Hydrodynamic approach to nuclear collisions at beam energy scan energies

Akihiko Monnai (KEK)

In collaboration with: Björn Schenke (BNL) and Chun Shen (Wayne)

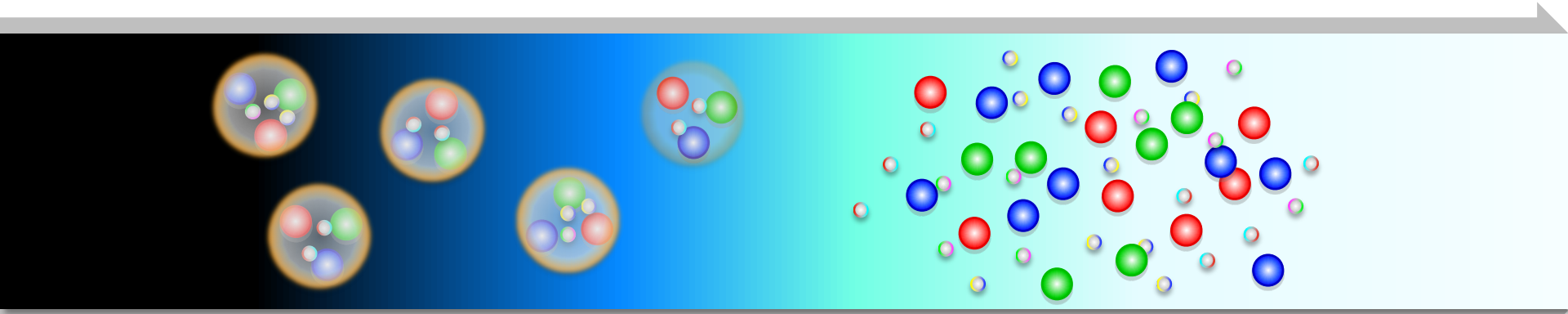
AM, B. Schenke, C. Shen, arXiv:1902.05095 [nucl-th]

Hadron Interactions and Polarization from Lattice QCD, Quark Model, and Heavy Ion Collisions

28th March 2019, Yukawa Institute for Theoretical Physics, Kyoto, Japan

Introduction

- The quark-gluon plasma (QGP)

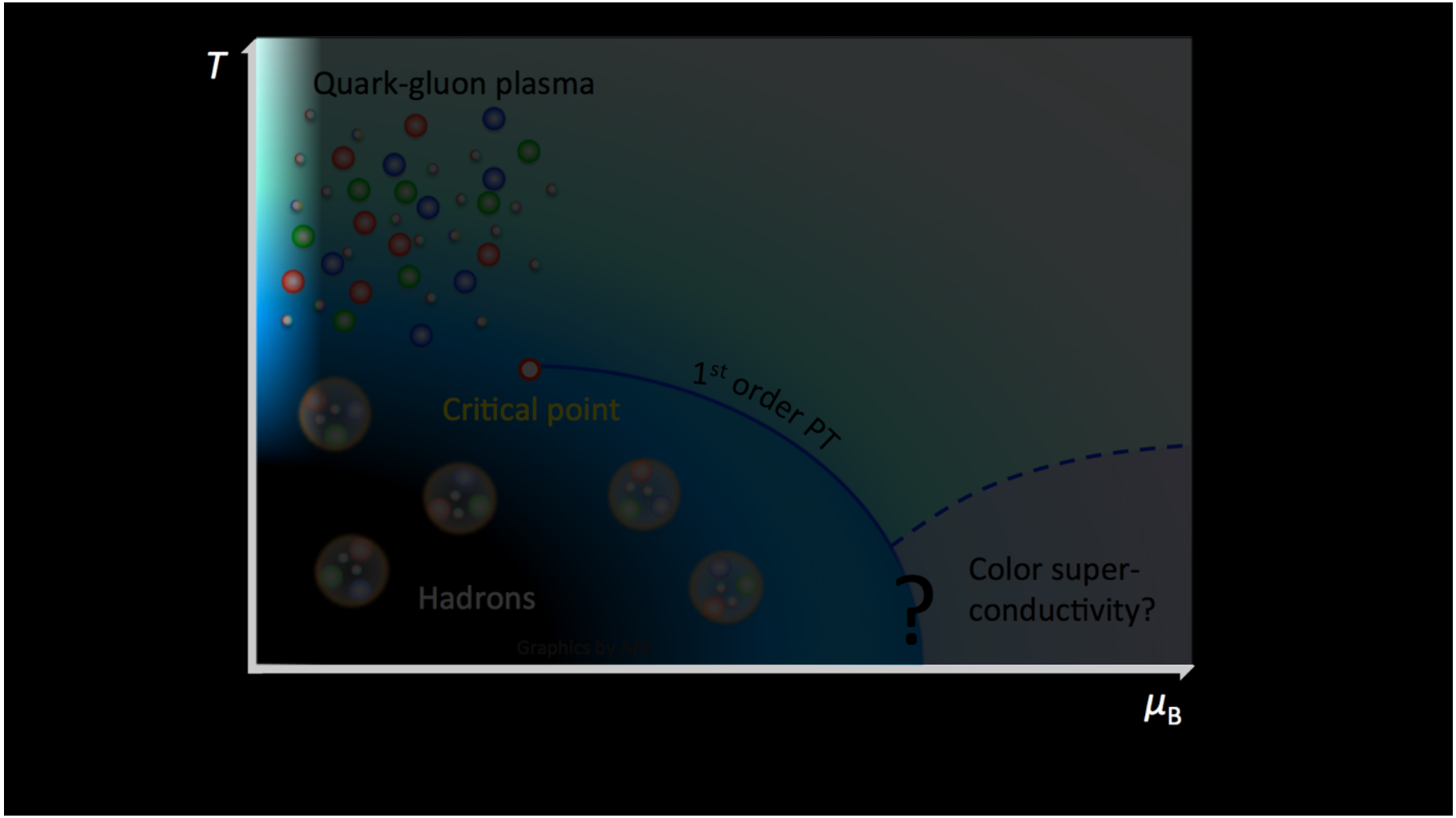


- A high-temperature phase of QCD (> 2×10^{12} K)
- Well-established theoretically by lattice QCD at vanishing μ_B and experimentally by nuclear collisions

- BNL Relativistic Heavy Ion Collider (RHIC)
- CERN Large Hadron Collider (LHC)

Introduction

- Little is known at finite density ("sign problem" of lattice QCD)



Introduction

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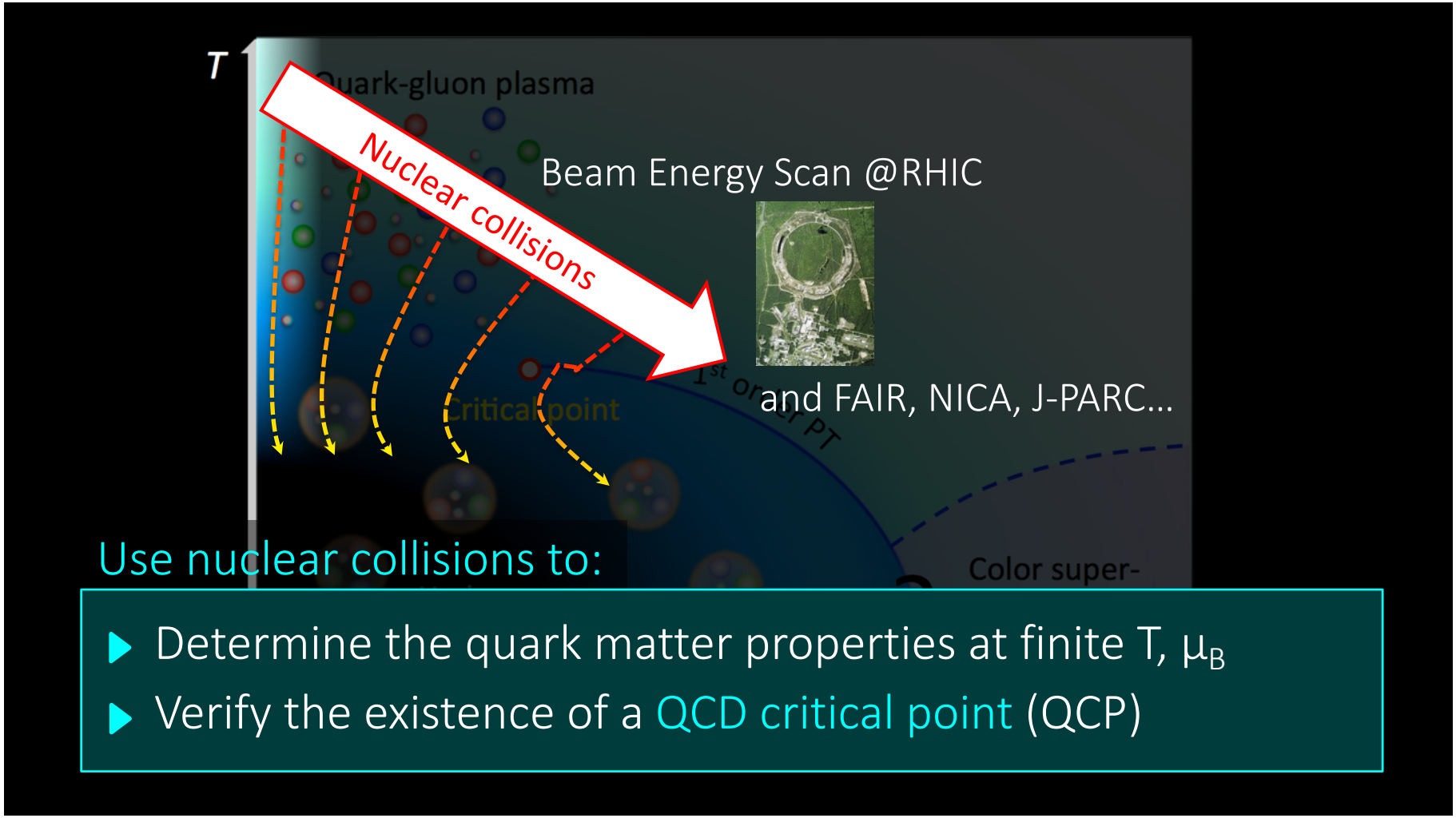


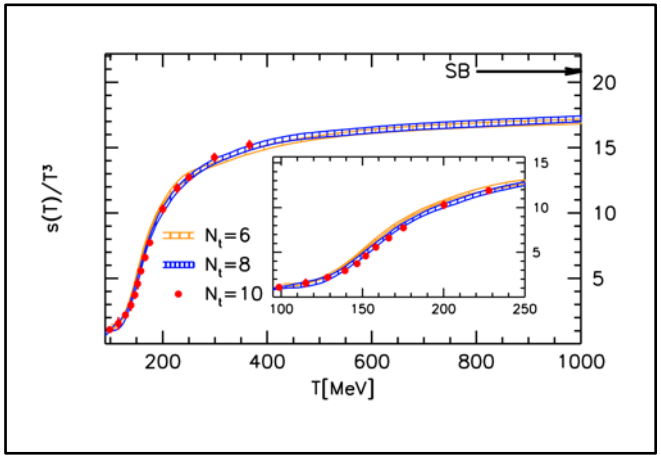
Use nuclear collisions to:

- Determine the quark matter properties at finite T, μ_B
- Verify the existence of a QCD critical point (QCP)

