Hydrodynamic approach to nuclear collisions at beam energy scan energies

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Hadron Interactions and Polarization from Lattice QCD, Quark Model, and Heavy Ion Collisions

28th March 2019, Yukawa Institute for Theoretical Physics, Kyoto, Japan
Introduction

- The quark-gluon plasma (QGP)

- A high-temperature phase of QCD (> $2 \times 10^{12}$ K)
- Well-established theoretically by **lattice QCD** at vanishing $\mu_B$ and experimentally by **nuclear collisions**
  - BNL Relativistic Heavy Ion Collider (RHIC)
  - CERN Large Hadron Collider (LHC)
Introduction

- Little is known at finite density ("sign problem" of lattice QCD)
Introduction

- Little is known at finite density ("sign problem" of lattice QCD)

Use nuclear collisions to:
- Determine the quark matter properties at finite $T$, $\mu_B$
- Verify the existence of a QCD critical point (QCP)
Introduction

- Modeling nuclear collisions

QCD properties

We need a “link” between fundamental QCD properties and experimental data of nuclear collisions

We consider the relativistic hydrodynamic model
Introduction

- Relativistic nuclear collisions

Nuclei (saturated gluons)

Hadronic transport
\[ \tau > 10 \text{ fm} \]
- Freeze-out

Hydrodynamic evolution
\[ \tau = 1-10 \text{ fm} \]
- Local equilibration

Glasma
\[ \tau < 1 \text{ fm} \]
- Collision

Color glass condensate
\[ \tau < 0 \text{ fm} \]
Introduction

- Relativistic nuclear collisions

![Diagram of nuclear collision with labels for different phases: Color glass condensate, Glasma, Hydrodynamic evolution, Hadronic transport, Freeze-out, Local equilibration, Collision, and Nuclei (saturated gluons).]
Introduction

- Relativistic nuclear collisions

Glasma (Longitudinal color magnetic & electric fields)

<table>
<thead>
<tr>
<th>Hadronic transport</th>
<th>( \tau &gt; 10 \text{ fm} )</th>
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<tbody>
<tr>
<td>Freeze-out</td>
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<table>
<thead>
<tr>
<th>Hydrodynamic evolution</th>
<th>( \tau = 1-10 \text{ fm} )</th>
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<tr>
<td>Local equilibration</td>
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<table>
<thead>
<tr>
<th>Glasma</th>
<th>( \tau &lt; 1 \text{ fm} )</th>
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<tbody>
<tr>
<td>Collision</td>
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</tbody>
</table>

| Color glass condensate  | \( \tau < 0 \text{ fm} \) |
Introduction

- Relativistic nuclear collisions

Hadronic transport
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Glasma
\( \tau < 1 \text{ fm} \)
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Color glass condensate
\( \tau < 0 \text{ fm} \)

QGP fluid
(After local thermalization)
Introduction

- Relativistic nuclear collisions

- Hadronic transport
  - $\tau > 10 \text{ fm}$
  - Freeze-out

- Hydrodynamic evolution
  - $\tau = 1-10 \text{ fm}$
  - Local equilibration

- Glasma
  - $\tau < 1 \text{ fm}$
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- Color glass condensate
  - $\tau < 0 \text{ fm}$
Introduction

- Relativistic nuclear collisions

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- Color glass condensate
  - $\tau < 0 \text{ fm}$
Introduction

- Evidence for the QGP fluid

Spatial anisotropy

Momentum anisotropy

Characterized by Fourier harmonics of azimuthal distribution

$$\frac{dN}{d\phi} = \frac{N}{2\pi} [1 + 2v_1 \cos(\phi - \Psi_1) + 2v_2 \cos(2\phi - 2\Psi_2) + 2v_3 \cos(3\phi - 3\Psi_3) + ...]$$

$$v_2 :$$ elliptic flow
Introduction

- Experimental data

Kolb et al., PLB 500, 232 (2001)

Consistent with the nearly-perfect liquid picture up to $p_T \sim 2$ [GeV]

- The QGP is strongly-coupled near the quark-hadron transition
- We may use hydrodynamics for an effective theory of QGP
Is it good at BES energies?

