Super-horizon evolution of the inflationary gravitational waves

Jinn-Ouk Gong

Korea Astronomy and Space Science Institute
Daejeon 34055, Korea

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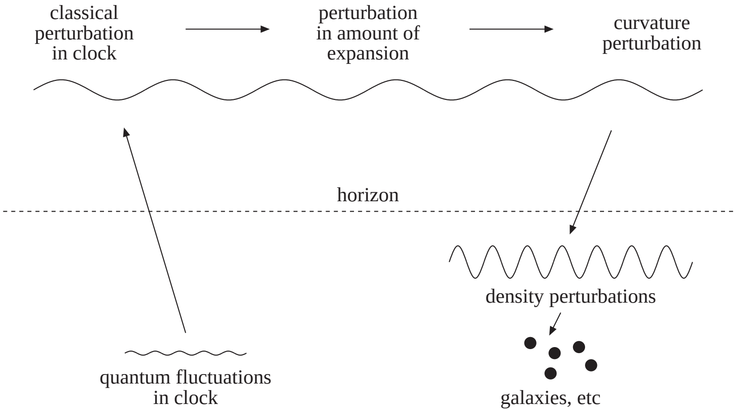
Outline

Generation and evolution of perturbations



Everything seems to be clearly understood

Quantum aspects of perturbations?

If the inflationary picture is the case...

- Quantum-to-classical transition?
- Quantum signature of perturbations?
- Effective theory description?

Important to test the inflationary paradigm

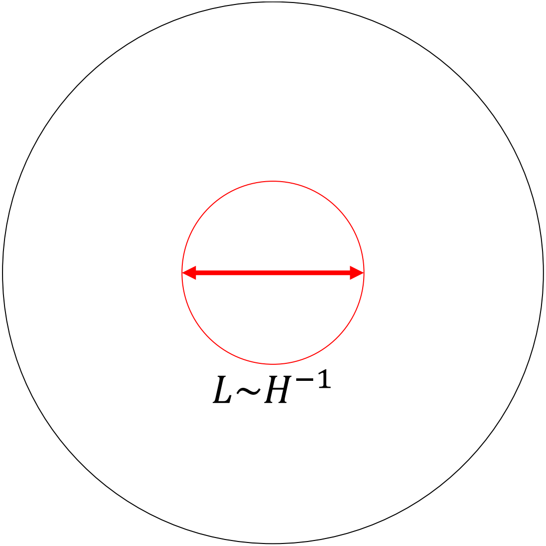
Why tensor perturbations?

- Persistent
- Well defined even during dS
- (For pure tensor) free from gauge

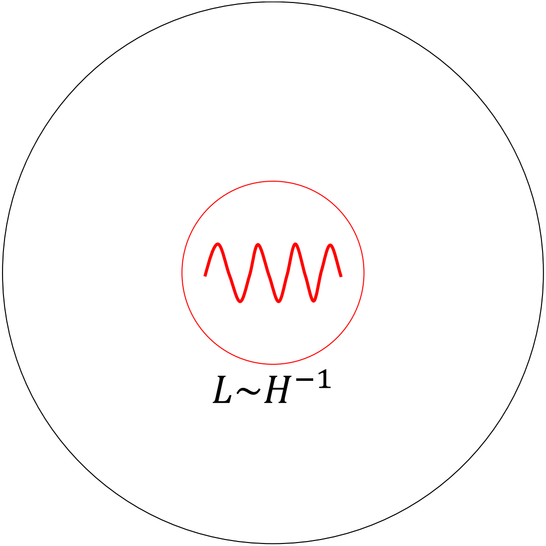
How pure tensor modes behave

1. on super-horizon scales,
2. keeping quantum nature?

