

Gravitational-wave astronomy in quantum gravity

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01/20– Quantum gravity (QG)

- QG is an umbrella name labeling theories attempting to quantize the gravitational force.
- Three main motivations: (1) **unify** the forces of Nature; (2) **resolve the singularities** of general relativity (big bang, black holes); (3) resolve the Λ problem.

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Dimensional argument: in homogeneous cosmology, with only the Planck scale ℓ_{Pl} and the Hubble scale H available, **perturbative** quantum corrections are of the form

$$(\ell_{\text{Pl}}H)^n \stackrel{\text{today}}{\equiv} (\ell_{\text{Pl}}H_0)^n \sim (10^{-60})^n, \quad n = 1, 2, 3, \dots$$

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... Hold on ...

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QG and **GWs**:

- Growing literature, but little or no knowledge about non-perturbative effects.
- Any imprint in GW production or propagation? Which theories?



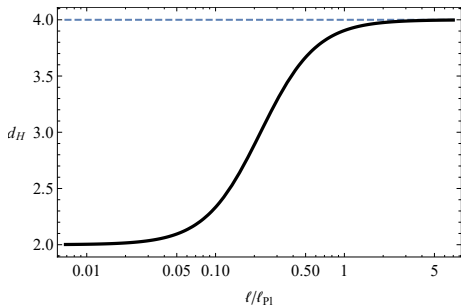
All theories of quantum gravity have something in common:

- **Dimensional flow**: Changing behaviour of correlation functions, spacetime with scale-dependent dimension d . $d < 4$ in the UV. **Universal** feature in QG [’t Hooft 1993; Carlip 2009; G.C. PRL 2010; Carlip 2017].
- **Fuzziness**: intrinsic uncertainty in measurements of times and distances.

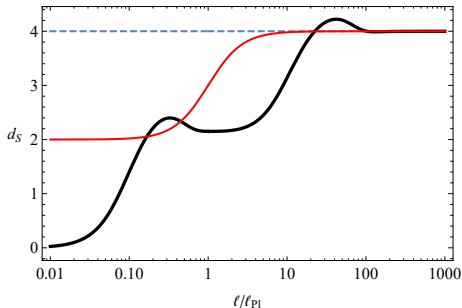
06/20– Dimensional flow

Universal **non-perturbative** effect (present at all scales): running **Hausdorff** and **spectral** dimensions.

d_H : scaling of volume



d_S : scaling of dispersion rel.



07/20- Effective dynamics in QG

Assumptions: continuum limit, effective action

$$S = \frac{1}{2} \int d\varrho(x) \varphi \mathcal{K} \varphi, \quad [\varrho] = -d_{\text{H}}, \quad [\varphi] = \frac{d_{\text{H}} - [\mathcal{K}]}{2}$$

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Return probability and spectral dimension

$$\mathcal{P}(\sigma) \propto \int d\tilde{\varrho}(k) e^{-\sigma \ell_*^{[\mathcal{K}]} \mathcal{K}(-k^2)}, \quad d_{\text{S}} := -2 \frac{d \ln \mathcal{P}(\sigma)}{d \ln \sigma} = 2 \frac{d_{\text{H}}^k}{[\mathcal{K}]}$$

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Anomalous scaling

$$\Gamma = \frac{d_{\text{H}}}{2} - \frac{d_{\text{H}}^k}{d_{\text{S}}}, \quad d_{\text{S}} \neq 0$$

08/20– UV geometry in QG

	d_H^{UV}	$d_H^{k,\text{UV}}$	d_S^{UV}	Γ_{UV}	$\Gamma_{\text{meso}} \gtrsim 1$
GFT/spin foams/LQG	2	4	$1 \leq \cdot < 4$	$-3 \leq \cdot < 0$	✓
Causal dynamical triangulations (phase C)	4	4	$3/2$	$-2/3$	
κ -Minkowski bicovariant ∇^2 (c.i.m.)	1	3	3	$-1/2$	
κ -Minkowski bicross-product ∇^2 (c.i.m.)	1	3	6	0	
Stelle gravit	4	4	2	0	
String theory (low-energy limit)	D	D	2	0	
Asymptotic safety	4	4	2	0	
Hořava–Lifshitz gravity	4	4	2	0	
κ -Minkowski relative-locality ∇^2 (c.i.m.)	1	3	$+\infty$	$1/2$	
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κ -Minkowski bicross-product ∇^2 (o.m.)	4	3	6	$3/2$	✓
κ -Minkowski relative-locality ∇^2 (o.m.)	4	3	$+\infty$	2	✓
Padmanabhan's non-local model	4	4	$+\infty$	2	✓

Strategy:

- 1 Concentrate on GW **propagation** (simpler than **production**).
- 2 **Model-independent relation** between **luminosity distance** d_L and **GW amplitude** h .
- 3 Use **standard sirens** to place **model-independent constraints** on this relation.
- 4 Apply them to **specific QG theories**.