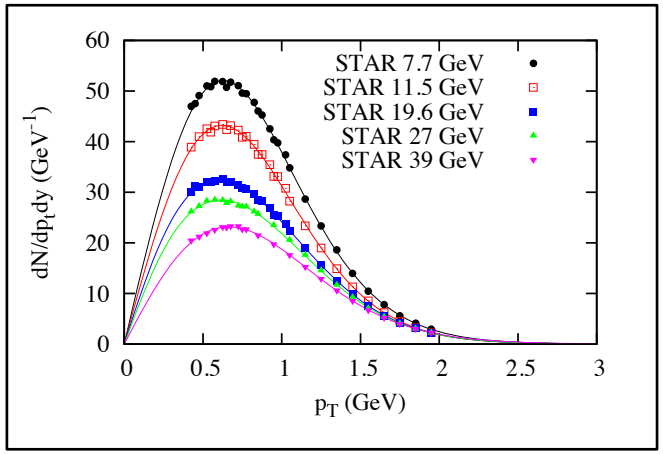
Introduction

- Modeling nuclear collisions

QCD properties



Experimental data

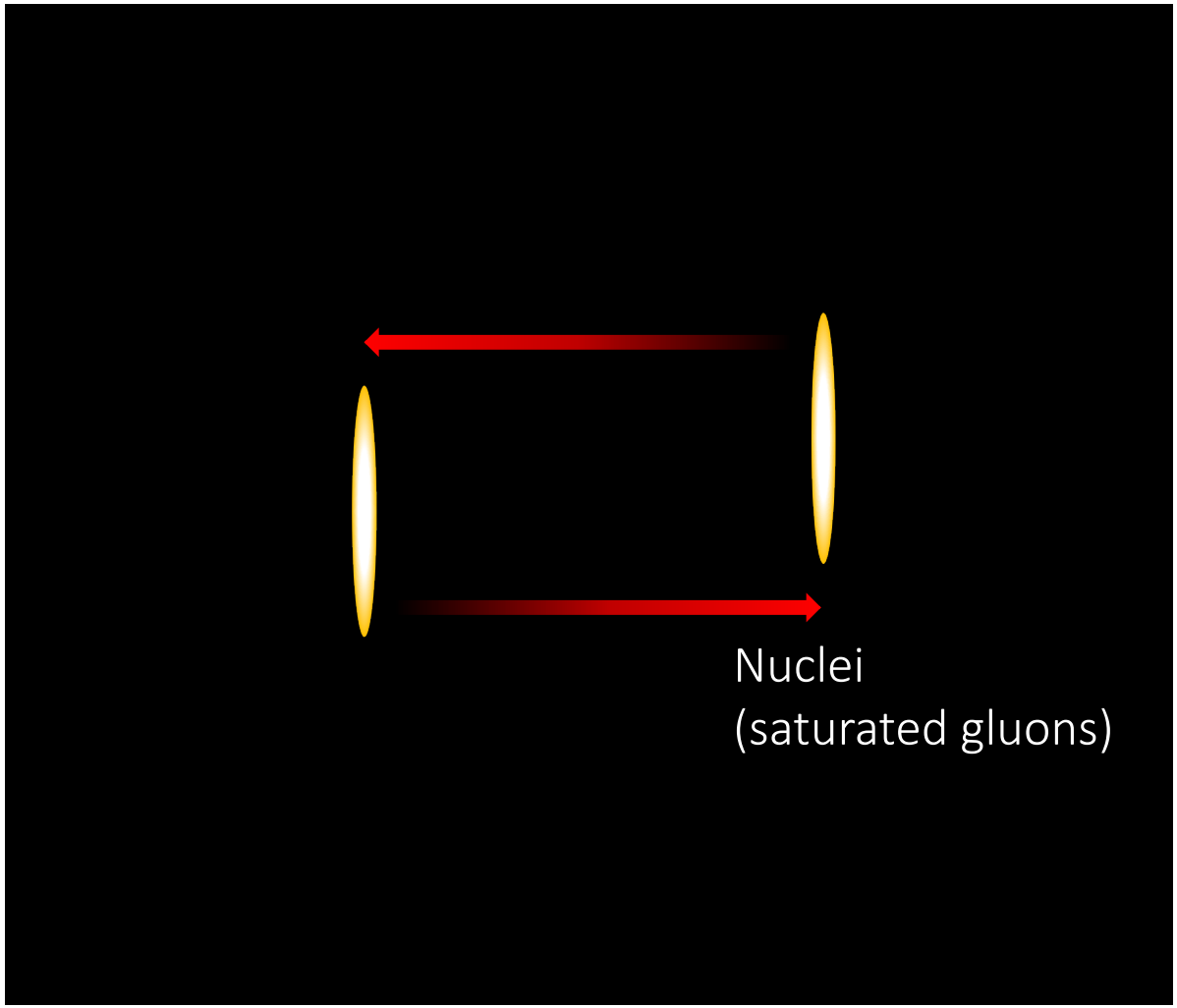


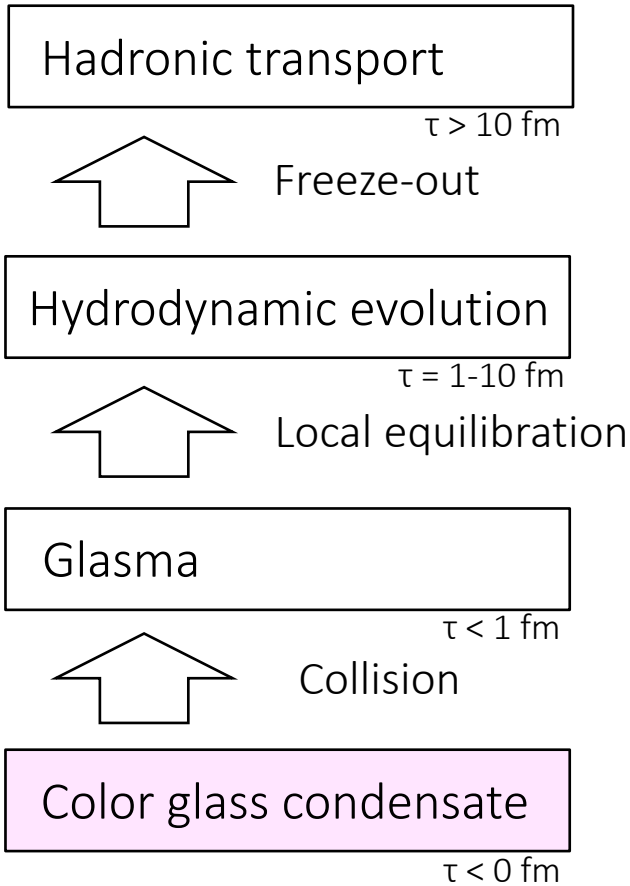
- We need a "link" between fundamental QCD properties and experimental data of nuclear collisions

⇨ We consider the relativistic hydrodynamic model

Introduction

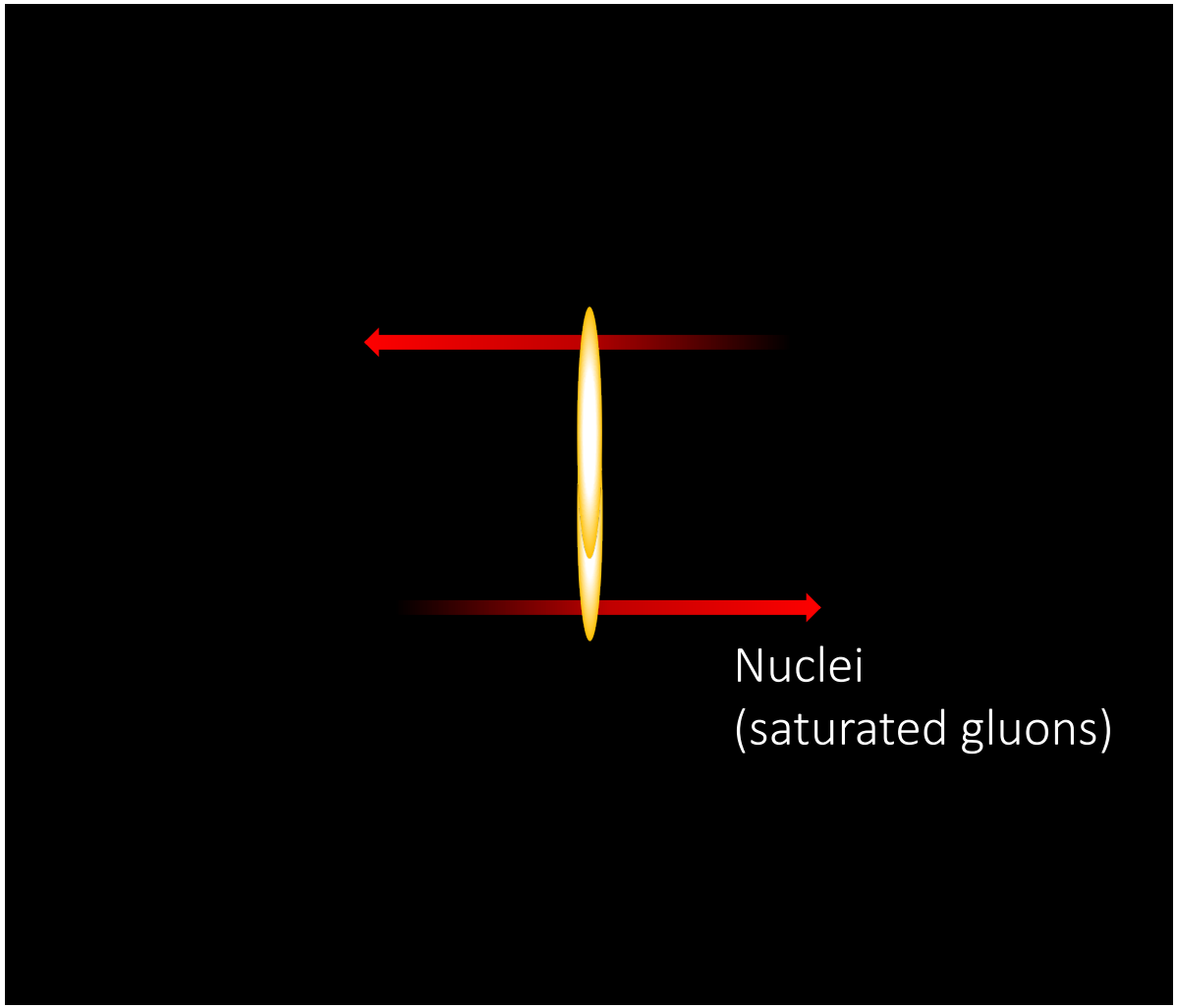
- Relativistic nuclear collisions

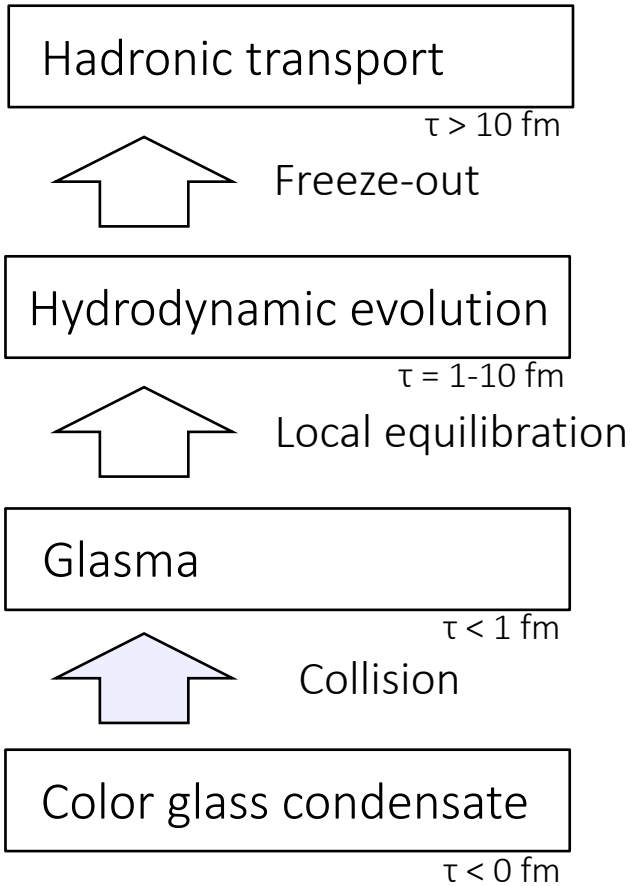




Introduction

- Relativistic nuclear collisions





Introduction

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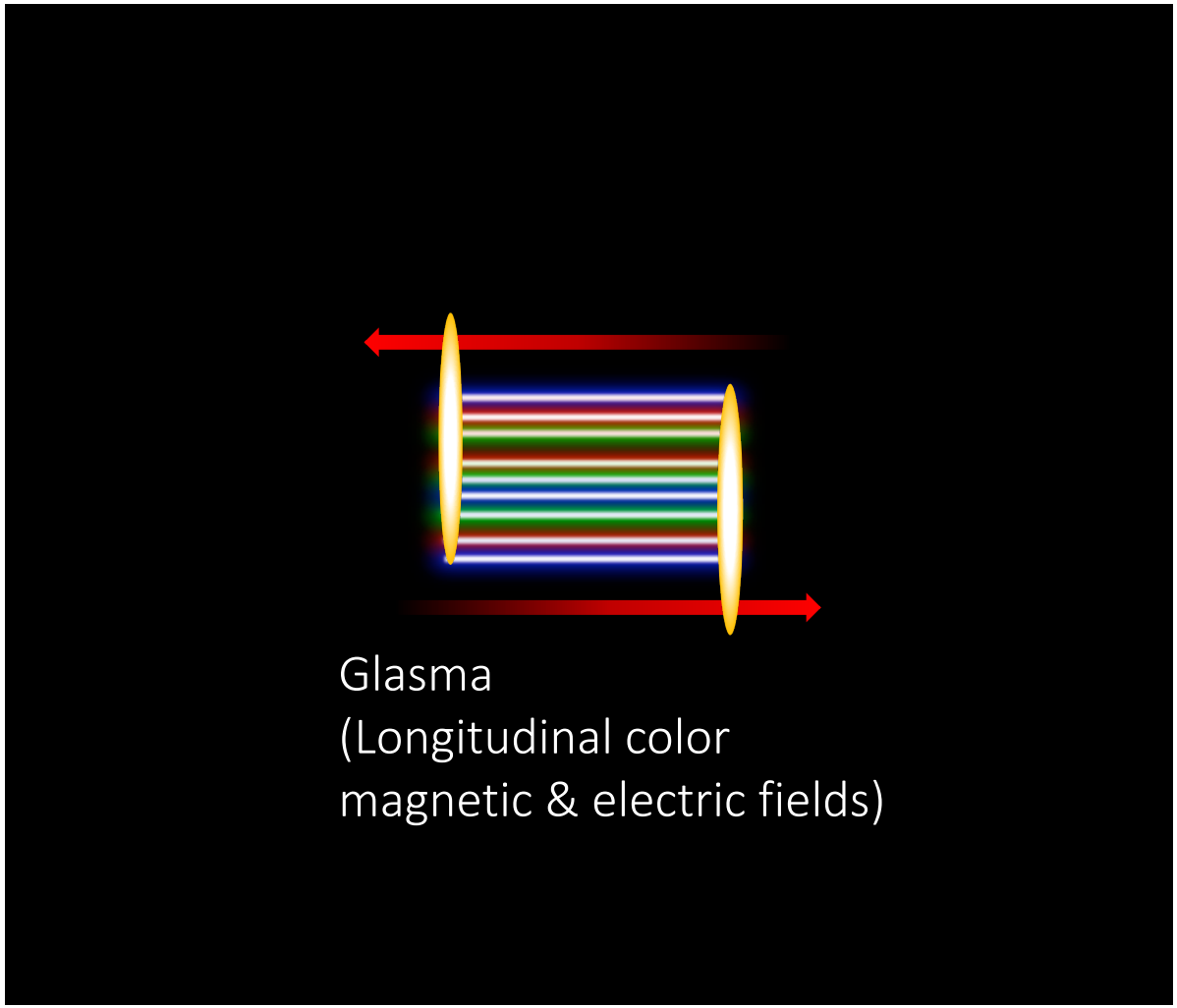
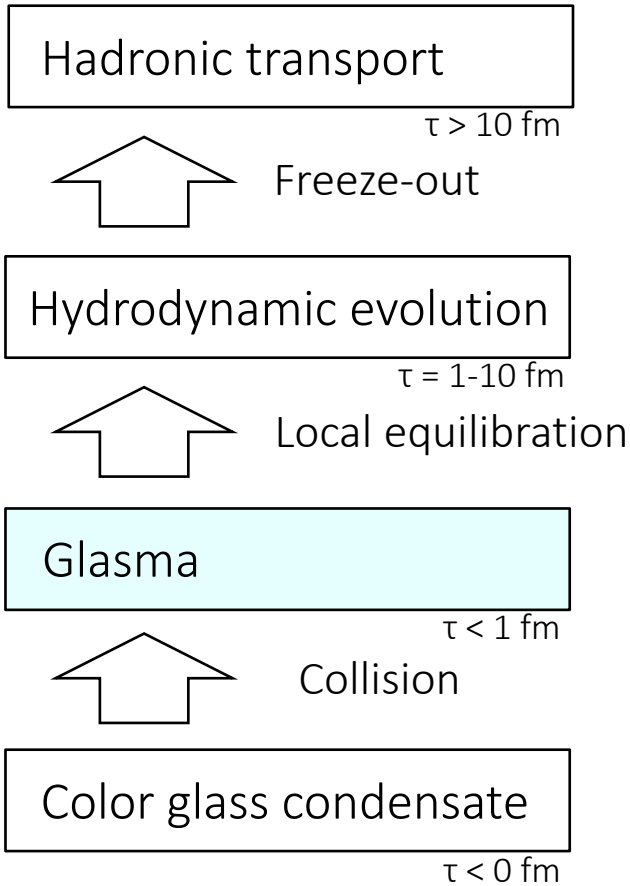
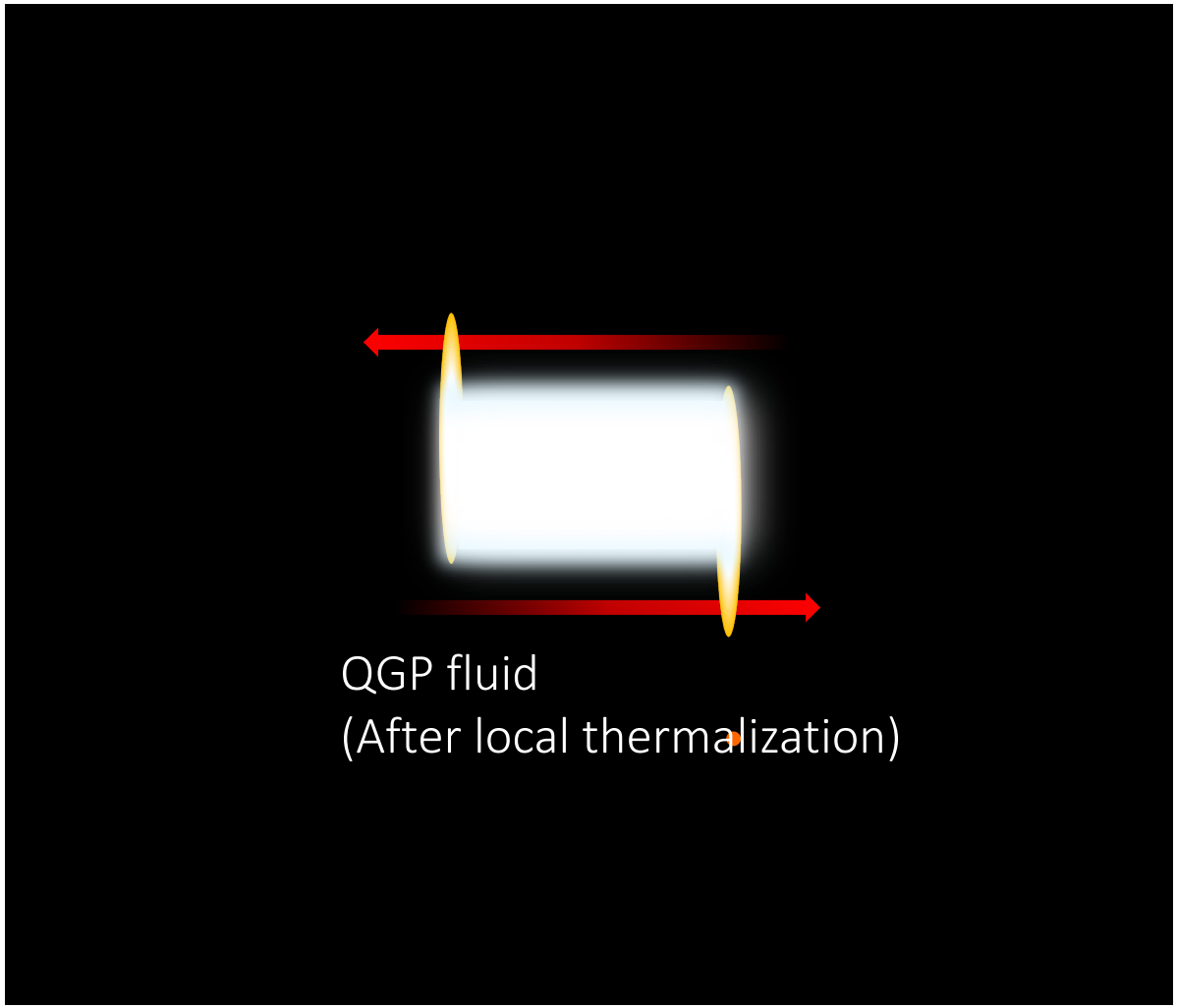
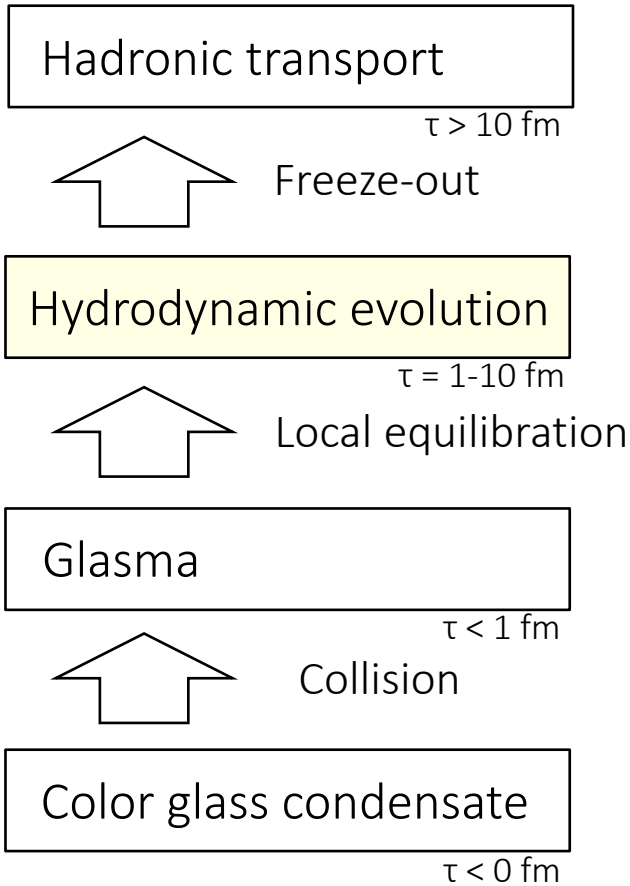




