THE EFFECT OF KOZAI-LIDOV MECHANISM ON PERIOD SHIFT AND GRAVITATIONAL WAVEFORM

ONGOING WORK

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INTRODUCTION

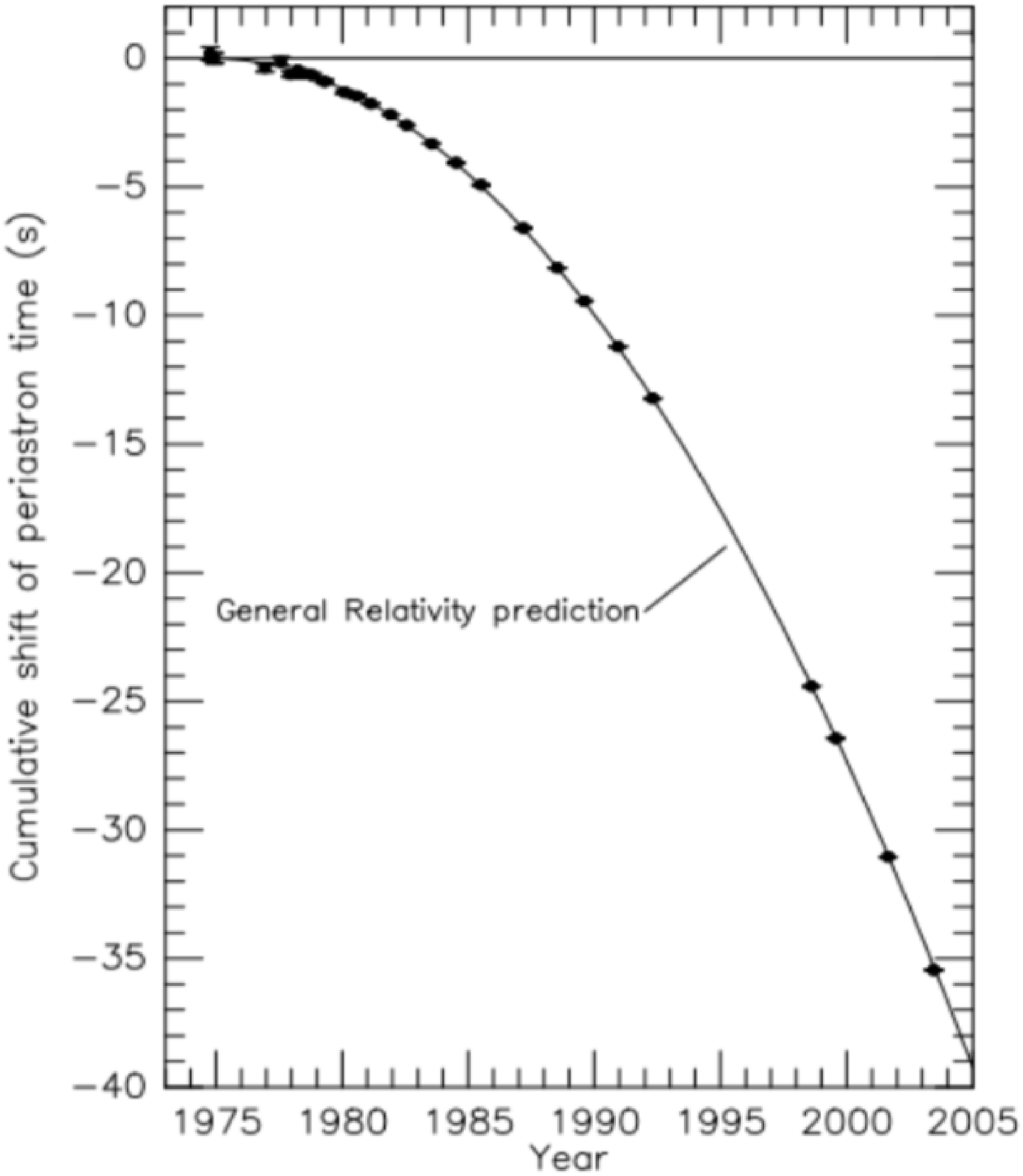
First Indirect Observation

Hulse Taylor Binary

Russell Hulse and Joseph Taylor, showed that

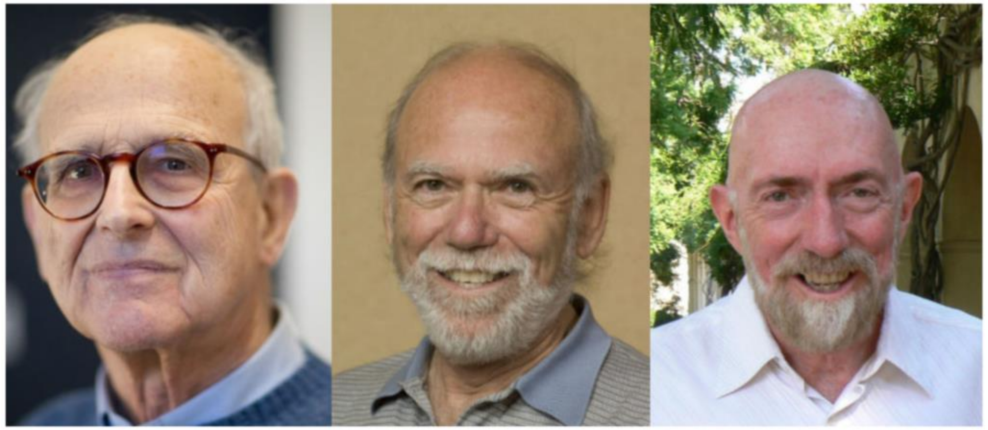
- over time, the neutron stars were gradually spiraling towards each other.
- energy loss agreed with the predicted energy that would be radiated by gravitational waves.

