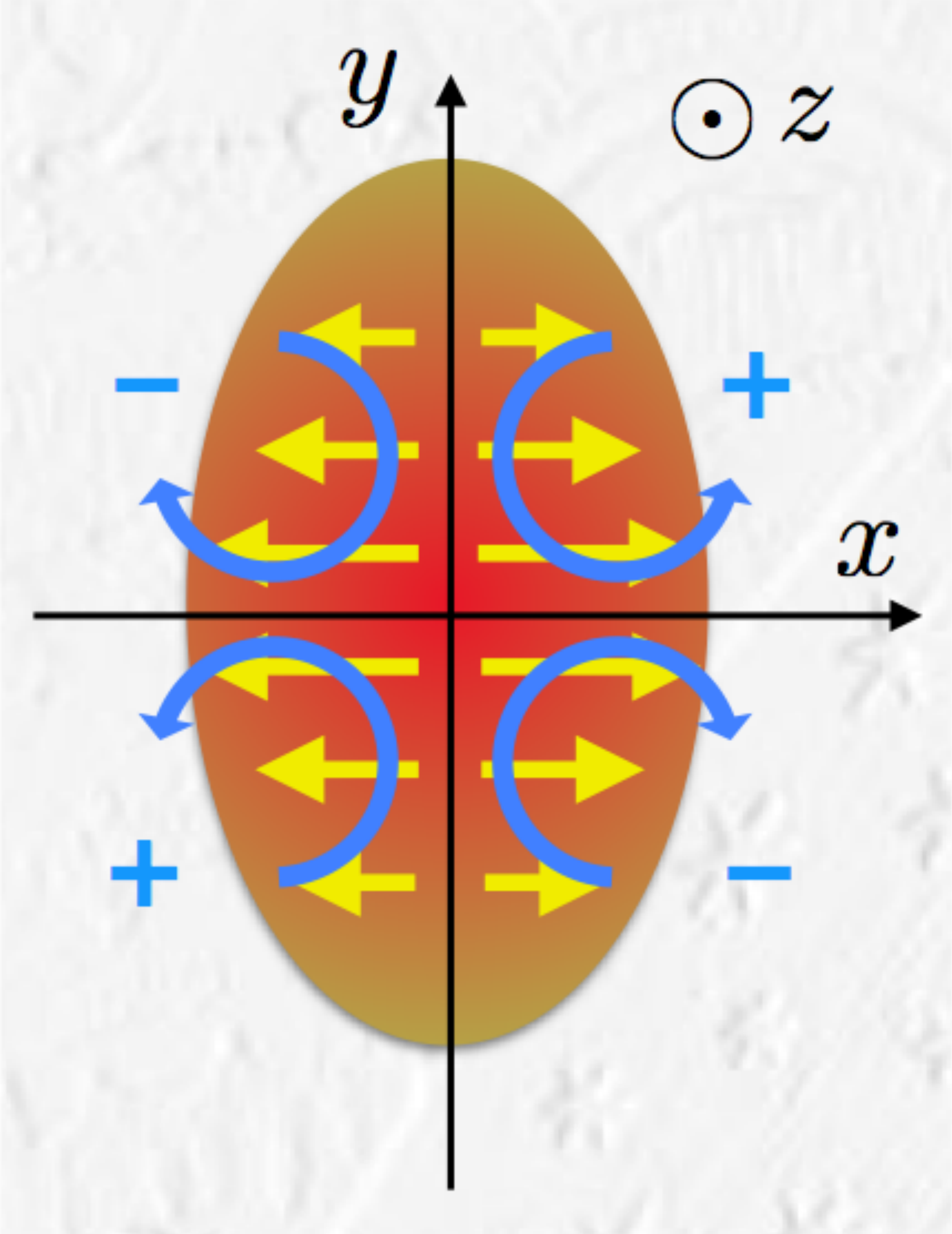
S. Voloshin, sQM2017 proceedings

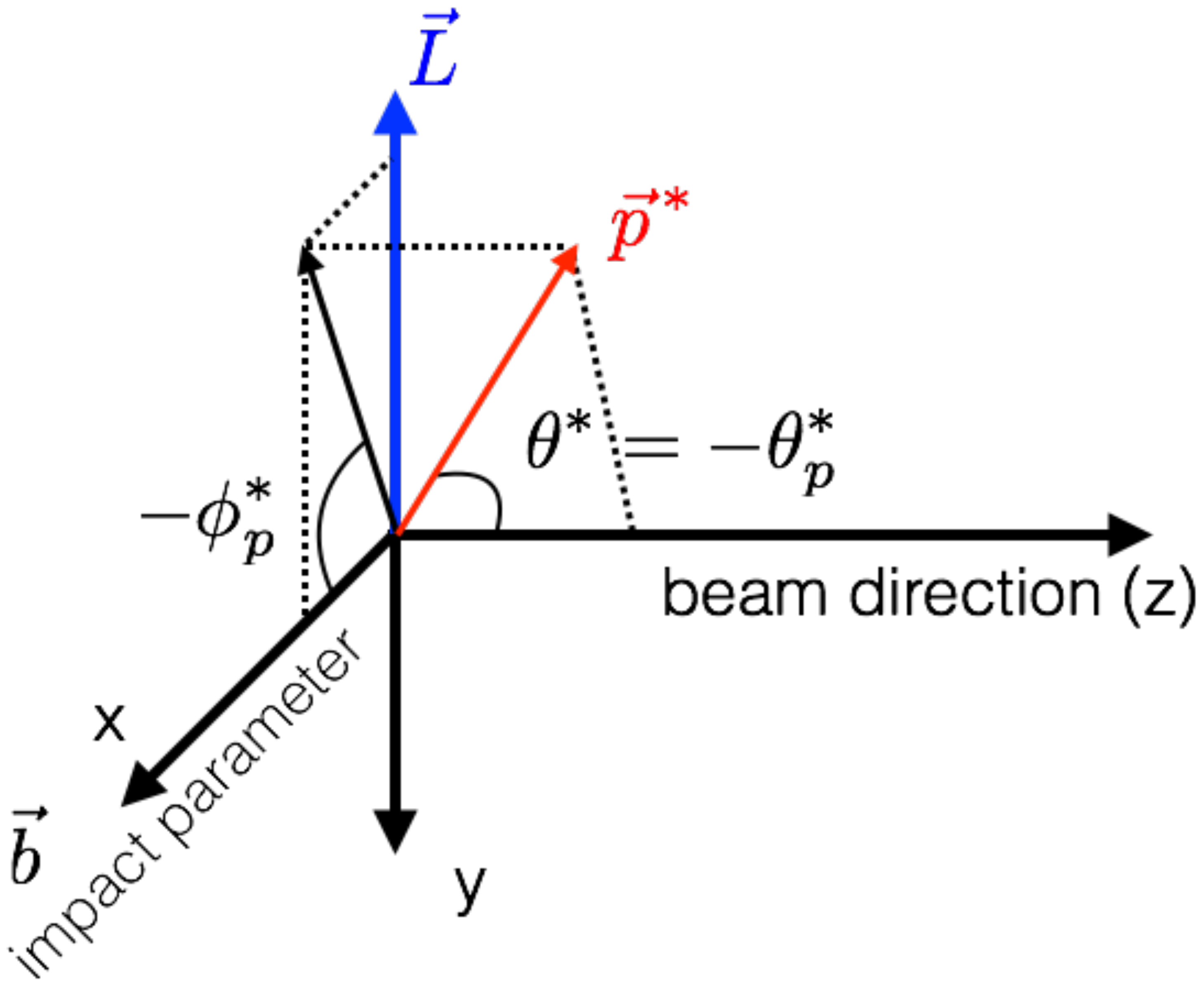


A similar energy dependence of dv₁/dy to the polarization!

Polarization along the beam direction

S. Voloshin, SQM2017
F. Becattini and I. Karpenko, PRL120.012302 (2018)





$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\rm H}\mathbf{P_H}\cdot\mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int \frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^* = \alpha_{\rm H} P_z \langle(\cos\theta_p^*)^2\rangle$$

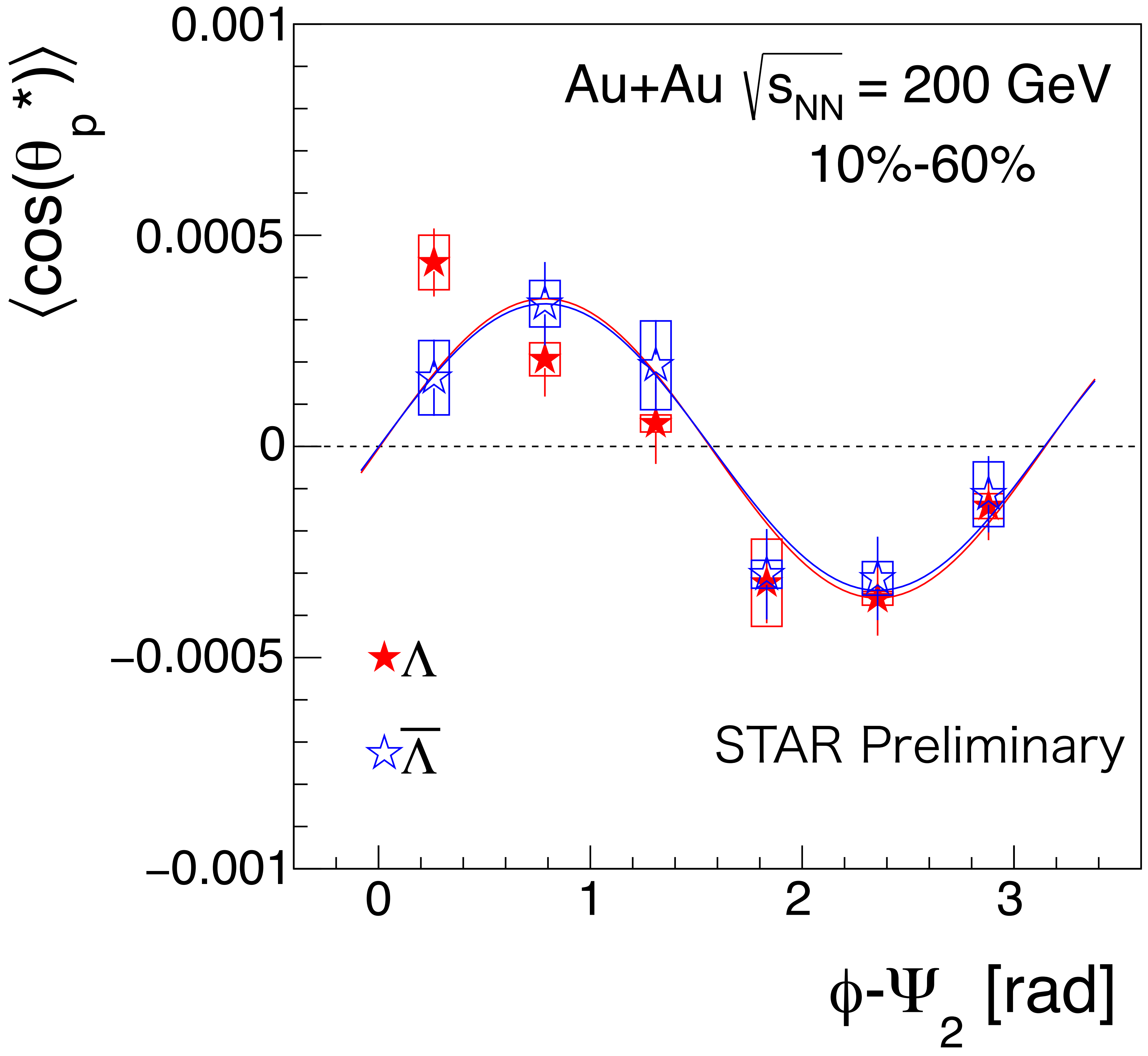
$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_{\rm H}\langle(\cos\theta_p^*)^2\rangle} = \frac{3\langle\cos\theta_p^*\rangle}{\alpha_{\rm H}}$$ (if perfect detector)

$\alpha_{\rm H}$: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

Stronger flow in in-plane than in out-of-plane could make local polarization along beam axis!

