Some aspects of f(R) gravity

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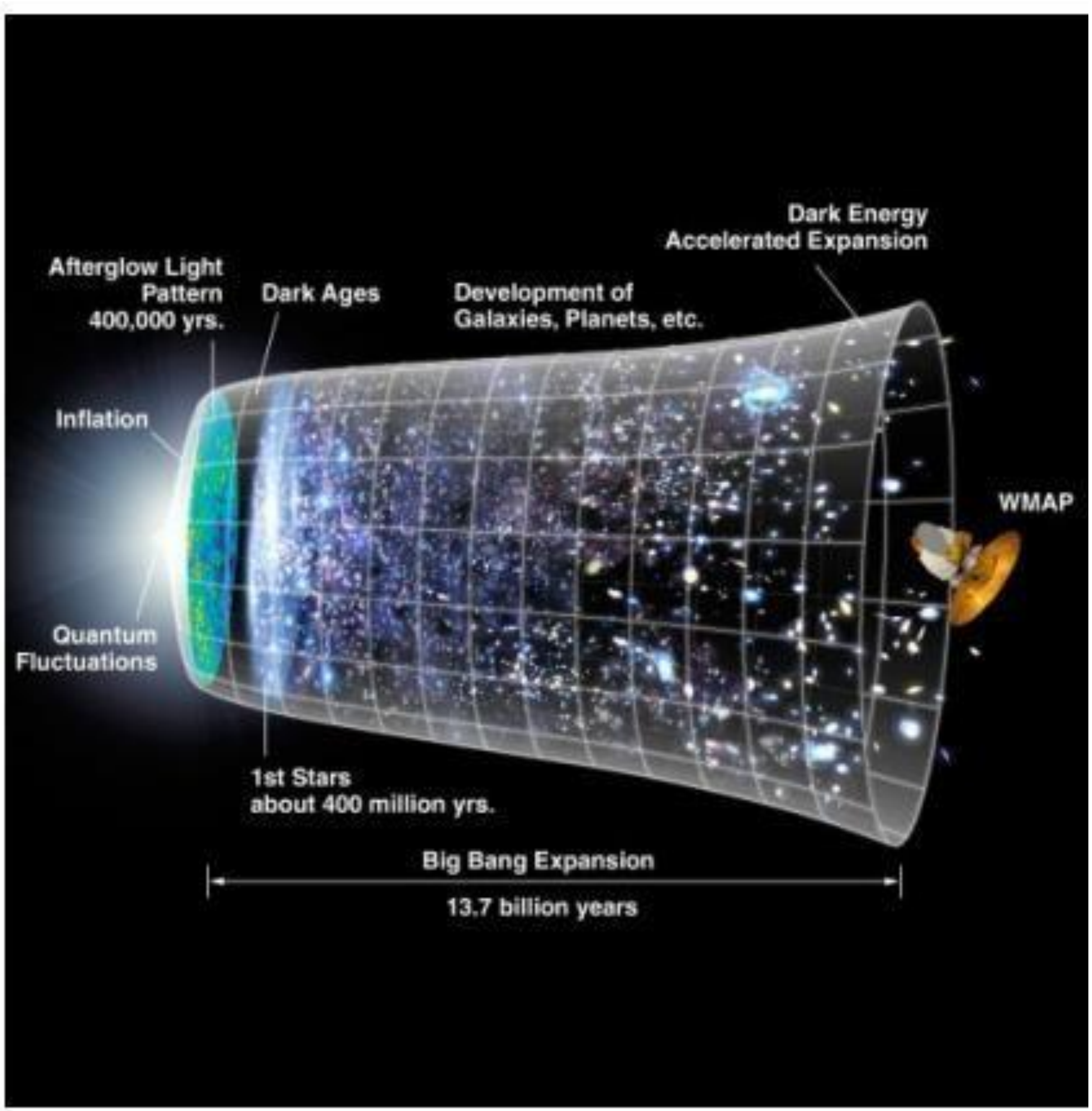
APWSW 2019 YITP, Kyoto