Nobel Prize 1993



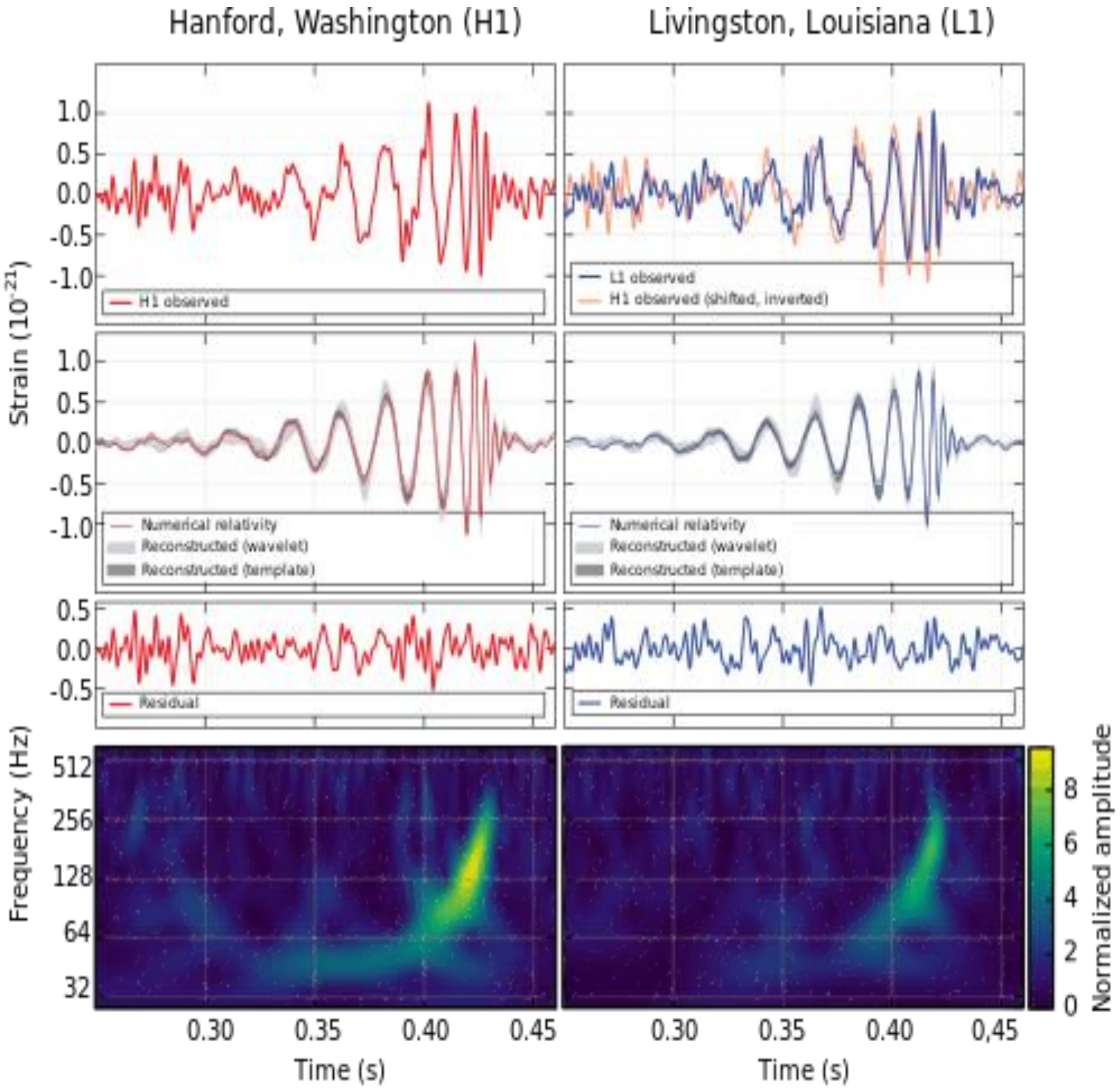
First Direct Observation
GW150914

The waveform, detected by both LIGO observatories, matched the predictions of general relativity for a gravitational wave emanating from the inward spiral, merger of a pair of black holes and the subsequent "ringdown" of the single resulting black hole.

