Shear viscosity bound violation in coupled SYK islands

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XHG，S.K.Jian(Tsinghua U)，Y.L.Wang (SHU), Z. Xian(ITP) and H. Yao(Tsinghua U),
arXiv:arXiv:1810.00669
W.H. Cai(SHU), XHG, GH. Yang(SHU), JHEP 1801 (2018) 076, arXiv:1711.07903

2019.02.15@YITP, Kyoto, Janpan

Outline

- Introduction and motivation
- Shear viscosity in holographic fluids: rotational and translational symmetric systems
- Shear viscosity in holographic solids: symmetries broken systems

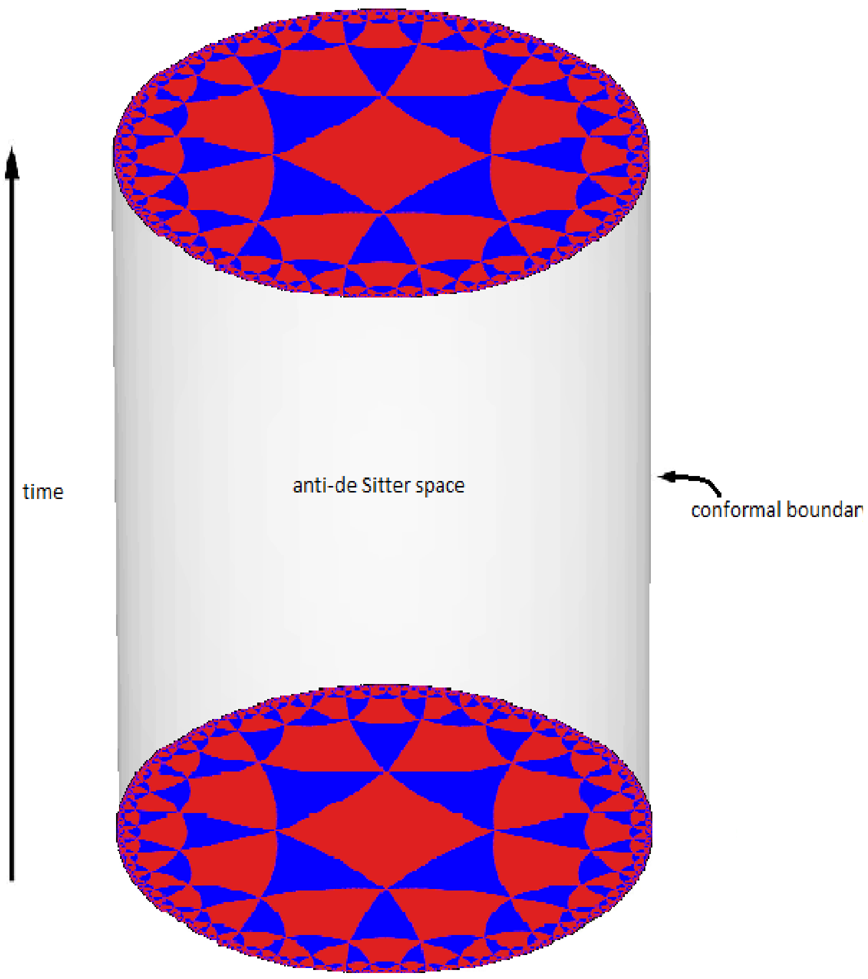
on Gravity side

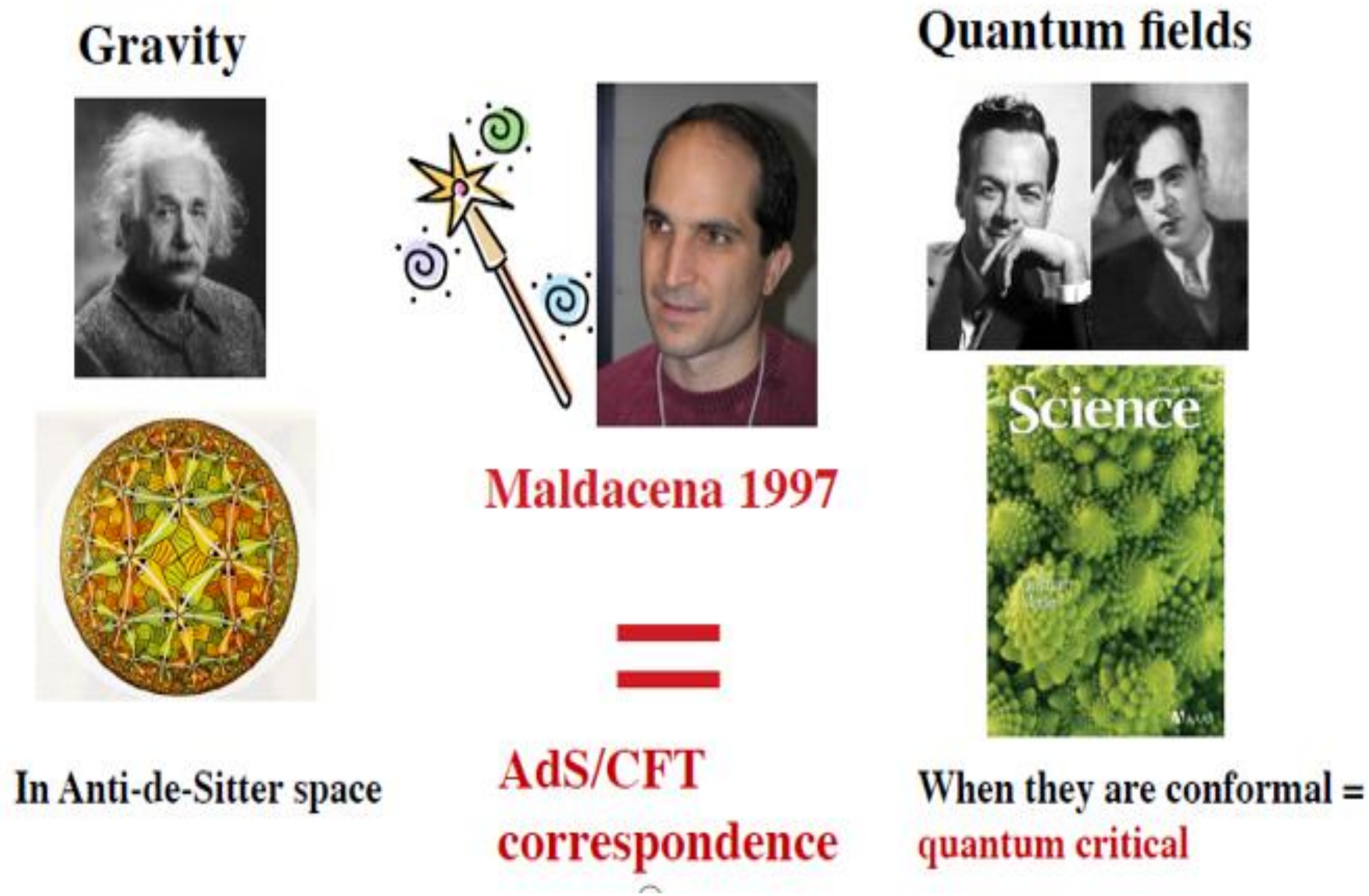
- Shear viscosity in quantum matters
- Shear viscosity in SYK islands (i.e. trnaslatinally invariant non-Fermi liquids)

on field theory side

- Discussion and conclusion

1.Gauge /gravity duality





Gravity

In Anti-de-Sitter space

Maldacena 1997

=

AdS/CFT correspondence

Quantum fields

When they are conformal = quantum critical

Useful tools for understanding strongly coupled quantum systems

AdS/CFT duality

Strongly coupled field theory

Black holes in AdS

Thermodynamics

Temp
Energy
Entropy

Black hole thermodynamics

Hawking temp
BH mass
BH entropy

- BH has the notion of temperature due to Hawking radiation.
- BH satisfies thermodynamic-like laws (0th-3rd)

Gravity/Fluid duality and shear viscosity bound

- Fluid mechanics is among the oldest and the most fundamental subjects in physics.
- A fundamental characterization of fluids is the shear viscosity that measures the resistance of a fluid to the shear stress.
- How small the shear viscosity can be ?
- Kovtun-Son-Starinets conjecture →

$$\boxed{\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}.}$$

- How universal is the KSS conjecture?
- How about KSS conjecture in SYK models?

on Gravity side

2. Shear viscosity in holographic fluids with rotational and translational symmetries

According to AdS/CFT

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

KSS bound

Some examples:

1. Conformal plasma (N=4 Super-YM) *Policastro, Son, Starinets*, hep-th/0104066
2. Nonconformal plasma, *Kovtun, Son, Starinets,* hep-th/0309213
3. Plasmas at finite chemical potential

Son, Starinets, hep-th/0601157; Saremi, 0601159;Maeda, et. al 0602010; Qun Wang, 2009……

J. W. Chen, Mei Huang, Y.H. Li, E. Nakano,D. L. Yang, Phys.Lett.B670:18-21,2008.

V. Hubney et al. 2007

….

