Super-horizon evolution of the inflationary gravitational waves

Jinn-Ouk Gong

Korea Astronomy and Space Science Institute
Daejeon 34055, Korea

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1 Introduction

2 Lindblad equation

3 Pure tensor cubic interaction

4 Reduced density matrix

5 Conclusions
Generation and evolution of perturbations

- Classical perturbation in clock
- Perturbation in amount of expansion
- Curvature perturbation

Quantum fluctuations in clock

Horizon

Density perturbations

Galaxies, etc.

Everything seems to be clearly understood
Quantum aspects of perturbations?

If the inflationary picture is the case...
- Quantum-to-classical transition?
- Quantum signature of perturbations?
- Effective theory description?

Important to test the inflationary paradigm
Why tensor perturbations?

- Persistent
- Well defined even during dS
- (For pure tensor) free from gauge

How pure tensor modes behave

1. on super-horizon scales,
2. keeping quantum nature?
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System and environment

$L \sim H^{-1}$

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System and environment

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\[ L \sim H^{-1} \]
System and environment

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System and environment

$L \sim H^{-1}$

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Lindblad equation

\[ \frac{d\rho_{\text{red}}}{d\tau} = -i[H, \rho_{\text{red}}] - \frac{1}{2} \sum (L_\mu^+ L_\mu \rho_{\text{red}} + \rho_{\text{red}} L_\mu^+ L_\mu - 2L_\mu \rho_{\text{red}} L_\mu^+) \]
Lindblad equation

\[
\frac{d\rho_{\text{red}}}{d\tau} = -i[H, \rho_{\text{red}}] - \frac{1}{2} \sum (L_\mu^\dagger L_\mu \rho_{\text{red}} + \rho_{\text{red}} L_\mu^\dagger L_\mu - 2L_\mu \rho_{\text{red}} L_\mu^\dagger)
\]

- Unitary evolution: von Neumann equation

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The Lindblad equation is given by:

$$\frac{d\rho_{\text{red}}}{d\tau} = -i[H, \rho_{\text{red}}] - \frac{1}{2} \sum \left( L^\dagger_\mu L_\mu \rho_{\text{red}} + \rho_{\text{red}} L^\dagger_\mu L_\mu - 2L_\mu \rho_{\text{red}} L^\dagger_\mu \right)$$

- **Unitary evolution:** von Neumann equation
- **Non-unitary evolution:** Lindblad operators
  - Due to the interaction between system and environment
    $$L_\mu \sim \left\langle \mathcal{E}_f \middle| H_{\text{int}} \middle| \mathcal{E}_i \right\rangle$$
  - Exponential decay of (some components of) $\rho_{\text{red}}$
  - Effective theory description

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Pure tensor interaction

\[ S_3^{(t)} = \int d\tau d^3 x d^2 m_{Pl} \left[ -\frac{1}{2} h_{ij} h_{jk} h'_{ki} - 2 \mathcal{H} h_{ij} h_{jk} h'_{ki} + 2 \left( 1 - \frac{\epsilon}{3} \right) h_{ij} h_{jk} h_{ki} ight. \\
\left. + h_{ij} \left( \frac{1}{4} h_{kl,i} h_{kl,j} + \frac{1}{2} h_{ik,l} h_{jl,k} - \frac{3}{2} h_{ik,l} h_{jk,l} \right) \right] \]

1. Most terms are not slow-roll suppressed
2. Pol tensor products with different combinations of indices
Cubic interaction Hamiltonian

\[ H_{\text{int},I}(\tau) = \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \frac{d^3 k_3}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(k_{123}) \sum_{\lambda_1,\lambda_2,\lambda_3} \]
\[ \times \left\{ h_0(\tau) a_{k_1}^{\lambda_1} a_{k_2}^{\lambda_2} a_{k_3}^{\lambda_3}(\tau_0) + h_1(\tau) \left[ a_{-k_1}^{\lambda_1\dagger} a_{k_2}^{\lambda_2} a_{k_3}^{\lambda_3}(\tau_0) + 2 \text{ perm} \right] + h.c. \right\} \]