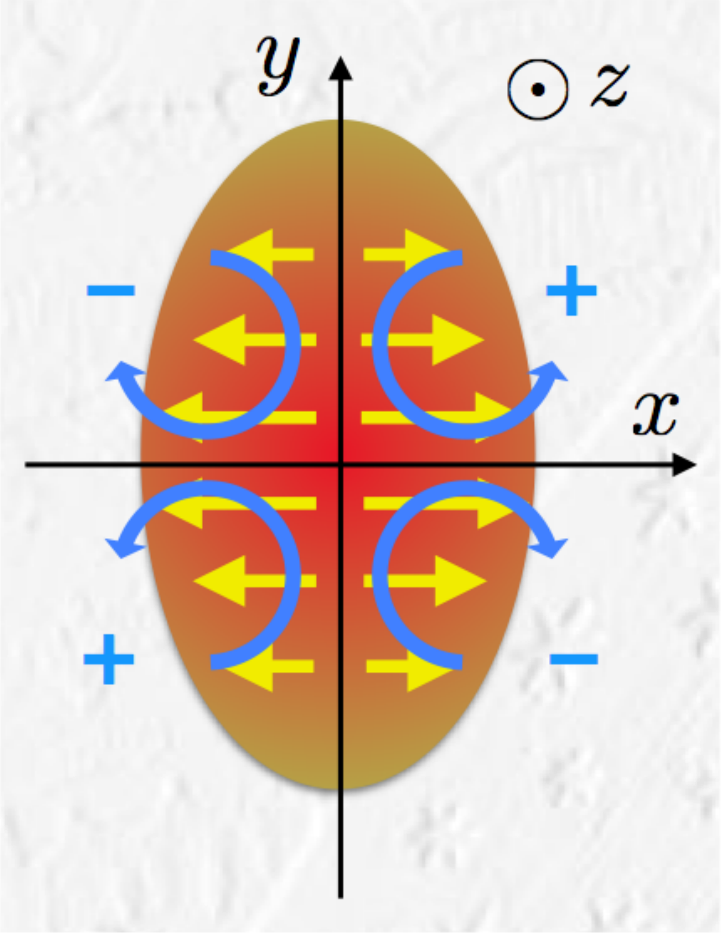
Longitudinal component, P_z, can be expressed with $\langle\cos\theta_p^*\rangle$.
$\langle(\cos\theta_p^*)^2\rangle$ accounts for an acceptance effect

Polarization along the beam direction

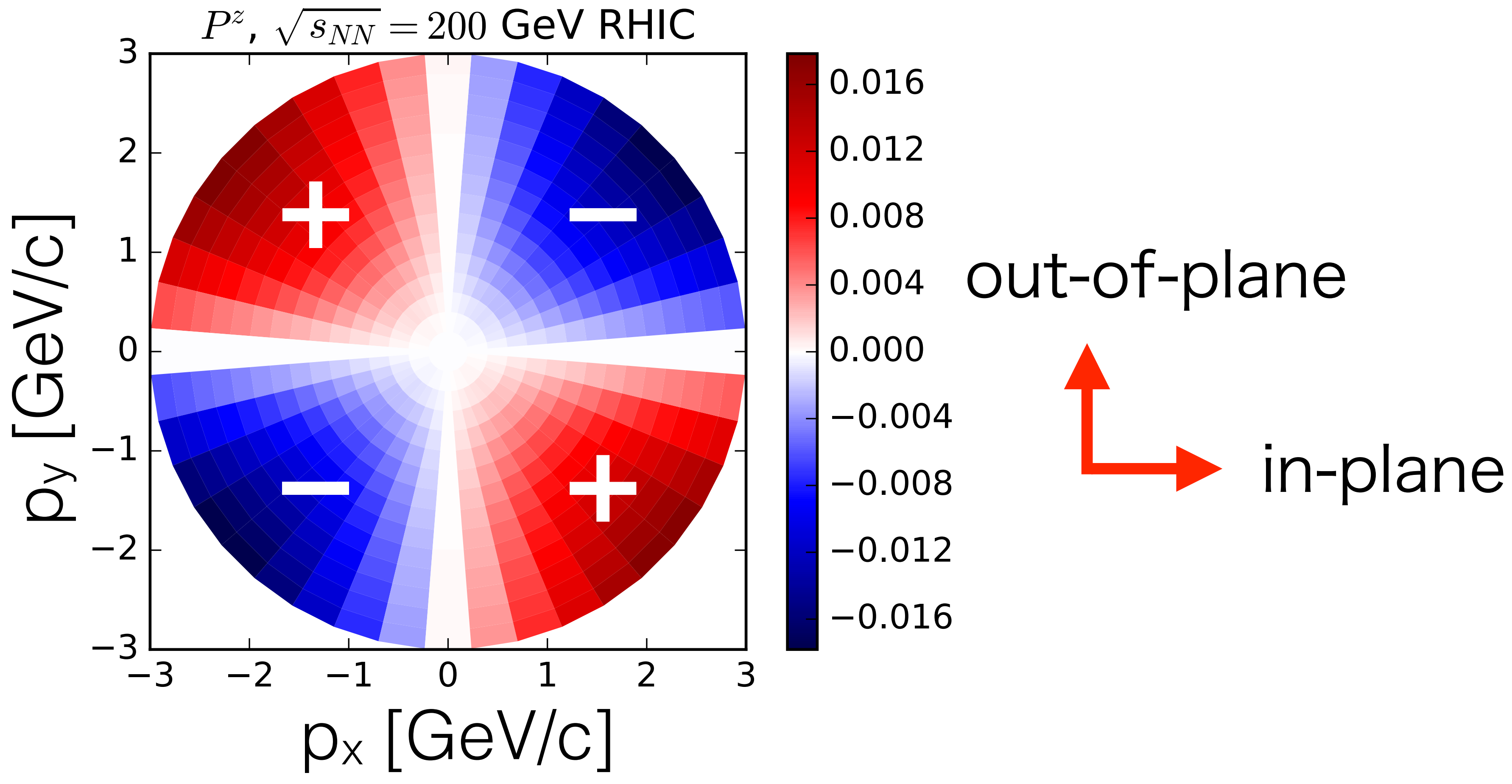


- Effect of Ψ2 resolution is not corrected here

S. Voloshin, SQM2017



Hydro calculation of P$_z$
F. Becattini and I. Karpenko,
PRL.120.012302 (2018)

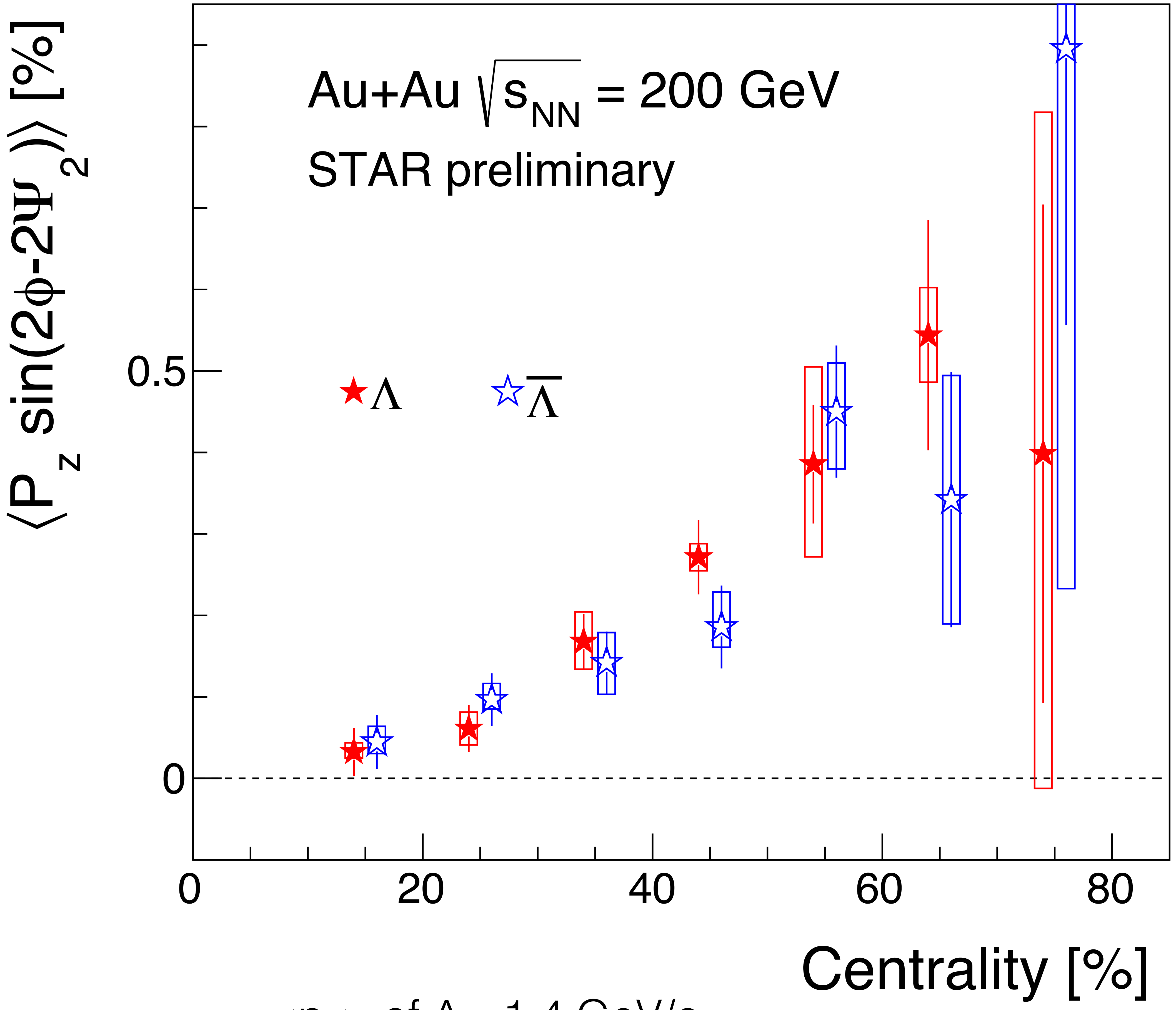


□ Sine structure as expected from the elliptic flow!

□ Opposite sign to hydrodynamic model and a transport model (AMPT)

- Hydro model: F. Becattini and I. Karpenko, PRL.120.012302 (2018)
- AMPT model: X. Xia, H. Li, Z. Tang, Q. Wang, arXiv:1803.0086

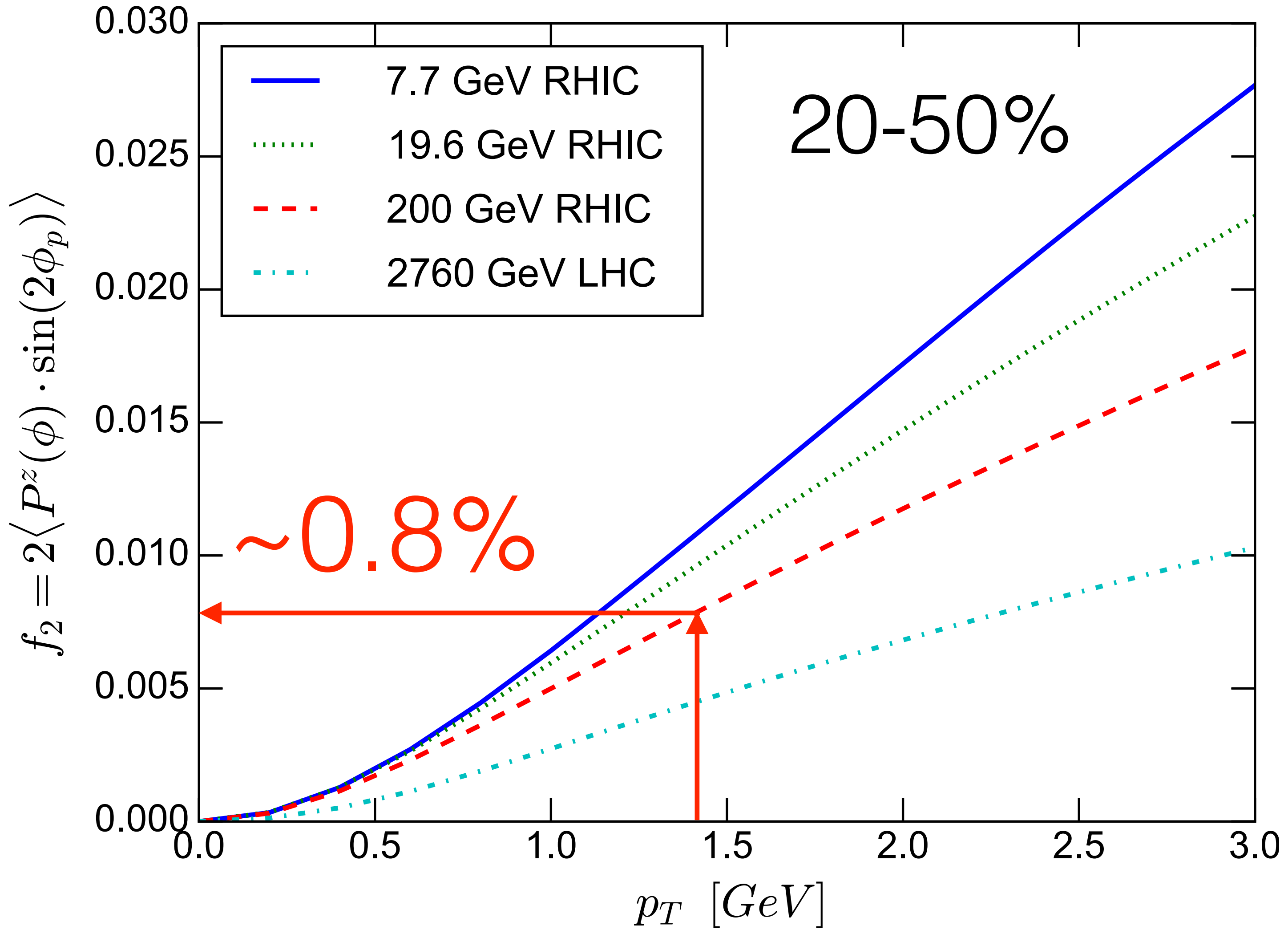
Centrality dependence of P_z modulation



<p_T> of Λ ~1.4 GeV/c
(0.5<p_T<6 GeV/c)

- Strong centrality dependence as in v_2
- Similar magnitude to the global polarization
- ~5 times smaller magnitude than the hydro and AMPT with the opposite sign!