Introduction

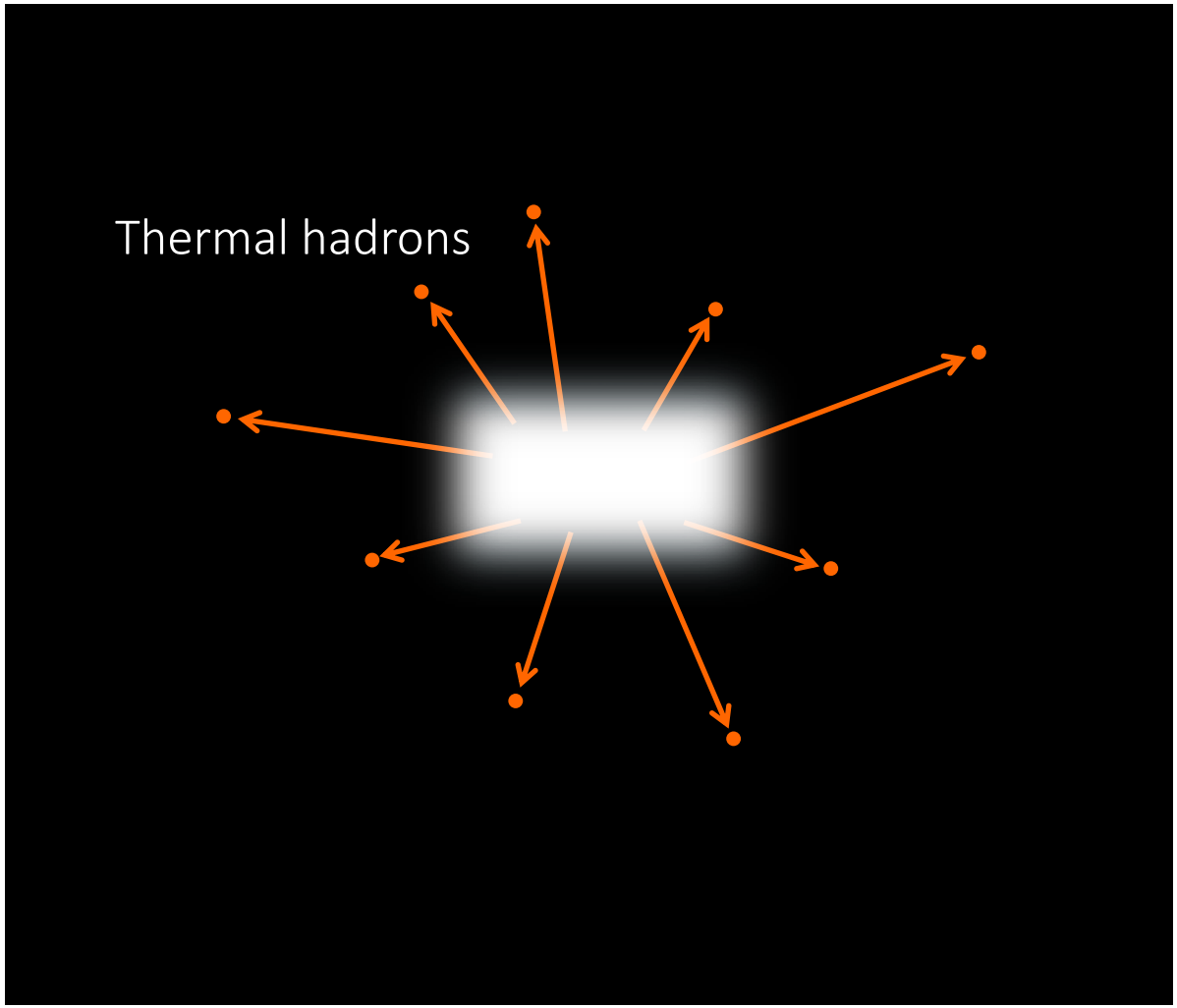
- Relativistic nuclear collisions

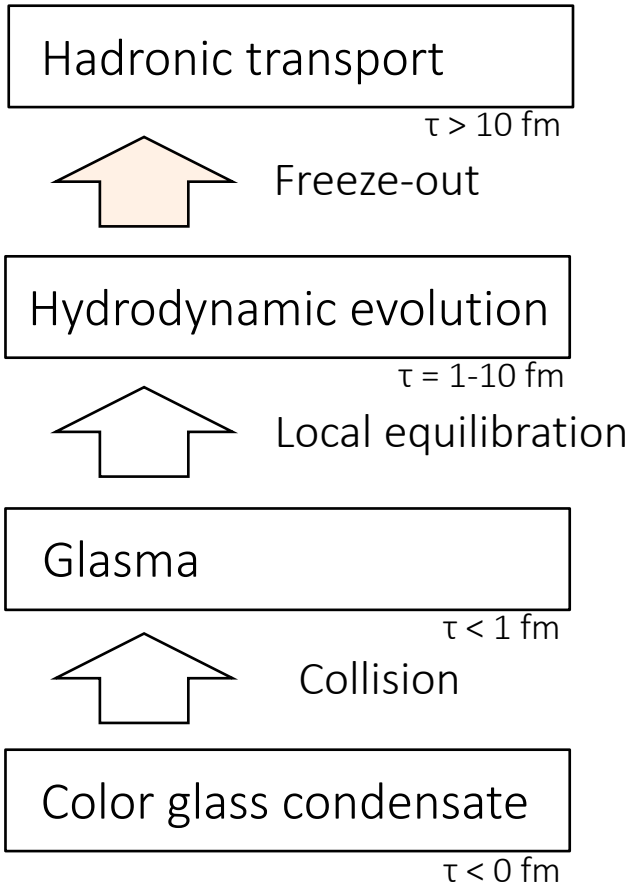




Introduction

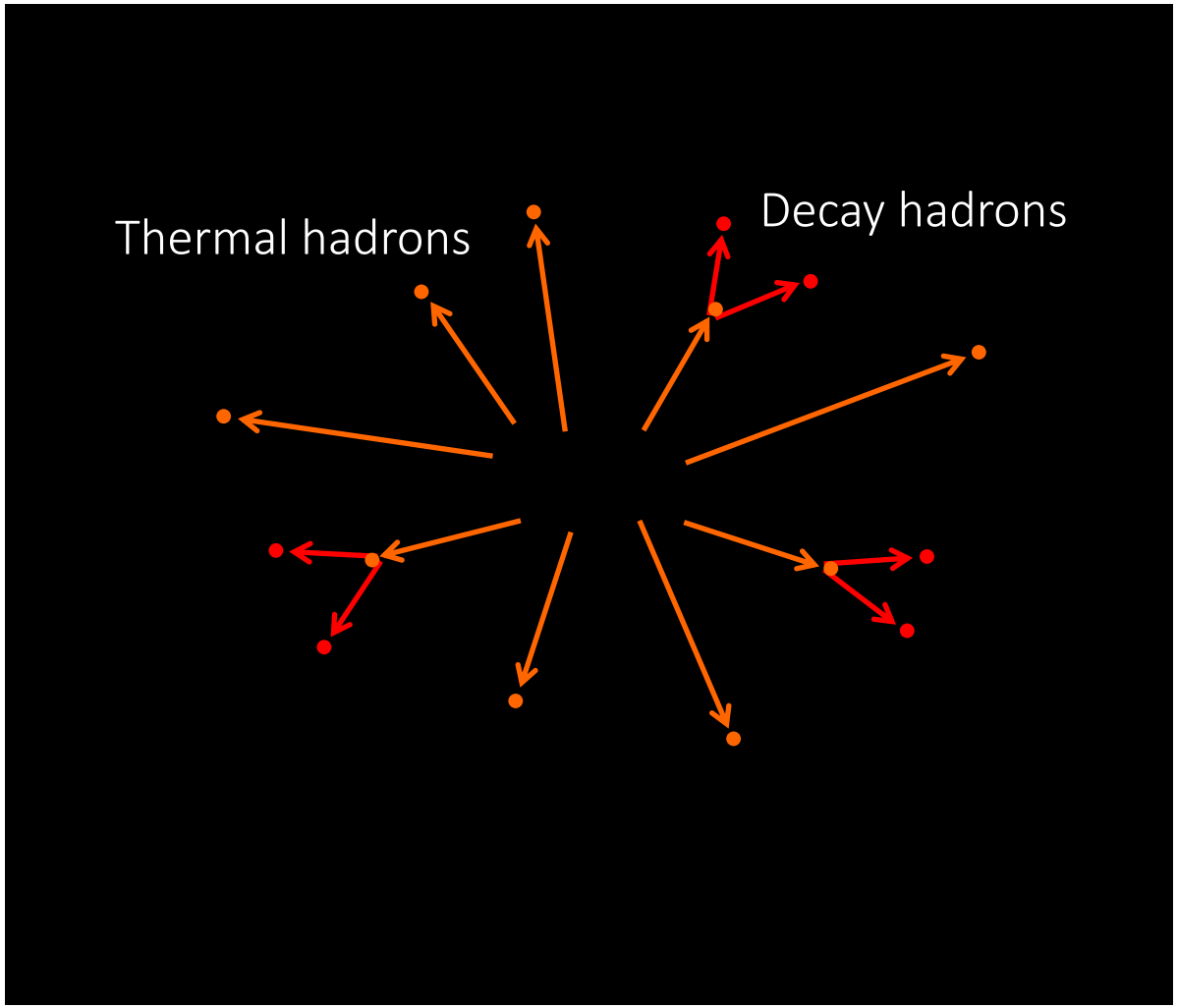
- Relativistic nuclear collisions

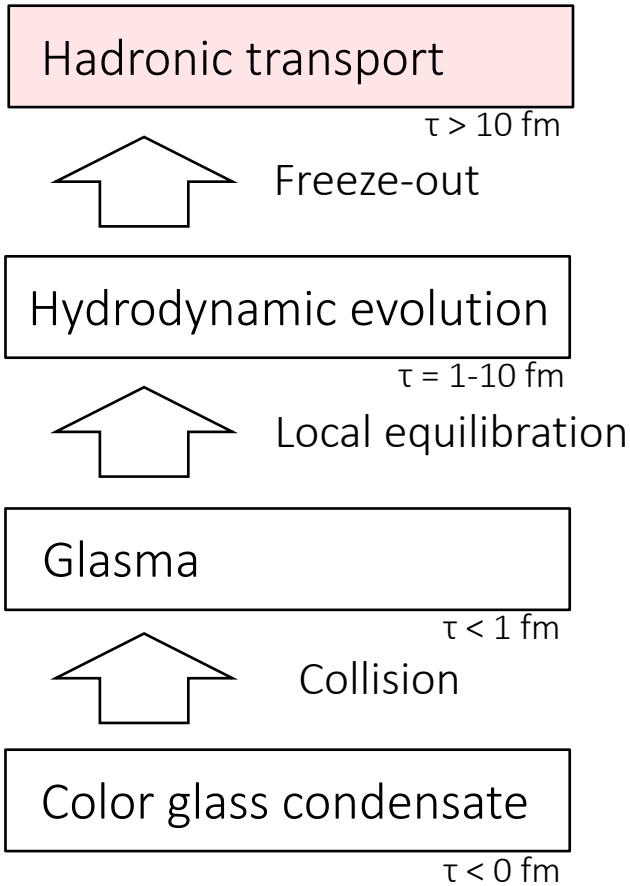




Introduction

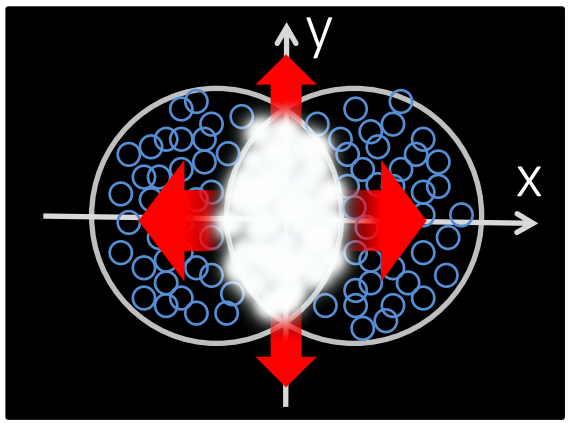
- Relativistic nuclear collisions





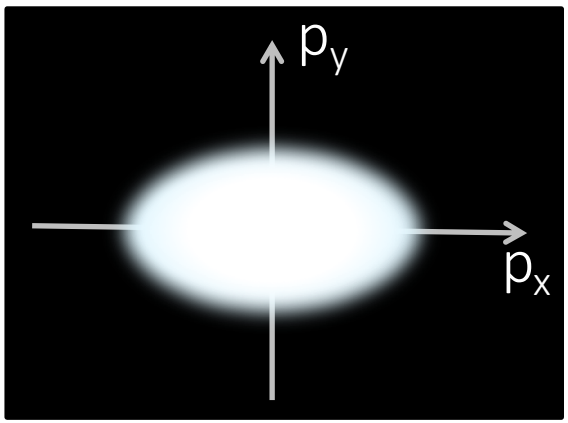
Introduction

- Evidence for the QGP fluid



Spatial anisotropy

In-medium
Interaction



Momentum anisotropy

- Characterized by Fourier harmonics of azimuthal distribution

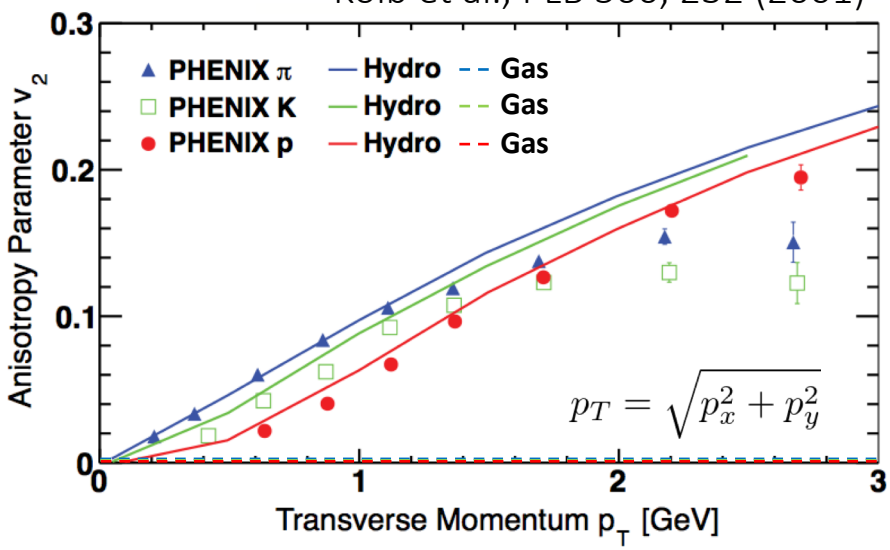