Einstein gravity+symmetric matter fields

KSS bound satisfied

KSS bound in higher curvature gravity

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{2\lambda}{D-3}[(D-1)-(D-3)a]\right).$$

KSS bound is slightly violated

Kats et al 2008; H. Liu; N. Ohta , PRD 2008; R.G. Cai 2009; ...

XHG, Sang-Jin Sin, 2008,2009

For tensor type perturbation, the D-dimensional Gauss-Bonnet gravity coupling

$$\lambda_{\rm causality} \leq \frac{D^4 - 10D^3 + 41D^2 - 92D + 96}{4(D^2 - 5D + 10)^2}.$$

$$\lambda = (D-3)(D-4)\alpha'/l^2,$$

R. G. Cai, Z. Y. Nie, N. Ohta, and Y. W. Sun,
Phys. Rev. D 79 (2009) 066004;

XHG， S.J. Sin JHEP 05 051 2009

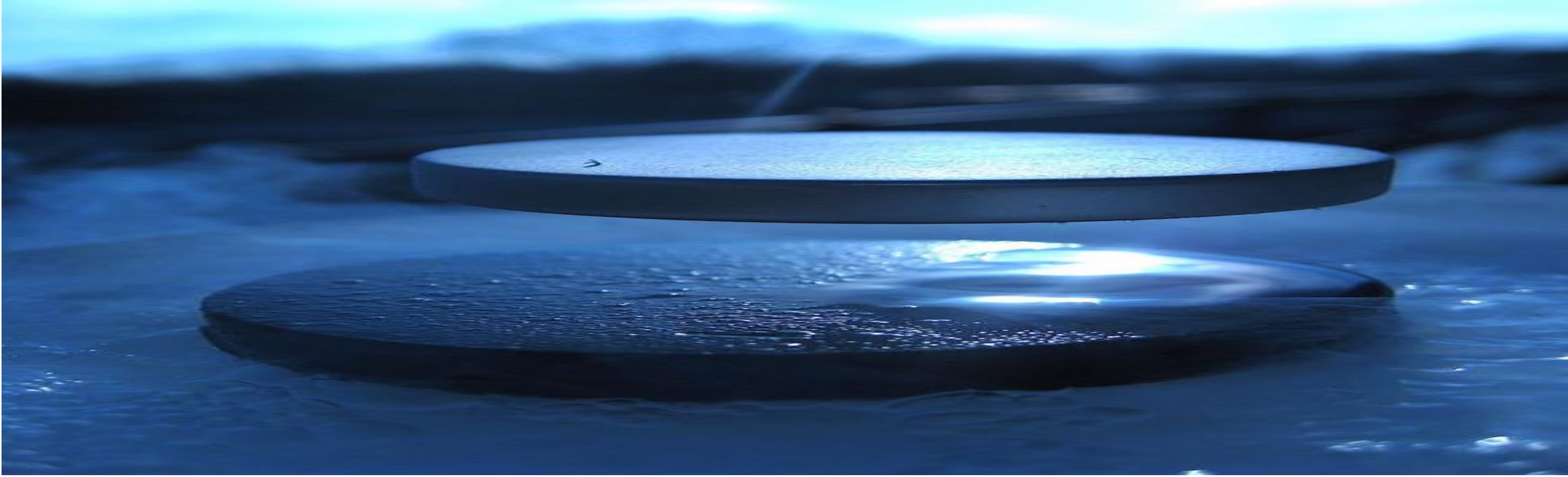
For more general cases: scalar type, vector type; GB and Lovelock gravity

$$-\frac{(D-3)(3D-1)}{4(D+1)^2} \leq \lambda \leq \frac{(D-3)(D-4)(D^2-3D+8)}{4(D^2-5D+10)^2}.$$

Bechul, Myers, JHEP 2009

on Gravity side

3. Shear viscosity in holographic solids: rotational or translational symmetries broken



3.1 Anisotropic black brane: momentum dissipation along z-direction

- Anisotropic black brane solution: prolate and oblate

Type IIB supergravity in Einstein frame

$$\mathcal{L} = \hat{R} * 1 - \frac{1}{2} d\hat{\phi} \wedge * d\hat{\phi} - \frac{1}{2} e^{2\hat{\phi}} \hat{F}_1 \wedge * \hat{F}_1 - \frac{1}{4} \hat{F}_5 \wedge * \hat{F}_5,$$

One can reduce the ten-dimensional supergravity action on five-dimensional spherical internal space S^5 by using the non-linear Kaluza-Klein reduction (Bremer, Duff, H. Lu et al arxiv: 9807051)

5-dimensional effective action

$$\mathcal{L} = \sqrt{-g}\left(R + 12 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2} e^{2\phi}(\partial\chi)^2 - \tfrac{1}{4} F_2^2\right)$$

L. Cheng, XHG, SJ. Sin JHEP 1407 (2014) 083; Phys.Lett. B734 (2014) 116-121
XHG, Y. Ling, SJ. Sin, Phys.Rev. D92 (2015) no.10, 106005

- The metric is assumed to be

$$ds^2 = \frac{e^{-\frac{1}{2}\phi}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx^2 + dy^2 + \mathcal{H}dz^2 + \frac{du^2}{\mathcal{F}}\right),$$

Anisotropic direction

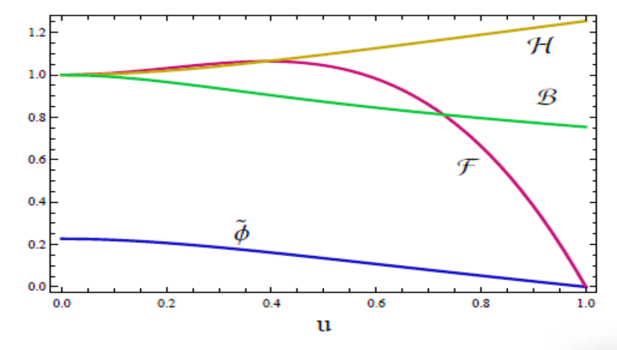
$$\phi = \phi(u), \quad A_t = A_t(u)\,, \qquad \chi = az$$

The asymptotical AdS_5 boundary conditions requires:

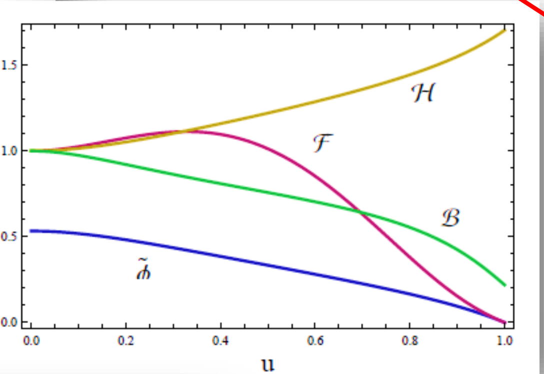
$$\phi(0) = 0, \quad \mathcal{F}(0) = \mathcal{B}(0) = 1, \quad \mathcal{H}(0) = 1.$$