- A historical point of view

![Diagram showing the discovery of a nearly-perfect fluid at RHIC and LHC across different years.](image)
Is it good at BES energies?

- A historical point of view

Discovery of a nearly-perfect fluid

\[ \sqrt{s_{NN}} \]

\( \text{Year} \)

1980 1990 2000 2010 2018

Bevalac AGS SPS RHIC RHIC-BES LHC

Not hydro Hydro Hydro

Akihiko Monnai (KEK), HIPLQH 2019, 28th March 2019
Is it good at BES energies?

- A historical point of view (around 2000)

\( \sqrt{s_{NN}} \) vs Year

- Ideal hydro
- Not hydro

1980 1990 2000 2010 2018

Bevalac AGS SPS RHIC
Is it good at BES energies?

- A historical point of view (around 2018)

Viscous hydro

Not hydro

Bevalac

AGS

SPS

RHIC

RHIC-BES

LHC

Shear viscosity: Csernai, Kapusta & McLerran, PRL 97, 152303 (2006)
Small systems and beam energy scan

- Similar but different physics

**Small systems**

- Temperature: large
- Volume: small

**Beam energy scan**

- Temperature: small
- Volume: large

- “Evidence of the QGP” such as jet quenching is more sensitive to volume, thermal photons to temperature
Λ polarization and beam energy scan

- Vorticity converted into spin

- Spin-orbit coupling + (possible) magnetic field effects

- More prominent at lower collision energies; a complete understanding of the background medium evolution is required
Overview

1. Introduction
2. Multiple charges
3. Summary and outlook
4. Diffusion and dissipation
2. Multiple charges

Conserved charges

- in relativistic nuclear collisions

Baryon number (B) (>
0 in total)

\[ p \quad n \]
+1 \quad +1

Electric charge (Q) (>
0 in total)

\[ p \quad n \]
+1 \quad 0

Strangeness (S) (= 0 in total)

\[ p \quad n \]
0 \quad 0

Essential in understanding particle-antiparticle ratios

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TABLE I: Left two columns: midrapidity yields of common particles from central Pb+Pb collisions measured by the NA49/SPS Collaboration [26-33] at \( \sqrt{s_{NN}} = 6.41 \text{ GeV}. \) Starred hadrons are not measured, but estimated from other hadrons.
Overview of hydro model

- with multiple charges

We start with construction of the QCD equation of state
Equation of state

- Construction
  - Lattice QCD: expansion up to the 4th order
    \[
    \frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{X_{l,m,n}^{B,Q,S}}{l!m!n!} \left( \frac{\mu_B}{T} \right)^l \left( \frac{\mu_Q}{T} \right)^m \left( \frac{\mu_S}{T} \right)^n
    \]
  - Match to hadron resonance gas (HRG) at lower T
    1. Taylor expansion is not reliable when the fugacity is large
    2. Agreement between lattice QCD and HRG is good in hadronic phase
Equation of state

- Construction

  - (Cont’d)

  3. EOS of hydrodynamic model should match EOS of kinetic theory for correct energy-momentum/charge conservation

\[
E_i \frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3} \int \sum p_i^\mu d\sigma_{\mu i} f_i \rightarrow u^\mu, T, \mu_B
\]

- Stefan-Boltzmann limits are used as anchors at very high T where lattice QCD data are scarce
Equation of state

Construction

- Connect to HRG at low T

\[
\frac{P}{T^4} = \frac{1}{2}[1 - f(T, \mu_J)] \frac{P_{\text{had}}(T, \mu_J)}{T^4} + \frac{1}{2}[1 + f(T, \mu_J)] \frac{P_{\text{lat}}(T, \mu_J)}{T^4} \quad J = \{B, Q, S\}
\]

where

\[
f(T, \mu_J) = \tanh[(T - T_c(\mu_B))/\Delta T_c] \\
T_c = 0.16 - 0.4 \times (0.139\mu_B^2 + 0.053\mu_B^4)(\text{GeV}) \quad \Delta T_c = 0.1T_c(0)
\]