$$\frac{dN}{d\phi} = \frac{N}{2\pi}[1 + 2v_1\cos(\phi - \Psi_1) + 2\boxed{v_2}\cos(2\phi - 2\Psi_2) + 2v_3\cos(3\phi - 3\Psi_3) + \ldots]$$

v_2 : elliptic flow

Introduction

- Experimental data

Kolb et al., PLB 500, 232 (2001)



Liquid: strong-coupling limit ✓

Gas: weak-coupling limit ✗

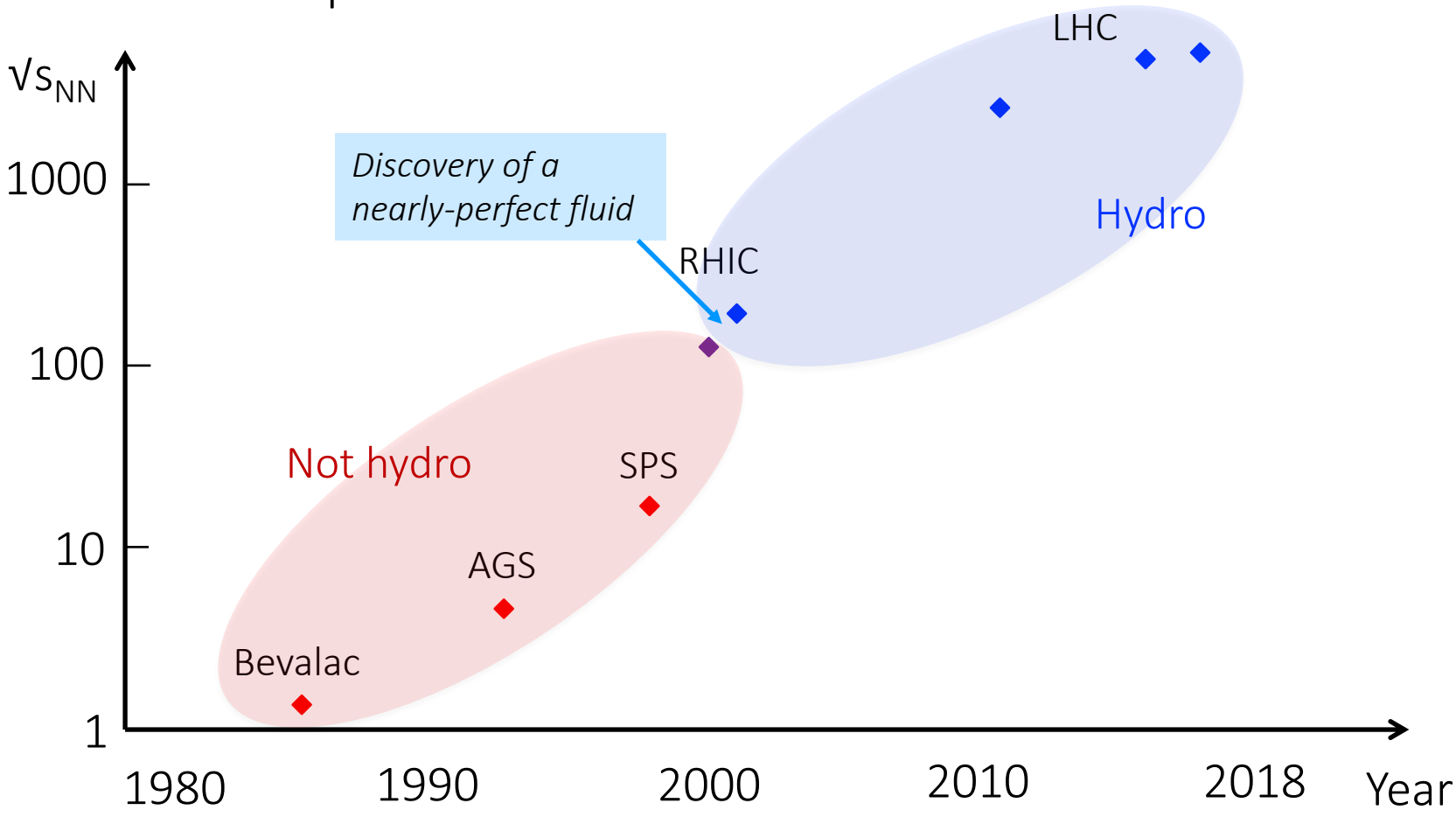
- Consistent with the nearly-perfect liquid picture up to $p_T \sim 2$ [GeV]

⇨

- The QGP is strongly-coupled near the quark-hadron transition
- We may use hydrodynamics for an effective theory of QGP

Is it good at BES energies?

- A historical point of view



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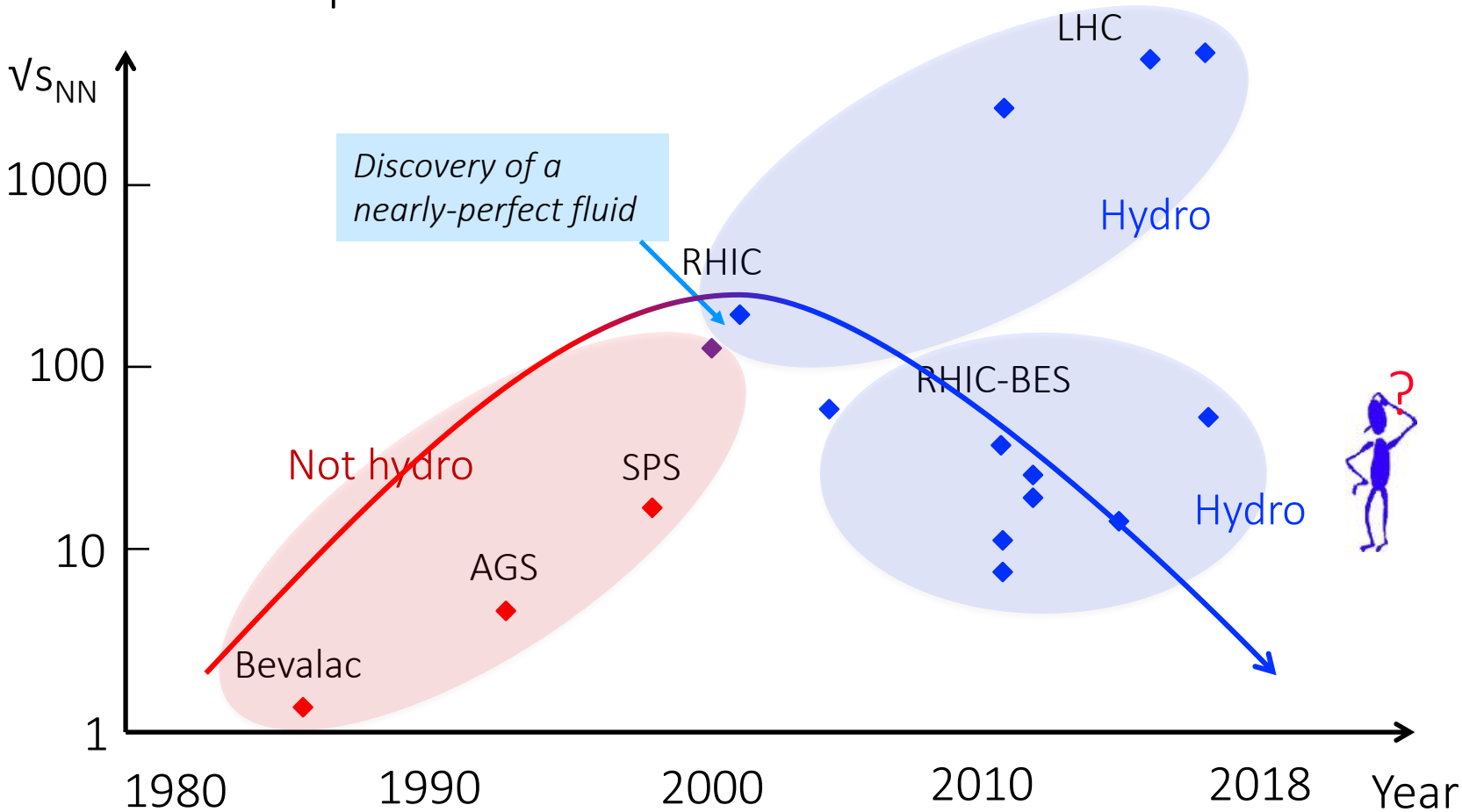
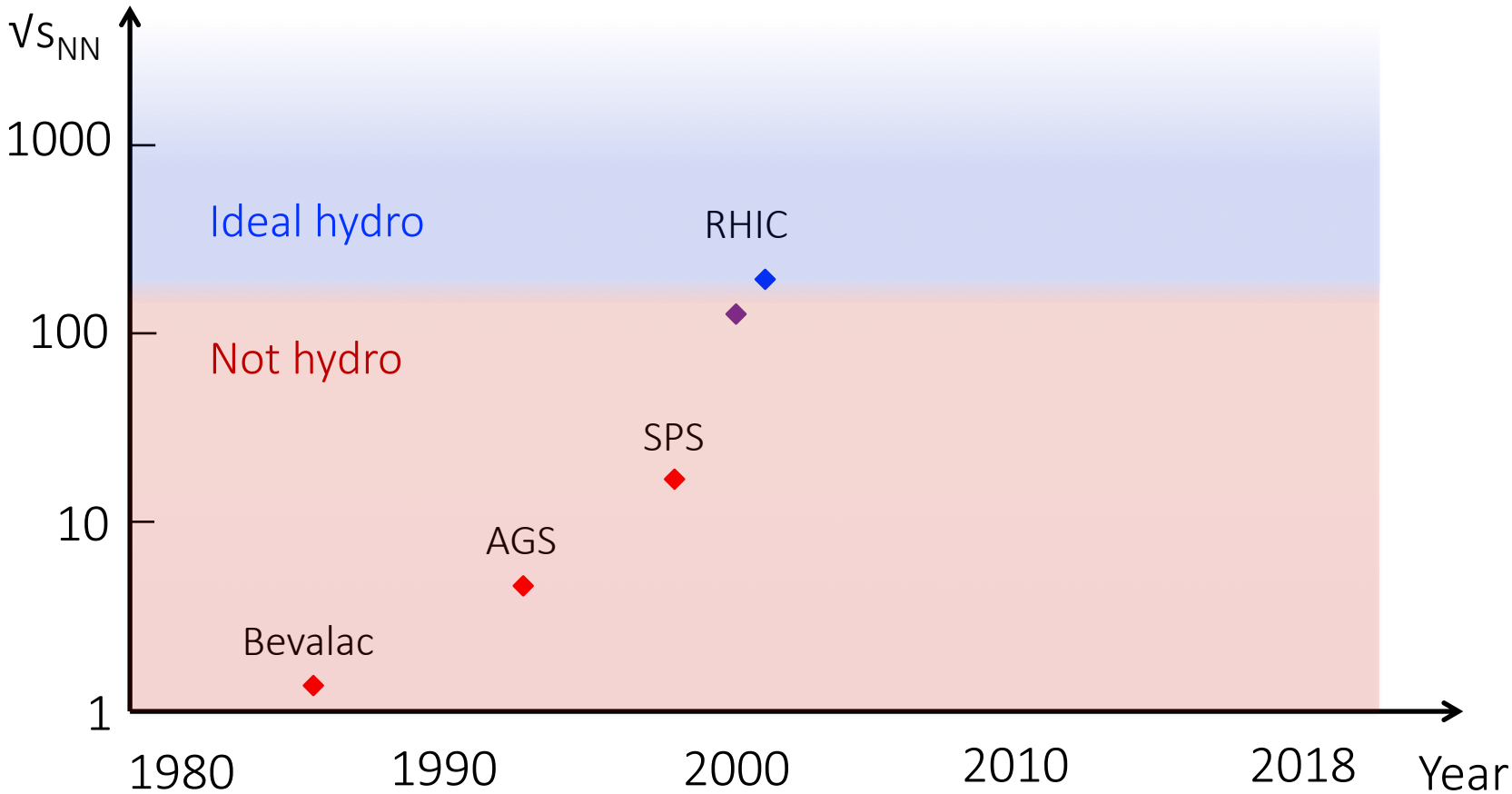