The horizon is defined by u= u_H at which F(u_H)=0

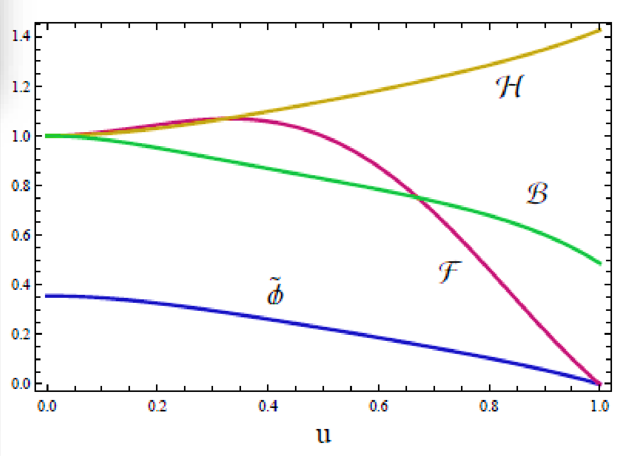
- Numerical solution



Charge increases



$a/T = 4.96, 17.73, 59.75$



- Analytic solution

$$\mathcal{F} = 1 - \left(\frac{u}{u_H}\right)^4 + \left[\left(\frac{u}{u_H}\right)^6 - \left(\frac{u}{u_H}\right)^4\right] q^2 + a^2 \mathcal{F}_2(u) + \mathcal{O}(a^4),$$

$$\mathcal{B} = 1 + a^2 \mathcal{B}_2(u) + \mathcal{O}(a^4),$$

$$\mathcal{H} = e^{-\phi(u)}, \quad \text{with} \quad \phi(u) = a^2 \phi_2(u) + \mathcal{O}(a^4),$$

$$q = \frac{u_H^3 Q}{2\sqrt{3}}$$

$$\begin{aligned}\mathcal{F}_2(u) = \frac{1}{24\sqrt{1+4q^2}u_{\mathrm{H}}^4}\Bigg\{ & 3(-4q^2u^6 + u_{\mathrm{H}}^6)\log\left(\frac{(1+\sqrt{1+4q^2})u^2 + 2u_{\mathrm{H}}^2}{(1-\sqrt{1+4q^2})u^2 + 2u_{\mathrm{H}}^2}\right) \\ & + u^2u_{\mathrm{H}}^2\Big[8\sqrt{1+4q^2}(-u^2 + u_{\mathrm{H}}^2) + u^2\Big(3\log\left(-2 - 2q^2 + 2\sqrt{1+4q^2}\right) \\ & + 5(-2+q^2)\log\left(-1 + 2q^2 + \sqrt{1+4q^2}\right) - 12q^2\log\left(-2 - 2q^2 + 2\sqrt{1+4q^2}\right) \\ & + 7(1+q^2)\Big(\log\left((-1 + 2q^2 - \sqrt{1+4q^2})(2q^2u^2 + (-1 + \sqrt{1+4q^2})u_{\mathrm{H}}^2)\right) \\ & - \log\left(2q^2u^2 - (1 + \sqrt{1+4q^2})u_{\mathrm{H}}^2\right)\Big)\Big)\Big]\Bigg\},\end{aligned}$$

Metric functions

$$\phi_2(u) = \frac{u_{\rm H}^2}{4\sqrt{1+4q^2}} \log\left(\frac{(1+\sqrt{1+4q^2})u^2 + 2u_{\rm H}^2}{(1-\sqrt{1+4q^2})u^2 + 2u_{\rm H}^2}\right),$$

$$\mathcal{B}_2(u) = \frac{u_{\rm H}^2}{24}\left(\frac{10u^2u_{\rm H}^2}{q^2u^4 - u^2u_{\rm H}^2 - u_{\rm H}^4} + \frac{1}{\sqrt{1+4q^2}} \log\left(\frac{(1+\sqrt{1+4q^2})u^2 + 2u_{\rm H}^2}{(1-\sqrt{1+4q^2})u^2 + 2u_{\rm H}^2}\right)\right).$$

η/s

In an anisotropic fluid with axial symmetry, there are two kind of shear viscosity: transverse and longitudinal shear viscosities

- Tensor perturbation $h_{xy}(t,z,u)$

$$0 = h_y^{x\prime\prime} - \frac{3}{u}h_y^{x\prime} + \frac{1}{2}\frac{\mathcal{H}'}{\mathcal{H}}h_y^{x\prime} + \frac{\mathcal{F}'}{\mathcal{F}}h_y^{x\prime} - \frac{3}{4}\phi' h_y^{x\prime} + \frac{\mathcal{B}'}{2\mathcal{B}}h_y^{x\prime} - \frac{k_z^2 h_y^x}{\mathcal{F}\mathcal{H}} + \frac{\omega^2 h_y^x}{\mathcal{F}^2\mathcal{B}}.$$

- Transverse shear viscosity

$$\eta_{xy,\,xy} = \left(-\mathcal{N}^{ty}\mathcal{N}^{uy}\right)^{\frac{1}{2}}\Big|_{u=u_H} = \frac{e^{-\frac{5\phi_H}{4}}}{2\kappa^2 u_{\mathrm{H}}^3} = \frac{s}{4\pi}.$$

KSS bound satisfied

- Tensor type perturbation h_{zx}

$$0 = h_z^{x\prime\prime} - \frac{3}{u}h_z^{x\prime} - \frac{1}{2}\frac{\mathcal{H}'}{\mathcal{H}}h_z^{x\prime} + \frac{\mathcal{F}'}{\mathcal{F}}h_z^{x\prime} - \frac{3}{4}\phi' h_z^{x\prime} + \frac{\mathcal{B}'}{2\mathcal{B}}h_z^{x\prime} - \frac{k_y^2 h_z^x}{\mathcal{F}} + \frac{\omega^2 h_z^x}{\mathcal{F}^2\mathcal{B}}.$$

- Longitudinal Shear viscosity

KSS bound violated

$$\eta_{xz,\,xz} = \frac{s}{4\pi\mathcal{H}(u_{\mathrm{H}})} \rightarrow \frac{\eta_{xz,\,xz}}{s} \begin{cases} < \dfrac{1}{4\pi} & \text{prolate} \\ > \dfrac{1}{4\pi} & \text{oblate} \end{cases}$$

3.2 Viscosity to entropy density ratio: isotropic systems

Background: Gauss-Bonnet+linear axions

$$S = \frac{1}{2\kappa^2}\int_M d^5x\sqrt{-g}\left(R - 2\Lambda + \frac{\lambda}{2}\mathcal{L}_{GB} - \frac{1}{2}\sum_{i=1}^{3}(\partial\phi_i)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right),$$

$$\mathcal{L}_{GB} = \left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right).$$

E.O. M.

$$\nabla_\mu\nabla^\mu\phi_i = 0,$$

$$\nabla_\mu F^{\mu\nu} = 0,$$

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left(R + 12 + \frac{\lambda}{2}(R^2 - 4R_{\rho\sigma}R^{\rho\sigma} + R_{\lambda\rho\sigma\tau}R^{\lambda\rho\sigma\tau})\right) \\ +\frac{\lambda}{2}\left(2RR_{\mu\nu} - 4R_{\mu\rho}R_\nu{}^\rho - 4R_{\mu\rho\nu\sigma}R^{\rho\sigma} + 2R_{\mu\rho\sigma\lambda}R_\nu{}^{\rho\sigma\lambda}\right) \\ -\sum_{i=1}^{3}\left(\frac{1}{2}\partial_\mu\phi_i - \frac{g_{\mu\nu}}{4}(\partial\phi_i)^2\right) - \frac{1}{2}\left(F_{\mu\lambda}F_\nu{}^\lambda - \frac{g_{\mu\nu}}{4}F_{\lambda\rho}F^{\lambda\rho}\right) = 0. \end{aligned}$$