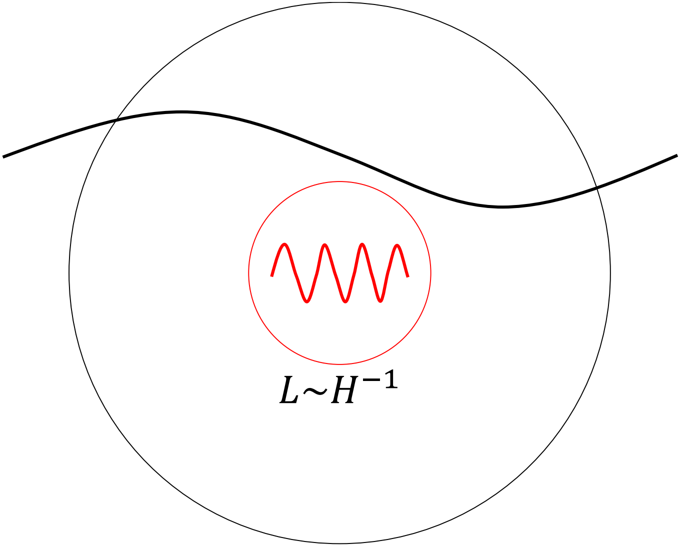
System and environment



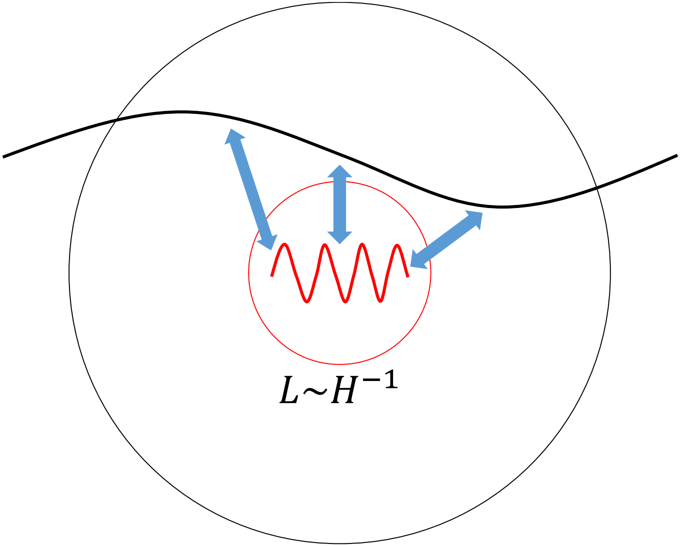
System and environment



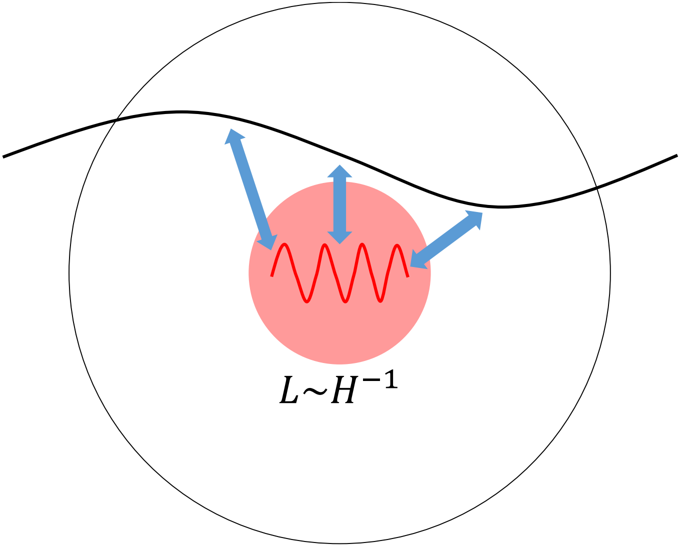
System and environment



System and environment



System and environment



Lindblad equation

$$\frac{d\rho_{\rm red}}{d\tau} = -i[H, \rho_{\rm red}] - \frac{1}{2}\sum\left(L_\mu^\dagger L_\mu \rho_{\rm red} + \rho_{\rm red} L_\mu^\dagger L_\mu - 2L_\mu \rho_{\rm red} L_\mu^\dagger\right)$$

Lindblad equation

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- Unitary evolution: von Neumann equation

Lindblad equation

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- Unitary evolution: von Neumann equation
- Non-unitary evolution: Lindblad operators
  1. Due to the interaction between system and environment

$$L_\mu \sim \left\langle \mathscr{E}_f \middle| H_{\rm int} \middle| \mathscr{E}_i \right\rangle$$