February 2019

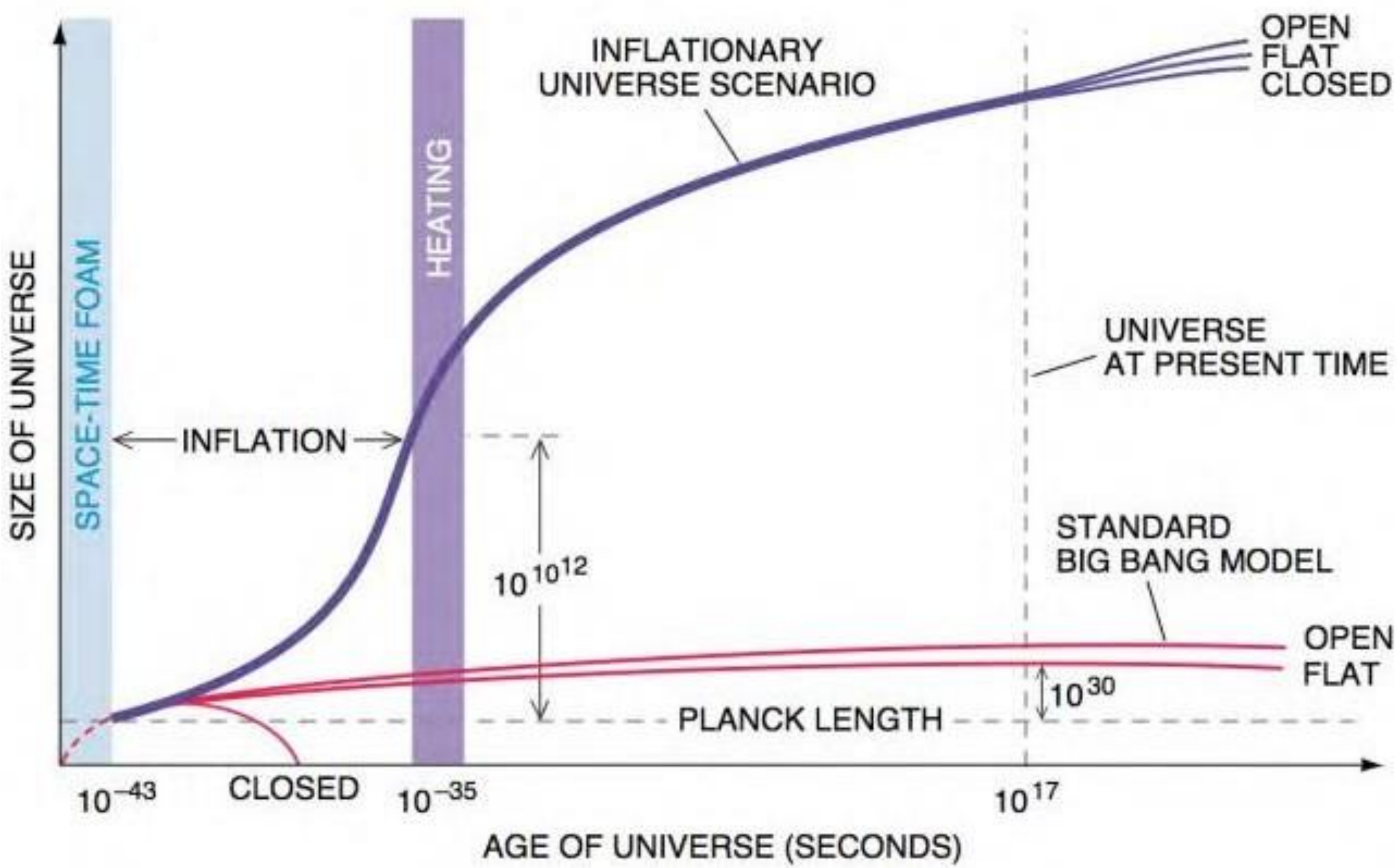
Plan

- Introduction
- The cosmic frontier
- f(R) connection?
- Outlook

History of Universe



Inflationary Universe



A Linde's talk

Big Bang

- Hubble expansion, light element abundance (BBN), leftover Black body radiation (CMB)
- What preceded Big Bang ?
- **DM/DE**, **Perturbation** evolution, **Inflation**
- Universe is homogeneous ?
- Isotropic Universe (same in all directions) ?
- Simultaneous expansion for all parts ?
- Universe is flat
- So many particle in Universe and it is so large