Y. Wang, XHG, Phys.Rev. D94 (2016) no.6, 066007

Viscosity to entropy density ratio

2.3 Low temperature expansion

$$T = \frac{\alpha}{\pi}\left(\frac{1}{\beta} - \frac{\beta}{8}\right) = 0.$$

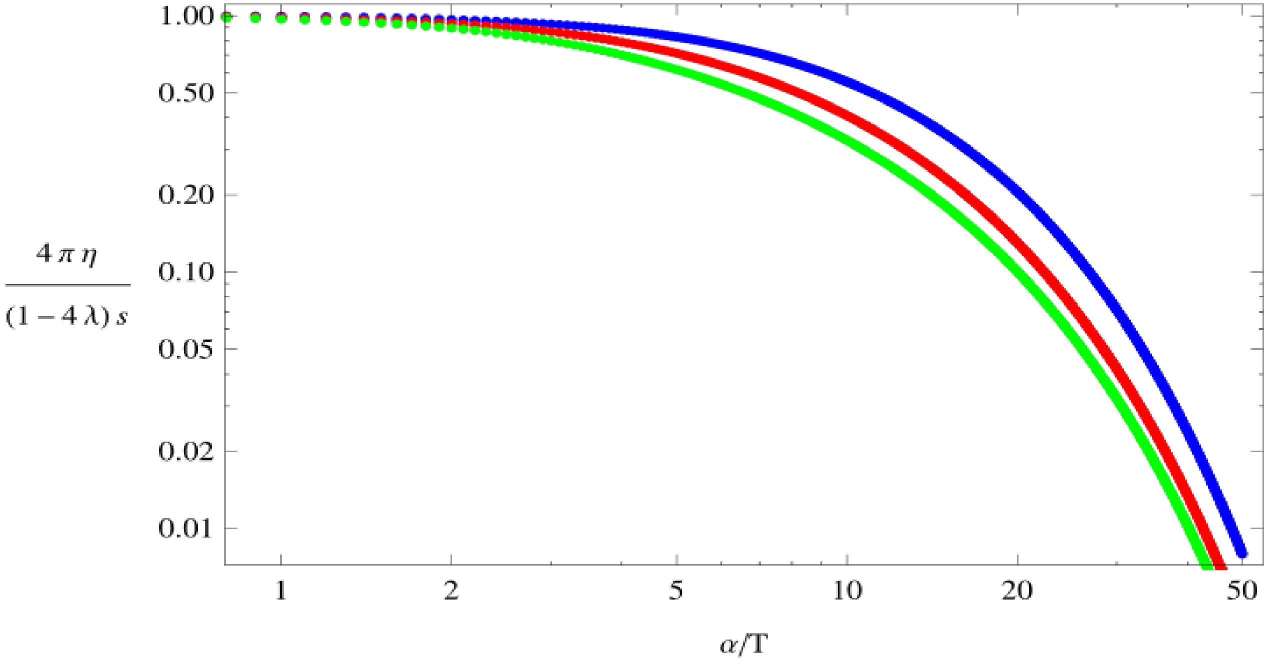
$$h(u) = \sum_{i=0}^{+\infty} \left(\beta^2 - 8\right)^i h_i(u).$$

$$4\pi\frac{\eta}{s} = 32\pi^2 \left(h_0'(1)\right)^2 \left(\frac{T}{\alpha}\right)^2$$

Viscosity to entropy density ratio at low temperature

2.4 Numerical results

As shown in Figure 2, we also give the chart in log-log plot for the numerical solutions of the equation in three dashed lines, with no less than 500 points on the each locus. The blue, red and green lines correspond to λ = −0.1,0.08,0.15 respectively.



Brief Summary

- In holographic solids, the KSS bound can be violated in a parameteric manner.

Einstein gravity+symmetric matter fields

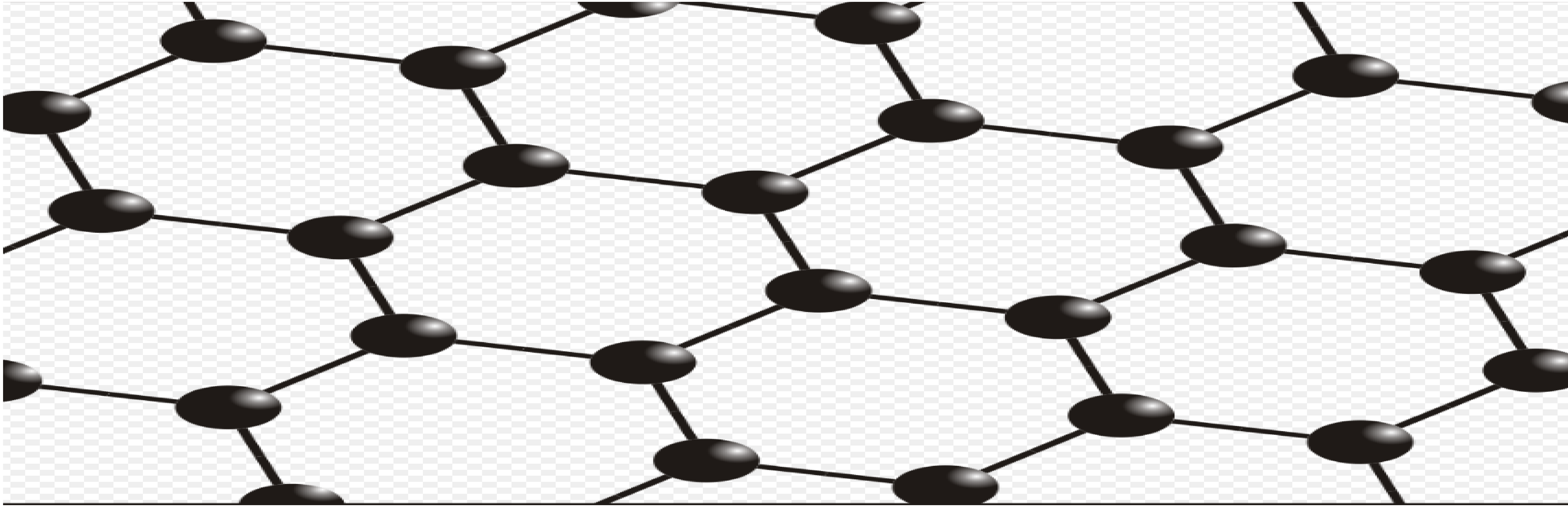
KSS bound satisfied

Einstein gravity+rotaional or translational symmetries broken matter fields or Massive gravity