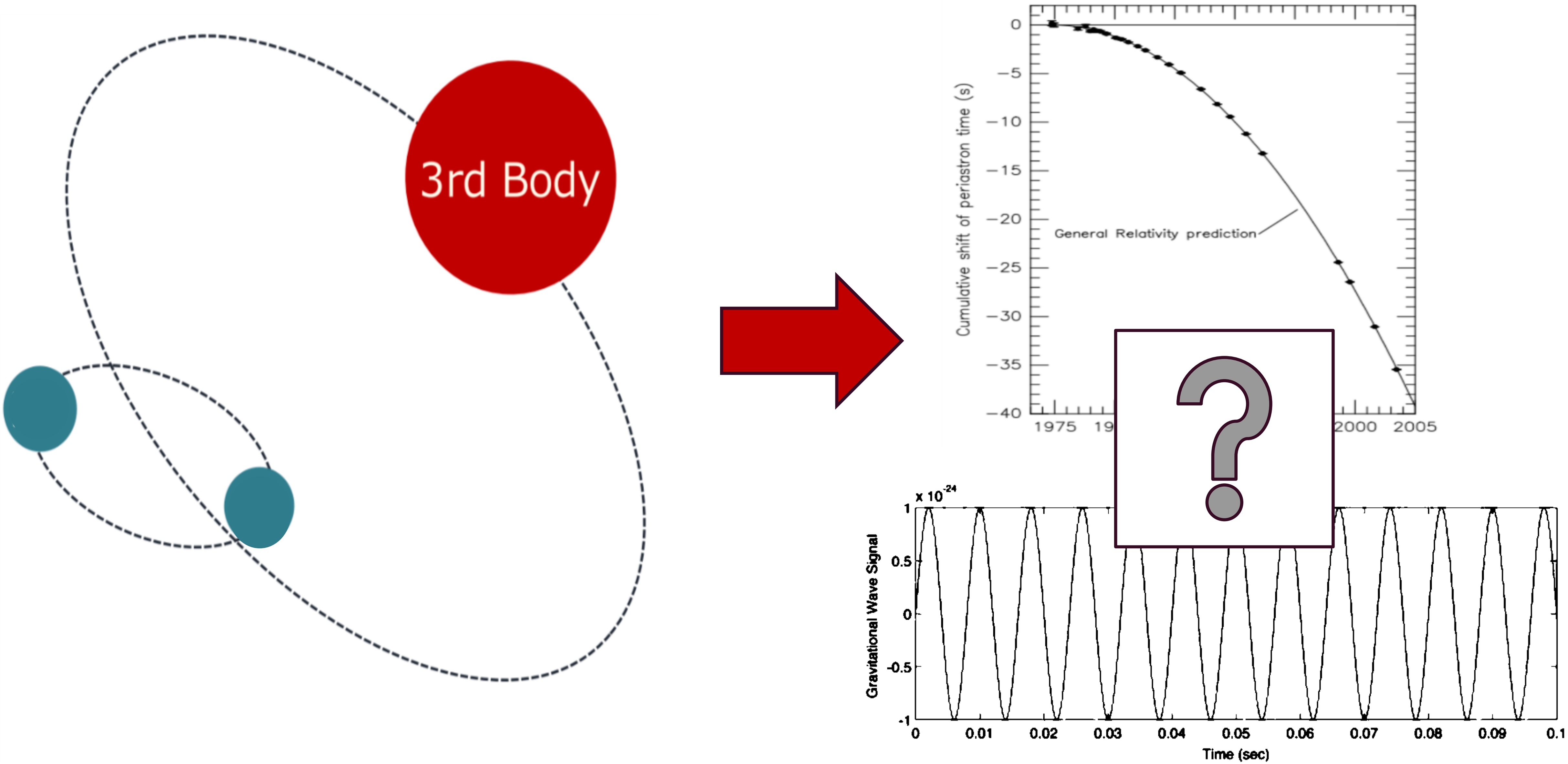


Rainer Weiss   Barry Barish   Kip Thorne

Nobel Prize 2017

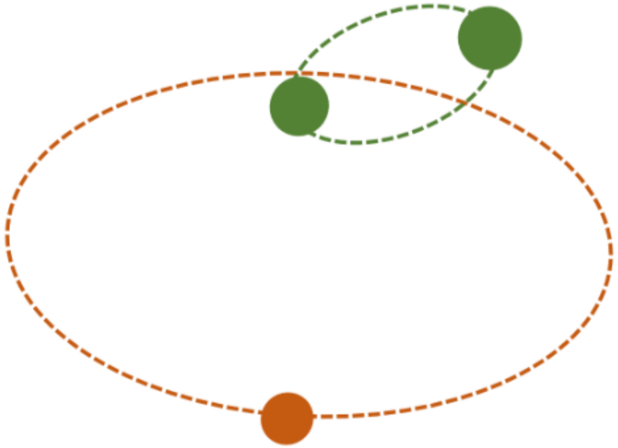


MOTIVATION

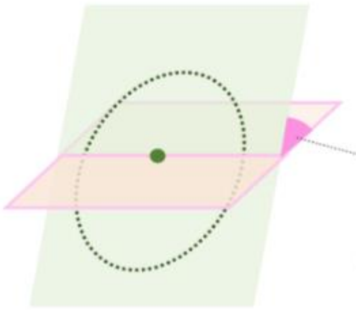

3rd Body
Cumulative shift of periastron time (s)
General Relativity prediction
0
−5
−10
−15
−20
−25
−30
−35
−40
1975
2000
2005
Gravitational Wave Signal
x 10^-24
1
0.5
0
-0.5
-1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Time (sec)
?

KOZAI - LIDOV MECHANISM

Dynamical phenomenon affecting the orbit of a binary system perturbed by a distant third body under certain conditions.