F. Becattini and I. Karpenko, PRL.120.012302 (2018)



Sign problem in P_z

Opposite sign to hydrodynamic model and AMPT model

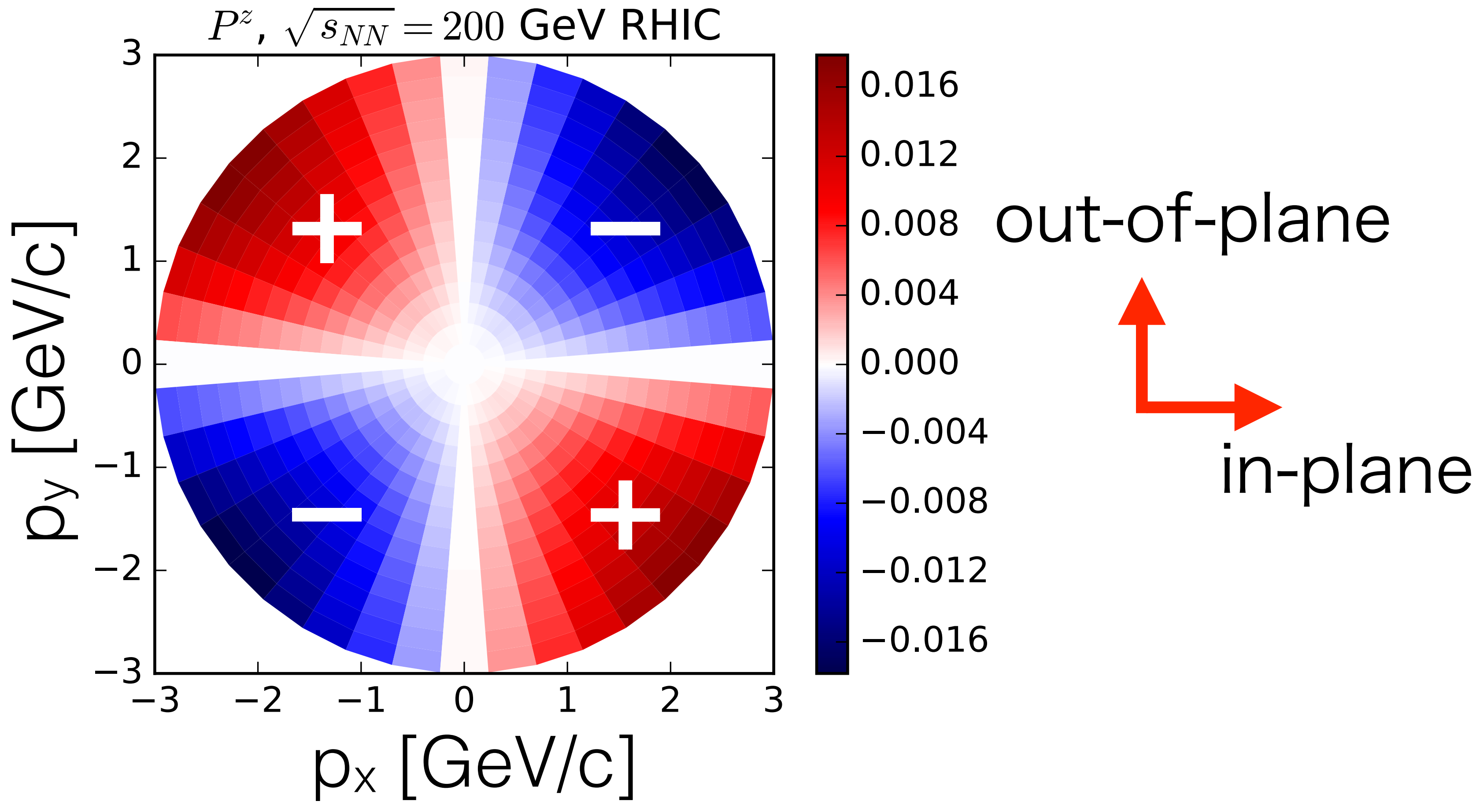
- F. Becattini and I. Karpenko, PRL.120.012302 (2018)
  3D viscous hydrodynamic model with UrQMD initial condition
  assuming a local thermal equilibrium
- AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

Same sign as chiral kinetic approach

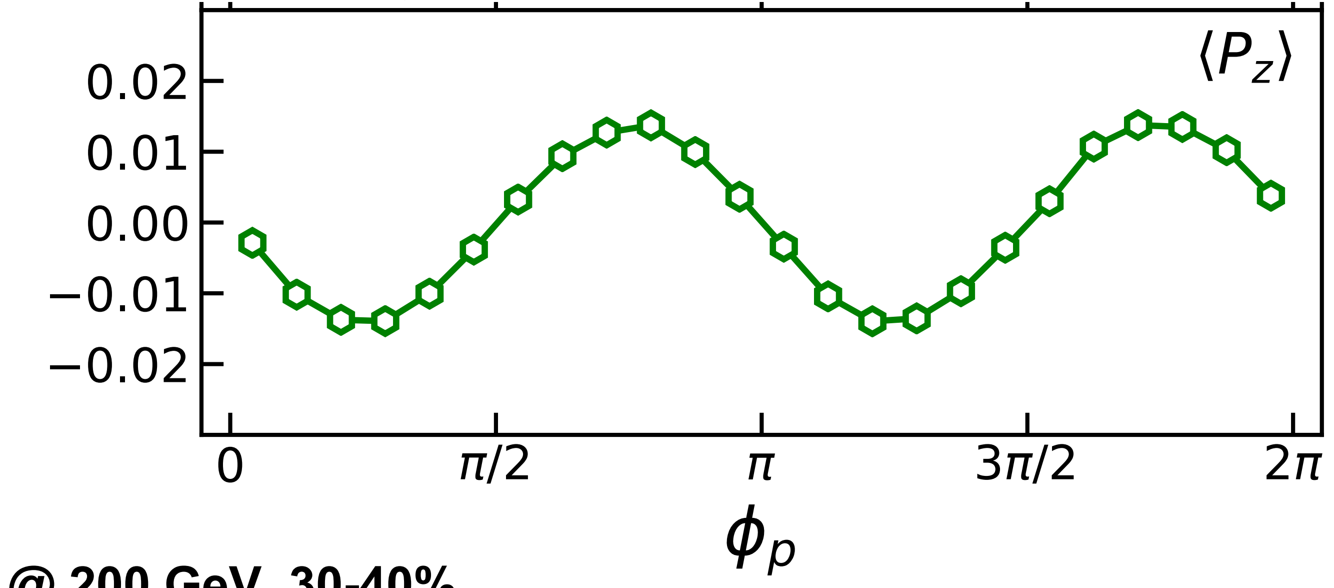
- Y. Sun and C.-M. Ko, arXiv:1810.10359
- Assuming non-equilibrium of spin degree of freedom
- Smaller quark scattering cross section changes the sign

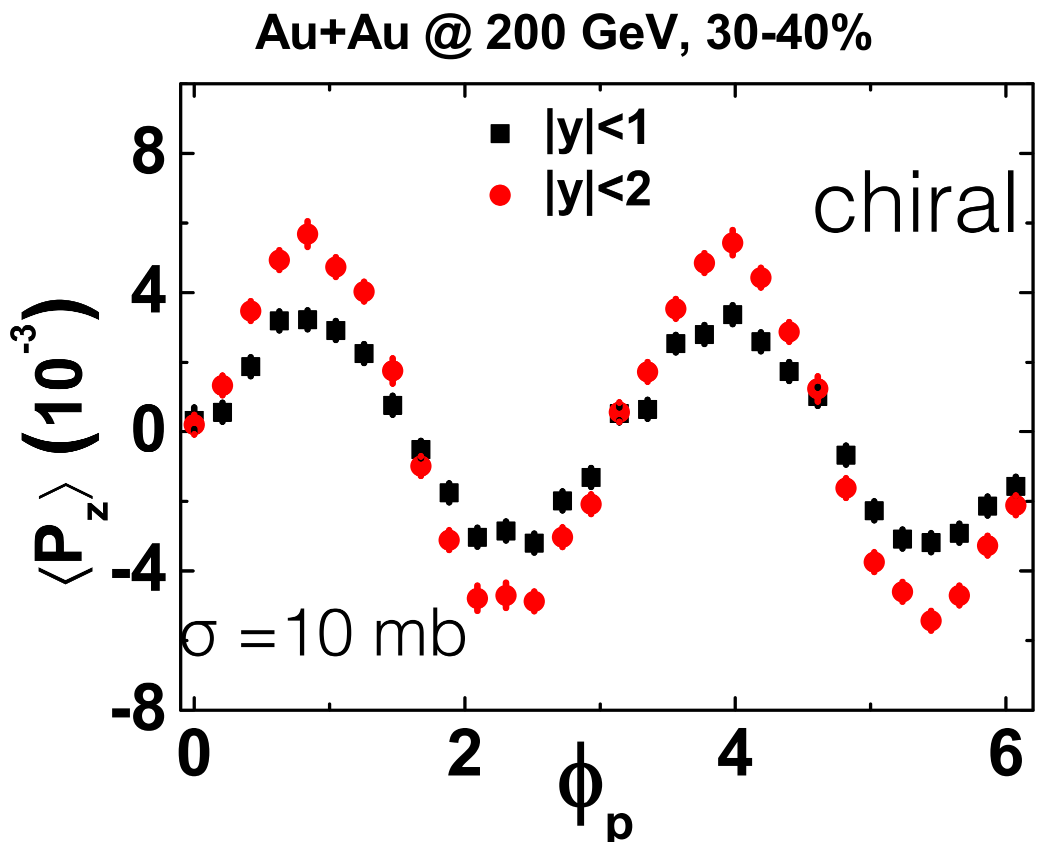
Suggest incomplete thermal equilibrium of spin degree of freedom as it develops later in time unlike the global polarization?

Hydrodynamic model



AMPT, Au+Au 200 GeV 20-50%





chiral kinetic approach

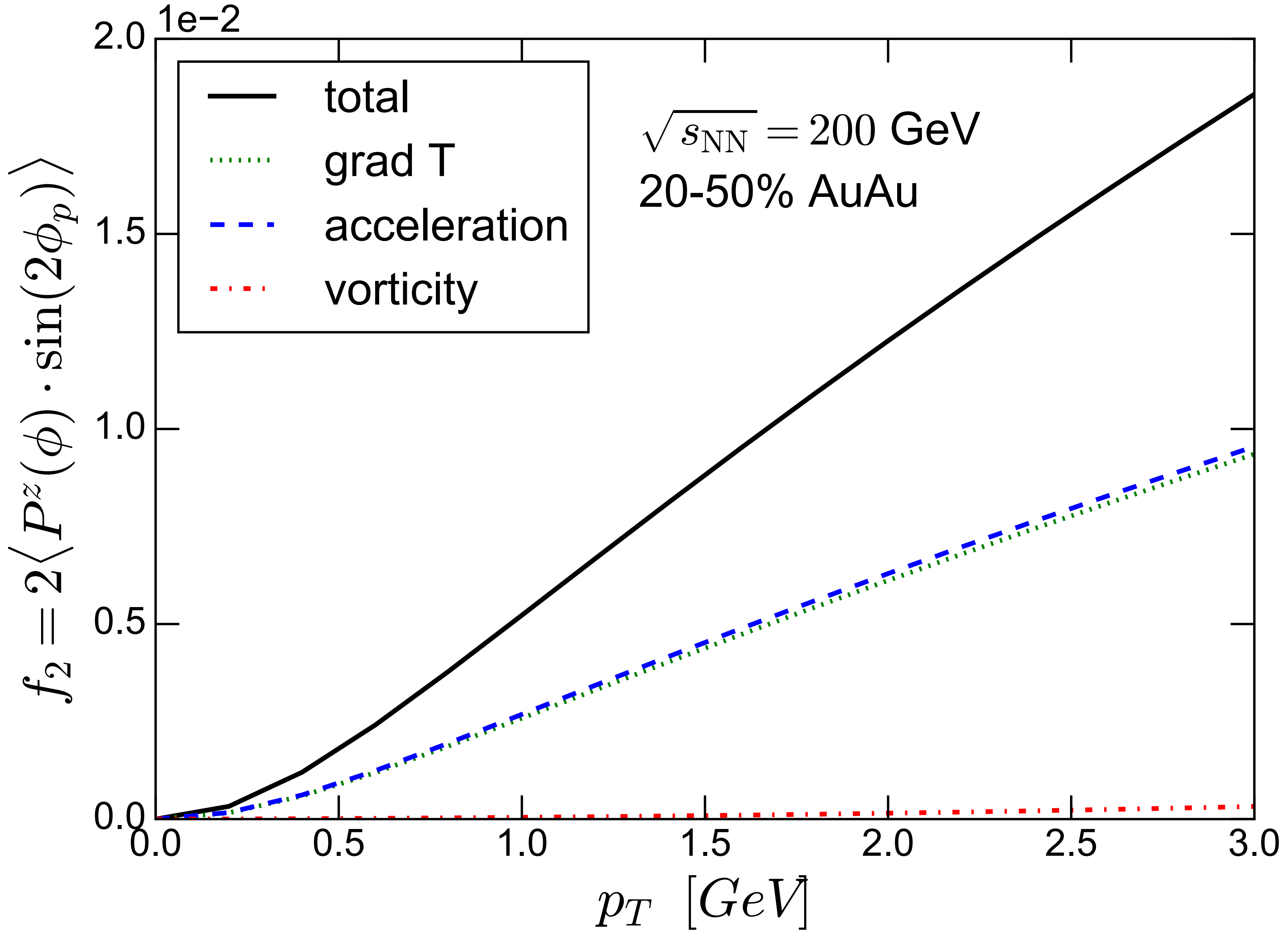
Contributions to P_z in hydro

I. Karpenko, QM2018

$$S^\mu \propto \varepsilon^{\mu\rho\sigma\tau}\varpi_{\rho\sigma}p_\tau = \varepsilon^{\mu\rho\sigma\tau}(\partial_\rho\beta_\sigma)p_\tau = \underbrace{\varepsilon^{\mu\rho\sigma\tau}p_\tau\partial_\rho\left(\frac{1}{T}\right)u_\sigma}_{\text{grad}T} + \underbrace{\frac{1}{T}2\left[\omega^\mu(u\cdot p) - u^\mu(\omega\cdot p)\right]}_{\text{“NR vorticity”}} + \underbrace{\varepsilon^{\mu\rho\sigma\tau}p_\tau A_\sigma u_\rho}_{\text{acceleration}}$$

temperature gradient **kinematic vorticity** **relativistic term**

Longitudinal quadrupole f_2:



P_z dominated by temperature gradient and relativistic term, but not by kinematic vorticity based on the hydro model.

How small is the kinematic vorticity?
Can we estimate it with the blast-wave model?