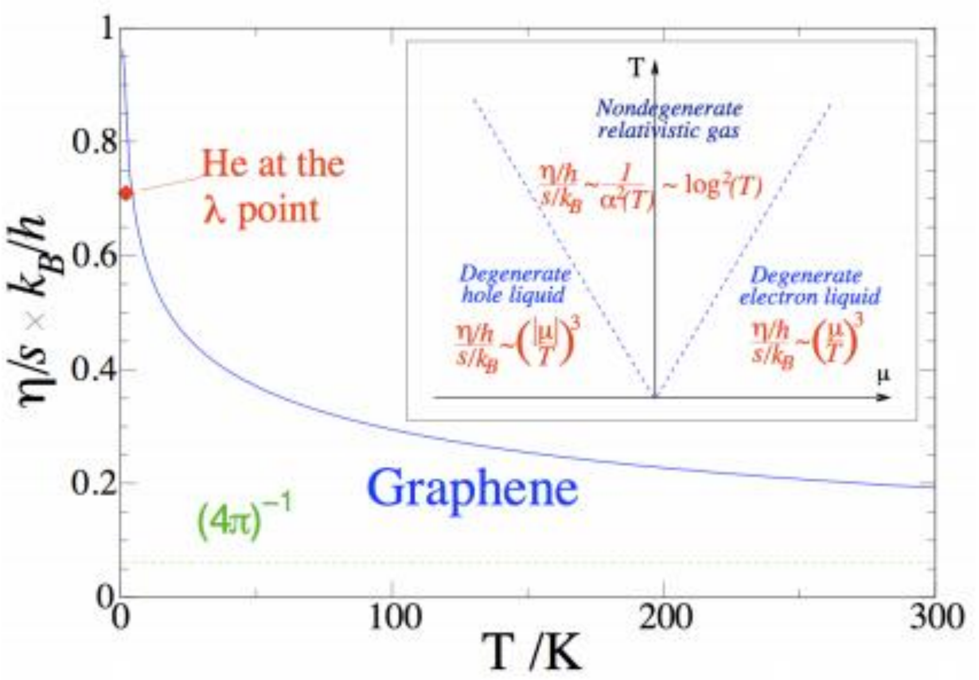
KSS bound severely violated

on field theory side

4. Shear viscosity in graphene, semimetals and unitary quantum gas



1). electron fluids in graphene



KSS bound satisfied

M. Muller, J. Schmalian, and L. Fritz,
Phys. Rev. Lett. 103, 025301 ,2009

2). Luttinger-Arikosov-Benelslavskii phase in quadratic semimetals

$$\frac{\eta}{s} = \frac{0.63}{\varepsilon^2}.$$

Setting $\varepsilon = 1$ gives $4\pi\eta/s = 8.0$

KSS bound satisfied

P. T. Dumitrescu, Phys. Rev. B 92, 121102 (2015)

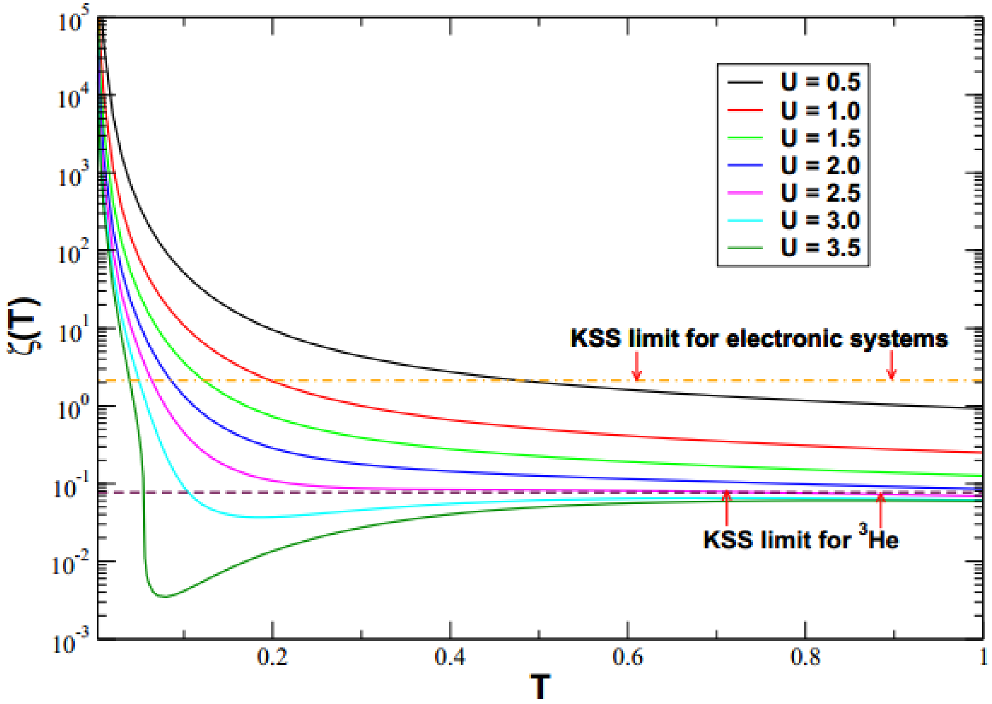
3). possible shear viscosity bound violation in unitary quantum gas

$$H = -t \sum_{\langle ij\rangle,\sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + \mathrm{H.c.}) - \mu \sum_{i,\sigma} n_{i\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

$$\zeta(T) \equiv \frac{\eta(T)}{s(T)} \frac{4\pi k_B}{\hbar} \left(\frac{m_b}{m}\right)^2$$

Dynamical mean field theory limit

$$H_{\mathrm{imp}} = \sum_{l,\sigma} (\tilde{\epsilon}_l - \mu) c^{\dagger}_{l\sigma} c_{l\sigma} + \sum_{l,\sigma} (V_l c^{\dagger}_{l\sigma} d_{0\sigma} + \mathrm{H.c.}) \\ -\mu \sum_{\sigma} n_{d0\sigma} + U n_{d0\uparrow} n_{d0\downarrow},$$

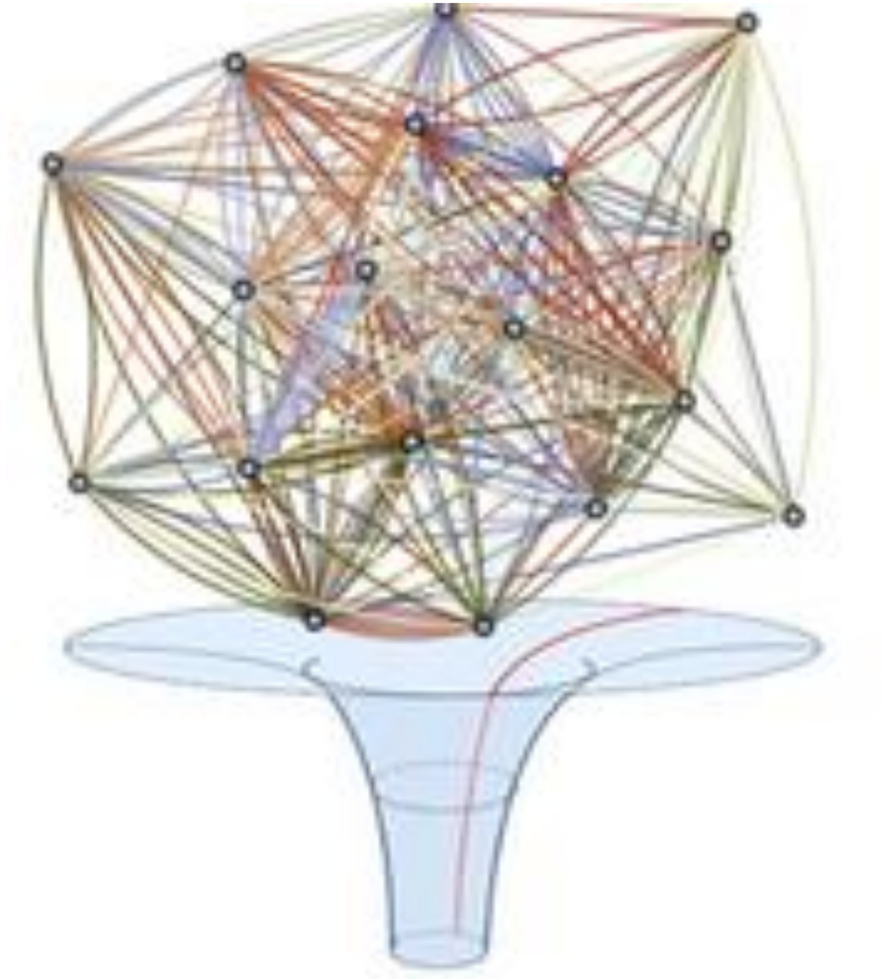


KSS bound may violated, but no gravity dual

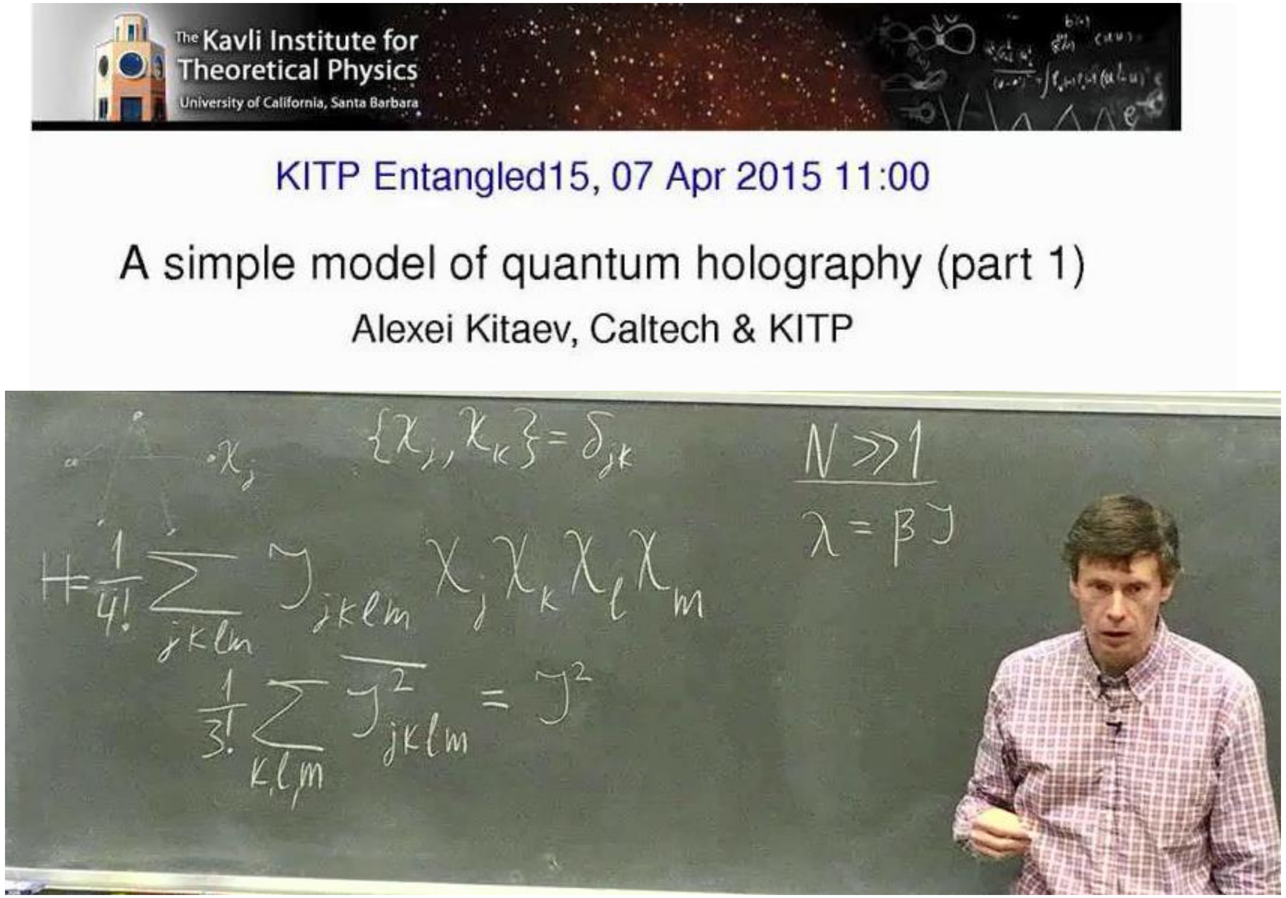
N. Pakhira and R. H. McKenzie, Phys. Rev. B 92, 125103, (2015).

on field theory side

5. Shear viscosity in coupled SYK Islands--translationally invariant non-Fermi liquid metals



Why Sachdev-Ye-Kitaev model ?



