Anisotropic magnetized dark energy Cosmological Model with skewness parameters

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Introduction

The accelerating behaviour of the expansion of present universe has been indicated by cosmological observations such as CMB[1, 2], clusters of galaxies [3] and type Ia supernovae [4, 5] etc.

It is believed that the reason of this acceleration may be due to the presence of an anti self attraction of matter.

Eventually, a violation to strong energy condition occurs as the acceleration is observed to be characterized by a hypothetical fluid of negative pressure and a positive energy density.

This violation signals a reverse gravitational impact and there results a transition from early declaration phase to late time acceleration phase.

Introduction

The cause of the late time cosmic acceleration is dubbed as dark energy (DE) and constitutes about 68.3% of energy density of the universe.

But the mystery is that, a little is known about DE: it violates strong energy condition and can cluster at large scale. Also, DE does not interact with baryonic matter and therefore making it difficult to detect. It dominates the present universe and was less effective in early time.

Cosmology and astrophysics inform about anisotropic features of DE, associating with the breakdown of global isotropy [6, 7].

we have focussed on the anisotropic behaviour of the universe in the considered DE model followed by few other effective dynamical analysis.

Introduction

The universe contains highly ionized matter and therefore cosmological models, having the matter field with contributions from magnetic field have significant impact on the dynamics of the universe.

The effect of magnetic field on the cosmic dynamics depends on the direction of the field lines [8, 9]. Magnetic field can affect the particle creation rate and expansion rate in anisotropic models.

The standard cosmological model is based upon the assumption of large scale isotropy and homogeneity of space. However, small scale anisotropies expected in the universe in view of the observations of temperature anisotropy in the CMB radiation data from WMAP and Planck.

Introduction

During early time of cosmic evolution, magnetic field has maximum effect in radiation dominated phase, producing large expansion anisotropies [10].

Planck data also show a slight redshit of the power spectrum from exact scale invariance. Though the standard ΛCDM model is well supported by different measurements, at least at low multipoles, it does not fit to the temperature power spectrum data.

Space-time

We have considered a spatially homogeneous and anisotropic Bianchi V space time in the form,

$$ds^2 = dt^2 - A_1(t)^2 dx^2 - e^{2\alpha x}[A_2(t)^2 dy^2 + A_3(t)^2 dz^2] \qquad (1)$$

The geometry of the considered space-time is represented by two equivalent transverse directions y and z and one different longitudinal direction z, along which the magnetic field is oriented.

Energy Momentum Tensor

The energy momentum tensor $T_{\mu\nu}$ consisting of two different components: the hypothetical DE fluid ($T^{de}_{\mu\nu}$) and the electromagnetic field ($E^{z}_{\mu\nu}$) as

$$T_{\mu\nu} = E^{(z)}_{\mu\nu} + T^{(de)}_{\mu\nu}. \tag{2}$$

$$\begin{aligned} T^{(de)}_{\mu\nu} &= diag[\rho_d, -p_{xd}, -p_{yd}, -p_{zd}] \\ &= diag[1, -\omega_{xd}, -\omega_{yd}, -\omega_{zd}]\rho_d \\ &= diag[1, -(\omega_d+\delta), -(\omega_d+\gamma), -(\omega_d+\eta]\rho_d, \end{aligned} \tag{3}$$

$$E^{(z)}_{\mu\nu} = \frac{1}{4\pi}\left[g^{sp}F_{\mu s}F_{\nu p} - \frac{1}{4}g_{\mu\nu}F_{sp}F^{sp}\right], \tag{4}$$

The magnetic field is due to an electric current produced along z-direction after quantizing the axis of magnetic field in the same direction.