Blast-wave model

- Hydro inspired model parameterized with freeze-out condition assuming the longitudinal boost invariance
  - Freeze-out temperature T_f
  - Radial flow rapidity ρ_0 and its modulation ρ_2
  - Source size R_x and R_y

$$\rho(r,\phi_s) = \tilde{r}[\rho_0 + \rho_2\cos(2\phi_b)]$$

$$\tilde{r}(r,\phi_s) = \sqrt{(r\cos\phi_s)^2/R_x^2 + (r\sin\phi_s)^2/R_y^2}$$

- Calculate vorticity at the freeze-out using the parameters extracted from spectra, v_2, and HBT fit
  -

$$\langle\omega_z\sin(2\phi)\rangle = \frac{\int d\phi_s \int r dr\, I_2(\alpha_t)K_1(\beta_t)\omega_z\sin(2\phi_b)}{\int d\phi_s \int r dr\, I_0(\alpha_t)K_1(\beta_t)}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right),$$

u: local flow velocity, I_n, K_n: modified Bessel functions

F. Retiere and M. Lisa, PRC70.044907 (2004)

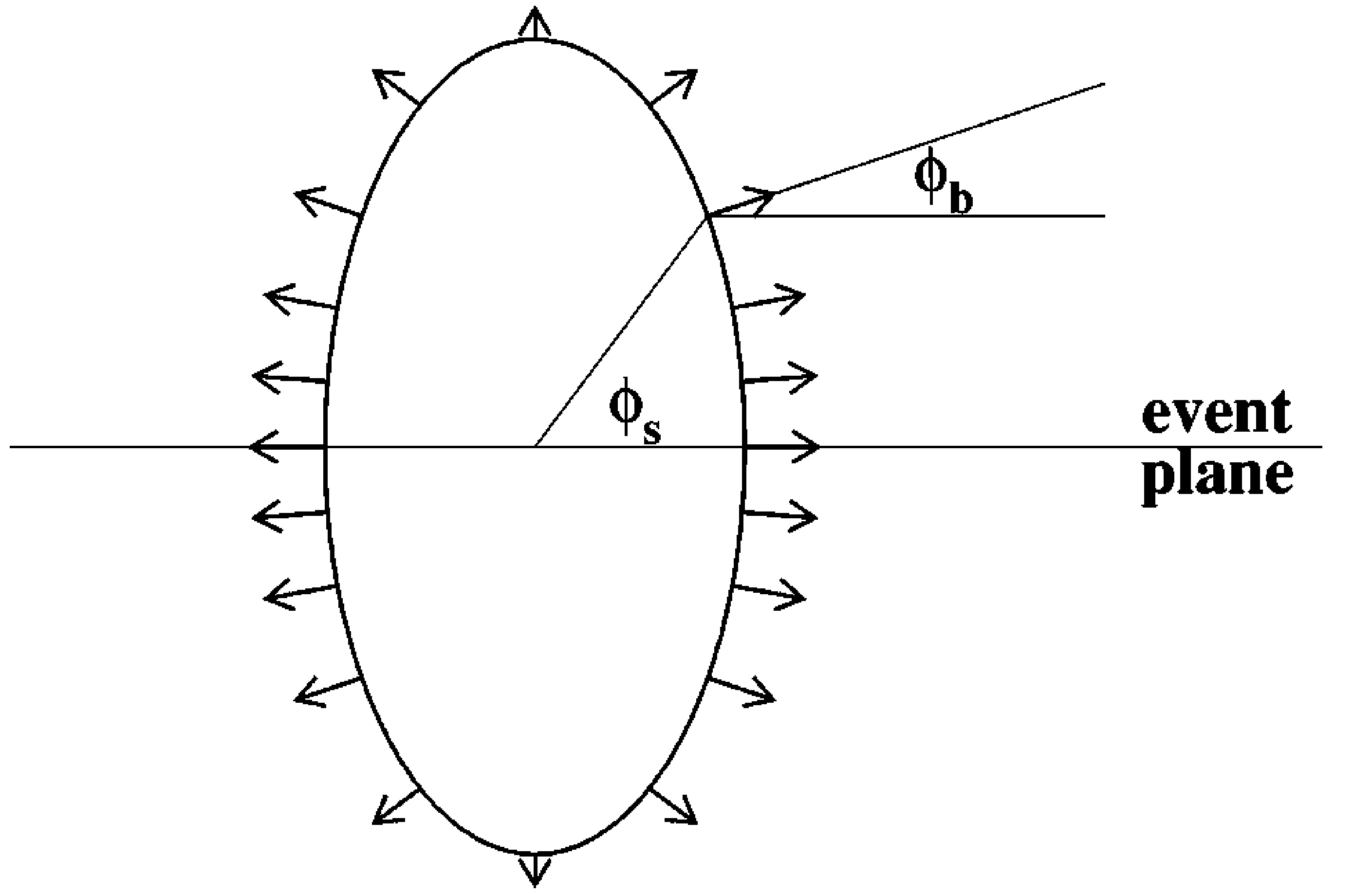
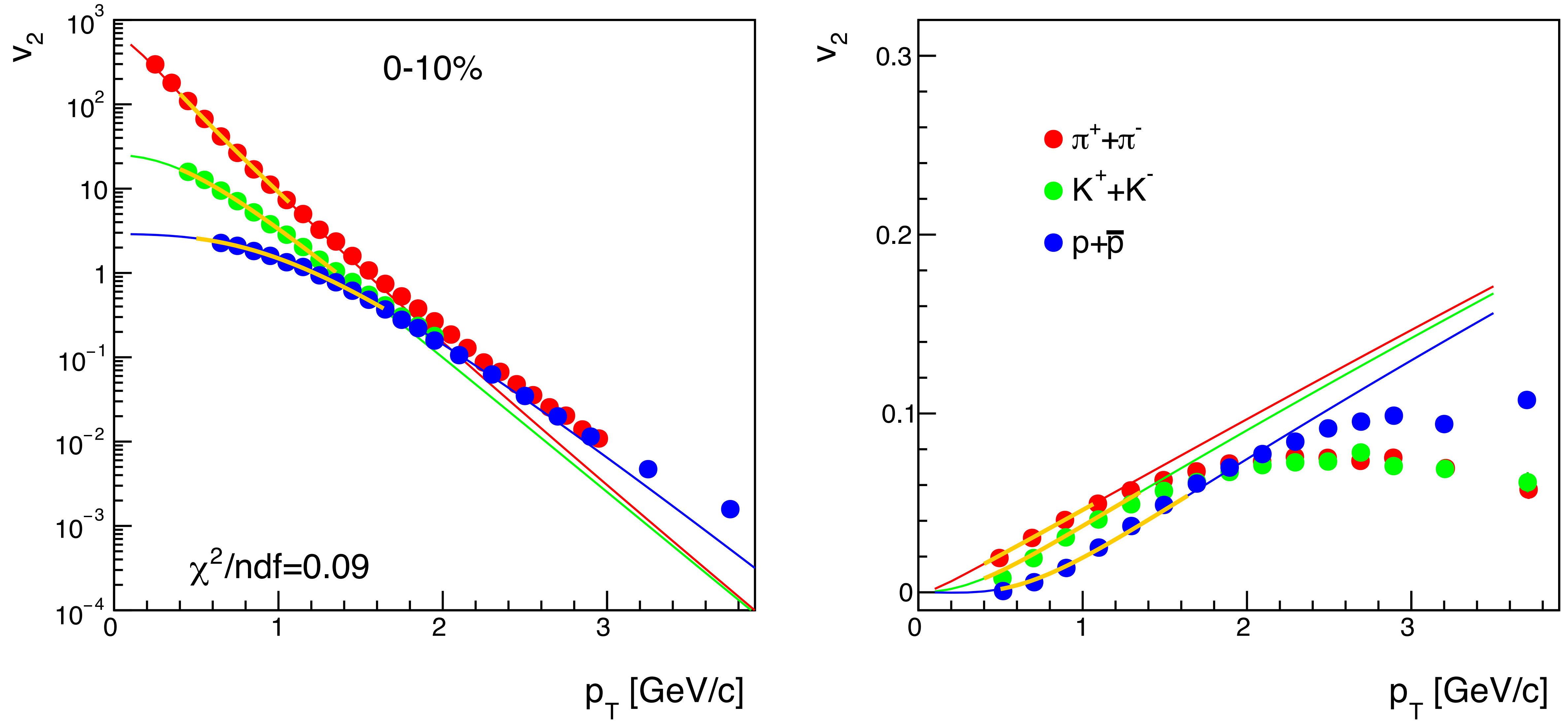


FIG. 2. Schematic illustration of an elliptical subshell of the source. Here, the source is extended out of the reaction plane ($R_y > R_x$). Arrows represent the direction and magnitude of the flow boost. In this example, $\rho_2 > 0$ [see Eq. (4)].

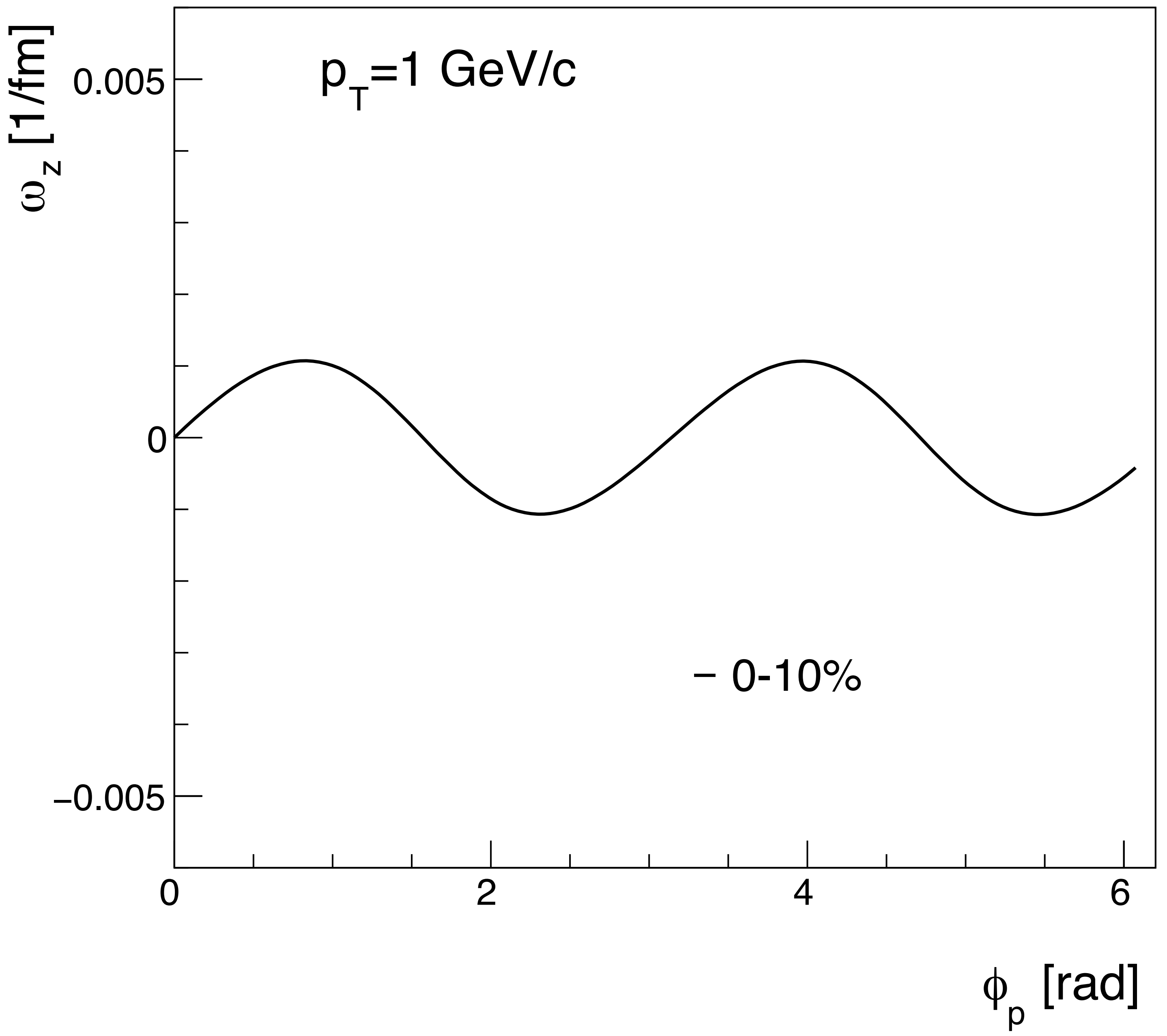
ϕ_s: azimuthal angle of the source element
ϕ_b: boost angle perpendicular to the elliptical subshell