The Kavli Institute for Theoretical Physics
University of California, Santa Barbara

KITP Entangled15, 07 Apr 2015 11:00

A simple model of quantum holography (part 1)

Alexei Kitaev, Caltech & KITP

χ_j

$\{\chi_j, \chi_k\} = \delta_{jk}$

$N \gg 1$

$\lambda = \beta J$

$$H = \frac{1}{4!} \sum_{jklm} J_{jklm} \chi_j \chi_k \chi_l \chi_m$$

$$\frac{1}{3!} \sum_{k,l,m} \overline{J^2_{jklm}} = J^2$$

properties

- a strongly correlated quantum system at low energy
- solvable at large N limit ($N \gg \beta J \gg 1$)
- quantum chaos (out-of-time correlations)
- connection of SYK to the AdS_2 horizon
  - an emergent conformal symmetry (reparametrization), which is spontaneously broken to the isometry group of $AdS_2 : SL(2, \mathbb{R})$

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1$$

  - the effective action

$$SYK \Rightarrow S_{\text{eff,4d,fluc,Schwarzian}} \Leftarrow boundary(AdS_2)$$

chaos

The wavefunction is scrambling by an initial perturbation.
The growth of chaos is characterized by Lyapunov exponent via out-of-time(OTO) correlation

$$exp\left(\frac{1}{\tau_L}(t-\frac{|x|}{v_B})\right)$$

bound in a quantum system

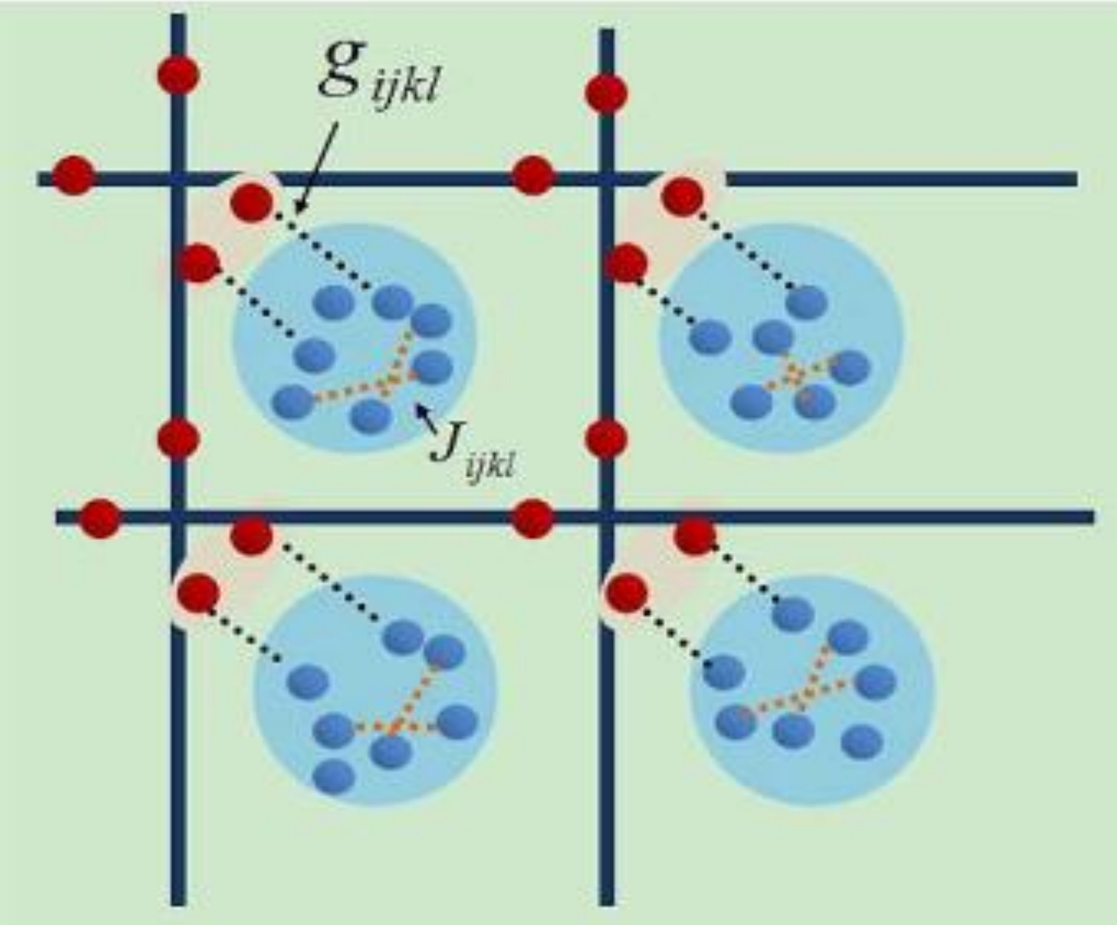
$$\tau_L \geq \frac{1}{2\pi}\frac{\hbar}{k_B T}$$

agrees with Einstein gravity

SYK is fastest possible. [Jensen (2016)]

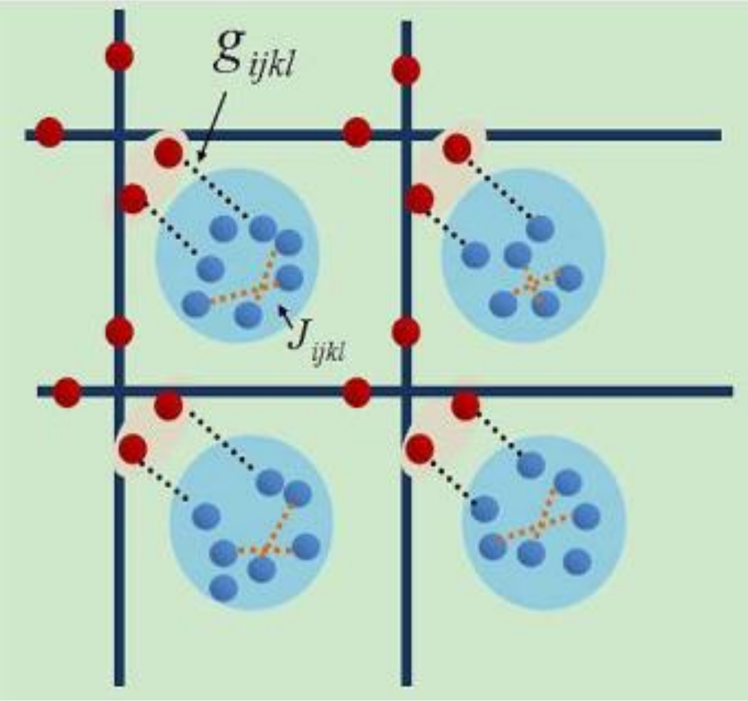
Our goal:

Is there a shear viscosity bound in a deformed SYK models, which is believed dual to a black hole in Einstein gravity?