Field Equations

$$\frac{\ddot{A}_2}{A_2}+\frac{\ddot{A}_3}{A_3}+\frac{\dot{A}_2\dot{A}_3}{A_2A_3}-\frac{\alpha^2}{A_1^2}=-(\omega_d+\delta)\rho_d-\mathcal{M} \quad (5)$$

$$\frac{\ddot{A}_1}{A_1}+\frac{\ddot{A}_3}{A_3}+\frac{\dot{A}_1\dot{A}_3}{A_1A_3}-\frac{\alpha^2}{A_1^2}=-(\omega_d+\gamma)\rho_d+\mathcal{M} \quad (6)$$

$$\frac{\ddot{A}_1}{A_1}+\frac{\ddot{A}_2}{A_2}+\frac{\dot{A}_1\dot{A}_2}{A_1A_2}-\frac{\alpha^2}{A_1^2}=-(\omega_d+\eta)\rho_d+\mathcal{M} \quad (7)$$

$$\frac{\dot{A}_1\dot{A}_2}{A_1A_2}+\frac{\dot{A}_2\dot{A}_3}{A_2A_3}+\frac{\dot{A}_3\dot{A}_1}{A_3A_1}-\frac{3\alpha^2}{A_2^2}=\rho_d-\mathcal{M} \quad (8)$$

$$2\frac{\dot{A}_1}{A_1}-\frac{\dot{A}_2}{A_2}-\frac{\dot{A}_3}{A_3}=0 \quad (9)$$

Energy Conservation Equation

The energy conservation equation, $T^{\mu\nu}_{;\nu} = 0$, for the combined anisotropic fluid,

$$\dot{\mathcal{M}} + 2\mathcal{M}(H_2 + H_3) + \dot{\rho_d} + 3\rho_d(\omega_d + 1)H + \rho_d(\delta H_1 + \gamma H_2 + \eta H_3) = 0, \quad (10)$$

where the energy conservation equation for electromagnetic field, $E^{ij}_{;j} = 0$ and dark energy fluid $T^{ij(de)}_{;j} = 0$, respectively expressed as:

$$\dot{\mathcal{M}} + 2\mathcal{M}(H_2 + H_3) = 0 \quad (11)$$

and

$$\dot{\rho_d} + 3\rho_d(\omega_d + 1)H + \rho_d(\delta H_1 + \gamma H_2 + \eta H_3) = 0 \quad (12)$$

Eqs. (8) and (12) yield,

$$\rho_d = \frac{\dot{A}_1\dot{A}_2}{A_1A_2} + \frac{\dot{A}_2\dot{A}_3}{A_2A_3} + \frac{\dot{A}_3\dot{A}_1}{A_3A_1} - \frac{3\alpha^2}{A_1^2} + \frac{k^2}{8\pi A_2^2A_3^2e^{4\alpha x}} \tag{13}$$

Eqn. (12) split into two parts: one corresponds to the deviation of equation of the state parameter and other is deviation free part:

$$\dot{\rho_d} + 3\rho_d(\omega_d + 1)H = 0, \tag{14}$$

$$\rho_d(\delta H_1 + \gamma H_2 + \eta H_3) = 0.$$

$$\delta\rho_d = -\frac{(\gamma H_x + \eta H_y)\rho_d}{H_z}. \tag{15}$$

Anisotropic Relation

In order to obtain an anisotropic relationship between the directional scale factors, we assume $A_2 = A_3^m$, $m \neq 1$ is the anisotropic factor.

The anisotropic parameter $\mathcal{A} = \frac{1}{3}\sum\left(\frac{\triangle H_i}{H}\right)^2 = \frac{2}{3}\left(\frac{m-1}{m+1}\right)^2$.

The directional scale factors, $A_1 = R, A_2 = R^{\frac{2m}{m+1}}, A_3 = R^{\frac{2}{m+1}}$.

Description of Skewness Parameters

With algebraic manipulations of eqns. (5)-(7) and (13), the skewness parameters ,

$$\delta = \frac{1}{\rho_d}\left[H(2,3)\epsilon(m)F(R) + 2H(3)\mathcal{M}(R,x;z)\right], \quad (16)$$

$$\gamma = -\frac{1}{\rho_d}\left[H(1,3)\epsilon(m)F(R) - 2H(3)\mathcal{M}(R,x;z)\right] \quad (17)$$

$$\eta = \frac{1}{\rho_d}\left[H(1,2)\epsilon(m)F(R) - 2H(2,1)\mathcal{M}(R,x;z)\right], \quad (18)$$