Hierarchical triplet : Binary + tertiary companion

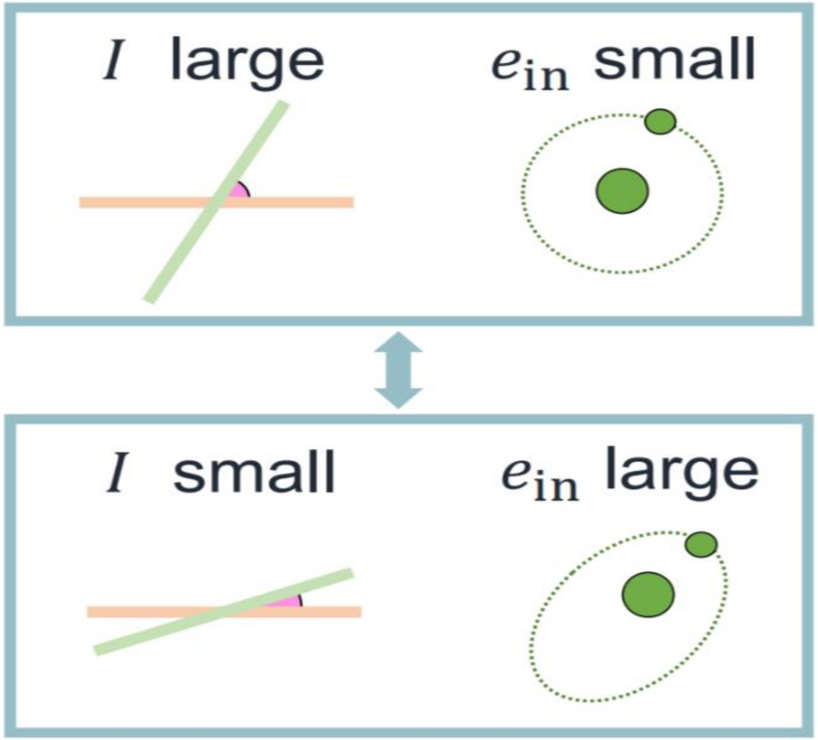


I : relative inclination

e : eccentricity

Newtonian TPQ

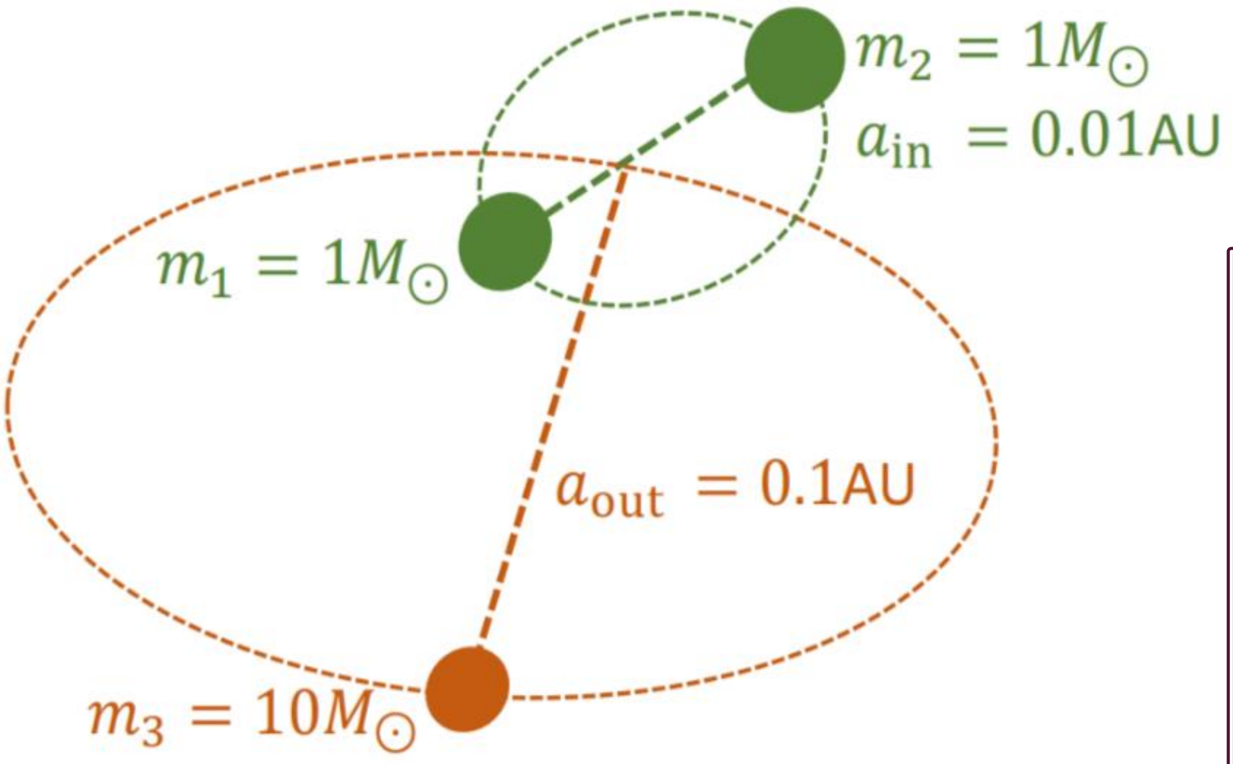
$$\Theta = \sqrt{1 - e_{\text{in}}^2}\cos I = const.$$



Leads to a periodic exchange between eccentricity and inclination.