- Coefficients at \( \tau \), operators at \( \tau_0 \)
- Sandwiched between \( |0\rangle_0 \), some operators directly work
- System-environment splitting

\[ \int \frac{d^3 k}{(2\pi)^3} a_k = \int_{k \in k_S} \frac{d^3 k}{(2\pi)^3} a_{k \in k_S} + \int_{k \in k_E} \frac{d^3 k}{(2\pi)^3} a_{k \in k_E} \]
On \( |0\rangle_S \equiv |0\rangle_{k < aH \text{ at } \tau_0} \)
On \( |0\rangle_E \equiv |0\rangle_{k > aH \text{ at } \tau_0} \)
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Quote from W. H. Zurek (1981):

... observable of the measured quantum system can be considered “recorded” by the apparatus. The basis that contains this record – the **pointer basis** of the apparatus – consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian.

\[
\left. \frac{d\rho_{\text{red}}}{d\tau} \right|_{ab} \equiv \left\langle a \left| \frac{d\rho_{\text{red}}}{d\tau} \right| b \right\rangle
\]
Squeezed state as the pointer basis

Lindblad equation can be schematically written as

\[
\frac{d\rho_{\text{red}}}{d\tau} \sim \sum_{m,n} \rho_{mn} U_0 a_1^\dagger a_2^\dagger \cdots a_m^\dagger |0\rangle_S \langle 0|_S a_1 a_2 \cdots a_n U_0^\dagger
\]

Seems natural basis: \( \{ U_0 |0\rangle_S, U_0 a_1^\dagger |0\rangle_S, U_0 a_1^\dagger a_2^\dagger |0\rangle_S \cdots \} \)

- We do not directly observe primordial perturbations

\[
C_{BB}^{\ell} \sim \int (\text{transfer function}) \times P_h(k)
\]

- Classicality not on individual solution but on stat properties

(Guth & Pi 1985, Polarski & Starobinsky 1996)
Cubic interactions

Sub-horizon $k_E \gtrsim aH$  \hspace{1cm} Super-horizon $k_S \lesssim aH$

$L = 1/(aH)$
Cubic interactions

Sub-horizon $k_E \geq aH$  Super-horizon $k_S \leq aH$

$L = 1/(aH)$

- All modes are in the environment or system sector
Cubic interactions

Sub-horizon $k_E \geq aH$  
Super-horizon $k_S \lesssim aH$

$L = 1/(aH)$

- All modes are in the environment or system sector
- 2 system and 1 environment: $k_1 \approx k_2$ and $|k_3| \approx 2|k_1|$
Cubic interactions

Sub-horizon $k_E \gtrsim aH$  
Super-horizon $k_S \lesssim aH$

$L = 1/(aH)$

- All modes are in the environment or system sector
- 2 system and 1 environment: $k_1 \approx k_2$ and $|k_3| \approx 2|k_1|$
- 1 system and 2 environment: $k_1 \approx -k_2$ and $k_3 \ll k_1 \approx k_2$
Triangular contributions

- \( EEE \): 0 from the beginning
- \( ISS \): Absorbed into unitary evolution
- \( ESS \): Flattened triangle
  - No clear distinction (we want \( k_E \gg aH \) and \( k_S \ll aH \))
  - Disappear in the enfolded limit
- \( (ISS)_{sq}, (EES)_{sq} \): Squeezed triangle
  - At least \( O(q^2/H^2) \)
  - Disappear at leading order
- \( EES \): Only non-zero contribution
Matrix notation of Lindblad equation

\[
\frac{d\rho_{\text{red}}}{d\tau} = \begin{pmatrix}
\mathcal{E}_{00} & 0 & \mathcal{E}_{02} & 0 & 0 & 0 & 0 \\
0 & \mathcal{E}_{11} & 0 & 0 & 0 & 0 & 0 \\
\mathcal{E}_{20} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} + h.c.
\]
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Conclusions

TO BE CONCLUDED LATER, BUT SOME PRELIMINARY REMARKS:

1. Studying quantum origin may be relevant
2. Pure tensor perturbations are of physical interest
3. Non-linear evolution allows system-environment interactions
   1. Lindblad equation: evolution of reduced density matrix
   2. Exponential decay of (some components of) $\rho_{\text{red}}$
   3. (Probably) no remaining quantum nature in gravitational sector