ω_z and P_z from the BW model

e.g. Blast-wave fit to spectra and v_2



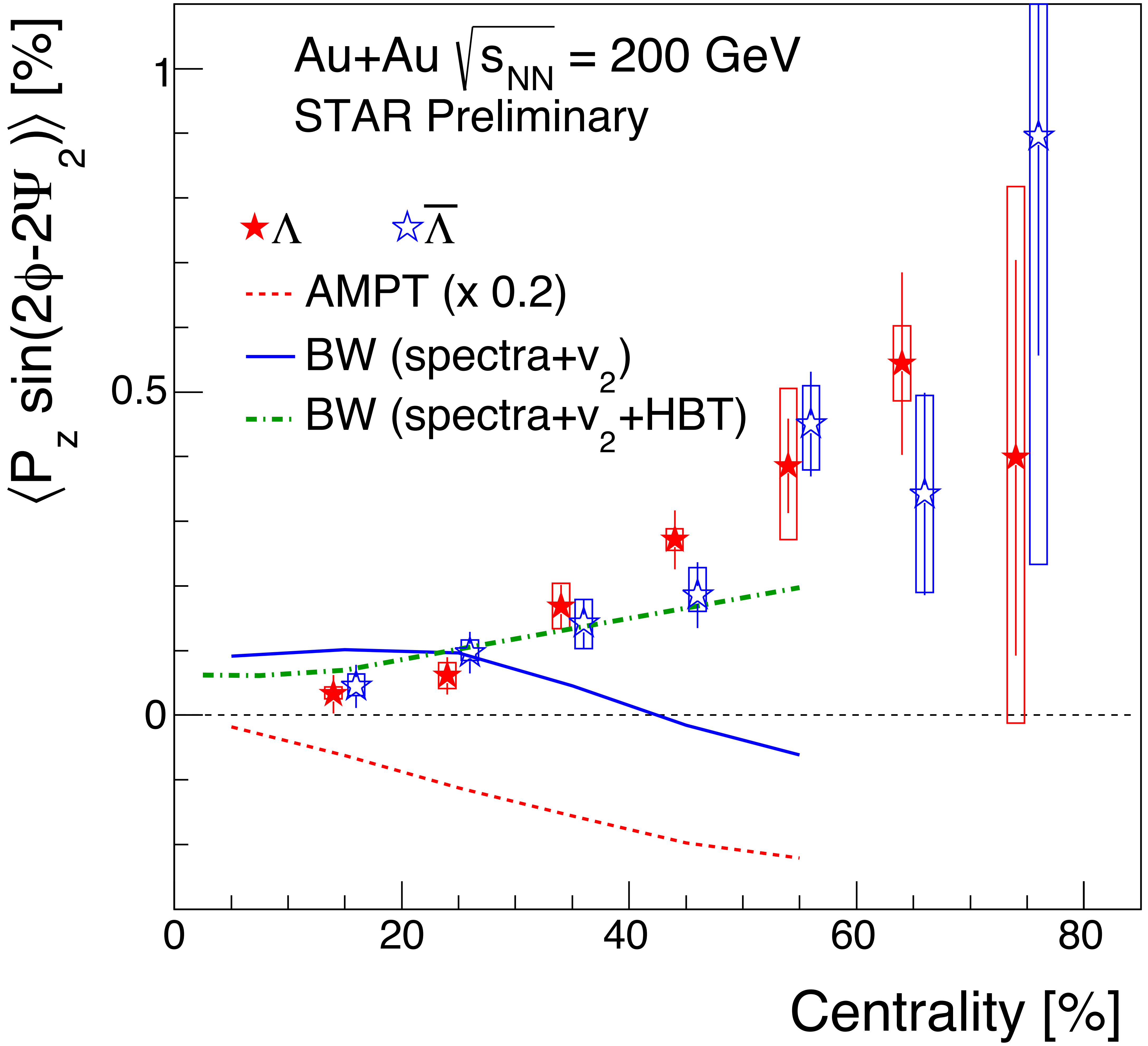
Data:
PHENIX, PRC69.034909 (2004)
PHENIX, PRC93.051902(R) (2016)



Calculated vorticity ω_z shows the sine modulation. Assuming a local thermal equilibrium, z-component of polarization is estimated as follows:

$$P_z \approx \omega_z/(2T)$$

P_z modulation from the BW model



X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

T. Niida, S. Voloshin, A. Dobrin, and R. Bertens,
in preparation

BW parameters obtained with HBT: STAR, PRC71.044906 (2005)

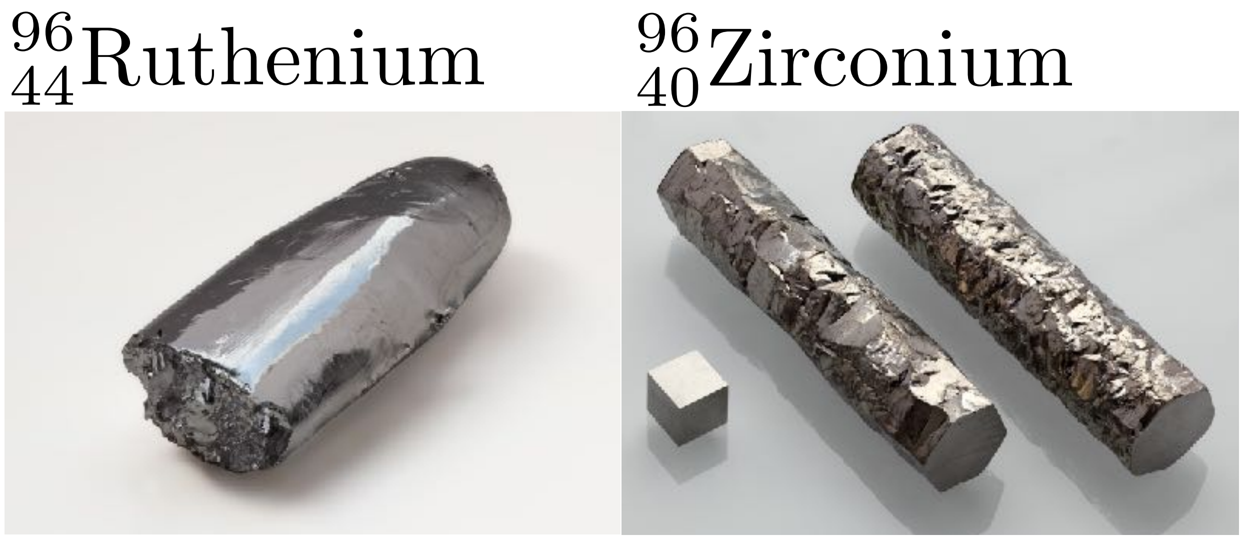
Summary

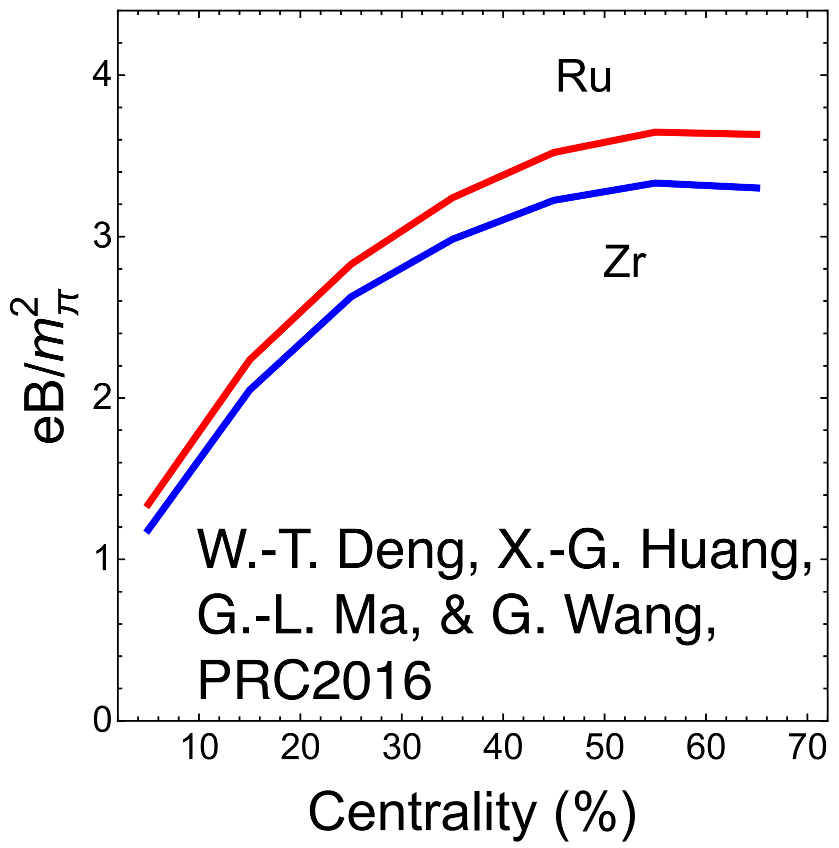
- Observation of positive Λ global polarization at $\sqrt{s_{NN}}$ = 7.7-62.4 GeV, and later at 200 GeV
  - Indicating the thermal vorticity of the system in HIC $\omega \sim 10^{22}\ s^{-1}$ (T=160 MeV)
  - Polarization decreases at higher energies, and
  - Larger signal in more peripheral collisions but no significant dependence on p_T
    → ***Quantitatively consistent with hydrodynamic and AMPT models***
  - Larger signal in in-plane than in out-of-plane
    → ***Disagree with hydrodynamic model***
  - Charge-asymmetry dependence (~2σ level) in the polarization
    → ***A possible relation to the axial current induced by B-field***

- Λ polarization along the beam direction at $\sqrt{s_{NN}}$ = 200 GeV
  - Quadrupole structure relative to the 2nd-order event plane, as expected from the elliptic flow
    → **Qualitatively consistent with a picture of the elliptic flow but agree/disagree among the data and theoretical calculations in the sign**
  - Strong centrality dependence as in the elliptic flow
  - The blast-wave model predicts the same sign and similar magnitude to the data

Outlook

- Isobar collision data (Ru+Ru, Zr+Zr) already taken in 2018!
  - Same mass number but different number of protons
    - →10% difference in the magnetic field
    - →More P_H splitting btw Λ and anti-Λ in Ru?



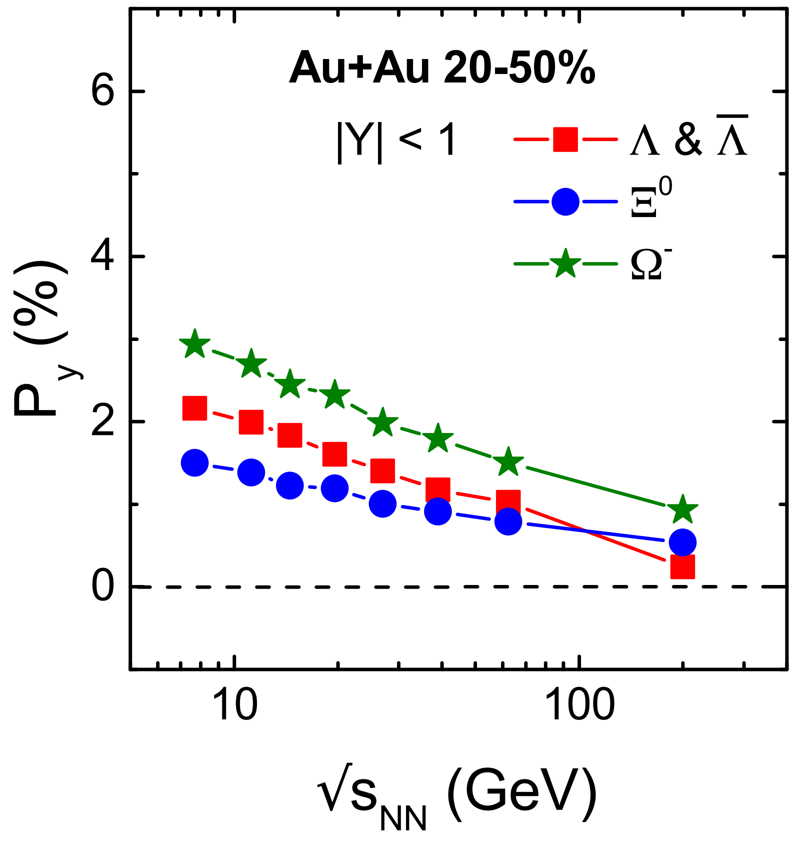


W.-T. Deng, X.-G. Huang, G.-L. Ma, & G. Wang, PRC2016

- New 27 GeV data taken in 2018! (x10 events with ~1.5 better EP resolution)
  - Possible probe of the magnetic field from Λ vs anti-Λ global polarization

D.-X. Wei *et al.*, arXiv:1810.00151

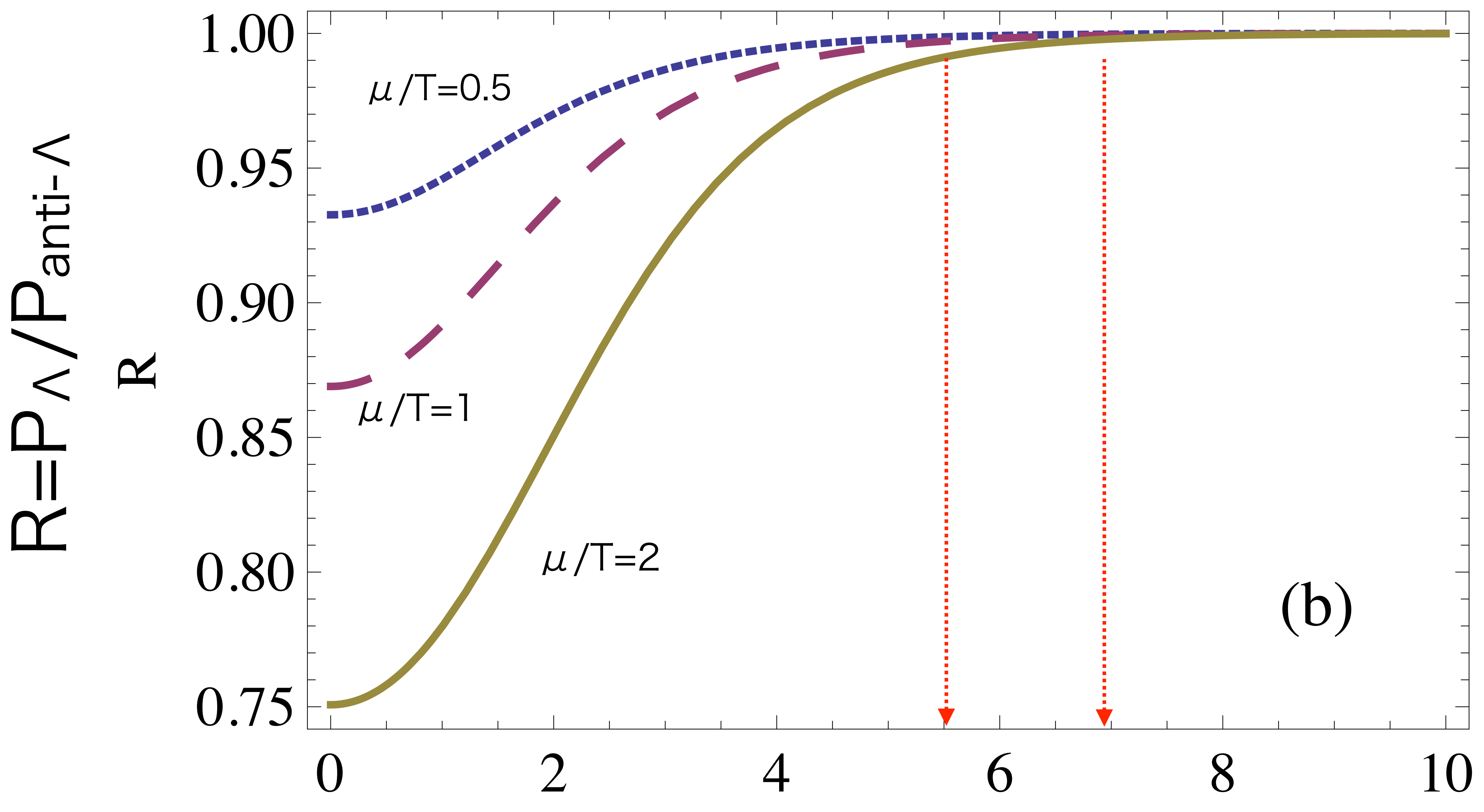
- Beam Energy Scan II (2019+) with STAR detector upgrade
  - x10 events for $\sqrt{s_{NN}}$ = 7.7-19.6 GeV (collider mode) + $\sqrt{s_{NN}}$ = 3-7.7 GeV (Fixed target)
  - How about at forward/backward rapidity? How about for multi-strangeness?