Cosmic Inflation

- Inflation solves some of the problems
- **Inflation: In fact provides an explanation for how the Universe could have been created out of matter less than one milligram**.
- Solves the issues like flatness, horizon and monopole problems.
- Simply a brilliant idea and of course surprising
- Expt. verification awaited…

Inflation

- The scalar field ϕ moves very slowly and that is why the potential energy essentially remains a constant for a fair amount of time.

$$\frac{\dot{a}}{a} = Const. = H \rightarrow a(t) = e^{Ht}$$

(this is termed as inflation)

Inflation and DE

- Inflation makes the Universe flat
- Adding a constant to the inflationary potential, one can get inflation as well as DE

$$V = \frac{1}{2}m^2\phi^2 + \Lambda$$

- (simplest model to explain Inflation and DE)

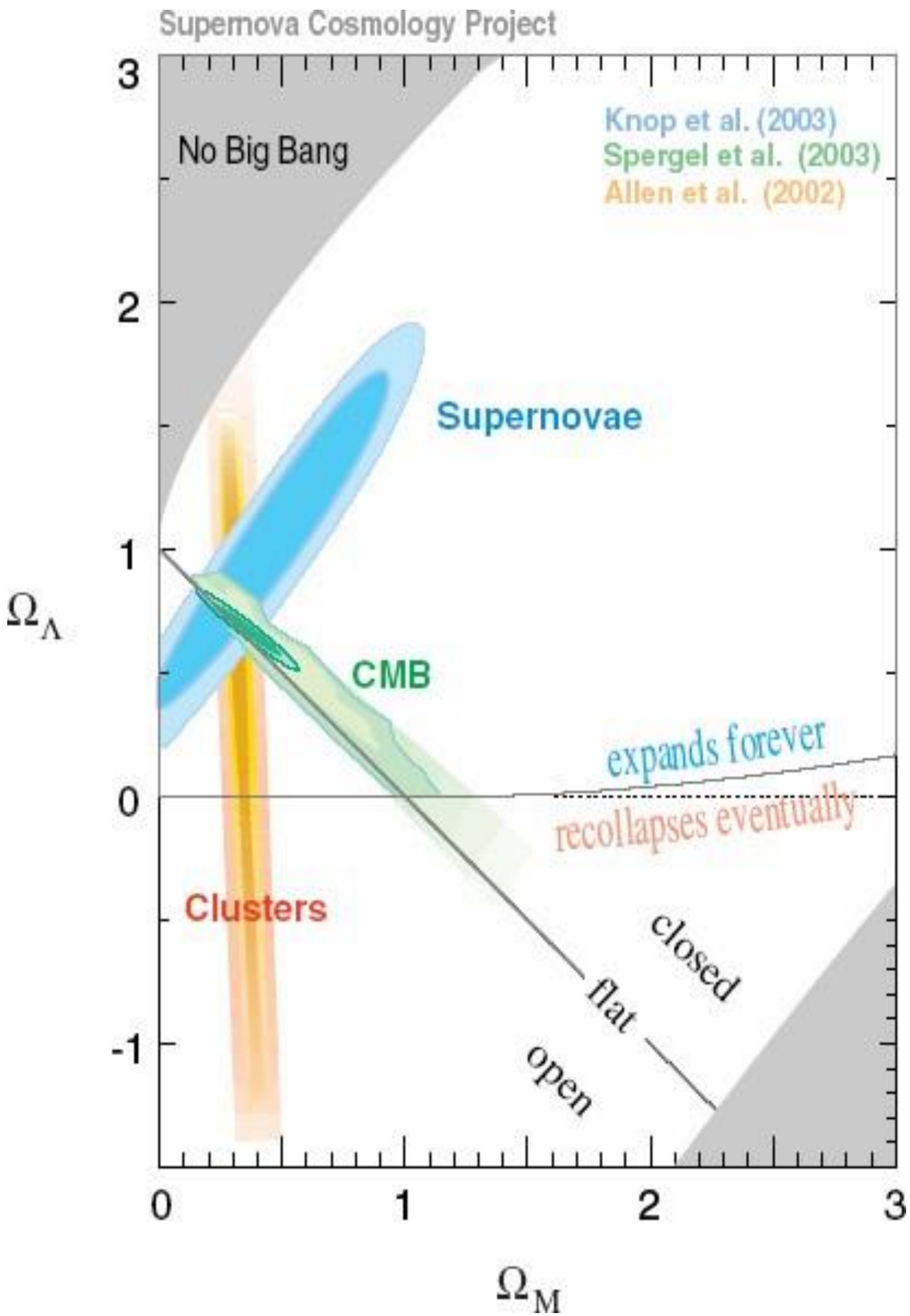
$$p = T - V(\phi)\ ;\ \rho = T + V(\phi)\ ;\ \omega = \frac{p}{\rho}$$