A translationally invariant model

- M flavors of conduction electrons c, M-> ∞
- N flavors of valence electrons f, N-> ∞
- Each site is identical
- The localized f-electrons act as a background 'bath'



$$\begin{aligned} H &= -t \sum_{<rr'>;i=1}^{M} (c^\dagger_{ri} c_{r'i} + h.c.) - \mu_c \sum_{r;i=1}^{N} f^\dagger_{ri} f_{ri} \\ &+ \frac{1}{NM^{1/2}} \sum_{r;i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl} f^\dagger_{ri} f_{rj} c^\dagger_{rk} c_{rl} + \frac{1}{N^{3/2}} \sum_{r;i,j,k,l=1}^{N} J_{ijkl} f^\dagger_{ri} f^\dagger_{rj} f_{rk} f_{rl}. \end{aligned}$$

$$<< J_{ijkl} J_{lkij} >>= \frac{J^2}{8} \qquad << g_{ijkl} g_{lkij} >>= g^2$$

A. Patel, et al. 1712.05026
D. Chowdhury, et al 1801.06178

Disorder averaged action

$$S = \int_0^\beta d\tau \left[\sum_{r;\ i=1}^{M} c_{ri}^\dagger(\tau)(\partial_\tau - \mu_c)c_{ri}(\tau) + \boxed{t} \sum_{\langle rr' \rangle;\ i=1}^{M} (c_{ri}^\dagger(\tau)c_{r'i}(\tau) + \text{h.c.}) + \sum_{r;\ i=1}^{N} f_{ri}^\dagger(\tau)(\partial_\tau - \mu)f_{ri}(\tau') \right]$$

$$- M\boxed{\frac{g^2}{2}} \sum_r \int_0^\beta d\tau d\tau' G_r^c(\tau - \tau')G_r^c(\tau' - \tau)G_r(\tau - \tau')G_r(\tau' - \tau)$$

$$- N\frac{J^2}{4} \sum_r \int_0^\beta d\tau d\tau' G_r^2(\tau - \tau')G_r^2(\tau' - \tau) - N\sum_r \int_0^\beta d\tau d\tau' \Sigma_r(\tau - \tau') \left(G_r(\tau' - \tau) + \frac{1}{N}\sum_{i=1}^{N} f_{ri}^\dagger(\tau)f_{ri}(\tau') \right)$$

$$- M\sum_r \int_0^\beta d\tau d\tau' \Sigma_r^c(\tau - \tau') \left(G_r^c(\tau' - \tau) + \frac{1}{M}\sum_{i=1}^{M} c_{ri}^\dagger(\tau)c_{ri}(\tau') \right)$$

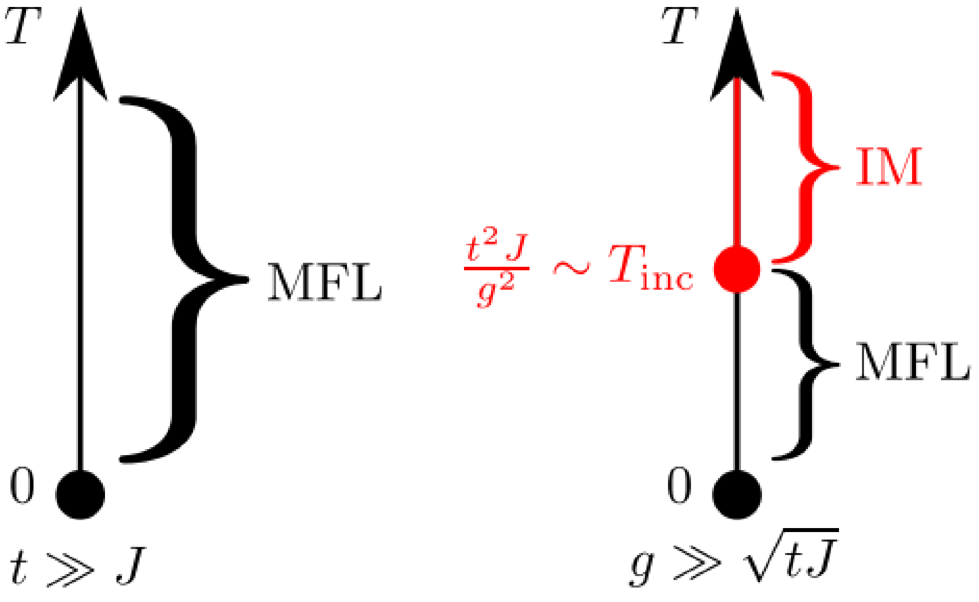
Two regimes:

parameter range for incoherent metal:

$$\mathrm{max}(\mathrm{T_{inc}}, \Lambda) \ll \mathrm{T} \ll \mathrm{J},$$

Entroy density for MFL: $\frac{g^2 M}{Jt^2}(T + T\ln\frac{J}{T})$

Entropy density for IM: $s = (N\frac{1}{J} + M\frac{J}{g^2})T$



In the large M, N limit, the saddle point equation

$$\Sigma(\tau-\tau')=-J^2G^2(\tau-\tau')G(\tau'-\tau)-\frac{M}{N}g^2G(\tau-\tau')G^c(\tau-\tau')G^c(\tau'-\tau)$$

$$\Sigma^c(\tau-\tau')=-g^2G^c(\tau-\tau')G(\tau-\tau')G(\tau'-\tau)$$

Green's function for f-fermions

$$G(i\omega_n)=\frac{1}{i\omega_n+\mu-\Sigma(i\omega_n)},$$

Green's function for c-fermions

$$G^c(i\omega_n)=\int\frac{d^dk}{(2\pi)^d}\frac{1}{i\omega_n-\epsilon_k+\mu_c-\Sigma^c(i\omega_n)}$$

Infinite bandwidth $t >> g, J$: marginal Fermi liquid regime

- The conduction electrons float on an effectively infinite deep Fermi sea
- In the M/N-->0, the Green's function of f-electrons is exactly of the incoherent form of the SYK model

$$G^f(\tau) = -\frac{\pi^{\frac{1}{4}} \cosh^{\frac{1}{4}}(2\pi\mathcal{E})}{J^{\frac{1}{2}}\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T\tau)}\right)^{\frac{1}{2}} e^{-2\pi\mathcal{E}T\tau}, \quad 0 \leq \tau \leq \beta.$$

- The Green's function of c-electrons

$$G^c(\mathbf{k}, i\omega) = \frac{1}{i\omega - \epsilon_k + \mu_c - \Sigma_{cf}(\mathbf{k}, i\omega)}.$$

$$= \nu(0)\int_{-\infty}^{+\infty} \frac{d\epsilon}{2\pi} \frac{1}{i\omega_n - \epsilon - \Sigma_{cf}(\mathbf{k}, i\omega)} = -\frac{i}{2}\nu(0)sgn(\omega).$$

$$G^c(\tau) = -\frac{\nu(0)T}{2\sin(\pi T\tau)}$$

- The self energy of c-electrons

$$\Sigma_{cf}(i\omega_n) = \frac{ig^2T}{2Jt\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\left(\frac{\omega_n}{T}\ln\left(\frac{2\pi Te^{\gamma_E-1}}{J}\right)+\frac{\omega_n}{T}\psi^{(0)}\left[0,-\frac{i\omega_n}{2\pi T}\right]+\pi\right);$$

- The free energy and entropy

$$F = \mathrm{Tr}[\log\, G_c^{-1}] + \mathrm{Tr}[\Sigma_{cf}\ G_c] + \mathrm{Tr}[\log\, G_f^{-1}] + \mathrm{Tr}[(\Sigma_f+\Sigma'_{cf})\ G_f] - \Phi_{\mathrm{LW}}[G_c, G_f].$$

$\Phi_{\mathrm{LW}}[G_c, G_f]$ is the Luttinger-Ward functional.

$$s^{MFL} \sim g^2MJ^{-1}t^{-2}(T+T\ln J/T)$$

M. Crisan and C. P. Moca, Journal of Superconductivity 9, 49 (1996)

Finite bandwidth: incoherent metal regime

- The Fermi energy and the bandwidth of the conduction electrons is comparable to the couplings

$$G^c(i\omega_n) \approx \frac{1}{2\pi(\mu_c - \Sigma_{cf}(i\omega_n))}. \qquad \mu_c - \Sigma_{cf} >> \omega \qquad T \gg T_{inc}$$

$$T_{inc} \equiv \frac{\Lambda^2 J}{g^2}$$

- Both c- and f-electrons are in fully incoherent metal regime

$$G^c(\tau) = -\frac{C_c}{\sqrt{1+e^{-4\pi\mathcal{E}_c}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E}_c T\tau}$$

- Self-energy

$$\Sigma_{cf}(i\omega_n) = \frac{iT^{\frac{1}{2}} g^2 \Lambda^{\frac{1}{2}} \nu^{\frac{1}{2}}(0)(-1)^{\frac{1}{4}}(1+e^{4\pi\mathcal{E}_c})^{\frac{1}{2}} e^{2\pi\mathcal{E}}}{\pi^{\frac{1}{4}} J^{\frac{1}{2}} 2^{\frac{3}{2}}(i+e^{2\pi\mathcal{E}_c})\cosh^{\frac{1}{4}}(2\pi\mathcal{E})} \frac{\Gamma(\frac{3}{4}+i\mathcal{E}_c+\frac{\omega_n}{2\pi T})}{\Gamma(\frac{1}{4}+i\mathcal{E}_c+\frac{\omega_n}{2\pi T})};$$