Back up

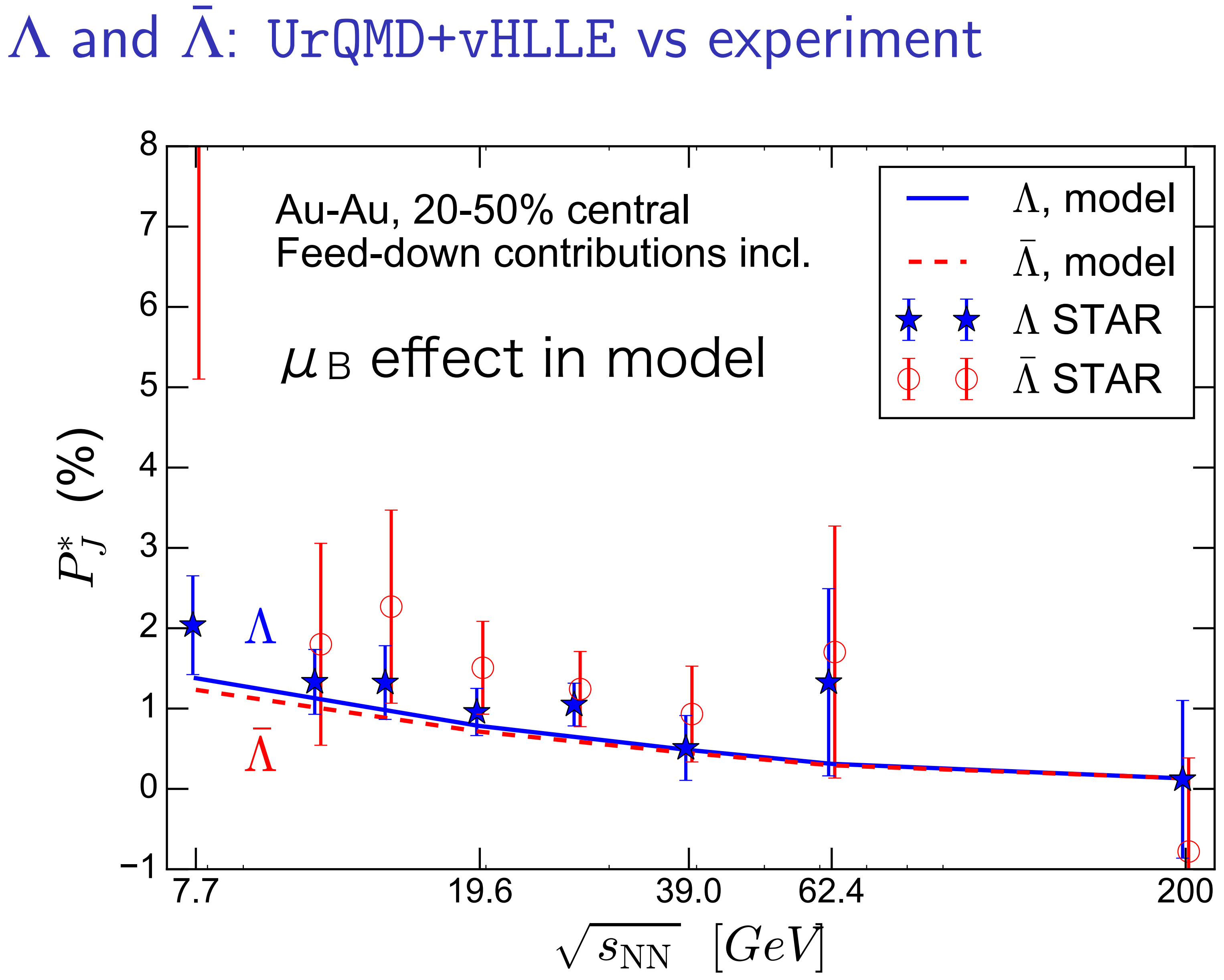
Effect of non-zero chemical potential

R. Fang, L. Pang, Q. Wang, and X. Wang, PRC94, 024904 (2016)



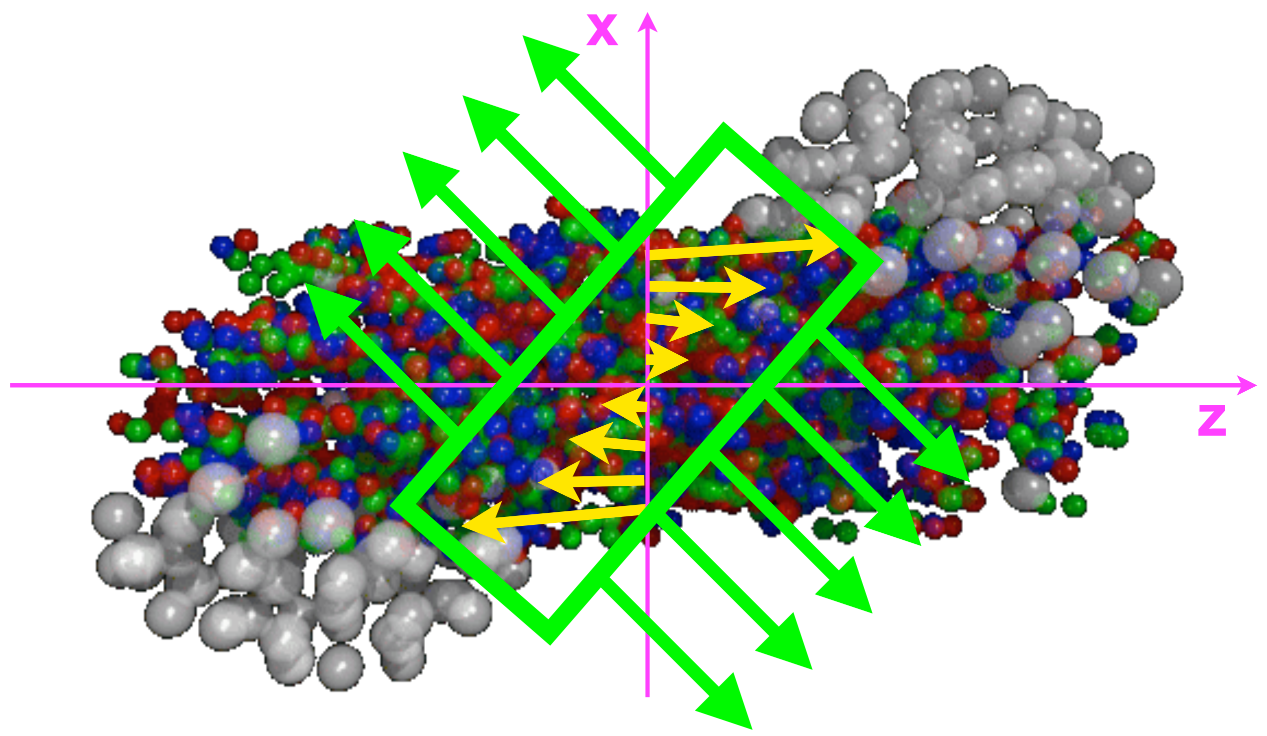
βm = m/T ~ 1.1GeV/(160-200)MeV
~ 5.5-6.8

Y. Karpenko, sQM2017



Non-zero chemical potential makes difference in polarization between Λ and anti-Λ, but the effect seems to be small.

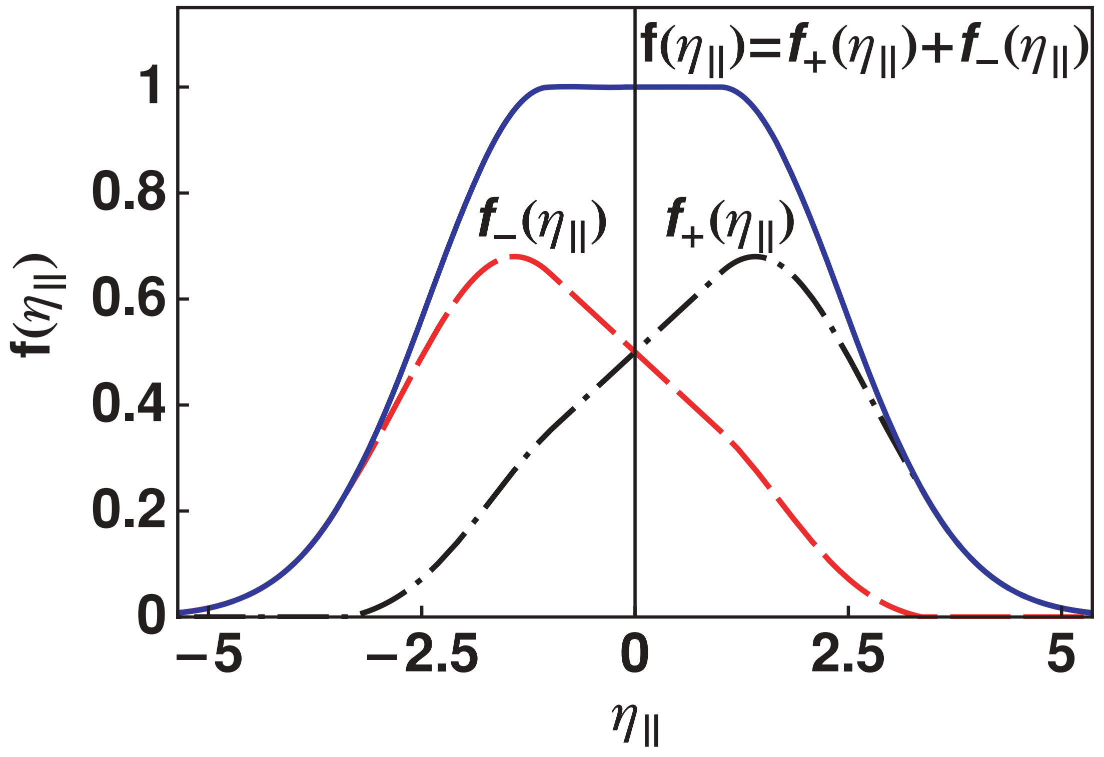
What’s the origin of v_1?



Vorticity is likely related to directed flow

Preferential emission of forward(backward) going participants results in the initial source tilt.
The initial tilt with an expansion leads to a vorticity, and creates η dependence of directed flow.