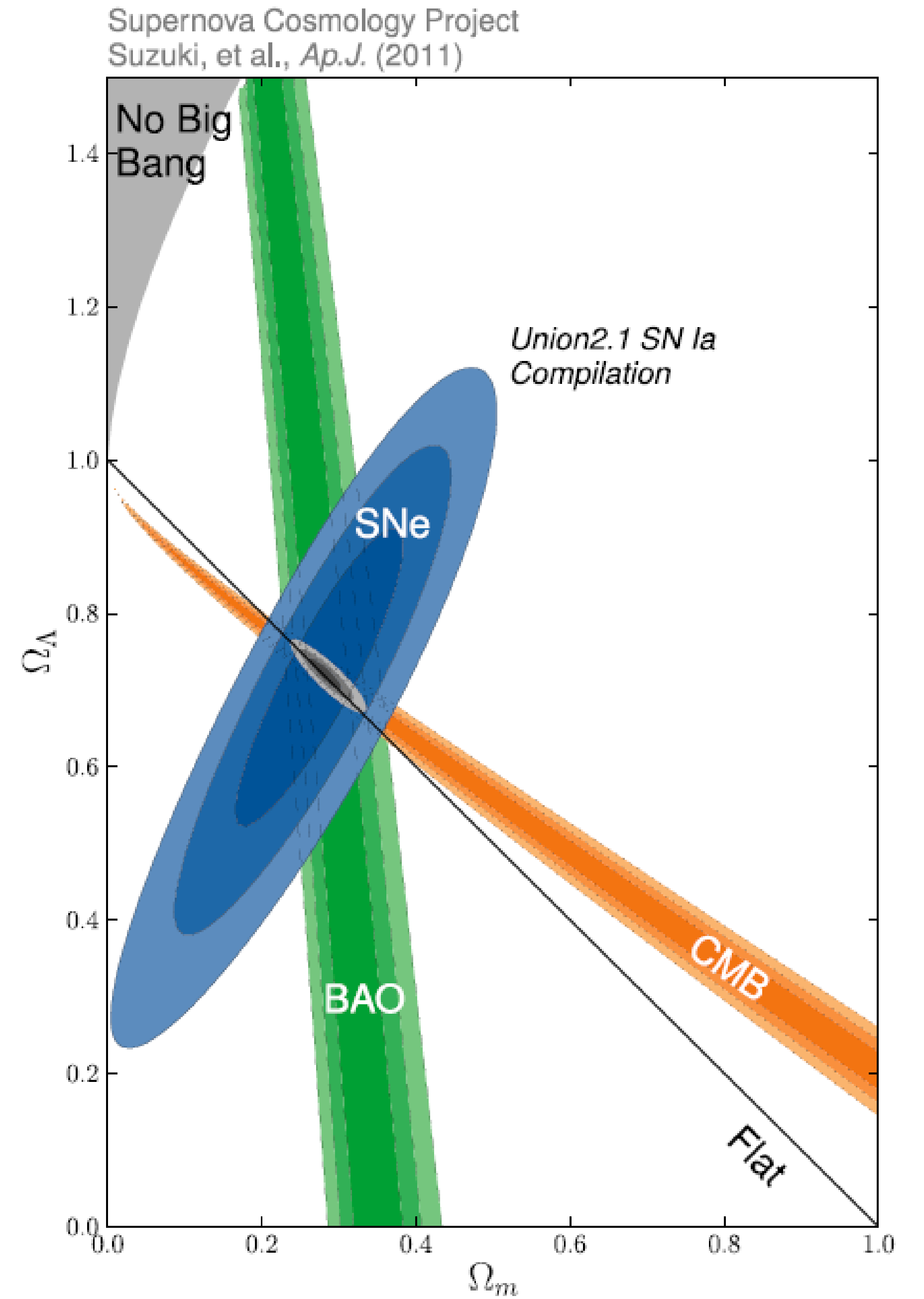
Challenging problem.. (DE)