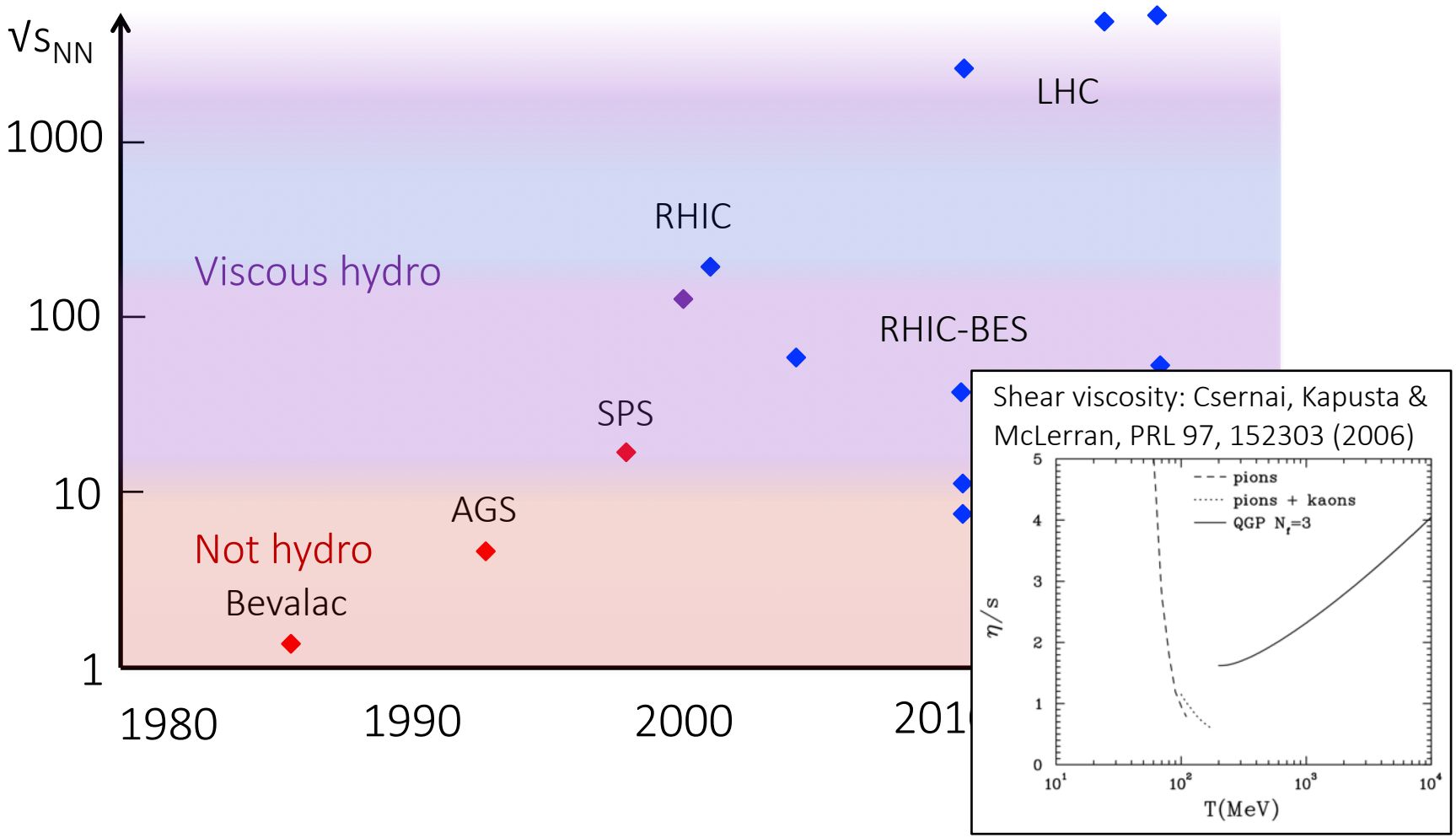
Is it good at BES energies?

- A historical point of view (around 2000)



Is it good at BES energies?

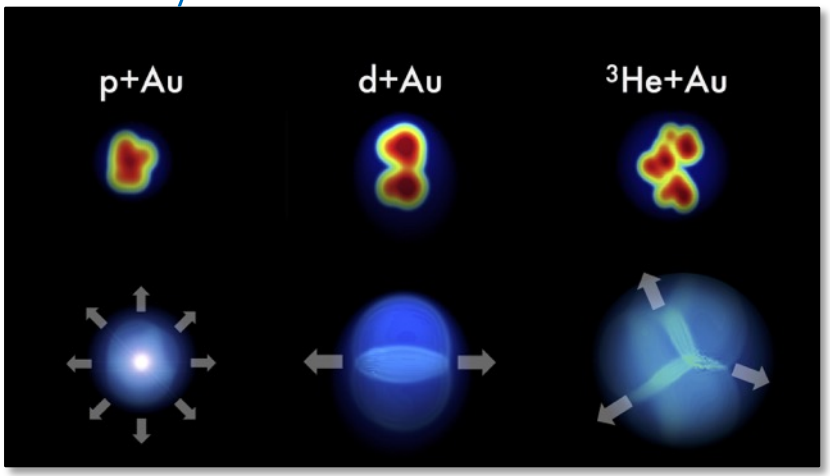
- A historical point of view (around 2018)



Small systems and beam energy scan

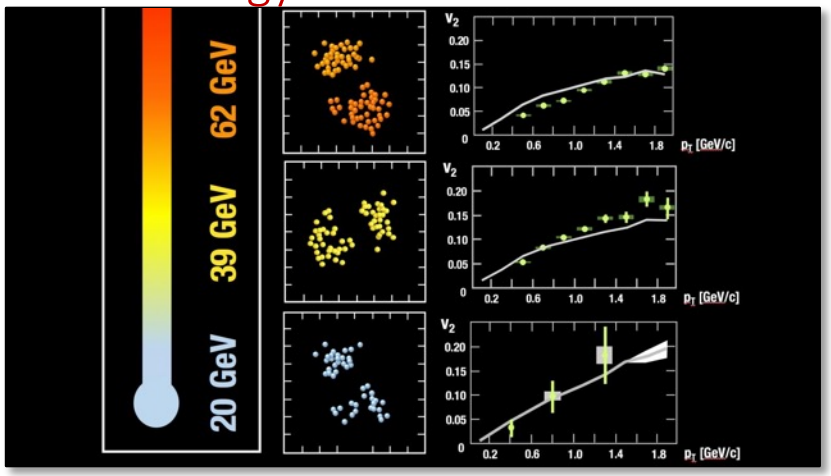
- Similar but different physics

Small systems



Temperature: large
Volume: small

Beam energy scan

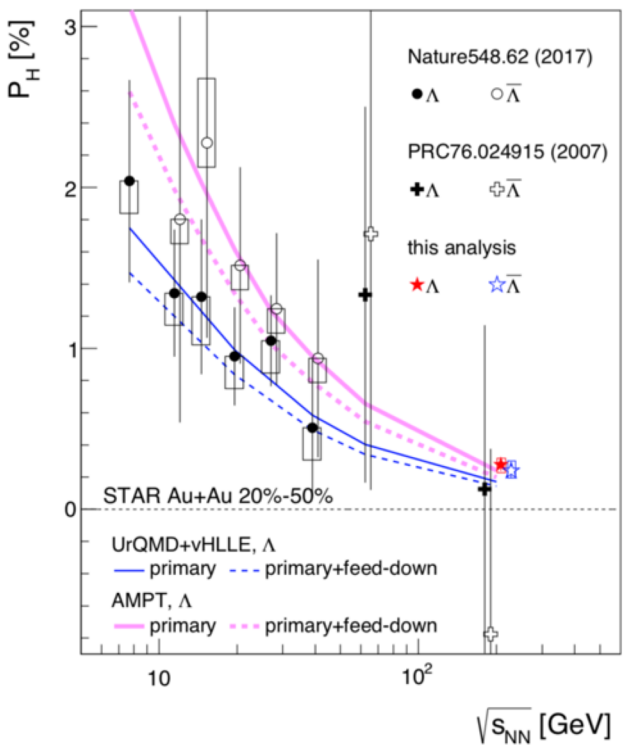


Temperature: small
Volume: large

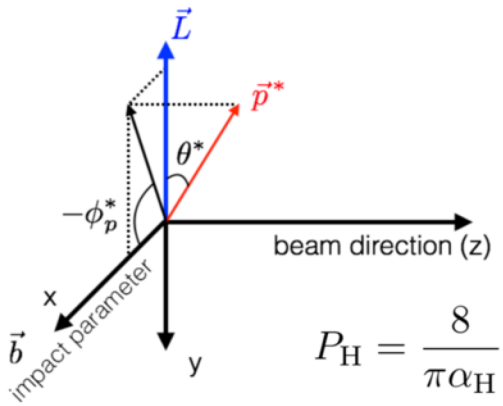
- "Evidence of the QGP" such as jet quenching is more sensitive to volume, thermal photons to temperature

Λ polarization and beam energy scan

- Vorticity converted into spin



- Spin-orbit coupling + (possible) magnetic field effects



$$P_{\mathrm{H}} = \frac{8}{\pi \alpha_{\mathrm{H}}} \frac{\langle \sin(\Psi_1 - \phi_p^*) \rangle}{\mathrm{Res}(\Psi_1)}$$

ϕ_p^*: ϕ of daughter proton in Λ rest frame

- More prominent at lower collision energies; a complete understanding of the background medium evolution is required

Overview

1. Introduction
2. Multiple charges
3. Summary and outlook
4. Diffusion and dissipation

2. Multiple charges

AM, B. Schenke, C. Shen, arXiv:1902.05095 [nucl-th]

Conserved charges

- in relativistic nuclear collisions

Baryon number (B)
(> 0 in total)

p n
+1 +1

Electric charge (Q)
(> 0 in total)

p n
+1 0

Strangeness (S)
(= 0 in total)

p n
0 0

Essential in understanding particle-antiparticle ratios

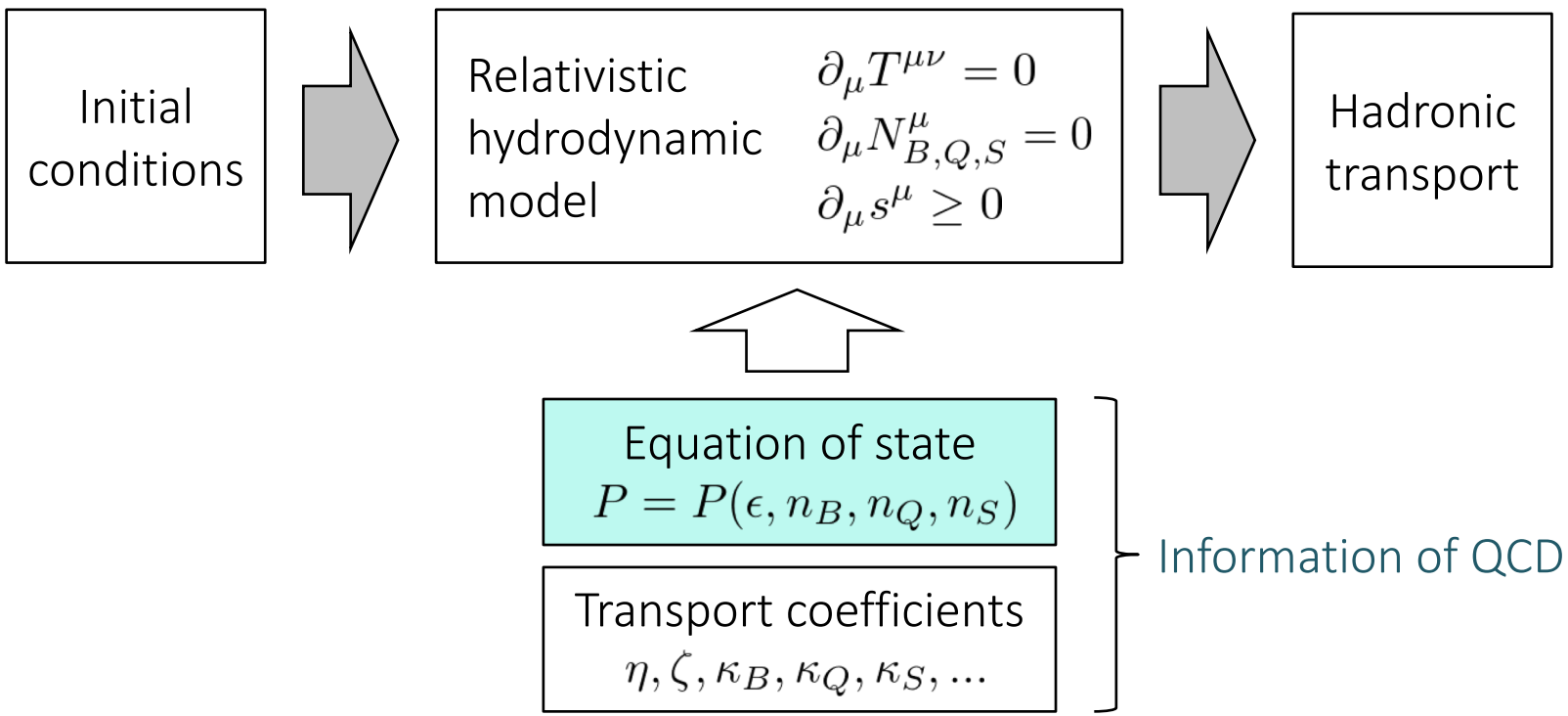
Dunlop et.al., PRC 84 044914

hadron	yield	u	d	s	$\bar{u}$	$\bar{d}$	$\bar{s}$
π^+	72.9	72.9				72.9	
π^-	84.8		84.8		84.8		
π^0 (*)	78.85	39.43	39.43		39.43	39.43	
ϕ	1.17			1.17			1.17
K^+	16.4	16.4					16.4
K^-	5.58			5.58	5.58		
K^0 (*)	10.99		10.99				13.84
$\bar{K}^0$ (*)	10.99			10.99		10.99	
p	46.1	92.2	46.1				
n (*)	70.84	70.84	141.68				
Λ	13.4	13.4	13.4	13.4			
Ξ^-	0.93		0.93	1.86			
$\bar{p}$	0.06				0.12	0.06	
$\bar{n}$ (*)	0.04				0.04	0.08	
$\bar{\Lambda}$	0.1				0.1	0.1	0.1
Sum		305.17	337.33	33	130.07	123.56	28.66

TABLE I: Left two columns: midrapidity yields of common particles from central Pb+Pb collisions measured by the NA49/SPS Collaboration [26–33] at $\sqrt{s_{NN}} = 6.41$ GeV. Starred hadrons are not measured, but estimated from other hadrons.