η/s in the MFL regime

- Shear viscosity

$$iG_R^{xy,xy}(\omega,\mathbf{p}) = \int dt d\mathbf{x} e^{i(\omega t - \mathbf{p}\cdot\mathbf{x})}\theta(t)\langle[T_{xy}(t,\mathbf{x}),T_{xy}(0,0)]\rangle.$$

$$\eta(T) = \frac{\pi}{a^d}\int_{-\infty}^{+\infty} d\omega \left(-\frac{\partial n_F(\omega}{\partial\omega}\right)\int_{-\infty}^{+\infty} d\epsilon\Theta_{xy}(\epsilon)A^2(\omega,\epsilon),$$

Θ_{xy} is the *transport density of states*

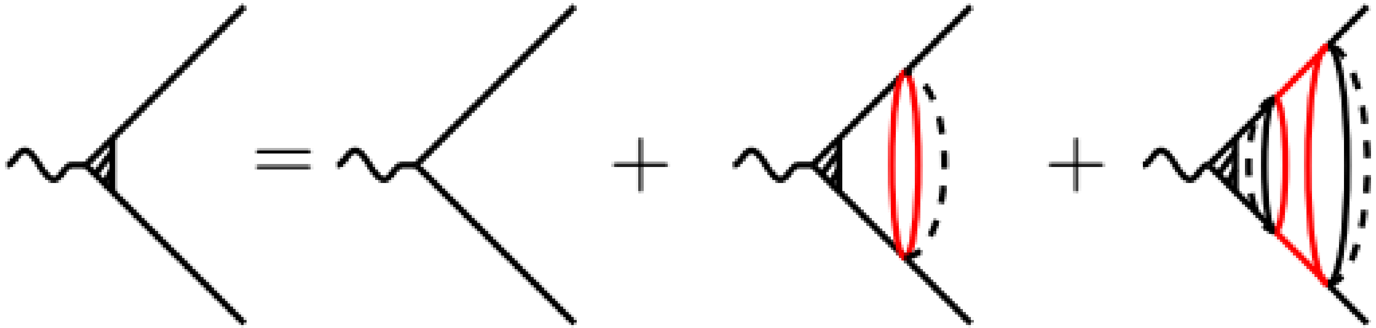
- Spectral function

$$A(\omega,\epsilon) = -\frac{1}{\pi}\mathfrak{Im}[G^c(i\omega_n \to \omega + i0^+)].$$

- For finite bandwidth, we considr the dispersion relation $\epsilon_k = \frac{k^2}{2m} - \frac{\Lambda}{2}$

both in the x and y directions

Vertex corrections



The ladder diagram shows the self-consistent equation for shear viscosity vertex. The black and red solid lines represent the Green′ s function of c fermions and f fermions, respectively. The dashed line represents disorder average and the shaded vertex represents full vertex.

The Feynman diagram for the calculation of <TxyTxy> at leading order in 1/N, where the vertex correction vanishes. The black lines represent the Green's function of c fermions.

- The shear viscosity

$$\eta_{\rm MFL}(T) = \frac{M\nu(0)}{64m^2T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\mathrm{sech}^2(\frac{\omega}{2T})\frac{1}{|\mathfrak{Im}\Sigma^{\rm MFL}_{cf}(\omega)|}$$

$$\approx 0.0300627\frac{Mt^2J}{g^2T}\cosh^{\frac{1}{2}}(2\pi\mathcal{E}).$$

$$\frac{\eta_{\rm MFL}}{\mathcal{S}^{\rm MFL}_c} \sim \cosh^{\frac{1}{2}}(2\pi\mathcal{E})\frac{J^2t^4}{g^4T^2\ln(\frac{J}{T})}.$$

KSS bound satisfied

In the zero temperature limit, eta/s becomes divergent.

- The ratio is larger than a constant

$$\eta_{\rm MFL}/\mathcal{S}^{\rm MFL}_c \gg 1/\ln(J/T_{\rm inc}) = 1/2\ln(g/t).$$ since $T \ll T_{inc},$

η/s in the IM regime

The ratio

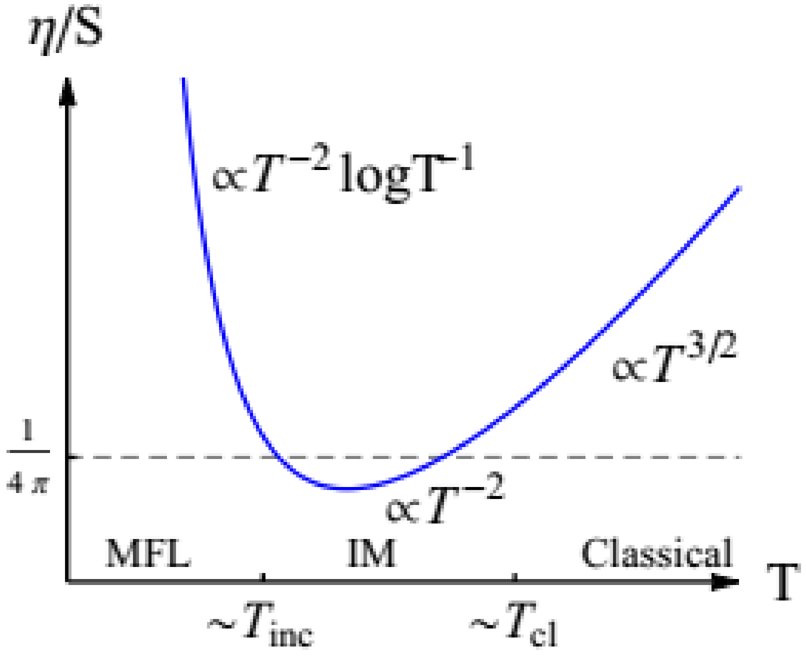
$$\eta_{\mathrm{IM}}(T) = \frac{M\pi^{\frac{1}{2}}}{24}\,\frac{\Lambda^2 J}{g^2 T}\,\frac{\cosh^{\frac{1}{2}}(2\pi\mathcal{E})}{\cosh(2\pi\mathcal{E}_c)}. \qquad \frac{\eta_{\mathrm{IM}}}{\mathcal{S}_c^{\mathrm{IM}}} \sim \frac{\cosh^{\frac{1}{2}}(2\pi\mathcal{E})}{\cosh(2\pi\mathcal{E}_c)}\,\frac{\Lambda^2}{T^2}. \longrightarrow 0.$$

Note that $\max(\mathrm{T}_{\mathrm{inc}}, \Lambda) \ll \mathrm{T} \ll \mathrm{J}$,

There exists a robust temperature window in the IM regime so that KSS bound violated in this regime.

KSS bound violated with a known gravity dual.
Translational invariant system.

The phase diagram



In local critical regime, the local interaction dominates over hoppings between different sites, and in turn dictates the scaling dimension of fermions. The local critical freedoms, i.e., the c fermions in our case, have scaling dimension 1/4.

The local criticality also renders the vertex correction vanishing, and leads to the spectral representation of shear viscosity

6. Discussion and outlook

- For a translationaly invariant non-Fermi liquid metals, the KSS bound is violated in the IM regime. But in the MFL regime, the KSS bound is satisfied as T->0.
- The violation of the KSS bound of the shear viscosity here is analogous to the deviation from the Mott-Ioffe-Regel limit in the incoherent metal regime.

$$\sigma = \frac{n\tau e^2}{m} \sim (k_F l) k_F^{d-2} \frac{e^2}{\hbar} \geq k_F^{d-2} \frac{e^2}{\hbar}$$

$$\eta = \frac{1}{5} n\hbar k_F l \Longrightarrow \quad \eta \geq n\hbar$$

- In Incoherent metal regime, the relation $k_F l \gg 1$ is broken
- Gravity dual of higher dimensional SYK models ? Einstein gravity+ symmetries?

Open questions

- With strong g, the relative particle number N_c - N_f symmetry can be spontaneously breaking such that the f fermions may contribute to the shear viscosity ?
- It seems that our results closely depend on the definition of the entropy density. If there is particle number symmetry breaking, do we need reconsidering the definition of the entropy density ?

Thank you for your attention !