- Several cosmological observations demonstrated that the Universe is expanding and is accelerating
- What is causing this acceleration?
- How can we learn more about this acceleration, the Dark Energy it implies, and the questions it raises?
- EOS only tells $w=-1$.

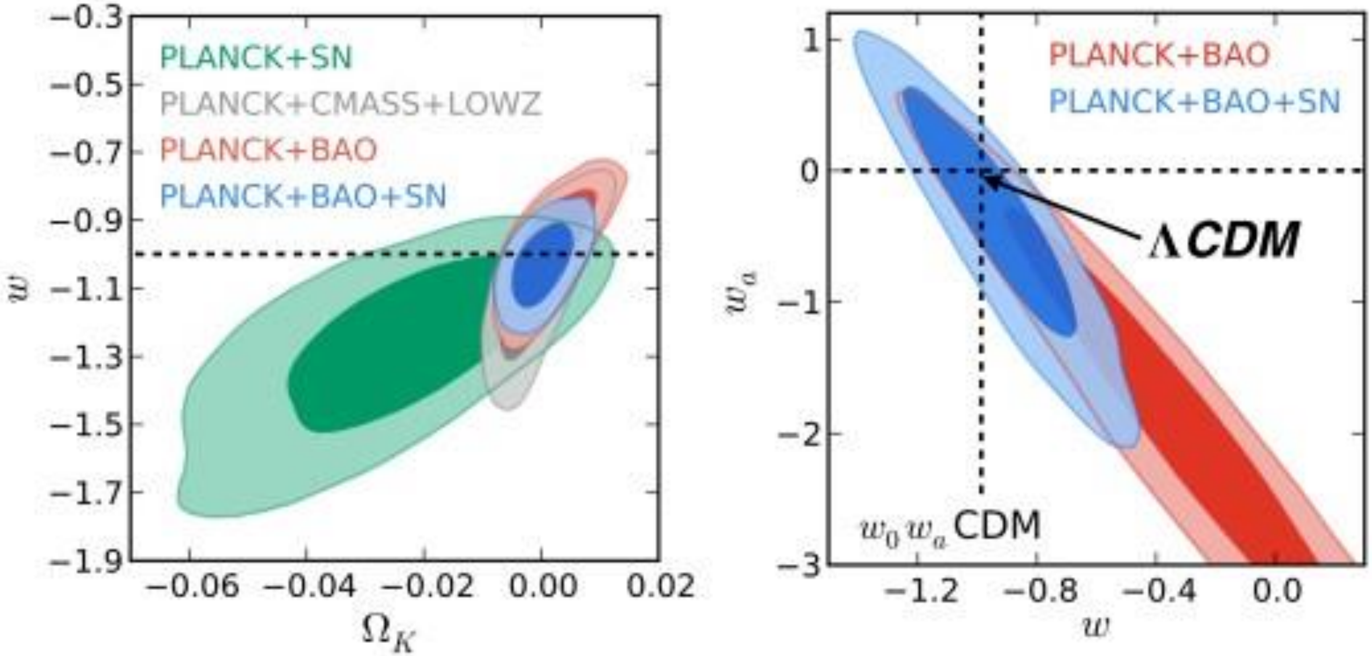
- Universe is accelerating ….