$F(R) = \left(\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2}\right)$, $\mathcal{M}(R,x;z) = \frac{M^2 R^{-2\left(\frac{3m+1}{m+1}\right)}}{8\pi e^{2\alpha x}} = \mathcal{M}$,
$H(2,3) = \frac{H_2 - H_3}{3H}$, $H(1,3) = \frac{H_1 + 2H_3}{3H}$, $H(1,2) = \frac{H_1 + 2H_2}{3H}$, $H(2,1) = \frac{H_2 - H_1}{3H}$ and $H(3) = \frac{H_3}{3H}$. The functional $\epsilon(m) = \frac{1-m}{1+m}$, measure of deviation from isotropic behaviour

Skewness Parameters without magnetic field

In the absence of magnetic field, the skewness parameters reduce to

$$\delta = \frac{1}{\rho_d}\Big[H(2,3)\epsilon(m)F(R)\Big]. \tag{19}$$

$$\gamma = -\frac{1}{\rho_d}\Big[H(1,3)\epsilon(m)F(R)\Big]. \tag{20}$$

$$\eta = \frac{1}{\rho_d}\Big[H(1,2)\epsilon(m)F(R)\Big]. \tag{21}$$

It is evident from the above equations that, in the absence of magnetic field and cosmic anisotropy, the skewness parameters vanish identically leaving behind a universe with isotropic expansion pervaded by an isotropic DE fluid.

The total effective DE density ρ and the effective EoS parameter ω,

$$\rho = 2\phi_1(m)\frac{\dot{R}^2}{R^2} - 3\frac{\alpha^2}{R^2} + \mathcal{M}(R, x; z) = \rho_d + \mathcal{M} \tag{22}$$

$$\omega\rho = -\frac{4\ddot{R}}{3R}\left[\phi_1(m)\right] - \frac{2}{3}\left[\frac{\dot{R}^2}{R^2}\phi_2(m) + \mathcal{M}(R, x; z)\left(\frac{1-3m}{3(m+1)}\right)\right] + \frac{\alpha^2}{R^2}, \tag{23}$$

$$\phi_1(m) = \frac{(m^2+4m+1)}{(m+1)^2} \text{ and } \phi_2(m) = \frac{(m^3+5m^2+5m+1)}{(m+1)^3}.$$

Cosmological Model with hybrid scale factor

Hybrid scale factor expressed through two adjustable parameters h_1 and h_2 as $R = e^{h_1 t} t^{h_2}$ i.e. $H = \left(h_1 + \frac{h_2}{t}\right)$.
The deceleration parameter, $q = -1 + \frac{h_2}{(h_1 t + h_2)^2}$.
The parameter h_2 can be constrained in the range $0 < h_2 < 1$ from simple arguments but h_1 can be constrained from a detailed analysis of $H(z)$ data.

$$A_1 = R = e^{h_1 t} t^{h_2}; A_2 = R^{\frac{2m}{m+1}} = (e^{h_1 t} t^{h_2})^{\frac{2m}{m+1}}; A_3 = R^{\frac{m}{m+1}} = (e^{h_1 t} t^{h_2})^{\frac{2}{m+1}}.$$

Consequently, the directional Hubble parameters are obtained as, $H_x = H$, $H_y = \left(\frac{2m}{m+1}\right) H$ and $H_z = \left(\frac{2}{m+1}\right) H$.

Dark energy density and EoS parameter with HSF

The effective DE density and the effective EoS parameter respectively reduced to,

$$\rho = 2(h_1 t + h_2)^2 t^{-2}\phi_1(m) - 3\alpha^2 e^{-2at} t^{-2b} + \mathcal{M} \tag{24}$$

$$\omega\rho = -\frac{4}{3}G(t)\phi_1(m) - \frac{2}{3}(h_1 t + h_2)^2 t^{-2}\phi_2(m) + \mathcal{M}(t,x;z)\left(\frac{1-3m}{m+1}\right) + \alpha^2 \tag{25}$$

$$G(t) = \frac{h_1^2 t^2 + 2h_1 h_2 t + h_2(h_2-1)}{t^2};\ \mathcal{M}(t,x;z) = \frac{M^2\left(e^{h_1 t} t^{h_2}\right)^{-2\left(\frac{3m+1}{m+1}\right)}}{24\pi e^{2\alpha x}}.$$