  2. Exponential decay of (some components of) $\rho_{\rm red}$
  3. Effective theory description

Pure tensor interaction

$$S_3^{(t)} = \int d\tau d^3x a^2 m_{\text{Pl}}^2 \left[-\frac{1}{2} h_{ij} h'_{jk} h'_{ki} - 2\mathcal{H} h_{ij} h_{jk} h'_{ki} + 2\left(1 - \frac{\epsilon}{3}\right) h_{ij} h_{jk} h_{ki} + h_{ij} \left(\frac{1}{4} h_{kl,i} h_{kl,j} + \frac{1}{2} h_{ik,l} h_{jl,k} - \frac{3}{2} h_{ik,l} h_{jk,l} \right) \right]$$

1. Most terms are not slow-roll suppressed
2. Pol tensor products with different combinations of indices

Cubic interaction Hamiltonian

$$H_{\mathrm{int},I}(\tau) = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})\sum_{\lambda_1,\lambda_2,\lambda_3}$$

$$\times\left\{h_0(\tau)a_{\boldsymbol{k}_1}^{\lambda_1}a_{\boldsymbol{k}_2}^{\lambda_2}a_{\boldsymbol{k}_3}^{\lambda_3}(\tau_0) + h_1(\tau)\left[a_{-\boldsymbol{k}_1}^{\lambda_1\dagger}a_{\boldsymbol{k}_2}^{\lambda_2}a_{\boldsymbol{k}_3}^{\lambda_3}(\tau_0) + 2\text{ perm}\right] + h.c.\right\}$$

- Coefficients at τ, operators at τ_0
- Sandwiched between $|0\rangle_0$, some operators directly work
- System-environment splitting

$$\int\frac{d^3k}{(2\pi)^3}a_{\boldsymbol{k}} = \underbrace{\int_{\boldsymbol{k}\in\boldsymbol{k}_{\mathscr{S}}}\frac{d^3k}{(2\pi)^3}a_{\boldsymbol{k}\in\boldsymbol{k}_{\mathscr{S}}}}_{\text{On }|0\rangle_{\mathscr{S}}\equiv|0\rangle_{k<aH\text{ at }\tau_0}} + \underbrace{\int_{\boldsymbol{k}\in\boldsymbol{k}_{\mathscr{E}}}\frac{d^3k}{(2\pi)^3}a_{\boldsymbol{k}\in\boldsymbol{k}_{\mathscr{E}}}}_{\text{On }|0\rangle_{\mathscr{E}}\equiv|0\rangle_{k>aH\text{ at }\tau_0}}$$

Pointer basis

Quote from W. H. Zurek (1981):

> *... observable of the measured quantum system can be considered "recorded" by the apparatus. The basis that contains this record – the* ***pointer basis*** *of the apparatus – consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian.*

$$\left.\frac{d\rho_{\rm red}}{d\tau}\right|_{ab} \equiv \left\langle a \left| \frac{d\rho_{\rm red}}{d\tau} \right| b \right\rangle$$

Squeezed state as the pointer basis

Lindblad equation can be schematically written as

$$\frac{d\rho_{\rm red}}{d\tau} \sim \sum_{m,n} \rho_{mn} U_0 a_1^\dagger a_2^\dagger \cdots a_m^\dagger |0\rangle_{\mathscr{S}} \langle 0|_{\mathscr{S}} a_{1'} a_{2'} \cdots a_n U_0^\dagger$$

Seems natural basis: $\left\{ U_0|0\rangle_{\mathscr{S}}, U_0 a_1^\dagger|0\rangle_{\mathscr{S}}, U_0 a_1^\dagger a_2^\dagger|0\rangle_{\mathscr{S}} \cdots \right\}$