Overview of hydro model

- with multiple charges



- We start with construction of the QCD equation of state

Equation of state

- Construction

  - Lattice QCD: expansion up to the 4th order

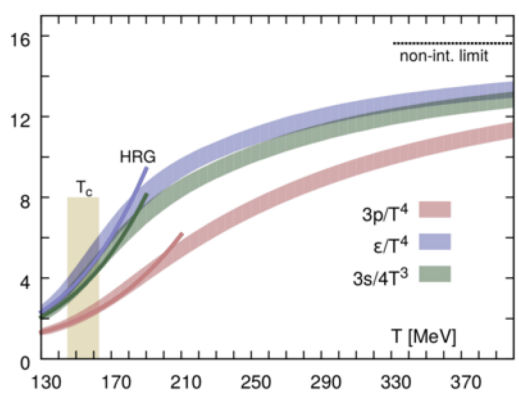
$$\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi^{B,Q,S}_{l,m,n}}{l!m!n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

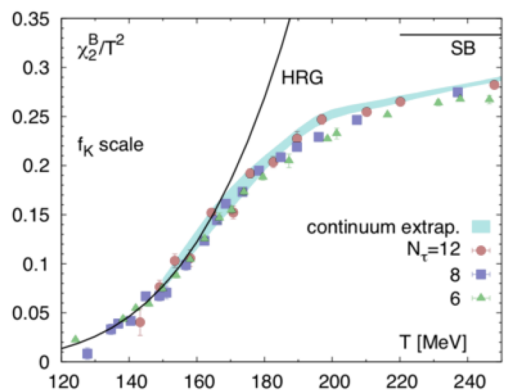
HotQCD: PRD 86, 034509 (2012); PRD 90, 094503 (2014); PRD 92, 073743 (2015)

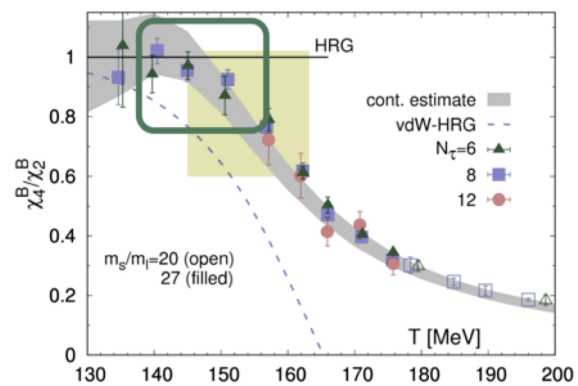
Wuppertal-Budapest: PLB 730, 99 (2013); JHEP 01, 138 (2012); PRD 92, 114505 (2015)

  - Match to hadron resonance gas (HRG) at lower T

    1. Taylor expansion is not reliable when the fugacity is large
    2. Agreement between lattice QCD and HRG is good in hadronic phase



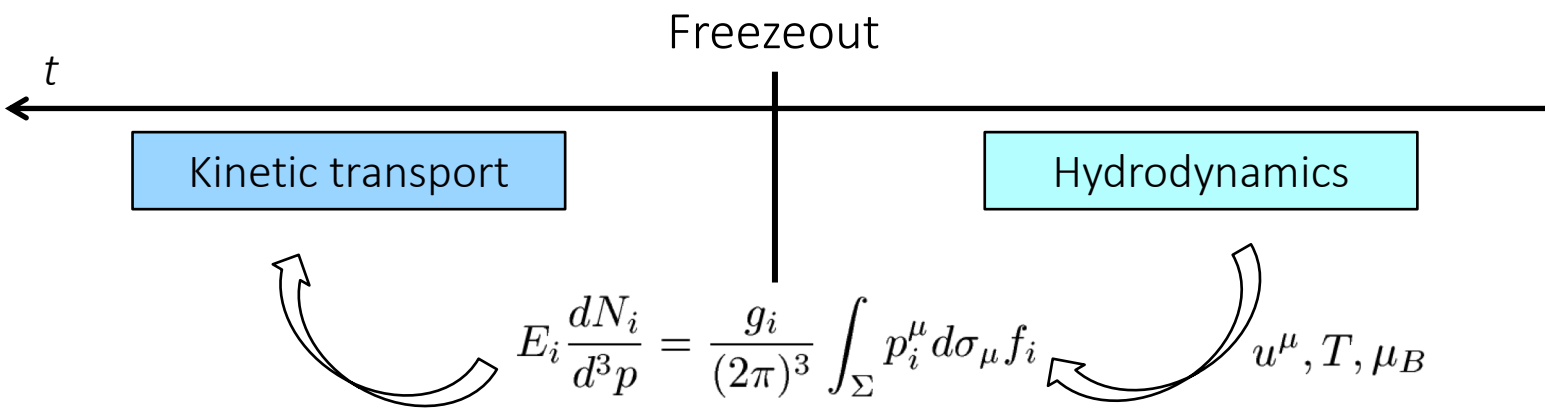




Equation of state

- Construction
  - (Cont'd)

    3. EOS of hydrodynamic model should match EOS of kinetic theory for correct energy-momentum/charge conservation



  - Stefan-Boltzmann limits are used as anchors at very high T where lattice QCD data are scarce

Equation of state

- Construction

  - Connect to HRG at low T

$$\frac{P}{T^4} = \frac{1}{2}[1 - f(T,\mu_J)]\frac{P_{\rm had}(T,\mu_J)}{T^4} + \frac{1}{2}[1 + f(T,\mu_J)]\frac{P_{\rm lat}(T,\mu_J)}{T^4} \qquad J = \{B, Q, S\}$$

where

$$\left.\begin{aligned} & f(T,\mu_J) = \tanh[(T - T_c(\mu_B))/\Delta T_c] \\ & T_c = 0.16 - 0.4 \times (0.139\mu_B^2 + 0.053\mu_B^4)({\rm GeV}) \\ & \Delta T_c = 0.1 T_c(0) \end{aligned}\right\} \text{Crossover-type EOS}$$

The dependences on sub-leading μ's are approximated to be small

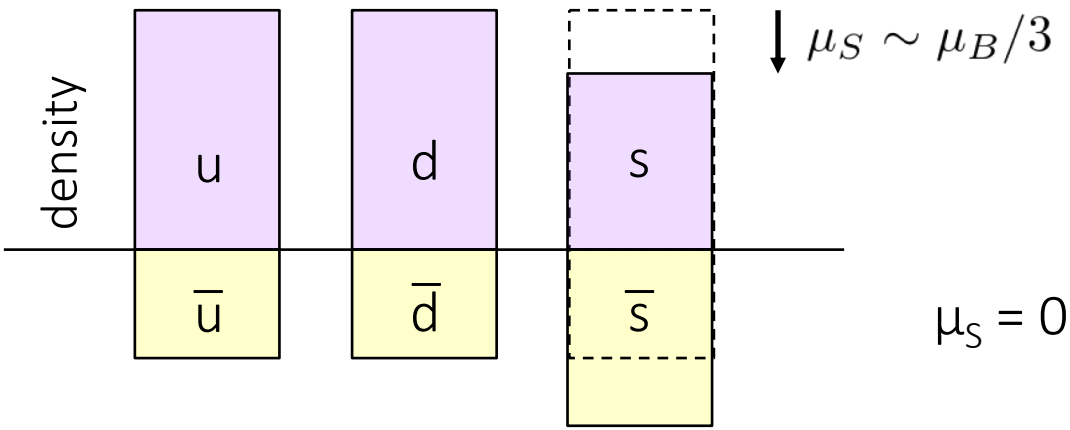
  - Parameters are chosen to satisfy thermodynamic conditions:

$$\frac{\partial^2 P}{\partial T^2} = \frac{\partial s}{\partial T} > 0\,, \quad \frac{\partial^2 P}{\partial \mu_J^2} = \frac{\partial n_J}{\partial \mu_J} > 0$$

Strangeness and charge densities

- Strange neutrality condition ($n_S = 0$)
  - μ_S is finite positive at $\mu_B > 0$ because of s quarks (or strange baryons)

s quark chemical potential: $\mu_s = \frac{1}{3}\mu_B + \frac{1}{3}\mu_Q - \mu_S = 0$



$\mu_S = 0$ leads to $n_S \neq 0$

The condition can be modified by initial fluctuations and diffusion

Strangeness and charge densities

- Charge-to-baryon ratio ($n_Q = c\, n_B$)
  - μ_Q is finite negative at $\mu_B > 0$ for neutron rich nuclei ($Z/A < 1/2$)

d quark abundance: $\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q > \mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$

Nucleus	Z/A
${}^{1}_{1}$H	1.000
${}^{2}_{1}$H	0.500
${}^{3}_{2}$He	0.667
${}^{27}_{13}$Al	0.481
${}^{63}_{29}$Cu	0.460
${}^{96}_{40}$Zr	0.417
${}^{96}_{44}$Ru	0.458
${}^{127}_{54}$Xe	0.425
${}^{197}_{79}$Au	0.401
${}^{208}_{82}$Pb	0.394
${}^{238}_{92}$U	0.387

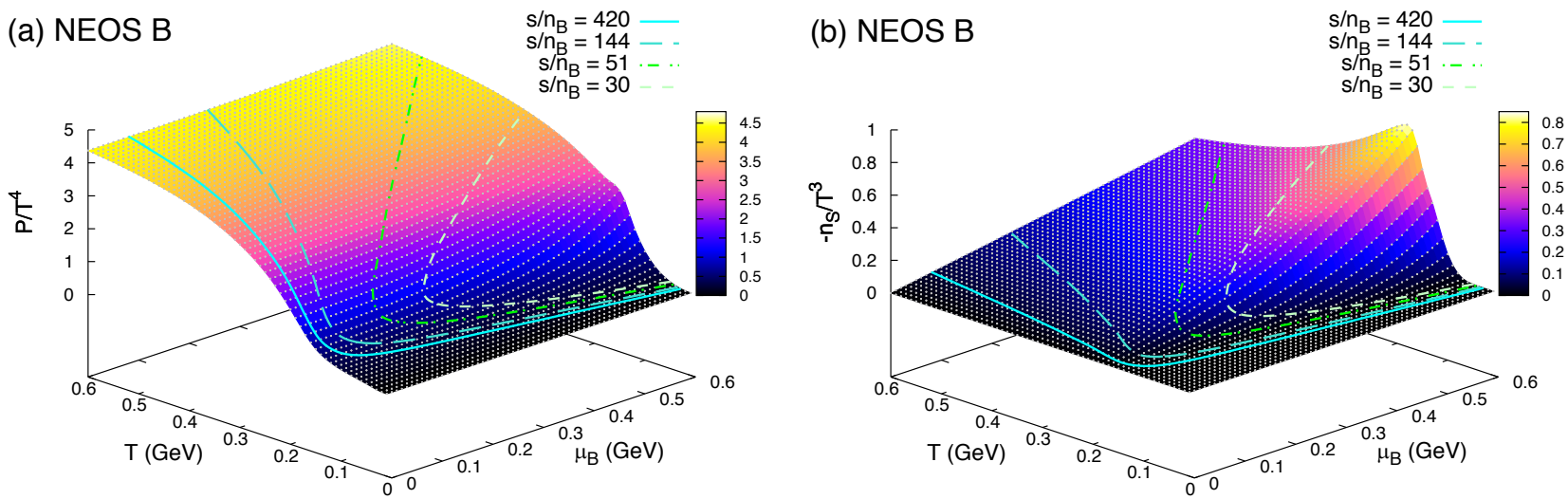
proton rich/neutral nuclei; $\mu_Q \geq 0$ for $\mu_B > 0$

relevant for background of isobars

$c \simeq 0.4$ for Au and Pb nuclei

Equation of state

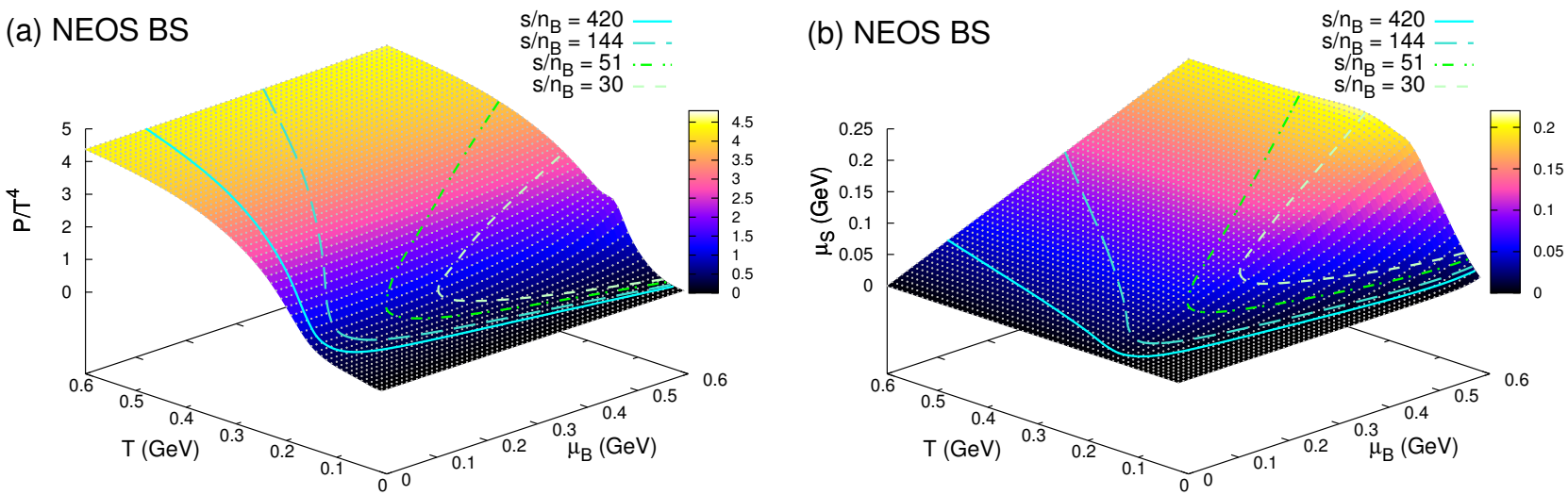
- $\mu_S = \mu_Q = 0$ (conventional; denoted as NEOS B)



