Horndeski Genesis: Strong Coupling and Absence Thereof

2019 YITP Asian-Pacific Winter School and Workshop on Gravitation and Cosmology, 11-15 February 2019

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based on work arXiv:1810.00465 with O. Evseev, O. Melichev and V. Rubakov

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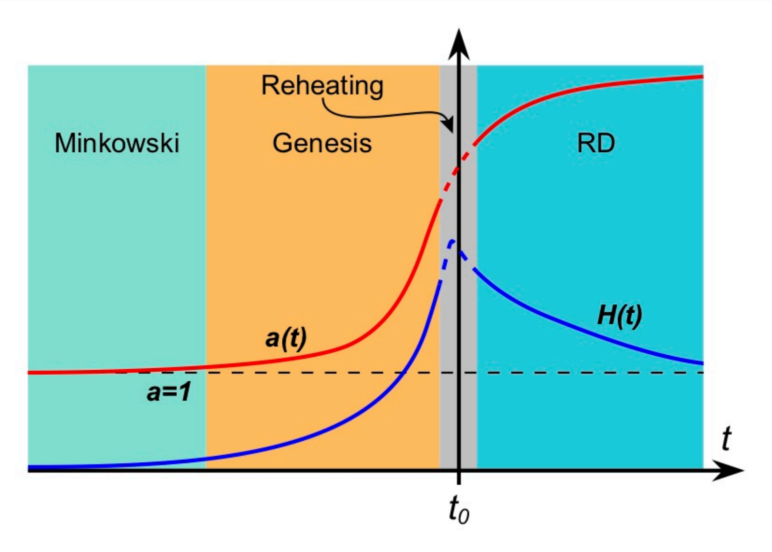
CONTENTS

MOTIVATION

- Inflation is now the strongest candidate of the early universe scenario that explains current cosmological observations consistently.
  Starobinsky'80, Guth'81, Steinhardt'82, Linde'83
- Non-singular stages in the early universe cannot only be something that replaces inflation, but also "early-time" completion of inflation just to get rid of the initial singularity.
  Vilenkin'92,
  Vilenkin, Borde'93
- We address whether healthy non-singular cosmologies can be implemented in the framework of general scalar-tensor theories.

GENESIS



Null Energy Condition

- If one uses general relativity to describe gravity, then an important characteristic is the null energy condition (NEC) for the matter energy-momentum tensor $T_{\mu\nu}$:

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0,$$

  for every null vector k^{μ}. Once the NEC holds in the cosmological context, then (assuming flat spatial sections) it follows from the Einstein equations that $dH/dt \leq 0$, where H is the Hubble parameter.
- This implies that there is a singularity in the past of the expanding universe. Therefore, one either modifies gravity or violates the NEC to build non-singular cosmology.

Let's violate NEC!

NEC violation could lead to singularity-free cosmology.
However, violating the NEC in a healthy manner turns out to be challenging.

Horndeski theory or Generalized Galileon

HORNDESKI THEORY OR GENERALIZED GALILEON

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

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Received: 10 *July* 1973

Abstract

Lagrange scalar densities which are concomitants of a pseudo–Riemannian metric-tensor, a scalar field and their derivatives of arbitrary order are considered. The most general second-order Euler–Lagrange tensors derivable from such a Lagrangian in a four-dimensional space are constructed, and it is shown that these Euler–Lagrange tensors may be obtained from a Lagrangian which is at most of second order in the derivatives of the field functions.

Full lagrangian

$$\begin{aligned}\mathcal{L}_H = {} & G_2(\phi, X) - G_3(\phi, X)\Box\phi + \\ & G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

and we use

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R,$$

where

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

$$\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi.$$

GENESIS MODEL

The ADM formalism and background solution

It is sufficient for our purposes to consider a subclass of Horndeski Lagrangians instead of the full one (ϕ and X are covariant variables):

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R.$$

Let us rewrite this Lagrangian in terms of ADM variables t and N:

$$\mathcal{L} = A_2(t, N) + A_3(t, N)K + A_4(K^2 - K_{ij}^2) + B_4(t, N)R^{(3)}.$$

Gauge is fixed with $Y_0 = const$:

$$e^{-\phi} = -\sqrt{2Y_0}t,$$

$$e^{\phi}\sqrt{\frac{Y_0}{X}} = N.$$

A subclass of Lagrangians in which the no-go theorem can be avoided (Tsutomu Kobayashi 2016) has the following form:

$$A_2 = M_{Pl}^4 f^{-2(\alpha+1)-\delta} a_2(N),$$
$$A_3 = M_{Pl}^3 f^{-2\alpha-1-\delta} a_3(N),$$
$$A_4 = -B_4 = -M_{Pl} f^{-2\alpha},$$

where α and δ are constant parameters satisfying:

$$2\alpha > 1 + \delta \ , \qquad \delta > 0 \ ,$$

and $f(t)$ is some function of time, which has the following asymptotics as $t \to -\infty$:

$$f \approx -ct, \qquad c = \text{const} > 0.$$

The background metric reads:

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 dx_i dx^i.$$

Avoiding the no-go theorem

An asymptotic solutions at early times ($t \to -\infty$) for Hubble parameter H, scale factor a and lapse function N are:

$$H \approx \frac{\chi}{(-t)^{1+\delta}} \ ,$$

$$a \approx 1 + \frac{\chi}{\delta(-t)^{\delta}}, \quad N \to 1 \ ,$$

$$\chi = \frac{\frac{2}{3}M_{Pl}^2 + \frac{c}{4}\,(2\alpha + 1 + \delta)\, M_{Pl}}{4(2\alpha + 1 + \delta)c^{2+\delta}} \ .$$

An important feature of this solution is that:

$$B_4(t, N),\ A_4(t, N) \to 0 \quad \text{при} \quad t \to -\infty,$$

and hence

$$G_4(\phi, X) \to 0 \quad \text{при} \quad t \to -\infty \ .$$

$G_4(\phi, X)$ is a coefficient multiplied by R so it has the sense of M_{Pl}^2:

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R.$$

$$G_4(\phi, X) \to 0 \quad \text{as} \quad t \to -\infty \ .$$

On the one hand, these are necessary conditions to avoid both ghost and gradient instabilities. On the other hand it signalizes that the strong coupling energy scale in this theory tends to zero as $t \to -\infty$.
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STRONG COUPLING REGIME

The scalar sector of perturbations

The perturbed metric for the scalar sector has the following form:

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right),$$

where

$$N = 1 + \alpha\,, \quad N_i = \partial_i \beta\,, \quad \gamma_{ij} = a^2 e^{2\zeta}\delta_{ij}\,.$$

Expanding the action up to the second order, one obtains the following expression for the quadratic action in the unitary gauge (Xian Gao et al. 2011)

$$S^{(2)}_{\alpha,\beta,\zeta} = \int N dt\, a d^3x \left[-3g_\zeta\left(\frac{a}{N}\dot{\zeta}\right)^2 + c_\zeta\left(\partial\zeta\right)^2 - 3a^2H^2 m_\alpha \alpha^2 + 2g_\zeta \partial\alpha\partial\zeta \right.$$
$$\left. + \,6\frac{a^2}{N}Hf_\alpha \alpha\dot{\zeta} + 2\frac{a}{N}g_\zeta \zeta\partial^2\beta - 2aHf_\alpha \alpha\partial^2\beta\right].$$

The early-time asymptotics of the latter coefficients are:

$$g_\zeta \sim c_\zeta \sim (-t)^{-2\alpha} ,$$
$$f_\alpha \sim (-t)^{-2\alpha} ,$$
$$m_\alpha \sim -(-t)^{-2\alpha+\delta} .$$

The fields α and β are constraint variables. One finds them by solving the constraint equations. The unconstrained quadratic action is:

$$S_\zeta^{(2)} = \int N dt\, a d^3x \left(\frac{\epsilon_s}{c_s^2} \frac{a^2}{N^2} \dot{\zeta}^2 - \epsilon_s (\partial\zeta)^2 \right),$$

$$\epsilon_s = \frac{1}{aN} \frac{d}{dt} \left(\frac{a g_\zeta^2}{H f_\alpha} \right) - c_\zeta, \quad c_s^2 = \frac{\epsilon_s}{3 g_\zeta} \left(1 - \frac{g_\zeta m_\alpha}{f_\alpha^2} \right)^{-1} .$$

$$\epsilon_s \sim (-t)^{-2\alpha+\delta}, \quad c_s^2 \sim (-t)^0 .$$

To figure out the strong coupling scale in the scalar sector, we have to go one step further and consider the cubic action.

$$S^{(3)}_{\zeta,\alpha,\beta} = \int Ndt\, ad^3x \Big\{ g_\zeta \Big[-9\frac{a^2}{N^2}\zeta\dot{\zeta}^2 + 2\frac{a}{N}\dot{\zeta}\Big(\zeta\partial^2\beta + \partial_i\zeta\partial^i\beta\Big) - \ldots$$

$$\ldots - \lambda_3 aH\alpha^2\Big(3\frac{a}{N}\dot{\zeta} - \partial^2\beta\Big) - \lambda_4\alpha^2\partial^2\zeta + \frac{\lambda_5}{2}(aH)^2\alpha^3\Big\},$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ are the functions of $g_\zeta, c_\zeta f_\alpha, m_\alpha, A_2, A_3, A_4, H$ and we find their asymptotic behaviour as $t \to \infty$ for our model.