Model + Timescales

Hierarchical triplet : Binary + tertiary companion



$$e_{in,0} = e_{out,0} = 0$$

$$I_0 = 60°$$

$$t_{KL} \simeq \frac{16}{15}\frac{a_{out}^3}{a_{in}^{3/2}}\sqrt{\frac{m_1}{Gm_3^2}}(1-e_{out}^2)^{\frac{3}{2}}$$

Kozai-Lidov Timescale

Time Scales for our Model

$$P_{in} = 0.258\ days$$
$$P_{out} = 3.334\ days$$
$$\tau_{KL} \sim 66\ days$$
$$\tau_{merger} \sim 10^9 years$$

$$P_{in} \ll P_{out} \ll \tau_{KL} \ll \tau_{merger}$$

Orbit Evolution Method

1st order post-Newtonian equation of motion

Einstein-Infeld-Hoffmann equation

Lorentz & Droste,, 1917

$$
\begin{aligned}
\frac{d\boldsymbol{v}_k}{dt} = & -G\sum_{n\neq k} m_n \frac{\boldsymbol{x}_k-\boldsymbol{x}_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|^3} \\
& \times\left[1-4G\sum_{n'\neq k}\frac{m_{n'}}{|\boldsymbol{x}_k-\boldsymbol{x}_{n'}|}-\sum_{n'\neq n}\frac{m_{n'}}{|\boldsymbol{x}_n-\boldsymbol{x}_{n'}|}\left\{1-\frac{(\boldsymbol{x}_k-\boldsymbol{x}_n)\cdot(\boldsymbol{x}_n-\boldsymbol{x}_{n'})}{2|\boldsymbol{x}_n-\boldsymbol{x}_{n'}|^2}\right\}+v_k^2\right. \\
& \left.+2v_n^2-4\boldsymbol{v}_k\cdot\boldsymbol{v}_n-\frac{3}{2}\left\{\frac{(\boldsymbol{x}_k-\boldsymbol{x}_n)\cdot\boldsymbol{v}_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|}\right\}^2\right] \\
& -G\sum_{n\neq k} m_n \frac{\boldsymbol{v}_k-\boldsymbol{v}_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|^3}(\boldsymbol{x}_k-\boldsymbol{x}_n)\cdot(3\boldsymbol{v}_n-4\boldsymbol{v}_k) \\
& -\frac{7}{2}G^2\sum_{n\neq k}\frac{m_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|}\sum_{n'\neq n}\frac{m_{n'}(\boldsymbol{x}_n-\boldsymbol{x}_{n'})}{|\boldsymbol{x}_n-\boldsymbol{x}_{n'}|}
\end{aligned}
$$

m_k, v_k, z_k are the mass, velocity and coordinates of k-th component of the system.

integrate with 6th order Implicit Runge-Kutta method

※no GW back reaction

Periastron Time Shift

We define the periastron time shift by $\Delta_p = T_N - P_0 N$

where T_N is the N-th periastron time.

Assuming the condition $|\int_0^t dt' \dot{P}(t')| \ll P_0$ as emission energy of GWs is not large.

We approximate Δ_p as $$\Delta_p \approx \frac{1}{P_0}\int_0^{T_N} dt \int_0^t dt' \dot{P}(t')$$

We then find,

$$\Delta_p = -\frac{192\pi}{5P_0}\left(\frac{G}{c^3}\frac{2\pi}{P_0}\right)^{5/3} m_1 m_2 (m_1+m_2)^{-1/3}\int_0^{T_N} dt \int_0^t dt' \left[1-\bar{e}^2(t')\right]^{-7/2}\left[1+\frac{73}{24}\bar{e}^2(t')+\frac{37}{96}\bar{e}^4(t')\right]$$

$\bar{e}(t)$ *is the eccentricity averaged over one orbital period of inner binary.*

Gravitational Waveform

We use Quadruople formula for the evolution of Gravitational Waveform.

$$[h_{ij}]_{quad} = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}\left(t - \frac{r}{c}\right)$$

$$Q^{ij} = M^{ij} - \frac{1}{3}\delta^{ij}M_{kk} = \int d^3x\, \rho(t,x)(x^i x^j - \frac{1}{3}r^2\delta^{ij})$$