- Thermodynamically consistent smooth EoS is obtained
- The strangeness neutrality condition is violated ($n_S < 0$)

Equation of state

- $n_S = 0$, $\mu_Q = 0$ (strangeness neutral; denoted as NEOS BS)



- A visible modification is observed at larger μ_B/T
- Finite positive μ_S is seen owing to the neutrality condition
- μ_B is becomes larger at large T for a given n_B

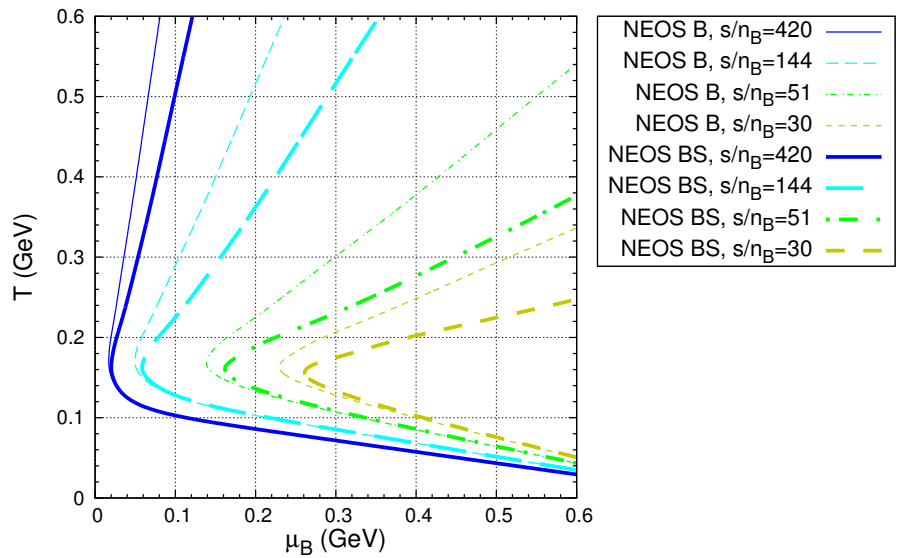
Equation of state

- Where you can probe on the μ_B-T plane

  - s/n_B is constant when entropy and net baryon number are conserved

$s/n_B = 420$	$\sqrt{s_{NN}} = 200$ GeV
$s/n_B = 144$	$\sqrt{s_{NN}} = 62.4$ GeV
$s/n_B = 51$	$\sqrt{s_{NN}} = 19.6$ GeV
$s/n_B = 30$	$\sqrt{s_{NN}} = 14.5$ GeV

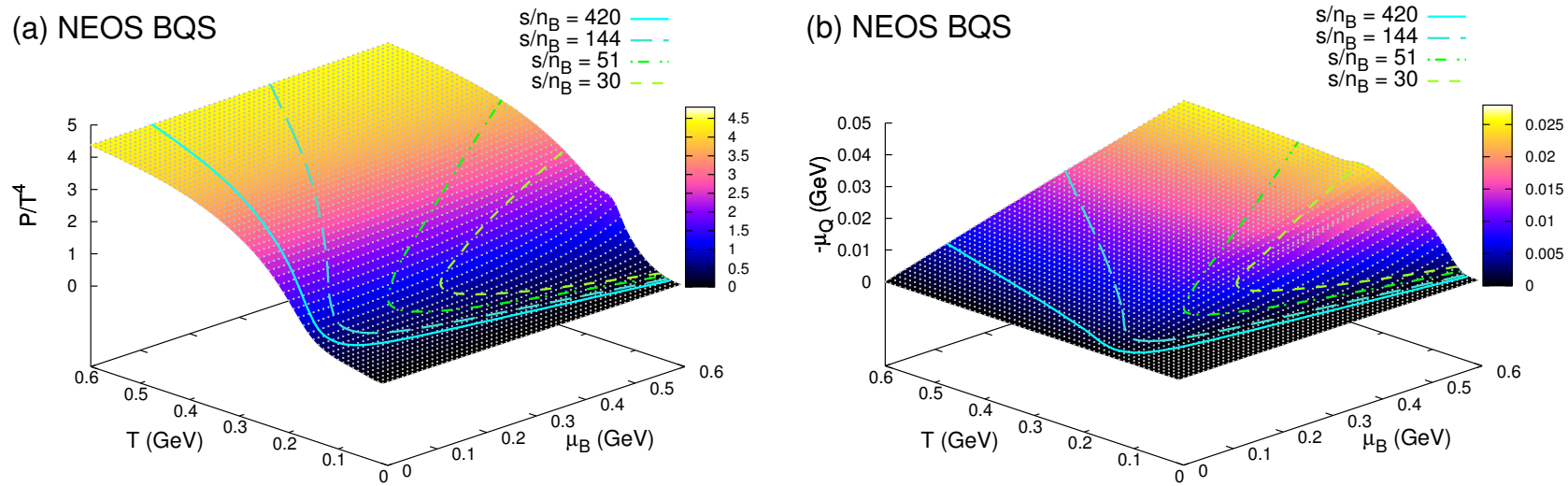
J. Gunther et. al., Nucl. Phys. A 967, 720 (2017)



  - If conformal, it is a straight line because $s \sim T^3$ and $n_B \sim \mu_B T^2$
    Larger μ_B is required to fix the s/n_B ratio when pions are dominant

Equation of state

- $n_S = 0$, $n_Q = 0.4n_B$ (realistic in HIC; denoted as NEOS BQS)



- Finite negative μ_Q owing to the condition $n_Q = 0.4n_B$
- The overall system is positively charged; μ_Q turns positive around $n_Q = 0.5n_B$

Stefan-Boltzmann limit

- Parton gas pressure

$$\frac{P}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{60} N_f + \frac{1}{2} \sum_{f=u,d,s} \left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2} \sum_{f=u,d,s} \left(\frac{\mu_f}{T}\right)^4$$

where $\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$, $\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$, $\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$

- The diagonal and off-diagonal susceptibilities are

$$\chi_2^B = \frac{1}{3}, \quad \chi_2^Q = \frac{2}{3}, \quad \chi_2^S = 1, \quad \chi_{1,1}^{B,Q} = 0, \quad \chi_{1,1}^{B,S} = -\frac{1}{3}, \quad \chi_{1,1}^{Q,S} = \frac{1}{3},$$

$$\chi_4^B = \frac{2}{9\pi^2}, \ \chi_4^Q = \frac{4}{3\pi^2}, \ \chi_4^S = \frac{6}{\pi^2}, \ \chi_{3,1}^{B,S} = -\frac{2}{9\pi^2}, \ \chi_{2,2}^{B,S} = \frac{2}{3\pi^2}, \ \chi_{1,3}^{B,S} = -\frac{2}{\pi^2},$$

$$\chi_{3,1}^{B,Q} = 0, \ \chi_{2,2}^{B,Q} = \frac{4}{9\pi^2}, \ \chi_{1,3}^{B,Q} = \frac{4}{9\pi^2}, \ \chi_{3,1}^{Q,S} = \frac{2}{9\pi^2}, \ \chi_{2,2}^{Q,S} = \frac{2}{3\pi^2}, \ \chi_{1,3}^{Q,S} = \frac{2}{\pi^2},$$

$$\chi_{2,1,1}^{B,Q,S} = \frac{2}{9\pi^2}, \ \chi_{1,2,1}^{B,Q,S} = -\frac{2}{9\pi^2}, \ \chi_{1,1,2}^{B,Q,S} = -\frac{2}{3\pi^2}$$

Stefan-Boltzmann limit

- The chemical potential ratio

$$\begin{pmatrix} n_B \\ n_Q \\ n_S \end{pmatrix} = T^2 \begin{pmatrix} \chi_2^B & \chi_{1,1}^{B,Q} & \chi_{1,1}^{B,S} \\ \chi_{1,1}^{B,Q} & \chi_2^Q & \chi_{1,1}^{Q,S} \\ \chi_{1,1}^{B,S} & \chi_{1,1}^{Q,S} & \chi_2^S \end{pmatrix} \begin{pmatrix} \mu_B \\ \mu_Q \\ \mu_S \end{pmatrix} + \mathcal{O}(\mu^3) \qquad n_J = \left.\frac{\partial P}{\partial \mu_J}\right|_{T,\mu_K}$$

NEOS B ($\mu_S = 0$ and $\mu_Q = 0$)

$$\mu_B \simeq 3n_B/T^2, \qquad \mu_S = 0 \qquad , \qquad \mu_Q = 0$$

NEOS BS ($n_S = 0$ and $\mu_Q = 0$)

$$\mu_B \simeq 4.5n_B/T^2, \; \mu_S \simeq 1.5n_B/T^2, \qquad \mu_Q = 0$$

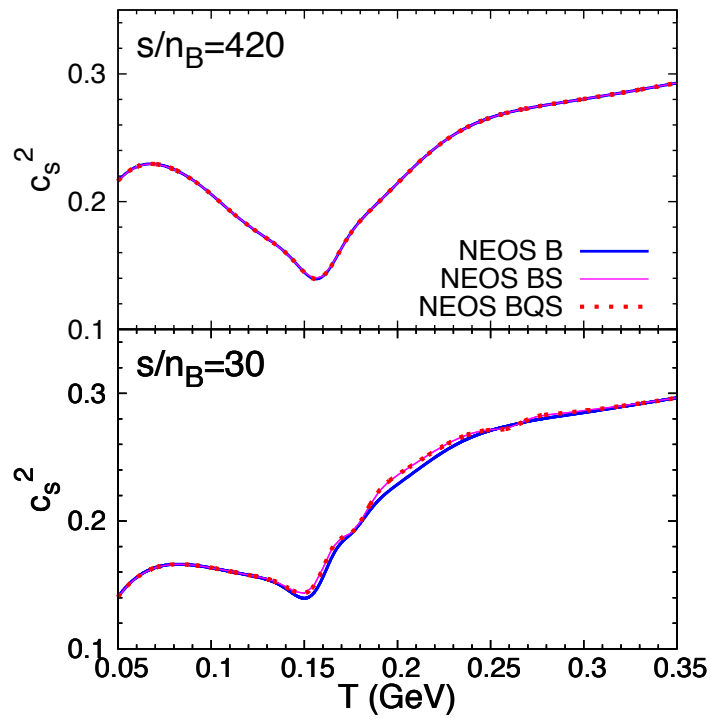
NEOS BQS ($n_S = 0$ and $n_Q = 0.4n_B$)

$$\mu_B \simeq 4.6n_B/T^2, \; \mu_S \simeq 1.6n_B/T^2, \; \mu_Q \simeq -0.2n_B/T^2$$

Sound velocity c_s

- s/n_B dependence

$$c_s^2 = \left.\frac{\partial P}{\partial e}\right|_{n_K} + \sum_J \frac{n_J}{e+P} \left.\frac{\partial P}{\partial n_J}\right|_{e,n_K} \quad J \neq K$$



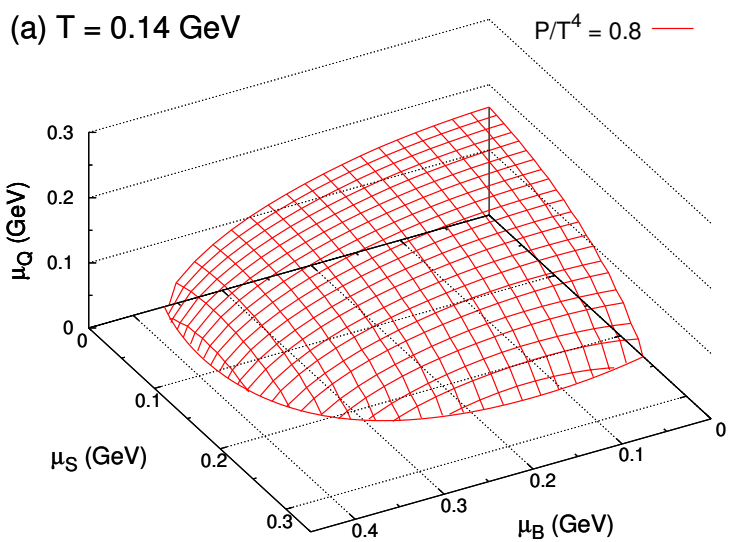
- In dense systems, c_s is suppressed at lower T
- The effect of strangeness neutrality becomes more apparent
- Finite n_s is relevant to c_s^2 in NEOS B

$$\left.\frac{\partial P}{\partial e}\right|_{n_B} + \frac{n_B}{e+P} \left.\frac{\partial P}{\partial n_B}\right|_{e} \neq c_s^2$$

The "conventional definition" w/o Q and S leads to underestimation

μ_B-μ_Q-μ_S space

- Constant pressure plane

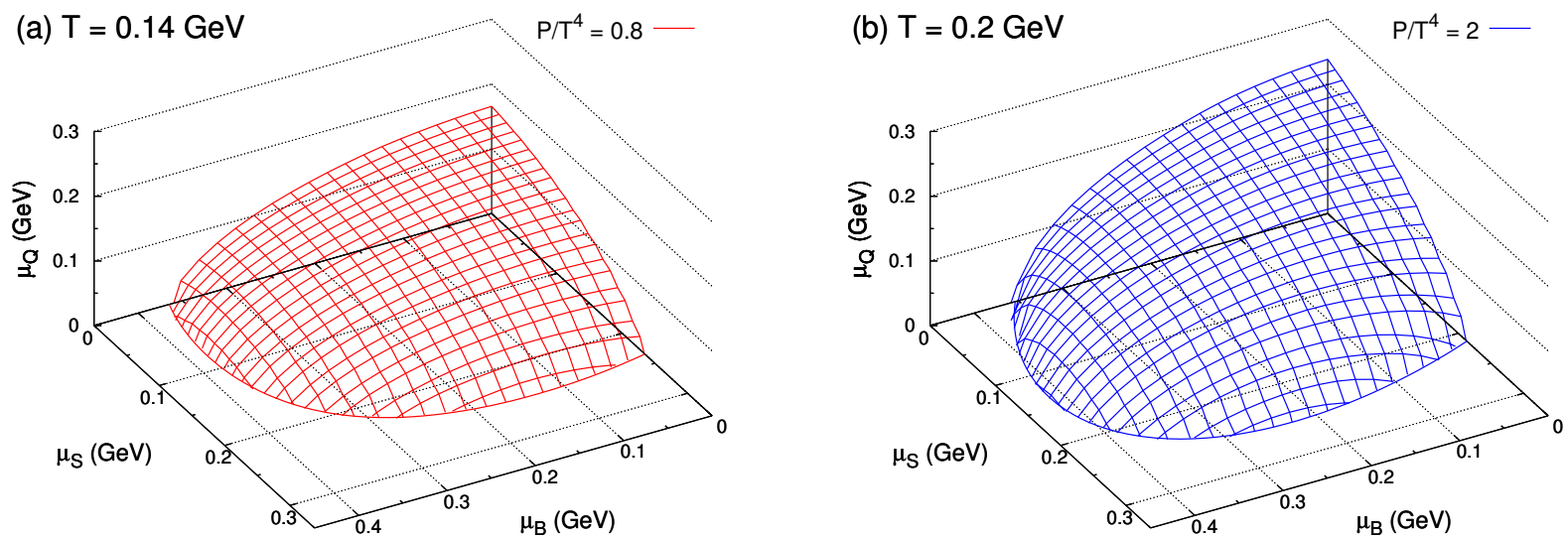


- The intercepts $P(\mu_B^{\rm int},0,0)=P(0,\mu_Q^{\rm int},0)=P(0,0,\mu_S^{\rm int})$