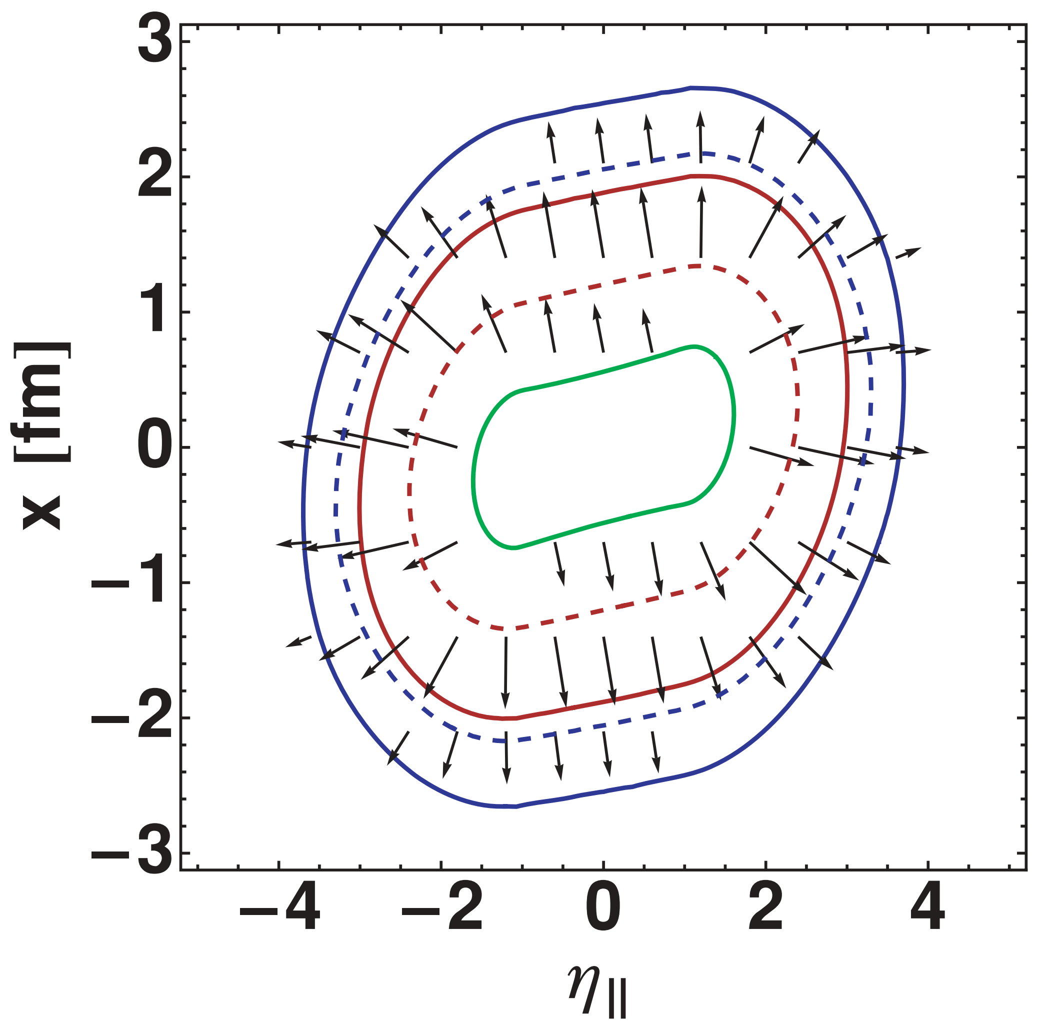
Bozek and Wyskiel, PRC81.054902 (2010)



$$f_+(\eta_{\|}) = f(\eta_{\|}) f_F(\eta_{\|})$$

$$f_F(\eta_{\|}) = \begin{cases} 0 & \eta_{\|} < -\eta_m \\ \frac{\eta_{\|}+\eta_m}{2\eta_m} & -\eta_m \le \eta_{\|} \le \eta_m \\ 1 & \eta_m < \eta_{\|} \end{cases}$$

wounded nucleon model



Hydro with the "tilted" source

F. Becattini et al., Eur. Phys. J. C (2015)75:406

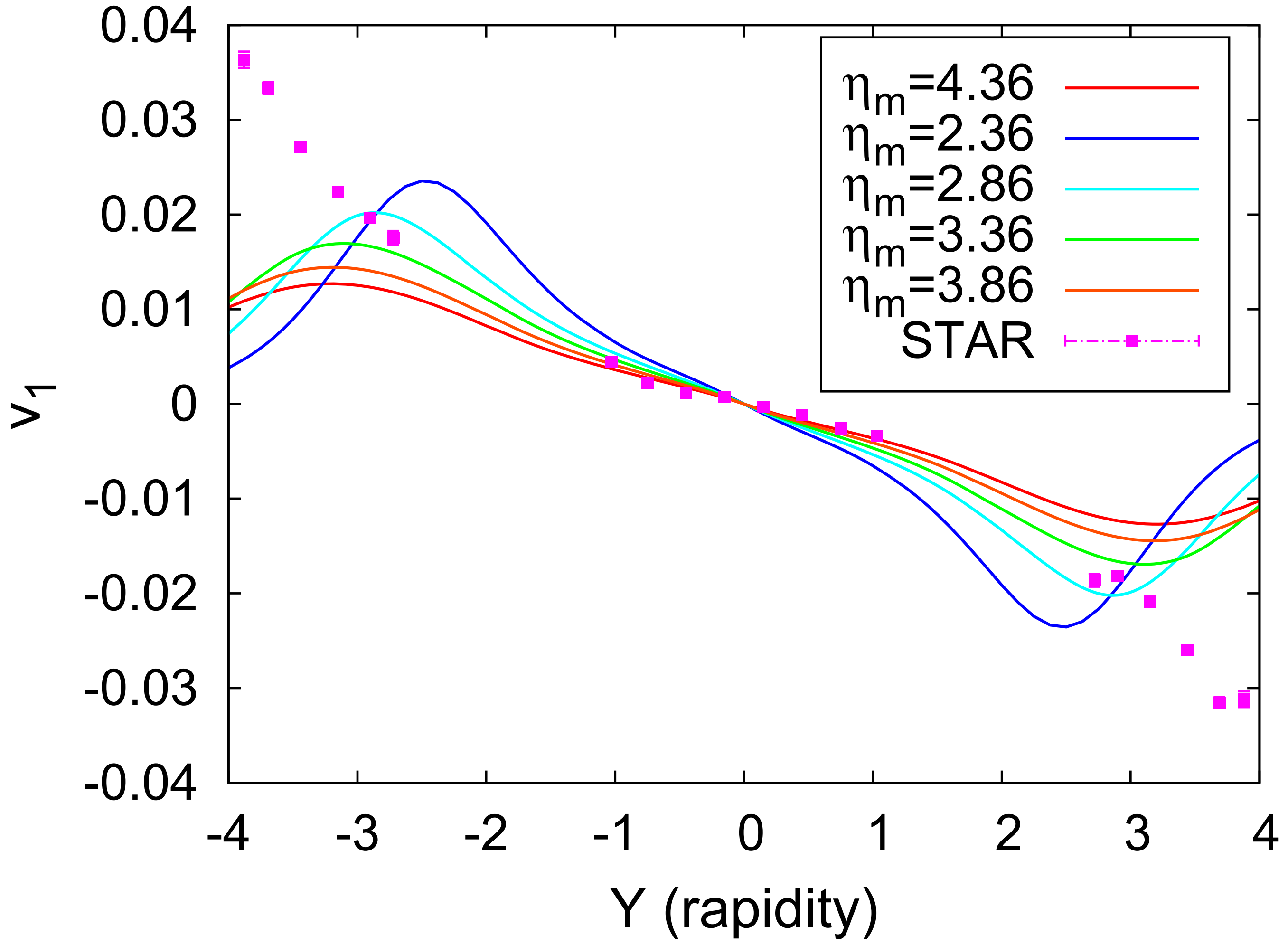


Fig. 6 Directed flow of pions for different values of η_m parameter with $\eta/s = 0.1$ compared with STAR data [22]

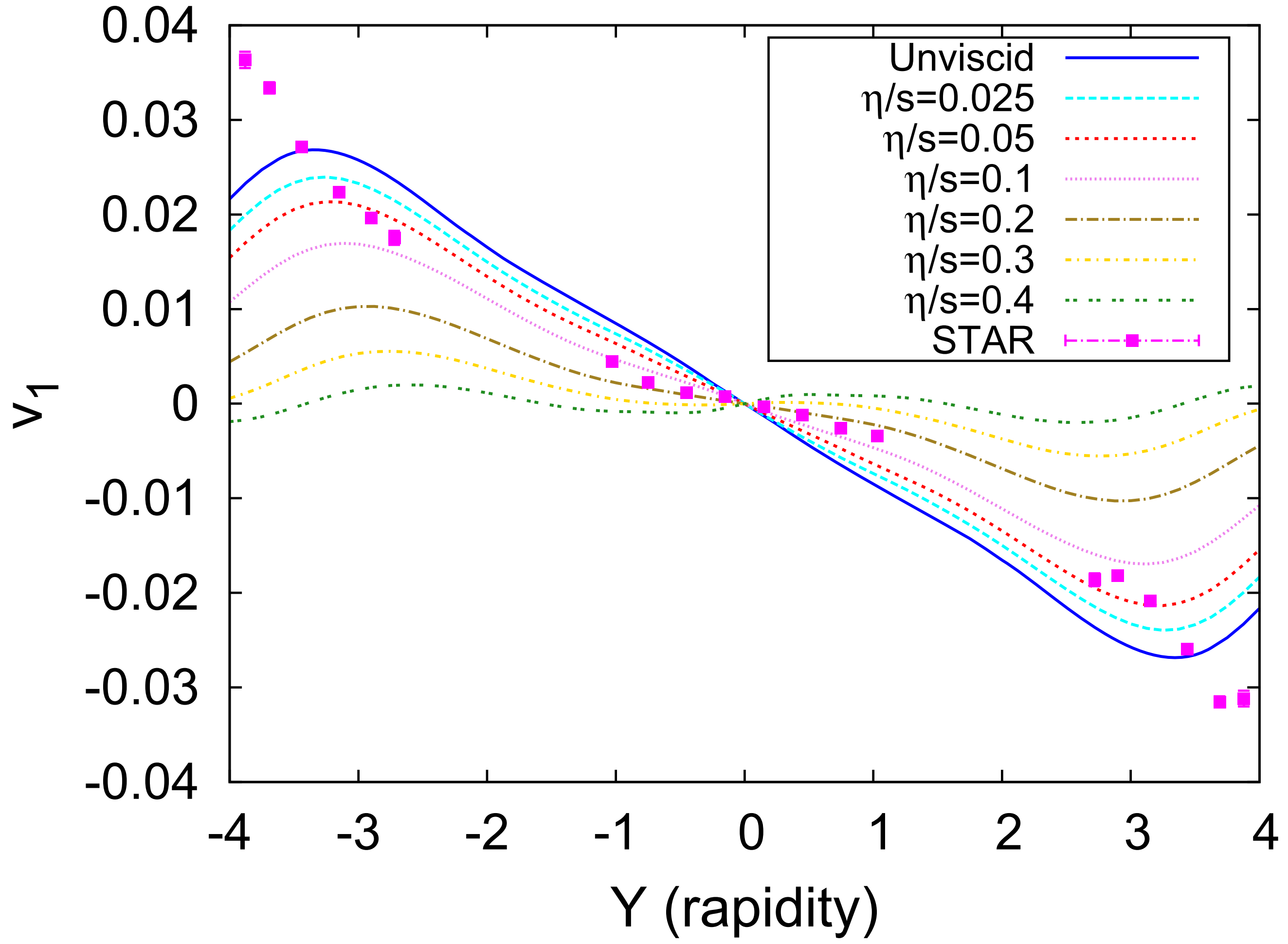


Fig. 7 Directed flow of pions for different values of η/s with $\eta_m = 2.0$ compared with STAR data [22]

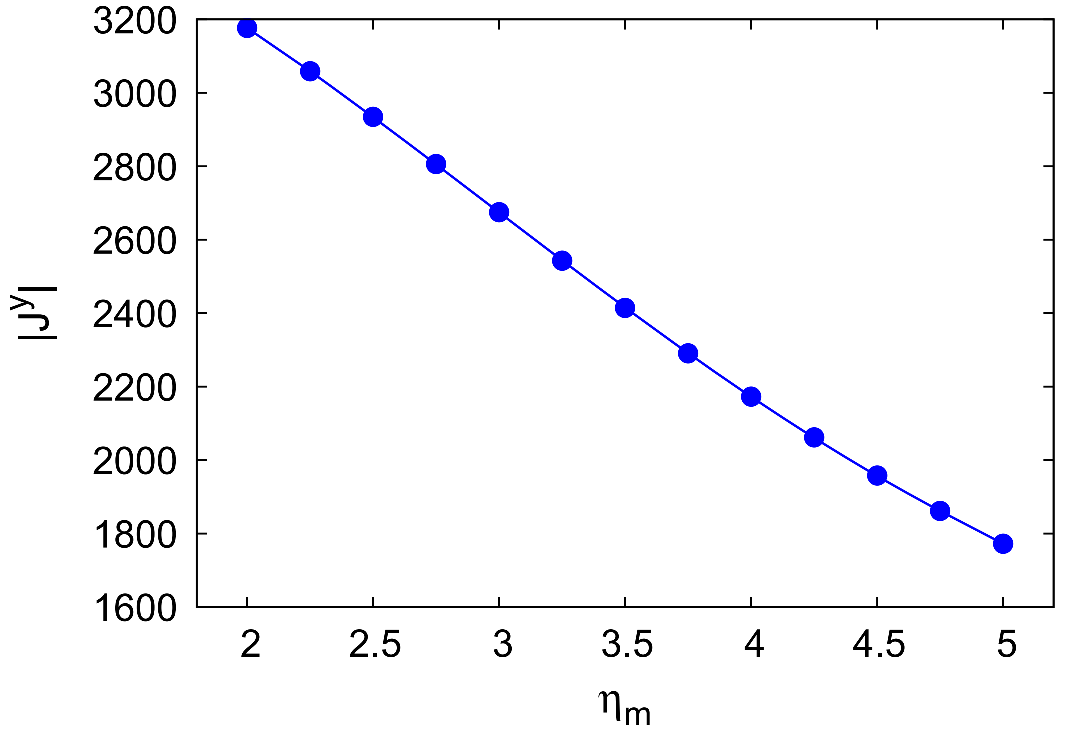


Fig. 9 Angular momentum (in $\hbar$ units) of the plasma with Bjorken initial conditions as a function of the parameter η_m

The tilted source which accounts for vorticity provides a better description of v_1!