We again solve the constraints in terms of α and β and obtain the following expression for unconstrained cubic action:

$$S_{\zeta}^{(3)} = \int Ndt\, ad^3x \left\{ \Lambda_1 \left(\frac{a}{N}\dot{\zeta} \right)^3 + ... + \Lambda_{18}\zeta\partial_i\partial_j\zeta\partial^i\partial^j\psi \right\},$$

where $\psi = \partial^{-2}(a\dot{\zeta}/N)$; $\Lambda_1 ... \Lambda_{18}$ are functions of $g_\zeta, c_\zeta f_\alpha, m_\alpha, A_2, A_3, A_4, H$, and hence of time t.

All of them have power-law behaviour at early times $t \to -\infty$:

$$\Lambda_i \sim (-t)^{x_i}.$$

The canonically normalized field

Every term $\mathbb{L}_i$ in the cubic Lagrangian ($i = \overline{1, 18}$) can be schematically written as follows:

$$\mathbb{L}_i = \Lambda_i \cdot \zeta^3 \cdot (\partial_t)^{a_i} \cdot (\partial)^{b_i} \ .$$

We naturally use the canonically normalised field π instead of ζ:

$$\pi = \sqrt{\epsilon_S}\zeta \propto |t|^{-\alpha+\delta/2}\zeta \ .$$

In terms of the canonically normalised field π:

$$\mathbb{L}_i = \tilde{\Lambda}_i \cdot \pi^3 \cdot (\partial_t)^{a_i} \cdot (\partial)^{b_i},$$

where

$$\tilde{\Lambda}_i = \Lambda_i \epsilon_S^{-3/2} = \Lambda_i |t|^{-\frac{3}{2}(\delta-2\alpha)} \sim |t|^{x_i - \frac{3}{2}(\delta-2\alpha)} \ .$$

No strong coupling condition

NO STRONG COUPLING CONDITION

By naive dimensional analysis (dimension of Λ_i is $[\Lambda_i] = 4 - a - b$ and $[\epsilon_s] = 2$) we immediately find that the strong coupling energy scale associated with the term $\mathbb{L}_i$ is:

$$E^{(i)}_{strong} \sim \tilde{\Lambda}_i^{-\frac{1}{a_i+b_i-1}} \sim |t|^{-\frac{x_i+3\alpha-3\delta/2}{a_i+b_i-1}} \ .$$

The inverse time scale of classical evolution is

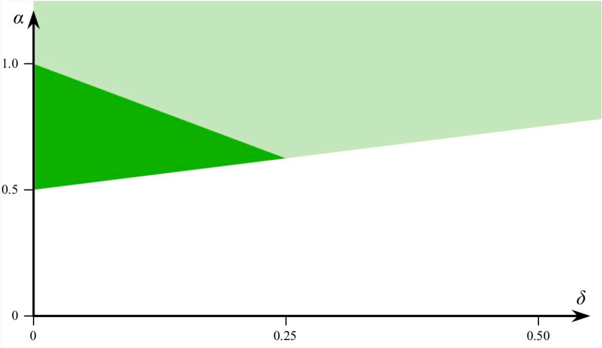
$$E_{class} \sim \frac{\dot{H}}{H} \sim |t|^{-1} \ . \tag{2}$$

The condition for legitimacy of the classical treatment of the early evolution, $E_{class} \ll E^{(i)}_{strong}$ for all i reads

$$x_i + 3\alpha - \frac{3}{2}\delta < a_i + b_i - 1 \ , \qquad i = \overline{1, 18} \ .$$

No strong coupling criterion

$$0 < \delta < \frac{1}{4}, \qquad 2 - 3\delta > 2\alpha > 1 + \delta.$$



CONCLUSION

Results

- We have studied the non-singular Genesis scenario in the framework of the Horndeski theory.
- It is possible to build stable Genesis for the very early times.
- But one of the options free of instabilities at all cosmological epochs is the one in which the early Genesis is naively plagued with strong coupling.
- We have shown that, indeed, despite the fact that the effective Plank mass tends to zero at early time asymptotics, the classical analysis is legitimate in a certain range of Lagrangian parameters.

Outlook

- Study all sectors of metric perturbations.
- Do a conformal transformations $\rightarrow$ interesting to understand the no strong coupling condition in another frame.


The presentation is over
Thank you for your attention
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$$S_{\zeta}^{(3)} = \int N dt\, a d^3x \left\{ \Lambda_1 \left(\frac{a}{N} \dot{\zeta} \right)^3 + \ldots\ldots + \Lambda_{18} \zeta \partial_i \partial_j \zeta \partial^i \partial^j \psi \right\}.$$

By inspecting the behaviour of Λ_i one finds that this combination is the smallest for $i = 1$, when:

$$\Lambda_1 \sim (-t)^{1-2\alpha+3\delta}\ , \quad a_1 = 3\ , \quad b_1 = 0\ , \quad a_1 + b_1 - x_1 = 2 + 2\alpha - 3\delta$$

$$0 < \delta < \frac{1}{4}, \qquad 2 - 3\delta > 2\alpha > 1 + \delta.$$