  $\mu_B^{\rm int}>\mu_S^{\rm int}>\mu_Q^{\rm int}$ in the hadronic phase because the lightest hadron to carry them are ordered in mass as $m_p>m_K>m_\pi$

μ_B-μ_Q-μ_S space

- Constant pressure plane

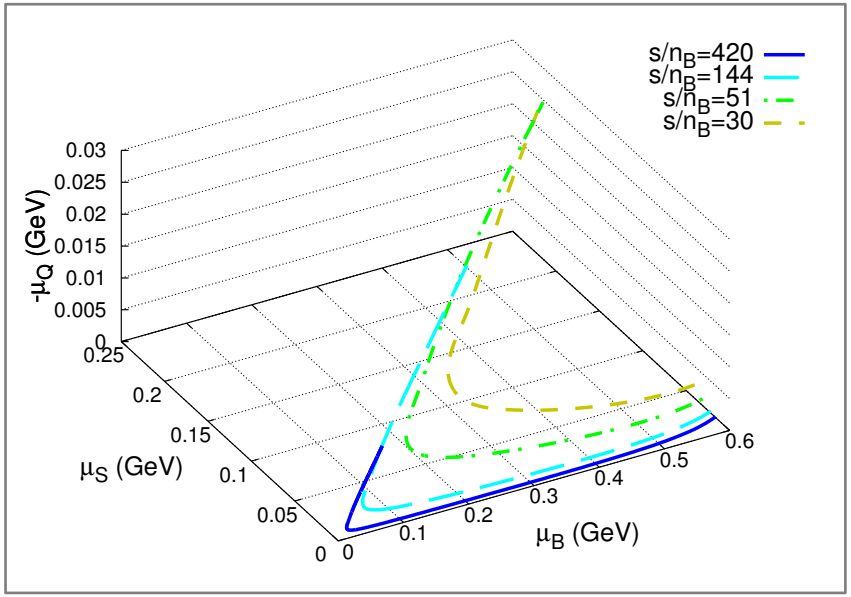


- The intercepts $P(\mu_B^{\rm int},0,0)=P(0,\mu_Q^{\rm int},0)=P(0,0,\mu_S^{\rm int})$

$\mu_B^{\rm int}>\mu_Q^{\rm int}>\mu_S^{\rm int}$ in the QGP phase as $\chi_2^B=1/3$, $\chi_2^Q=2/3$, $\chi_2^S=1$ in the parton gas limit implies $\mu_B^{\rm int}/3\sim 2\mu_Q^{\rm int}/3\sim\mu_S^{\rm int}$

μ_B-μ_Q-μ_S space

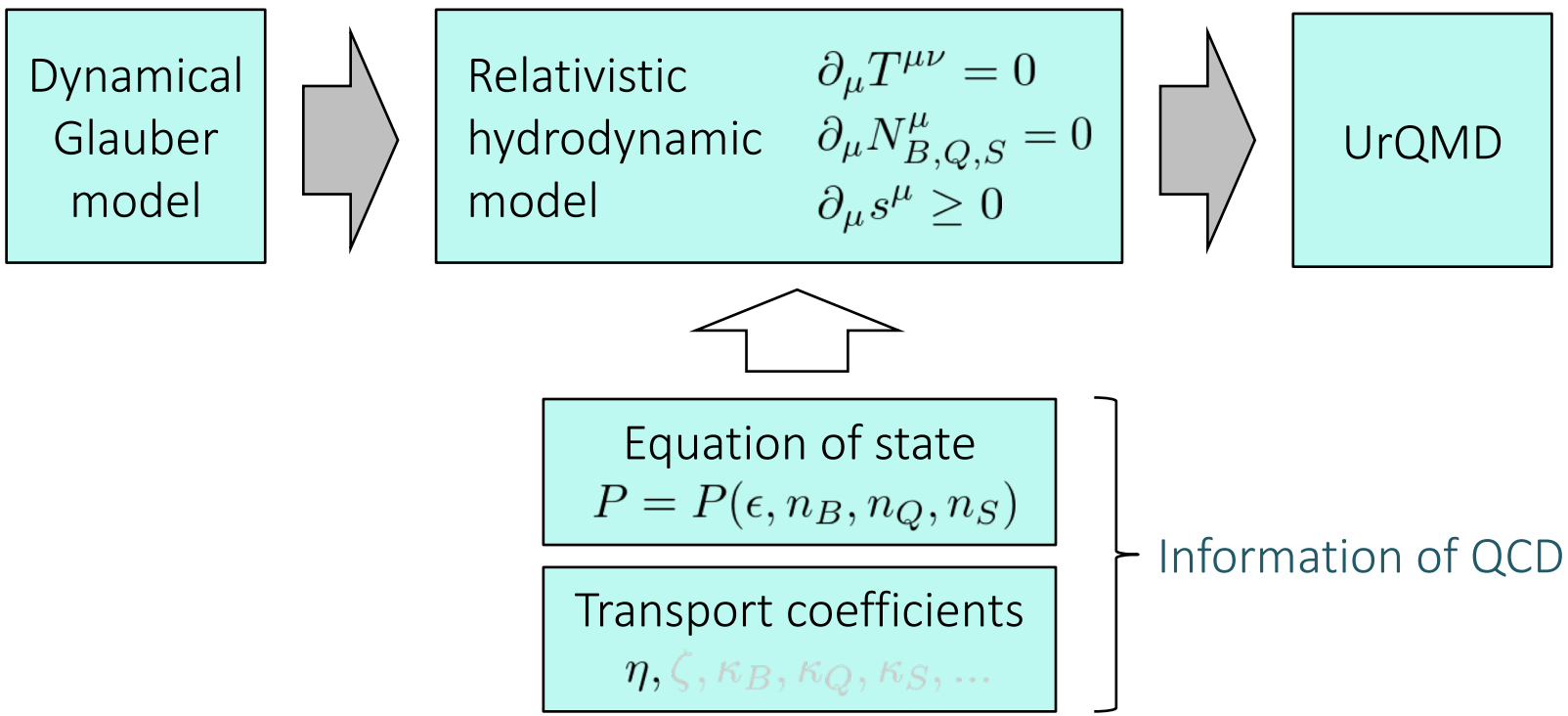
- Exploration of the QCD phase diagram



- We do not explore the μ_B-T plane in the BES experiments but a certain slice in the μ_B-μ_Q-μ_S-T hyperplane
- This may well affect the QCD critical point search

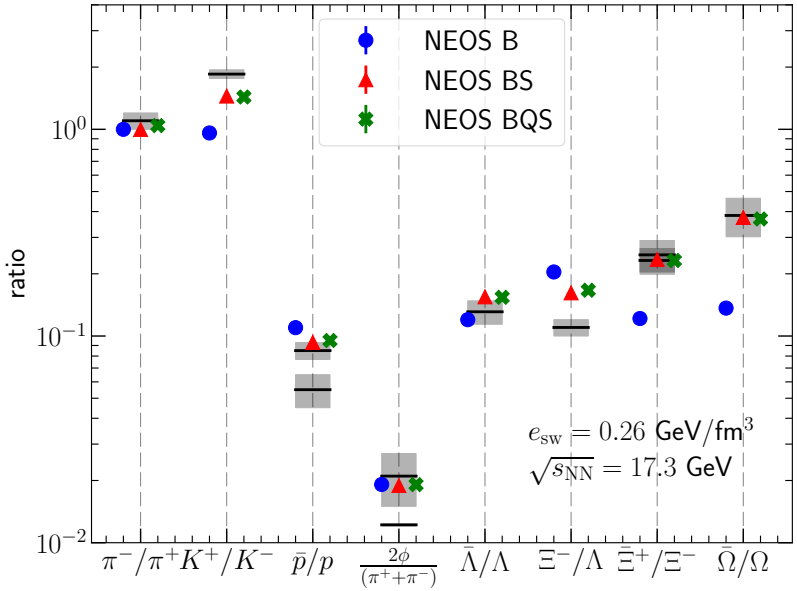
Hydrodynamic model

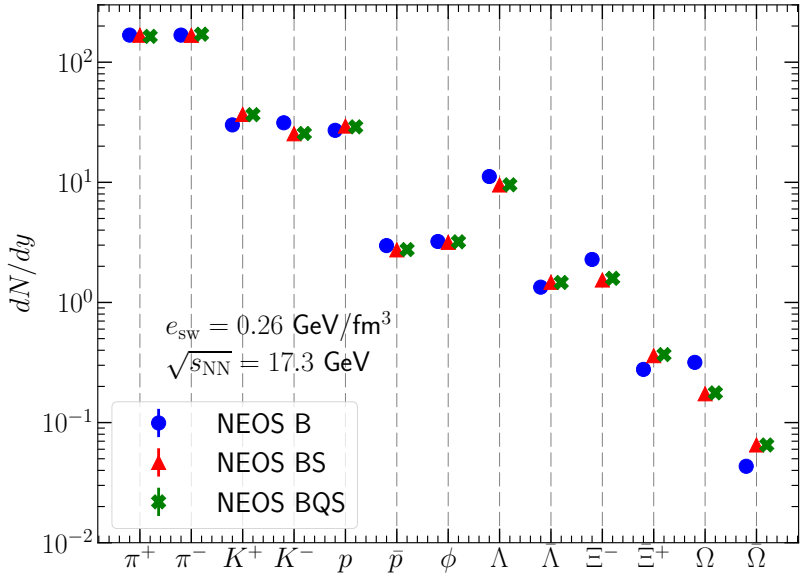
- with multiple charges



Hydrodynamic results

- 3+1 D viscous hydro + UrQMD for Pb-Pb 17.3 GeV in SPS

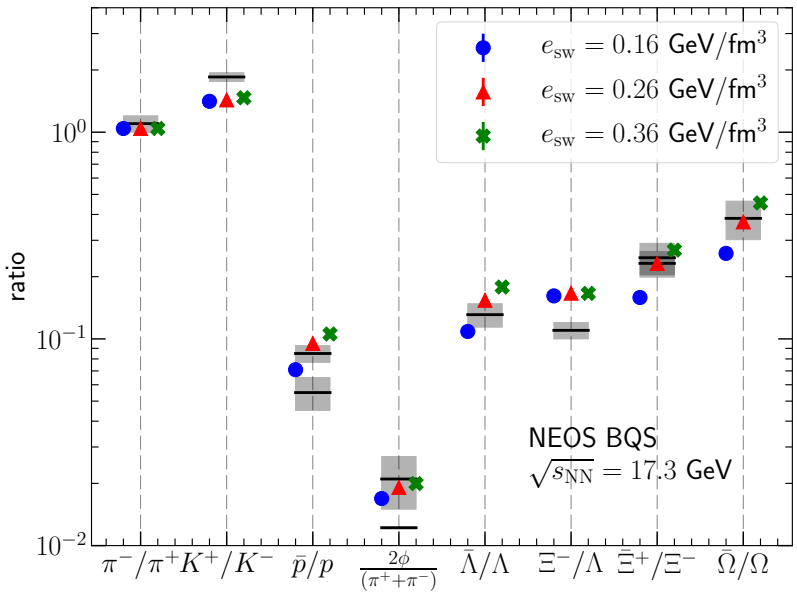


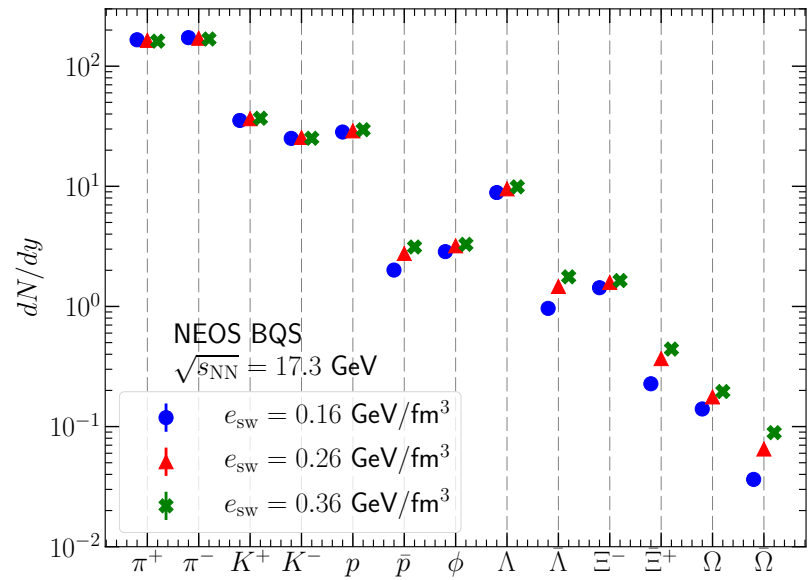


- Strange neutrality visibly improves description of strange hadrons
- Charge-to-baryon ratio fixing has small effects; π-/π+ ratio (> 1) is improved

Hydrodynamic results

- Switching temperature dependence





- The preferred switching to UrQMD is 0.16-0.26 GeV/fm^3
- Effects of chemical potential becomes larger for lower e_{sw}

3. Summary and outlook

Summary and outlook

- The QCD matter at finite density poses us challenges:

- Interplay of multiple charges (B, Q, S) are important
  - Equation of state is constructed
  - Strangeness neutrality condition leads to finite positive μ_S, and realistic charge-to-baryon ratio for Au/Pb to finite negative μ_Q
  - Particle-to-antiparticle ratios are described better in hydro model
- Estimation of baryon, strangeness and charge diffusion including cross-coupling currents
- p_T spectra, flow harmonics and rapidity distribution
- Realistic EoS for small systems as well as isobar experiments

Summary and outlook

- Our QCD equation of state model NEOS is publicly available: https://sites.google.com/view/qcdneos/home