Crossover-type EOS

The dependences on sub-leading \(\mu\)'s are approximated to be small

- Parameters are chosen to satisfy thermodynamic conditions:

\[
\frac{\partial^2 P}{\partial T^2} = \frac{\partial s}{\partial T} > 0, \quad \frac{\partial^2 P}{\partial \mu^2_J} = \frac{\partial n_J}{\partial \mu_J} > 0
\]
Strangeness and charge densities

- Strange neutrality condition ($n_S = 0$)

  - $\mu_S$ is finite positive at $\mu_B > 0$ because of $s$ quarks (or strange baryons)

  
  \[ \mu_S = \frac{1}{3} \mu_B + \frac{1}{3} \mu_Q - \mu_S = 0 \]

  
  The condition can be modified by initial fluctuations and diffusion

\[ \mu_S = 0 \text{ leads to } n_S \neq 0 \]

The condition can be modified by initial fluctuations and diffusion
Strangeness and charge densities

- Charge-to-baryon ratio \((n_Q = c \, n_B)\)

  - \(\mu_Q\) is finite negative at \(\mu_B > 0\) for neutron rich nuclei \((Z/A < 1/2)\)

\[
\mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q > \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q
\]

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Z/A</th>
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<tbody>
<tr>
<td>(^1)H</td>
<td>1.000</td>
</tr>
<tr>
<td>(^2)H</td>
<td>0.500</td>
</tr>
<tr>
<td>(^3)He</td>
<td>0.667</td>
</tr>
<tr>
<td>(^{27})Al</td>
<td>0.481</td>
</tr>
<tr>
<td>(^{63})Cu</td>
<td>0.460</td>
</tr>
<tr>
<td>(^{96})Zr</td>
<td>0.417</td>
</tr>
<tr>
<td>(^{96})Ru</td>
<td>0.458</td>
</tr>
<tr>
<td>(^{127})Xe</td>
<td>0.425</td>
</tr>
<tr>
<td>(^{197})Au</td>
<td>0.401</td>
</tr>
<tr>
<td>(^{208})Pb</td>
<td>0.394</td>
</tr>
<tr>
<td>(^{238})U</td>
<td>0.387</td>
</tr>
</tbody>
</table>

- proton rich/neutral nuclei; \(\mu_Q \geq 0\) for \(\mu_B > 0\)
- relevant for background of isobars
- \(c \approx 0.4\) for Au and Pb nuclei
Equation of state

- $\mu_s = \mu_Q = 0$ (conventional; denoted as NEOS B)

- Thermodynamically consistent smooth EoS is obtained
- The strangeness neutrality condition is violated ($n_S < 0$)
Equation of state

- $n_S = 0$, $\mu_Q = 0$ (strangeness neutral; denoted as NEOS BS)

A visible modification is observed at larger $\mu_B/T$

- Finite positive $\mu_S$ is seen owing to the neutrality condition
- $\mu_B$ is becomes larger at large $T$ for a given $n_B$
Equation of state

Where you can probe on the $\mu_B$-$T$ plane

$\frac{s}{n_B}$ is constant when entropy and net baryon number are conserved

- $\frac{s}{n_B} = 420$, $\sqrt{s_{NN}} = 200$ GeV
- $\frac{s}{n_B} = 144$, $\sqrt{s_{NN}} = 62.4$ GeV
- $\frac{s}{n_B} = 51$, $\sqrt{s_{NN}} = 19.6$ GeV
- $\frac{s}{n_B} = 30$, $\sqrt{s_{NN}} = 14.5$ GeV


If conformal, it is a straight line because $s \sim T^3$ and $n_B \sim \mu_B T^2$

Larger $\mu_B$ is required to fix the $s/n_B$ ratio when pions are dominant
Equation of state

- $n_S = 0, n_Q = 0.4n_B$ (realistic in HIC; denoted as NEOS BQS)

- Finite negative $\mu_Q$ owing to the condition $n_Q = 0.4n_B$

- The overall system is positively charged; $\mu_Q$ turns positive around $n_Q = 0.5n_B$
Stefan-Boltzmann limit