- Type Ia Supernovae observations (SNe Ia)
- Cosmic Microwave Background Radiation (CMBR)
- Cluster of Galaxies (Large scale structure)


Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
expands forever
recollapses eventually
Clusters
closed
flat
open
Ω_Λ
Ω_M


Supernova Cosmology Project
Suzuki, et al., *Ap.J.* (2011)
No Big Bang
Union2.1 SN Ia Compilation
SNe
BAO
CMB
Flat
Ω_Λ
Ω_m

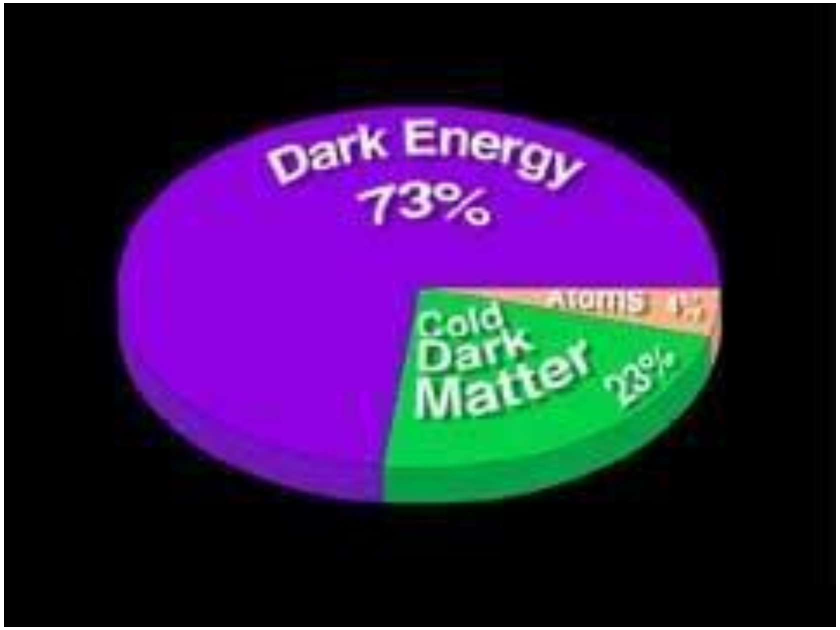
Dark Energy: Expansion History Constraints



- Constraints consistent with a **Λ*CDM*** cosmology (i.e., a constant dark energy equation of state, w=-1), even after allowing for a non-zero curvature (Ω_K) or evolving dark energy (w_a)

Talk by M. Vargas

Observations

- Dark Energy: 73%
- Dark Matter: 23%
- Baryons: 4%
- Massive neutrinos : 0.1%
- 

$\Omega_M = \rho_M/\rho_c$

$\Omega_\Lambda = \rho_\Lambda/\rho_c$

$\rho_c = 3H^2/8\pi$

H= Hubble parameter

Dark Energy

- **Dark Energy**: Most embarrassing observation in Physics – A. Einstein
- Is it Cosmological Constant?
- Is it a Failure of GR?
- Novel property of matter?
- Many ideas have been proposed

Einstein's Eqn.

- Einstein Equation: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

(Testable theory of the Universe)