Skewness Parameters with HSF

Eqns. (16) - (18) can be obtained with HSF as,

$$\delta = -\frac{2}{3\rho_d}\left[\left(\frac{m-1}{m+1}\right)\epsilon(m)F(t) - \mathcal{M}(t,x;z)\right] \tag{26}$$

$$\gamma = \frac{1}{3\rho_d}\left[\left(\frac{m+5}{m+1}\right)\epsilon(m)F(t) + 2\mathcal{M}(t,x;z)\right] \tag{27}$$

$$\eta = -\frac{1}{3\rho_d}\left[\left(\frac{5m+1}{m+1}\right)\epsilon(m)F(t) + (3m+1)\mathcal{M}(t,x;z)\right] \tag{28}$$

Where $F(t) = \frac{3h_1^2t^2+6h_1h_2t+3h_2^2-h_2}{t^2}$.

The pressure anisotropies in the form of skewness parameters depend explicitly on the magnetic field through the quantity $\mathcal{M}$. The quantity $\mathcal{M}$ is considered to evolve with cosmic time and therefore, its influence on the physical parameters dynamically changes with time.

EoS Parameters vs. z

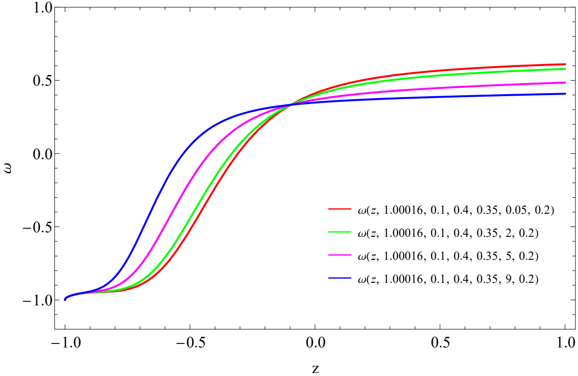

Figure: Effective EoS parameter versus z for representative values of the parameter $\mathcal{M}$ with $m = 1.0001633$, $h_1 = 0.1$, $h_2 = 0.4$, $\alpha = 0.35$

EoS Parameters vs. z

The EoS parameter decreases in an early epoch with an increase in the magnetic field. However, with the increase in cosmic time, the decrement of ω slows down with increase in $\mathcal{M}$ and behave in a reverse trend at $z = -0.096$.

At $z = -0.096$, the effective EoS parameter is independent of the quantity $\mathcal{M}$ and acquires a value of $\omega = 0.329$. EoS parameter decreases more rapidly after the transit point. In general, for low value of the magnetic field, the decrement of ω is much rapid compared to higher values of $\mathcal{M}$.

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At a late phase of cosmic time, the effective EoS parameter becomes independent of the quantity $\mathcal{M}$ and all the curves of ω merge to a single point and behave like a cosmological constant with $\omega = -1$.

Hence, magnetic field has a substantial role to play at an early phase. However, its influence can not be ignored at late phase of cosmic evolution.

Effective EoS

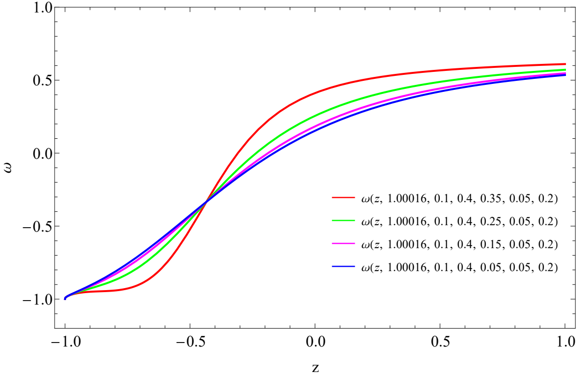

Figure: Variation of effective EoS parameter versus z for representative values of the parameter α with $m = 1.0001633,\ h_1 = 0.1,\ h_2 = 0.4,\ \mathcal{M} = 0,05$

Effective EoS

In the figure, we have shown the effect of α on ω for four different choices. ω remains in the quintessence region for all considered values of α but the behavioural switching over red shift of the universe shifts towards ΛCDM line. It can be noted that the transition starts from the point $(-0.436, -0.341)$.