10/20– Luminosity distance d_L^{EM}

Flux = power per unit area

$$F =: \frac{L}{4\pi(d_L^{\text{EM}})^2}$$

Proper distance $r = \tau_0 - \tau(z)$, redshift $1 + z = a_0/a$.

$$d_L^{\text{EM}} = \frac{a_0^2}{a} r = (1+z) \int_{t(z)}^{t_0} \frac{dt}{a} = (1+z) \int_0^z \frac{dz'}{H(z')}$$

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Possible modifications in QG: integration measure (**distances**) and/or Hubble parameter (**dynamics**).

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Local wave zone $r \gg s \sim \lambda \sim \frac{1}{\omega}$: leading term of a $\omega r \gg 1$ expansion of $h(t, r)$.

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Scaling argument from the retarded Green function:

$$[h/\kappa] = \Gamma \quad \Rightarrow \quad h \simeq \frac{\kappa \mathcal{F}(t, r)}{(r^2)^{\Gamma/2}} \stackrel{\Gamma=1}{\propto} \frac{1}{r}.$$

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Cosmological propagation (flat FLRW): $r \rightarrow ar$ rule from covariance:

$$h \propto \frac{1}{(d_L^{\text{EM}})^{\Gamma}} \quad \Rightarrow \quad h^{\text{UV}} \simeq \frac{1}{(d_L^{\text{EM}})^{\Gamma_{\text{UV}}}}, \quad h^{\text{IR}} \simeq \frac{1}{d_L^{\text{EM}}}$$

12/20– Master formula

$$h \propto \frac{1}{d_L^{\text{GW}}}, \quad d_L^{\text{GW}} = d_L^{\text{EM}} \left[1 \pm |\gamma - 1| \left(\frac{d_L^{\text{EM}}}{l_*} \right)^{\gamma-1} \right]$$

12/20– Master formula

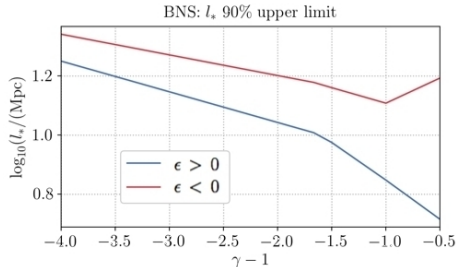
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Very similar to models with extra dimensions [Deffayet, Menou 2007; Pardo et al. 2018; Abbott et al. 2018]

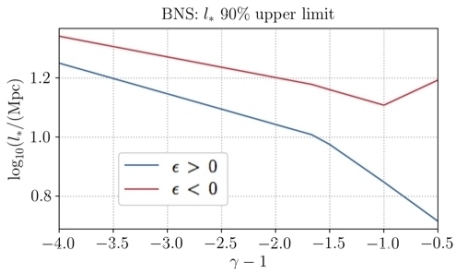
$$h \propto \frac{1}{d_L^{\text{GW}}}, \quad d_L^{\text{GW}} = d_L^{\text{EM}} \left[1 + \left(\frac{d_L^{\text{EM}}}{R_c} \right)^{n_c} \right]^{\frac{D-4}{2n_c}}$$

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$\gamma = \Gamma_{\text{UV}} < 1$: UV regime of QG unobservable.

14/20– QG and standard sirens. 2

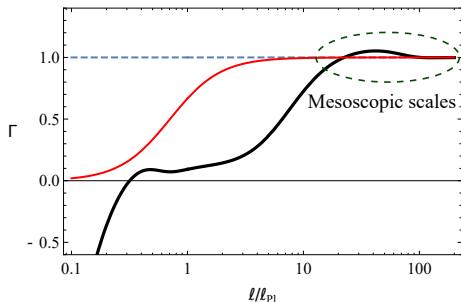
GW interferometers: $\Delta d_L/d_L \sim 0.01 - 0.2$. Detectable QG effect if $\gamma \gtrsim 1$, even when $\ell_* = O(\ell_{\text{Pl}})$:

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Theories where $\gamma = \Gamma_{\text{meso}} \gtrsim 1$:

- Non-commutative κ -Minkowski spacetime

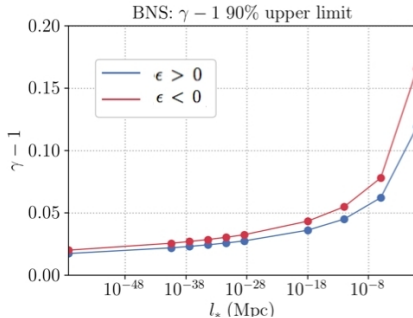
$$\Gamma_{\text{meso}} \simeq 1 + \frac{5}{96\pi} \frac{\ell_{\text{Pl}}^2}{\ell^2} \sim 1 + 10^{-120}.$$

- Padmanabhan's model near BH horizon

$$\Gamma_{\text{meso}} \simeq 1 + \frac{5\pi}{2} \frac{\ell_{\text{Pl}}^2}{\ell^2} \sim 1 + 10^{-120}.$$

- QGs with discrete pre-geometries: GFT, spin foams, LQG.

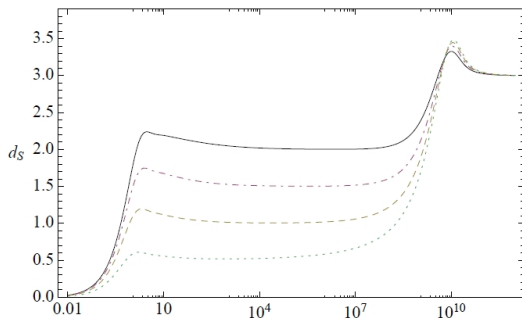
Γ_{meso} **strongly state dependent.**



$$\Gamma_{\text{meso}} - 1 < 0.02, \quad d_S^{\text{meso}} - 4 < 8\%$$

17/20– Future work: testing GFT/spin foams/LQG

Numerical analysis of dimensional flow [G.C., Oriti, Thürigen 2013,2014,2015]



Γ_{meso} can be calculated from realistic quantum states of geometry.

18/20– Excluding some LQC semi-classical states

Effective-dynamics LQC. Quantum corrections δ_{PI} can be large
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Mukhanov equation ($w_k := a_{\text{LQC}} h_k$):

$$w_k'' + \left[(1 + 2\alpha_0 \delta_{\text{Pl}}) k^2 - \frac{a_{\text{LQC}}''}{a_{\text{LQC}}} \right] w_k = 0, \quad a_{\text{LQC}} := a \left(1 - \frac{\alpha_0}{2} \delta_{\text{Pl}} \right)$$

$$\frac{d_L^{\text{GW}}(z)}{d_L^{\text{EM}}(z)} = \frac{a}{a_{\text{LQC}}} \simeq 1 + \frac{1}{2} \alpha_0 \delta_{\text{Pl}}$$

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But the correction is constrained to be too small from the
 GW170817 + GRB170817A bound on propagation speed:

$$\alpha_0 \delta_{\text{Pl}} < 10^{-15}.$$

19/20– Strain noise and QG

Uncertainty/fuzziness at Planck scales $\ell_* = \ell_{\text{Pl}}$ (intrinsic QG noise) no greater than the strain noise:

	S ($\text{Hz}^{-1/2}$)	f (Hz)	α
LIGO/Virgo/KAGRA	10^{-23}	10^2	< 0.47
LISA	10^{-20}	10^{-2}	< 0.54
DECIGO	10^{-23}	10^{-1}	< 0.47

Model-independent bound on the small-scale Hausdorff dimension of spacetime:

$$d_{\text{H}}^{\text{UV}} < 1.9.$$

- Different QGs have common non-perturbative effects present at all scales (dimensional flow).
- In most theories, dimensional flow is not observable in standard-sirens observations. Possible exception: [GFT/spin foams/LQG](#).
- [Model-independent bounds](#) are possible and go beyond back-of-the-envelope arguments about quantum corrections.
- Future work?
 - LQC in hybrid quantization (avoids speed bound);
 - full GFT/spin foams/LQG states of quantum geometry;
 - production of GWs in QG.

ご清聴ありがとうございました

Thank you

Muchas gracias

Grazie

Muito obrigado

Kiitos paljon

Danke schön

Merçi beaucoup

Спасибо