Chiral Magnetic Effect (CME)

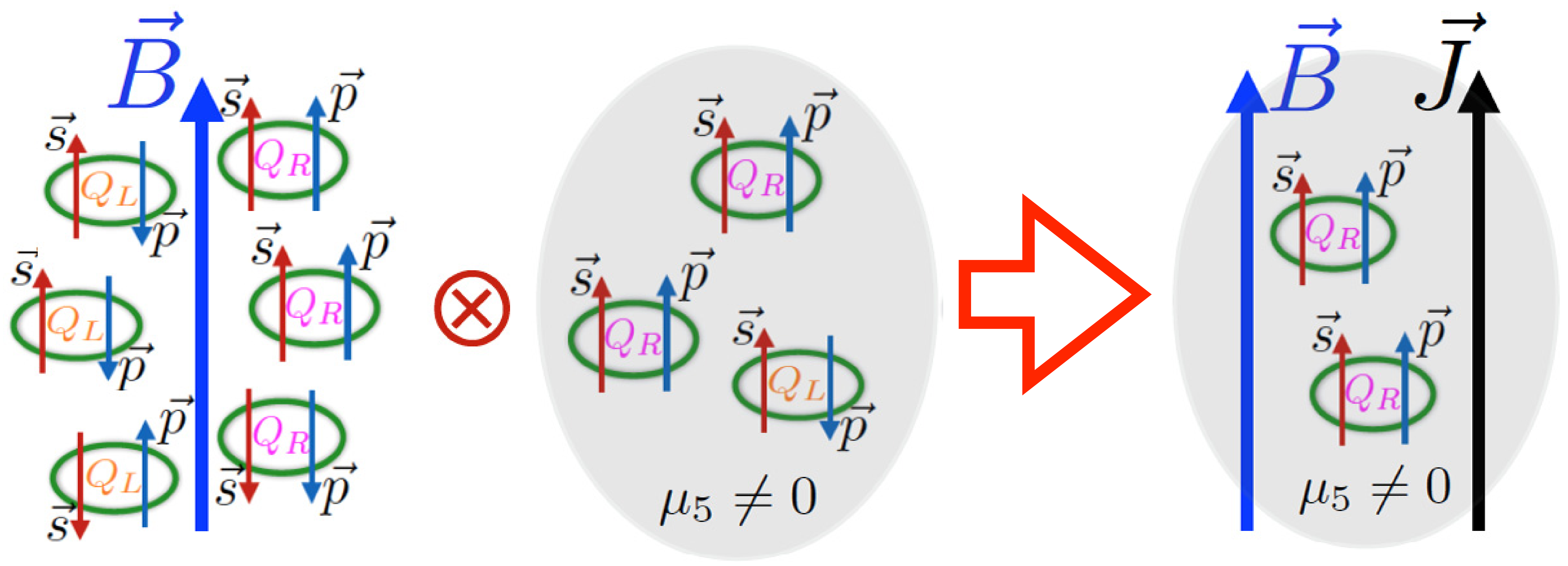
D. Kharzeev, R. Pisarski, M. Tytgat, PRL81, 512 (1998)
D. Kharzeev, PPNP75(2014)133-151

Magnetic field + massless quarks + chirality imbalance

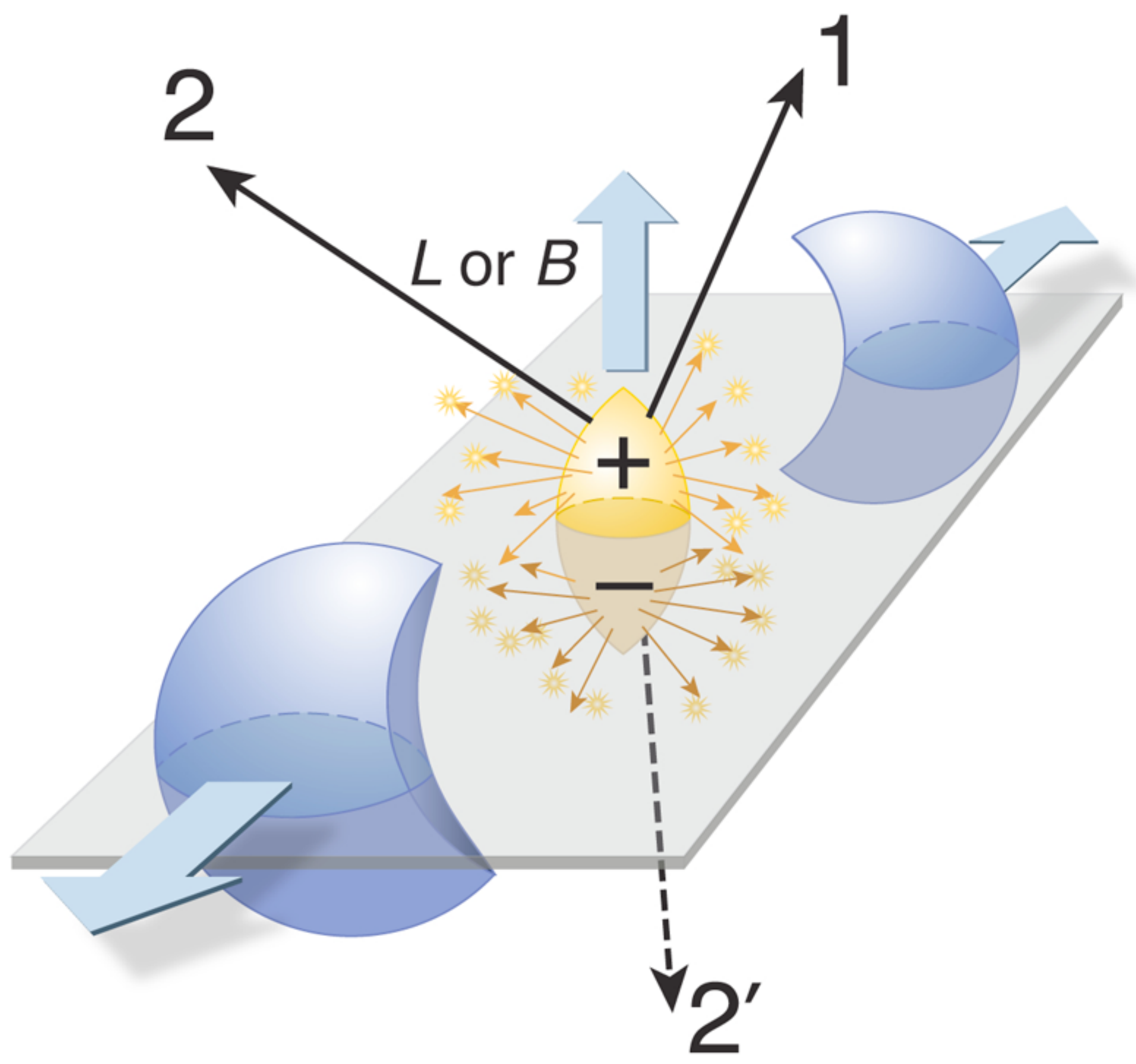
spin alignment (opposite direction for opposite sign)

spin and momentum in (anti-)parallel for right(left)-handed quarks

right-handed quarks ≠ left-handed quarks

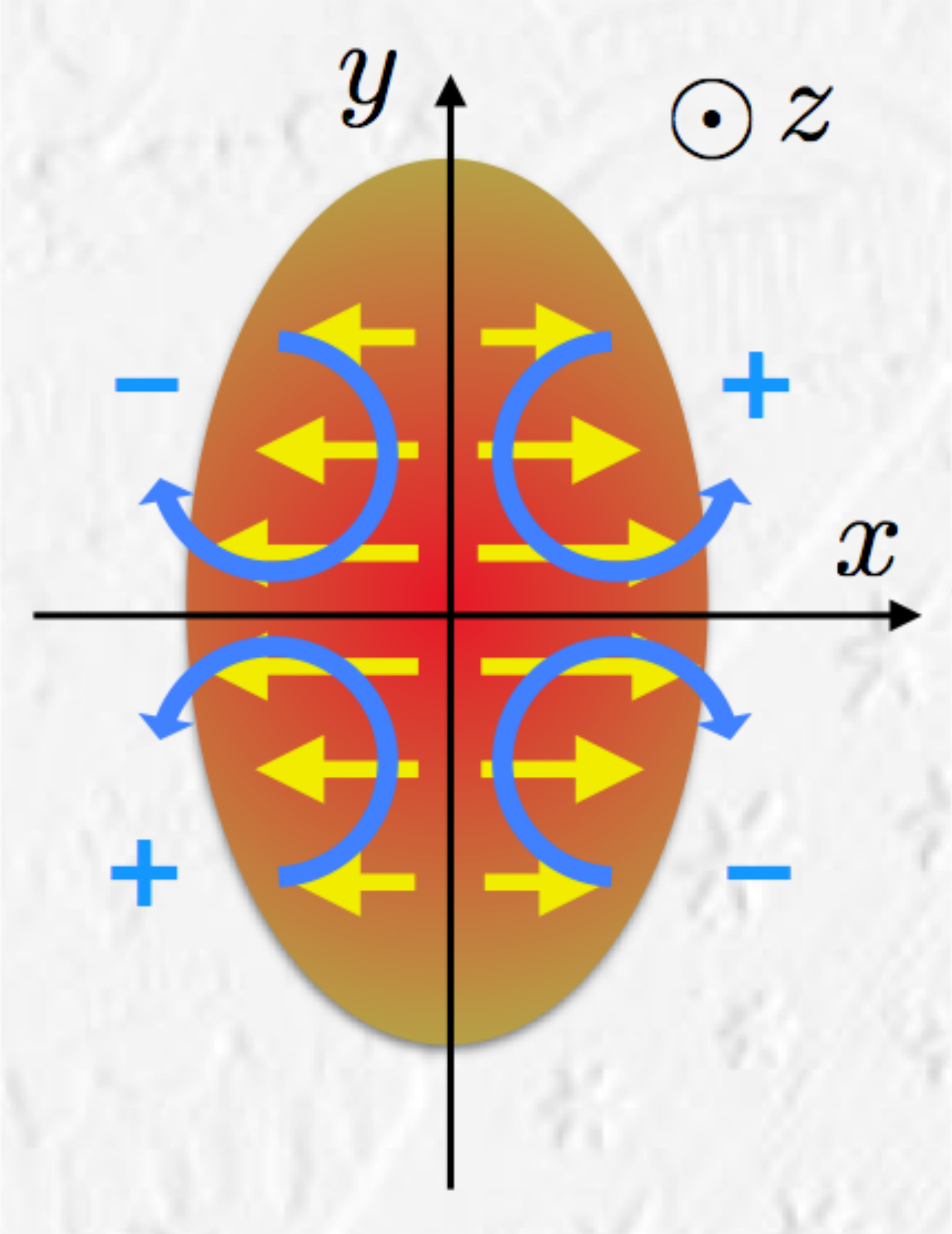


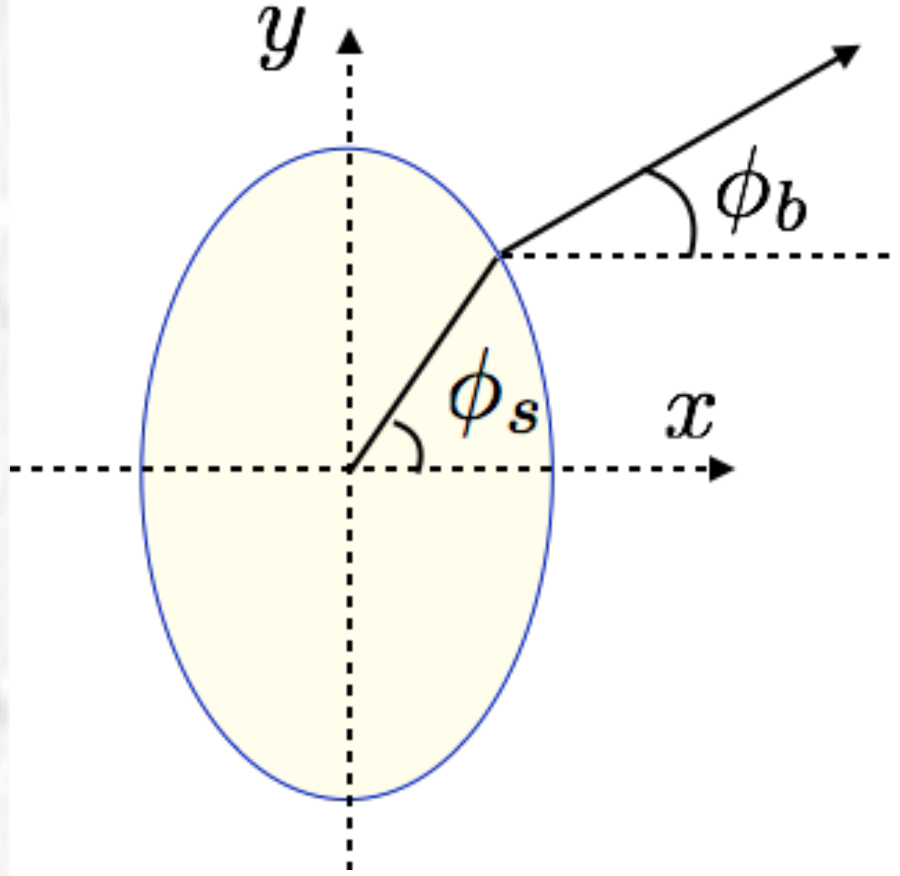
$$\vec{\mathbf{J}} \propto (Qe)\mu_5 \vec{\mathbf{B}}$$



Induction of electric current along the magnetic field, called *Chiral Magnetic Effect (CME)*

Blast-wave parameterization





S. Voloshin, arXiv:1710.08934

$$r_{max} = R[1 - a\cos(2\phi_s)],$$

$$\rho_t = \rho_{t,max}[r/r_{max}(\phi_s)][1 + b\cos(2\phi_s)] \approx \rho_{t,max}(r/R)[1 + (a+b)\cos(2\phi_s)].$$

$$\omega_z = 1/2(\nabla \times \mathbf{v})_z \approx (\rho_{t,nmax}/R)\sin(n\phi_s)[b_n - a_n].$$

a_n: spatial anisotropy
b_n: flow anisotropy
R: reference source radius
ρ_t: transverse flow velocity

Quadrupole or sine structure of ω_z is expected.

Systematic uncertainties

Case of 200 GeV as an example

- Event plane determination: ~22%
- Methods to extract the polarization signal: ~21%
- Possible contribution from the background: ~13%
- Topological cuts: <3%
- Uncertainties of the decay parameter: ~2% for Λ, ~9.6% for anti-Λ
- Extraction of Λ yield (BG estimate): <1%

Also, the following studies were done to check if there is no experimental effect:

- Two different polarities of the magnetic field for TPC
- Acceptance effect
- Different time period during the data taking
- Efficiency effect