Behaviour of ω is same as compared to different magnetic coefficient values, where as, with the increase in the value of α, the increment in ω is more rapid. Also, the value of ω for $\alpha = 0.35$(red line) is more for higher values of α and lies very similar to ω for for a suitable magnetic coefficient value.

Skewness parameter versus t

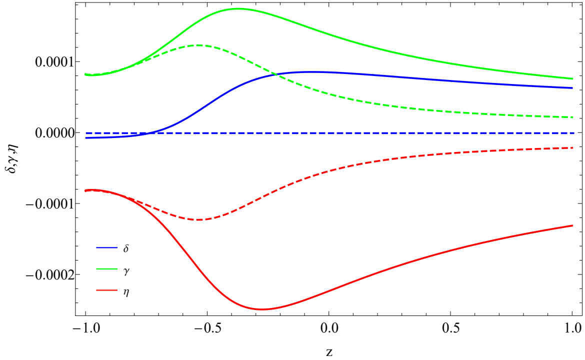

Figure: Skewness parameter versus t for the suitable discussed value of the parameter $m = 1.0001633$, $h_1 = 0.1$, $h_2 = 0.4$, $\alpha = 0.35$, $\mathcal{M} = 0, 05$. Dotted lines indicate the situation in the absence of magnetic fluid.

SKEWNESS PARAMETER VERSUS t

The figure depicts the evolutionary behaviour of skewness parameters in both absence and presence of magnetic field. The dotted lines of skewness parameters, indicating the absence of magnetic field, non evolving in the early phase of cosmic time but evolves uniformly towards late cosmic time of evolution.

The anisotropic parameter γ assumes a positive value and shows increasing trend towards present epoch. But η takes a negative value through out the evolution, having a decreasing trend towards present time. But both parameters retain their non-evolving behaviour towards late phase.

The parameter δ, indicates that the effective anisotropic pressure along x-direction of the universe, is independent of the cosmic expansion. The dotted lines express the anisotropic situation of the universe where only DE matter is involved.

Skewness parameter versus t

The presence of magnetic fluid drives a drastic change, particularly, in the early epoch. The solid lines clearly indicate the dominance of magnetic fluid during early time of evolution. Due to this effect, δ (blue solid line) starts evolving, assuming positive value and changes sign at a red shift value as we move forward to late epoch. The reason behind this sensitivity of δ to presence of magnetic fluid may be due to the consideration of normalising the average scale factor along x-direction ($H = H_1$).

Similarly, γ and η also show irregular evolutionary behaviour due the presence of matter. Whereas, one can clearly notice that η(red solid line) is most affected in comparison to the rest two anisotropic pressure parameters. This situation occurs due to the fact that we have designed the magnetic fluid to align along z-direction.

State Finder Diagnostic

To show the present DE hybrid model is physically realistic, the state finder diagnostic pair (r, s), where r is the jerk parameter and s characterizes the property of DE need to be assessed. Comparing with the standard ΛCDM model, this diagnosis decides the acceptability of corresponding model.

These parameters continuously evolve with time from big bang time $(t \to 0)$ to large value at late time $(t \to \infty)$. The values of these parameters can be calculated as,

$$r = 1 - \frac{3h_2}{(h_1 t + h_2)^2} + \frac{2h_2}{(h_1 t + h_2)^3}, \tag{29}$$

$$s = -\frac{6h_2(h_1 t + h_2) + 4h_2}{6h_2(h_1 t + h_2) - 9(h_1 t + h_2)^3}. \tag{30}$$

R-S PAIR

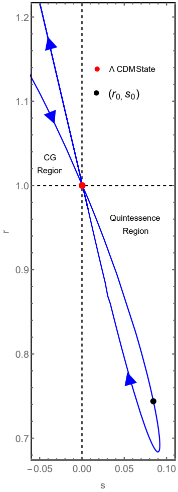

Figure: Variation of state finder parameters 'r' versus 's'

R-S

Figure depicts the evolutionary behaviour of state finder parameters r and s for the DE universe along with ΛCDM universe on rs- plane. The directions of the rs trajectory are represented by arrow marks in the diagram.