where $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = G_{\mu\nu}$

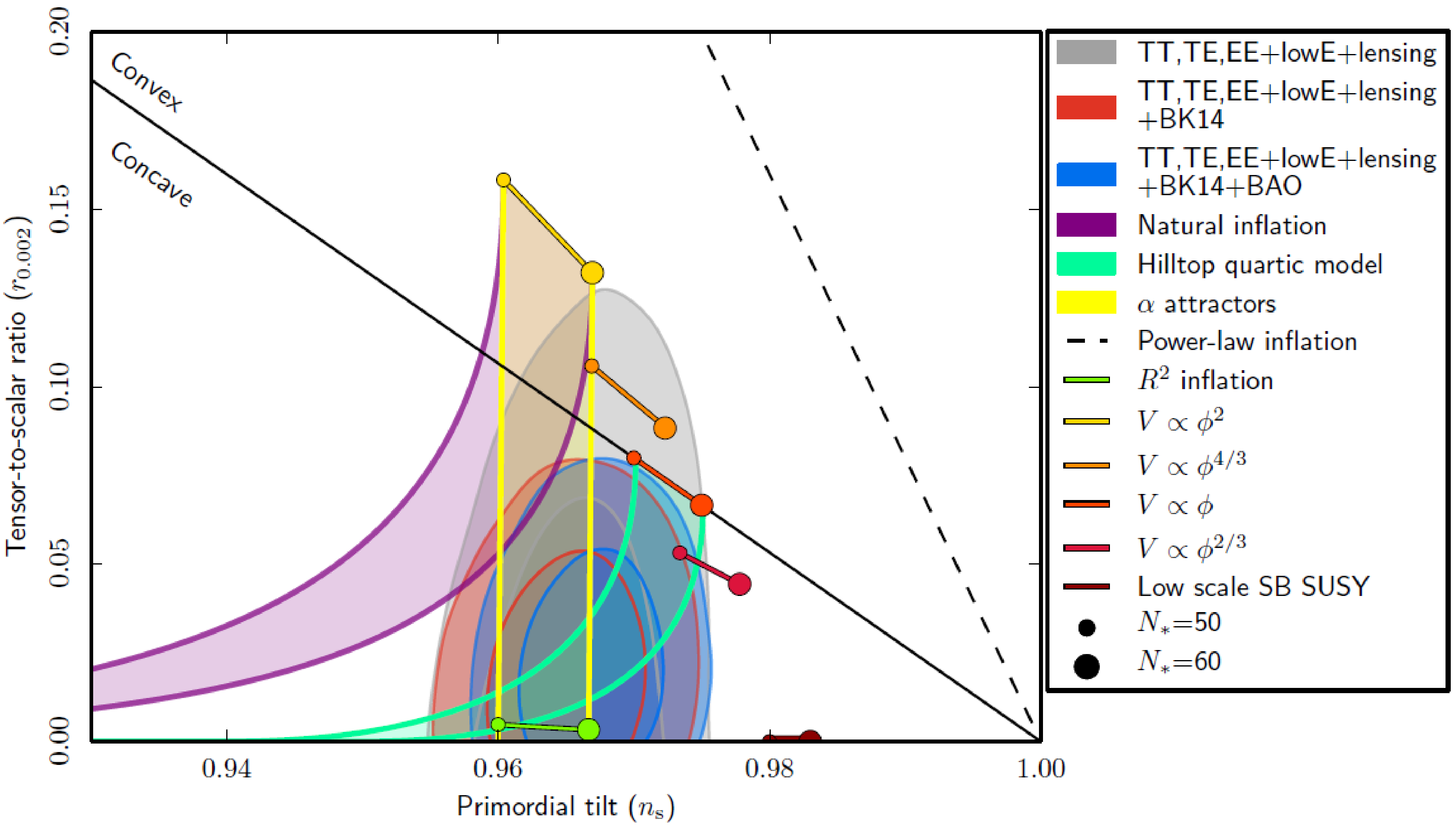
- and $R = g_{\mu\nu} R^{\mu\nu}$ (Ricci scalar)

* GR is well tested, but not unique

Is there any alternate option?

CMB constraints on Inflation models

Planck Collaboration (2018): Y Akrami et.al.



f(R) gravity

- **Inflation** and **Dark Energy**
  (Cosmic acceleration but different energy scales)
  (energy density differ by ~10^{120})
- Modify the gravity sector -> **modify** $\mathbf{G}_{\mu\nu}$
  (f(R) gravity model..)

