Gauge Field Mimetic Cosmology

Mohammad Ali Gorji
IPM, Tehran

In collaboration with
H. Firouzjahi, S. A. Hosseini Mansoori, and S. Mukohyama

2019 YITP Asian-Pacific Winter School and Workshop on Gravitation and Cosmology

February 13, 2019

Outline

- The conformal mode of gravity can be isolated through a singular conformal transformation [A. Chamseddine and V. Mukhanov (JHEP,2013)]

$$g_{\mu\nu} = \left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\tilde{g}_{\mu\nu} \tag{1}$$

where $g_{\mu\nu}$ is the physical metric and $\tilde{g}_{\mu\nu}$ is the auxiliary metric. This transformation implies $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$, and, in the absence of any matter field, we have

$$S = \int dtd^3x\sqrt{-g}\left[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1)\right] \tag{2}$$

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- In spatially flat FLRW spacetime, the energy density provided by the constraint term behaves like dark matter.
- The model, however, cannot provide any propagating mode at the level of perturbations! The sound speed vanishes for the curvature perturbation.

- In order to have nonzero sound speed for the curvature perturbations, it is suggested to add a higher derivative term to the action [A. Chamseddine, V. Mukhanov, A. Vikman (JCAP,2014)]

$$S = \int dtd^3x\sqrt{-g}\left[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + \frac{1}{2}\gamma(\Box\phi)^2\right]$$

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- Adding another HD term $\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$ and a coupling between the curvature and second derivative of the scalar field such as $\Box\phi R$ can make the setup stable. [S. Hirano, S. Nishi, T. Kobayashi (JCAP,2017)], [Y. Zheng, L. Shen, Y. Mou, M. Li (JCAP,2017)], [M. A. G, S. A. Hosseini Mansoori, H. Firouzjahi (JCAP,2017)]

Multiple-Field Extension

- The disformal transformation is defined as

$$g_{\mu\nu} = A(\phi, \tilde{X})\tilde{g}_{\mu\nu} + B(\phi, \tilde{X})\partial_\mu\phi\partial_\nu\phi \tag{3}$$

with $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. For the conformal case $B = 0$, the singular limit uniquely gives $A = \tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$ which is nothing but the mimetic transformation.

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- The two-field mimetic scenario can be obtained by looking at the singular limit of the two-field disformal transformation which implies the constraint [H. Firouzjahi, M. A. G, S. A. Hosseini Mansoori, A. Karami, T. Rostami (JCAP,2018)]

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- The two-field mimetic scenario provides healthy entropy perturbations while the curvature perturbation is still frozen with vanishing sound speed.

p-form Mimetic Gravity

- Implementing a gauge field instead of scalar field, we find the following singular conformal transformation [M. A. G, S. Mukohyama, H. Firouzjahi, S. A. Hosseini Mansoori (JCAP,2018)]

$$g_{\mu\nu} = \left(-\tilde{g}^{\rho\alpha}\tilde{g}^{\sigma\beta}F_{\alpha\beta}F_{\rho\sigma}\right)^{\frac{1}{2}}\ \tilde{g}_{\mu\nu} \quad \rightarrow \quad g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}F_{\mu\nu} = -1 \qquad (5)$$

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- We extend the setup to the case of a p-form potential $\mathcal{A}$ with the associated field strength $\mathcal{F} = d\mathcal{A}$ so that the action is given by

$$S_p = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \lambda_p\big(\langle\mathcal{F},\mathcal{F}\rangle \pm 1\big)\right] \tag{6}$$

where $\langle\mathcal{F},\mathcal{F}\rangle$ is the internal product which gives $\partial^{\mu}\phi\partial_{\mu}\phi$ and $F_{\mu\nu}F^{\mu\nu}$ for $p = 0$ and $p = 1$ cases respectively.

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- The case $p = 2$ turns out to be dual to the $p = 0$ case with the strong/week like duality $\lambda_0 \leftrightarrow \frac{1}{\lambda_2}$.

p=1: Mimetic SU(2) Gauge Theory

The $p = 1$ model with $SU(2)$ gauge symmetry for the internal field space is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \lambda\Big(\mathrm{Tr}(F^a_{\mu\nu}F_a^{\mu\nu}) + 1\Big)\right], \tag{7}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon_{abc}A^b_\mu A^c_\nu$. Defining the energy density parameters

$$\Omega_\lambda = \frac{2\lambda}{3H^2}, \qquad \Omega_r = \frac{4\lambda}{H^2 g^2}\frac{(A^2-k)^2}{a^4}, \qquad \Omega_k = -\frac{k}{a^2H^2}. \tag{8}$$

In the spatially curved FLRW background, EoM are

$$\Omega_k + \Omega_\lambda + \Omega_r = 1, \qquad \Omega_k^T = \Omega_k + \Omega_\lambda. \tag{9}$$

The total spatial curvature is determined by the two components: Ω_λ is the spatial curvature coming from the mimetic sector and Ω_k is the standard geometrical spatial curvature.

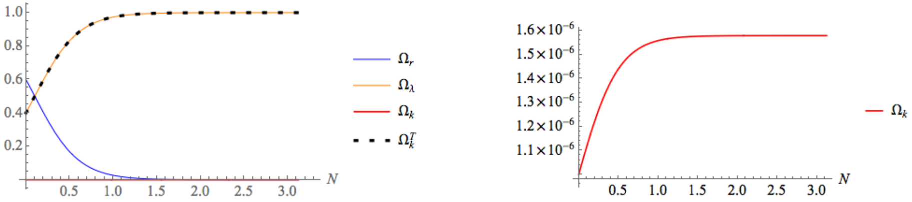

Figure: The negative branch for $k < 0$, $\Omega_r = 0.6$, and $\Omega_k = 10^{-6}$.