- Parton gas pressure

\[
\frac{P}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{60} N_f + \frac{1}{2} \sum_{f=u,d,s} \left( \frac{\mu_f}{T} \right)^2 + \frac{1}{4\pi^2} \sum_{f=u,d,s} \left( \frac{\mu_f}{T} \right)^4
\]

where \( \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q \), \( \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q \), \( \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S \)

- The diagonal and off-diagonal susceptibilities are

\[
\begin{align*}
\chi_2^B &= \frac{1}{3}, \quad \chi_2^Q = \frac{2}{3}, \quad \chi_2^S = 1, \quad \chi_{1,1}^{B,Q} = 0, \quad \chi_{1,1}^{B,S} = -\frac{1}{3}, \quad \chi_{1,1}^{Q,S} = \frac{1}{3}, \\
\chi_4^B &= \frac{2}{9\pi^2}, \quad \chi_4^Q = \frac{4}{3\pi^2}, \quad \chi_4^S = \frac{6}{\pi^2}, \quad \chi_{3,1}^{B,S} = -\frac{2}{9\pi^2}, \quad \chi_{2,2}^{B,S} = \frac{2}{3\pi^2}, \quad \chi_{1,3}^{B,S} = -\frac{2}{\pi^2}, \\
\chi_{3,1}^{B,Q} &= 0, \quad \chi_{2,2}^{B,Q} = \frac{4}{9\pi^2}, \quad \chi_{1,3}^{B,Q} = \frac{4}{9\pi^2}, \quad \chi_{3,1}^{Q,S} = \frac{2}{9\pi^2}, \quad \chi_{2,2}^{Q,S} = \frac{2}{3\pi^2}, \quad \chi_{1,3}^{Q,S} = \frac{2}{\pi^2}, \\
\chi_{2,1,1}^{B,Q,S} &= \frac{2}{9\pi^2}, \quad \chi_{1,2,1}^{B,Q,S} = -\frac{2}{9\pi^2}, \quad \chi_{1,1,2}^{B,Q,S} = -\frac{2}{3\pi^2}
\end{align*}
\]
Stefan-Boltzmann limit

- The chemical potential ratio

\[
\begin{pmatrix}
  n_B \\
  n_Q \\
  n_S 
\end{pmatrix} = T^2 \begin{pmatrix}
  \chi_{2,B} \\
  \chi_{1,1,Q} \\
  \chi_{1,1,S} \\
  \chi_{1,1,B} \\
  \chi_{2,Q} \\
  \chi_{1,1,S} \\
  \chi_{1,1,B} \\
  \chi_{2,S} \\
\end{pmatrix} \begin{pmatrix}
  \mu_B \\
  \mu_Q \\
  \mu_S 
\end{pmatrix} + \mathcal{O}(\mu^3)
\]

\[n_J = \left. \frac{\partial P}{\partial \mu_J} \right|_{T,\mu_K}\]

NEOS B (\(\mu_S = 0\) and \(\mu_Q = 0\))

- \(\mu_B \approx 3n_B/T^2\), \(\mu_S = 0\), \(\mu_Q = 0\)

NEOS BS (\(n_S = 0\) and \(\mu_Q = 0\))

- \(\mu_B \approx 4.5n_B/T^2\), \(\mu_S \approx 1.5n_B/T^2\), \(\mu_Q = 0\)

NEOS BQS (\(n_S = 0\) and \(n_Q = 0.4n_B\))

- \(\mu_B \approx 4.6n_B/T^2\), \(\mu_S \approx 1.6n_B/T^2\), \(\mu_Q \approx -0.2n_B/T^2\)
Sound velocity $c_s$

- $s/n_B$ dependence

\[ c_s^2 = \left. \frac{\partial P}{\partial e} \right|_{n_K} + \sum_J \frac{n_J}{e + P} \left. \frac{\partial P}{\partial n_J} \right|_{e, n_K} J \neq K \]

- In dense systems, $c_s$ is suppressed at lower $T$.