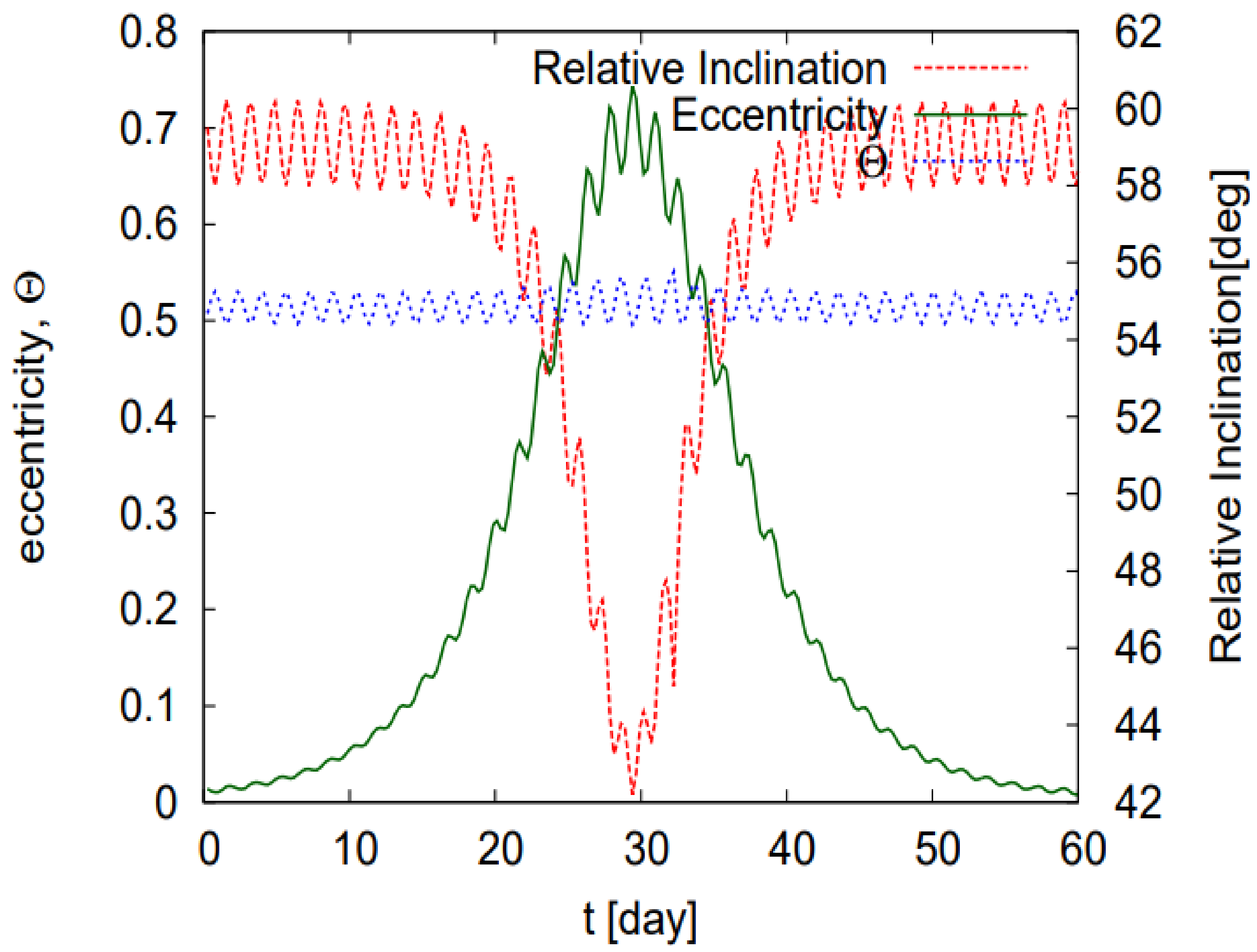
Q_{ij} *is the Quadrupole tensor.*

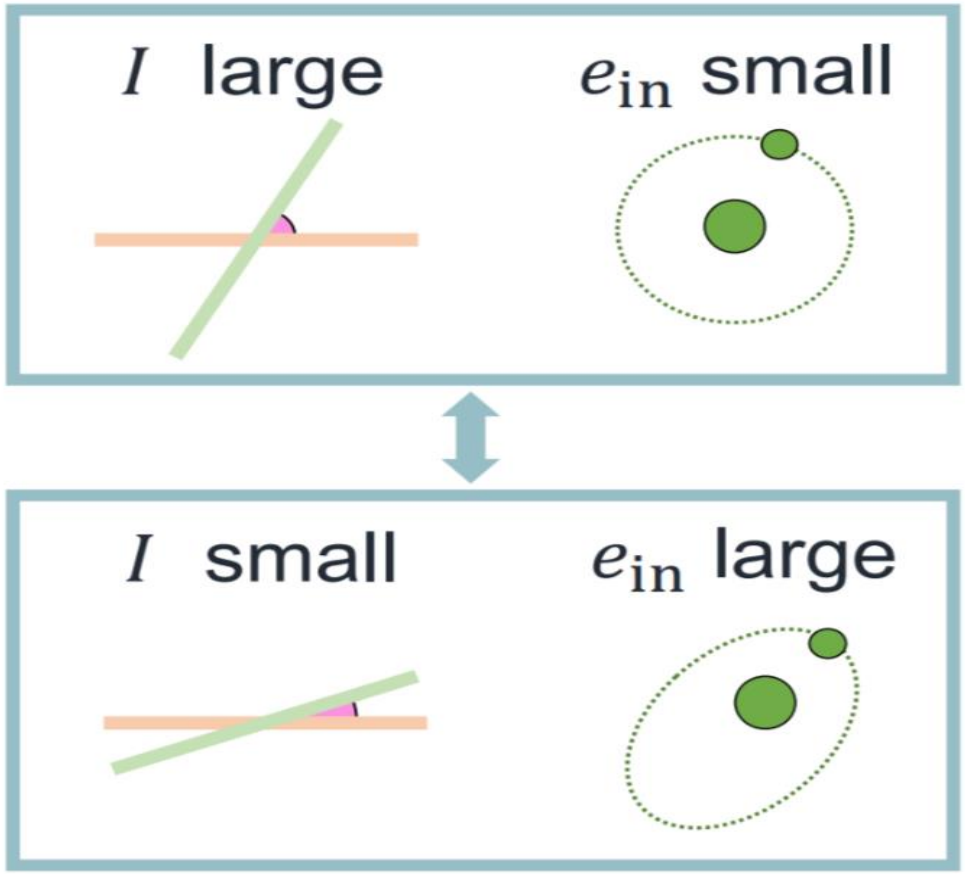
M_{ij} *is the second mass moment.*