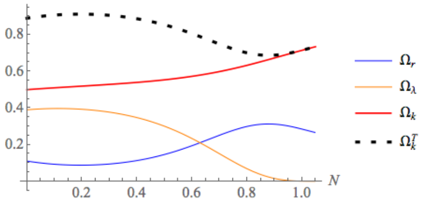

Figure: Positive branch for $k < 0$, $\Omega_r = 0.11$, and $\Omega_k = 0.5$

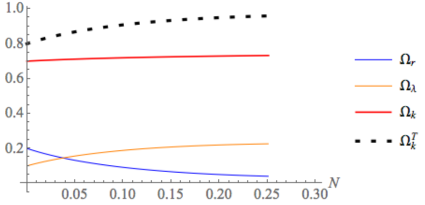

Figure: Negative branch: $k < 0$, $\Omega_r = 0.2$, and $\Omega_k = 0.7$

Summary

- The standard mimetic gravity provides dark matter-like energy density component while it is unstable at the level of perturbations. We need to add a coupling between the curvature and the second derivative of the mimetic scalar field to make the setup stable.

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- The standard mimetic gravity provides dark matter-like energy density component while it is unstable at the level of perturbations. We need to add a coupling between the curvature and the second derivative of the mimetic scalar field to make the setup stable.
- Implementing a gauge field rather than scalar field, we found that the model provides energy density component which behaves like the spatial curvature in the spatially flat FLRW background and also it is stable.
- In the spatially curved FLRW background, we then find extra spatial curvature energy density which coming from the mimetic sector only at the dynamical level. If we have cosmological observations that constraint the geometrical and dynamical spatial curvatures separately, we may find an observational consequences of the model.

Thank You

Two-Field Disformal Transformation

$$g_{\mu\nu} = A\tilde{g}_{\mu\nu} + B\,\phi_{,\mu}\phi_{,\nu} + C\,\psi_{,\mu}\psi_{,\nu} + D\left(\phi_{,\mu}\psi_{,\nu} + \psi_{,\mu}\phi_{,\nu}\right), \tag{10}$$

where A, B, C, D are given functions of ϕ, ψ, X, Y, Z where X, Y, Z are defined as

$$\begin{cases} X \equiv \tilde{g}^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\,, \\ Y \equiv \tilde{g}^{\mu\nu}\psi_{,\mu}\psi_{,\nu}\,, \\ Z \equiv \tilde{g}^{\mu\nu}\phi_{,\mu}\psi_{,\nu}\,. \end{cases} \tag{11}$$

$$\left(\frac{\partial g_{\mu\nu}}{\partial \tilde{g}_{\alpha\beta}} - \lambda^{(n)}\,\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu}\right)\xi^{(n)}_{\alpha\beta} = 0\,, \tag{12}$$

Instabilities

$$S = \int dt d^3x \sqrt{-g}\left[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + f(\Box\phi)\right] \tag{13}$$

For a special functional form of $f(\Box\phi)$, nonsingular universe found. The corresponding Friedmann equation has the same form as is suggested by Loop Quantum Cosmology.

$$S^{(2)} = \int dt\, d^3k\, a^3(-c_s^2)\left[\dot{\mathcal{R}}^2 - \frac{c_s^2 k^2}{a^2}\mathcal{R}^2\right] \tag{14}$$

with the sound speed $c_s^2 = \frac{f'}{2-3f'}$. Demanding $c_s^2 > 0$ to avoid the gradient instability, the system finds ghost instability!

Maxwell Term

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \lambda'\big(2\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + 1\big) + 2\Lambda\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})\right], \quad (15)$$

Performing the field redefinition

$$\lambda' \to \lambda = \lambda' - \Lambda \quad (16)$$

we find

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \lambda\big(2\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + 1\big) - \Lambda\right]. \quad (17)$$

The constant parameter *colorblue*Λ plays the roles of the the cosmological constant.

Spatial Curvature from p=1 Case

In order to find isotropic solution, we consider $O(3)$ symmetry for the internal field space

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \lambda\Big(\sum_{a=1}^{3} F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + 1\Big) - \frac{1}{4}\mathcal{E}\sum_{a=1}^{3} F^{(a)}_{\mu\nu}F^{(a)\mu\nu}\right], \tag{18}$$

in the isotropic configuration

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j\,, \qquad with \qquad A^{(a)}_{\mu} = A(t)\,\delta^a_\mu \tag{19}$$

The background equation is

$$3H^2 = \rho_{\lambda_1} + \rho_\Lambda\,, \qquad with \qquad \rho_{\lambda_1} = -\frac{3K_{\rm eff}}{a^2} \;\; and \;\; \rho_\Lambda = -\frac{\mathcal{E}}{12} \tag{20}$$

where $K_{\rm eff}$ is an integration constant.

p=1: cosmological perturbations in spatially flat FLRW

In the case of $p = 1$, the mimetic term provides energy density like the spatial curvature at the background level. We therefore find flat $K_{\rm eff} = 0$, closed $K_{\rm eff} = 1$ and open $K_{\rm eff} = -1$ de Sitter universes even if we consider spatially flat FLRW metric.

After fixing all gauge freedoms, imposing the mimetic constraint and integrating out the non-dynamical modes, we left with two scalar modes, two vector modes, and four tensor modes

$$S^{(2)} = S_S^{(2)}(\delta Q, U) + S_V^{(2)}(U_a) + S_T^{(2)}(h_{ij}, t_{ij})\,. \qquad (21)$$

In the case of closed de Sitter universe, the scalar, vector and tensor modes become ghost. For the flat and open case, however, all modes are healthy.