The two domains in this plot $\{r < 1, s > 0\}$ and $\{r > 1, s < 0\}$ characterize quintessence region and Chaplygin gas region (CG), respectively. Hence, the top left part of the trajectory represents CG behaviour and the right bottom part represents quintessence behaviour of our dark energy model. Both parts, eventually, met at the (red) point $\{r, s\} = \{1, 0\}$, the ΛCDM state of universe.

Although, the trajectory starts evolving from CG region, the present state (black dot) of our dark energy universe remains in the quintessence region. The black dot in the figure depicts the present value of (r; s); falling in preferred range of current observational data.

R-Q

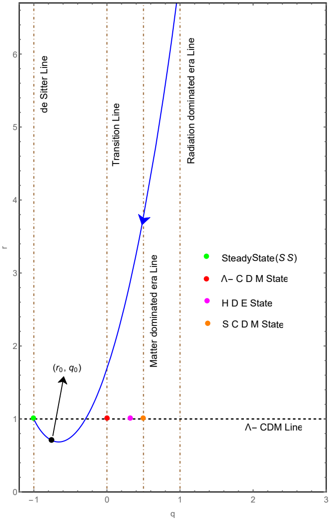

Figure: Evolution of state finder parameter 'r' w.r.t. deceleration parameter 'q'

R-Q

The geometrical behaviour of the model is presented in the figure. The corresponding evolutionary behaviour is investigated, comparing with ΛCDM model, and are plotted in the *rq*- plane. For similar kinematics, this is an effective way to compare and differentiate different cosmological models. Vertical lines stand for different eras of cosmic evolution, starting from BBN (big bang nucleosynthesis) to de-Sitter state ($q \sim -1$).

It is observed that the present DEH model evolves from a radiation dominated era to the de Sitter phase. Moreover, the ΛCDM line has been demonstrated in detail, showing transformation of trajectory from a matter dominated universe, known as standard cold dark matter (SCDM) state of universe, to steady state universe (SS) as time unfolds. The SCDM state, steady state (SS) model and holographic dark energy model with future event horizon correspond to fixed points $\{r, q\} = \{1, 0.5\}$, $\{r, q\} = \{1, 1\}$ and $\{r, q\} = \{1, \frac{1}{3}\}$, respectively in the plot.

R-Q

This reveals the phase transition of our model through $r-q$ trajectory as q changes its sign from positive to negative, starting from pre-SCDM state of universe (radiation dominated era). As time evolves, the (r,q) values gradually decrease and attain its minimum location. Then it starts to increase and the trajectory leads the path towards SS. This uplifting of the trajectory indicates that our suggested dark energy model might behave as SS universe at late epoch.

Several different models have different evolution trajectories whereas evolutionary behaviour in those models remain in the range $q \lesssim 0.5$, approaching to same future (de Sitter universe). But in the DEH model developed here, the universe may be described from the primordial nucleosynthesis time of universe [22]. According to general relativity the evolution of the universe start from dust dominated era ($q=0.5$ and $r=1$) whereas in our presented DEH model the universe starts evolving from the BBN times.

Conclusion

In the present work, we have constructed an anisotropic DE cosmological model in the framework of GR and Bianchi type V space time. In order to study the evolution of skewness parameters and the EoS parameters, we have developed the DE models in two fluid situations. The EoS parameter of DE evolves within the range predicted by the observations.

Study on skewness parameters shows that, the universe is mostly dominant by magnetic fluid than any other source of matter at an early phase. But in the present epoch, the DE dominates the universe. This may be attributed to the current accelerated expansion of the universe.

Conclusion

In the present models,as expected, the matter energy density and DE density remains positive. Therefore, some energy conditions are satisfied, which in turn imply that the derived models are physically realistic.

Moreover, it is showed that, there remain pressure anisotropies even at the late phase of cosmic evolution, though in the early phase pressure was assumed to be isotropic.

From the present model, we have shown that, magnetic field has substantial effect on the dynamics of the universe. At an early epoch, the impact is somewhat larger but we can not ignore the effect of magnetic field at a later cosmic phase around the present epoch.

This work will appear in IJMPD

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Thank You