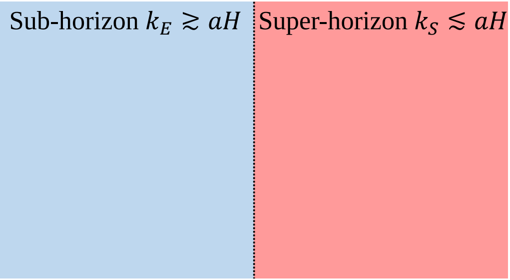
- We do not directly observe primordial perturbations

$$C_\ell^{BB} \sim \int (\text{transfer function}) \times P_h(k)$$

- Classicality not on individual solution but on stat properties

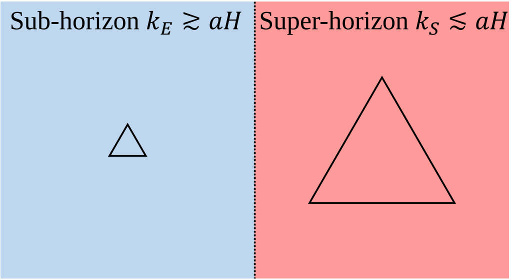
(Guth & Pi 1985, Polarski & Starobinsky 1996)

Cubic interactions



$$L = 1/(aH)$$

Cubic interactions



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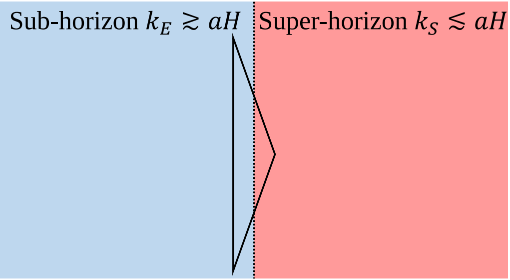
- All modes are in the environment or system sector

Cubic interactions



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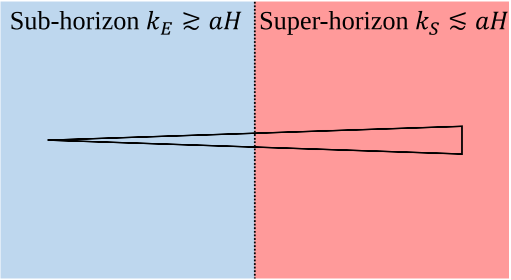
- All modes are in the environment or system sector
- 2 system and 1 environment: $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$ and $|\boldsymbol{k}_3| \approx 2|\boldsymbol{k}_1|$

Cubic interactions



$L = 1/(aH)$

- All modes are in the environment or system sector
- 2 system and 1 environment: $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$ and $|\boldsymbol{k}_3| \approx 2|\boldsymbol{k}_1|$
- 1 system and 2 environment: $\boldsymbol{k}_1 \approx -\boldsymbol{k}_2$ and $k_3 \ll k_1 \approx k_2$

Triangular contributions

- $\mathscr{E}\mathscr{E}\mathscr{E}$: 0 from the beginning
- $\mathscr{S}\mathscr{S}\mathscr{S}$: Absorbed into unitary evolution
- $\mathscr{E}\mathscr{S}\mathscr{S}$: Flattened triangle
  - No clear distinction (we want $k_{\mathscr{E}} \gg aH$ and $k_{\mathscr{S}} \ll aH$)
  - Disappear in the enfolded limit
- $(\mathscr{S}\mathscr{S}\mathscr{S})_{\rm sq}$, $(\mathscr{E}\mathscr{E}\mathscr{S})_{\rm sq}$: Squeezed triangle
  - At least $\mathscr{O}\left(q^2/\mathscr{H}^2\right)$
  - Disappear at leading order
- $\mathscr{E}\mathscr{E}\mathscr{S}$: Only non-zero contribution

Matrix notation of Lindblad equation

$$\frac{d\rho_{\rm red}}{d\tau} = \begin{pmatrix} \mathfrak{E}_{00} & 0 & \mathfrak{E}_{02} & 0 & 0 & 0 & 0 \\ 0 & \mathfrak{E}_{11} & 0 & 0 & 0 & 0 & 0 \\ \mathfrak{E}_{20} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + h.c.$$

Conclusions

TO BE CONCLUDED LATER, BUT SOME PRELIMINARY REMARKS:

1. Studying quantum origin may be relevant
2. Pure tensor perturbations are of physical interest
3. Non-linear evolution allows system-environment interactions
   1. Lindblad equation: evolution of reduced density matrix
   2. Exponential decay of (some components of) $\rho_{\rm red}$
   3. (Probably) no remaining quantum nature in gravitational sector