OR

- Modify the matter sector -> **modify** $\mathbf{T}_{\mu\nu}$
  (scalar field model..)
- If one includes Cosmological constant (Einstein):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

f(R)

- In the simplest generalization of General Relativity one can write the action:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_M$$

(In GR f(R) = R so df/dR = 1)

- In the modified gravity scenario: $\frac{df}{dR} \neq 1$
- Let us consider f(R) = a R^2
- Since the de Sitter solution in f(R) gravity gives (df/dR)R – 2 f = 0 it so happens that f(R) = aR^2 gives rise to de Sitter expansion. (Starobinsky 1980)

Dark Energy and Modified Gravity

- Dark Energy: About 70% of the energy density today consists of Dark Energy, which is responsible for Cosmic acceleration.
- The simplest one is the Cosmological Constant ($w = p/\rho = -1$)
- => If the cosmological constant originates from a vacuum energy then it is in fact much more larger than the scale of the Dark Energy

- Other dynamical DE models, where $w \neq -1$
- i) Modification of the matter sector:

  ii) Modification of gravity:

  f(R) gravity model, scalar-tensor theory..

Here we will consider the simplest one: f(R) model of gravity for Dark Energy

DE and Modified Gravity

- Modification of Gravity can give rise to observational signatures, DE equation of state, impact of LSS,CMB etc., which one can see on large scales.
- In small scales, the modification may not be significant and may be very close to the GR predictions (with small corrections) in case of Solar system experiments.

f(R) Inflation

- An example is the Starobinsky model of Inflation, with the account of a correction quadratic in the Ricci scalar in the modified framework, and of an exponential potential in the scalar field framework.
- Where $f(R) = R + R^2/6M^2$
- (during the inflation the R^2 term dominates, which actually give de Sitter like expansion)

Starobinsky model

- Since we have introduced the model, let us see;
- A) when $R^2/(6M^2) \gg R$: Inflationary expansion
- B) when $R^2/(6M^2) \sim R$: End of Inflation
- C) when $R^2/(6M^2) \ll R$: This is called the Reheating stage, where the scalar R oscillates around the minimum value of R=0.
- One can then discuss the inflation and reheating scenarios.

$f(R) = R + a R^m + b R^n$

- Dark Energy models with f(R) have been considered:
- (Alternative cosmologically viable f(R) model exists by Amendola et al, Amendola-Tsujikawa, Hu-Sawicki, Starobinsky)
- $f(R) = R + \alpha R^n - \beta R^{2-n}$ (Artymowski+Lalak)
- (α and β are positive constants)
- Let us consider $f(R) = R + \alpha R^n$, and to obtain Inflation one must satisfy:

$$n \in \left[\tfrac{1}{2}(1+\sqrt{3}), 2\right]$$

$f(R) = R + \alpha R^n - \beta R^{2-n}$

- Let us consider $\alpha \gg 1,\ \beta \ll 1$ and $\alpha\beta << 1$ This means that $\alpha R^n >> R >> \beta R^{2-n}$ during inflation.
- The last term will not affect inflation
- The Einstein frame scalar potential has a minimum

$$R_{min} = \left(\frac{\sqrt{1+4(2-n)n\alpha\beta}-1}{2(2-n)\alpha}\right)^{\frac{1}{n-1}} \simeq (n\beta)^{\frac{1}{n-1}}\left(1-\frac{(2-n)n\alpha\beta}{n-1}\right)$$

- The value of V at the minimum for small β:

$$V(\varphi_{min}) \simeq \frac{n}{8(n-1)^2}(n\beta)^{\frac{1}{n-1}}\left(n-1-n^2\alpha\beta\right) \sim \frac{1}{2}\beta^{\frac{1}{n-1}}$$

- The energy density for DE $\sim \beta^{\frac{1}{n-1}}$ ($\beta <<< 1$)
- The existence of stable minimum is the keypoint.

OUTLOOK

- Standard Model of Particle Physics complete
- No trace of BSM, DARK Matter….
- Flavor Connection? LHC input?
- Cosmology at center stage
- Gravitational Waves detected
- Lessons from INFLATION
- f(R) can relate inflation and DE
- Exciting time ahead!!!!!

Thank You!

References:

- A Starobinsky (1980, 2007)
- S Nojiri and S Odintsov (2003, 2007)
- A de Felice and S Tsujikawa (2010)
- Artimowski and Lalak (2015)
- K Bambah et al (2012)
- Takahasi and Yokoyama (2015)