- The effect of strangeness neutrality becomes more apparent.

- Finite $n_s$ is relevant to $c_s^2$ in NEOS B

\[ \left. \frac{\partial P}{\partial e} \right|_{n_B} + \frac{n_B}{e + P} \left. \frac{\partial P}{\partial n_B} \right|_e \neq c_s^2 \]

The “conventional definition” w/o Q and S leads to underestimation.
\[ \mu_B - \mu_Q - \mu_S \text{ space} \]

- Constant pressure plane

**The intercepts** \( P(\mu_B^{\text{int}}, 0, 0) = P(0, \mu_Q^{\text{int}}, 0) = P(0, 0, \mu_S^{\text{int}}) \)

\[ \mu_B^{\text{int}} > \mu_S^{\text{int}} > \mu_Q^{\text{int}} \] in the hadronic phase because the lightest hadron to carry them are ordered in mass as \( m_p > m_K > m_\pi \)
\( \mu_B - \mu_Q - \mu_S \) space

- Constant pressure plane

\[ (a) \quad T = 0.14 \text{ GeV} \]
\[ \frac{P}{T^4} = 0.8 \]

\[ (b) \quad T = 0.2 \text{ GeV} \]
\[ \frac{P}{T^4} = 2 \]

The intercepts \( P(\mu_B^{\text{int}}, 0, 0) = P(0, \mu_Q^{\text{int}}, 0) = P(0, 0, \mu_S^{\text{int}}) \)

\( \mu_B^{\text{int}} > \mu_Q^{\text{int}} > \mu_S^{\text{int}} \) in the QGP phase as \( \chi_2^B = 1/3 \), \( \chi_2^Q = 2/3 \), \( \chi_2^S = 1 \) in the parton gas limit implies \( \mu_B^{\text{int}} / 3 \sim 2\mu_Q^{\text{int}} / 3 \sim \mu_S^{\text{int}} \)
**$\mu_B$-$\mu_Q$-$\mu_S$ space**

- Exploration of the QCD phase diagram

- We do not explore the $\mu_B$-$T$ plane in the BES experiments but a certain slice in the $\mu_B$-$\mu_Q$-$\mu_S$-$T$ hyperplane.

- This may well affect the QCD critical point search.
Hydrodynamic model

- with multiple charges

Dynamical Glauber model → Relativistic hydrodynamic model

\[ \partial_\mu T^{\mu\nu} = 0 \]
\[ \partial_\mu N^{\mu}_{B,Q,S} = 0 \]
\[ \partial_\mu s^\mu \geq 0 \]

UrQMD

Equation of state
\[ P = P(\epsilon, n_B, n_Q, n_S) \]

Transport coefficients
\[ \eta, \zeta, \kappa_B, \kappa_Q, \kappa_S, ... \]

Information of QCD
Hydrodynamic results

- 3+1 D viscous hydro + UrQMD for Pb-Pb 17.3 GeV in SPS

- Strange neutrality visibly improves description of strange hadrons

- Charge-to-baryon ratio fixing has small effects; $\pi^-/\pi^+$ ratio ($>1$) is improved
Hydrodynamic results

- Switching temperature dependence

The preferred switching to UrQMD is $0.16 - 0.26$ GeV/fm$^3$

Effects of chemical potential becomes larger for lower $e_{SW}$
3. Summary and outlook
Summary and outlook

- The QCD matter at finite density poses us challenges:
  - Interplay of multiple charges (B, Q, S) are important
    - Equation of state is constructed
    - Strangeness neutrality condition leads to finite positive \( \mu_S \), and realistic charge-to-baryon ratio for Au/Pb to finite negative \( \mu_Q \)
    - Particle-to-antiparticle ratios are described better in hydro model
  
- Estimation of baryon, strangeness and charge diffusion including cross-coupling currents
  
- \( p_T \) spectra, flow harmonics and rapidity distribution
  
- Realistic EoS for small systems as well as isobar experiments
Summary and outlook

- Our QCD equation of state model NEOS is publicly available: https://sites.google.com/view/qcdneos/home