Energy spectra is evaluated as

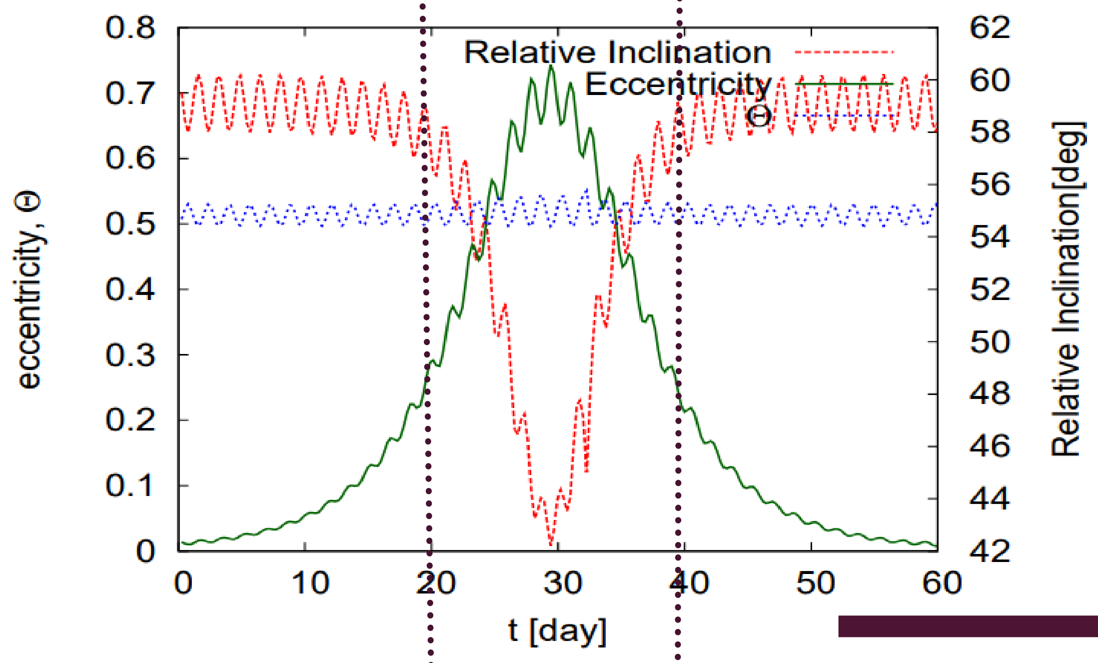
$$\frac{dE}{d\omega} = \frac{G\,\omega^6}{5\,\pi\,c^5}\,\tilde{Q}_{ij}\,(\omega)\tilde{Q}^*{}_{ij}(\omega)$$

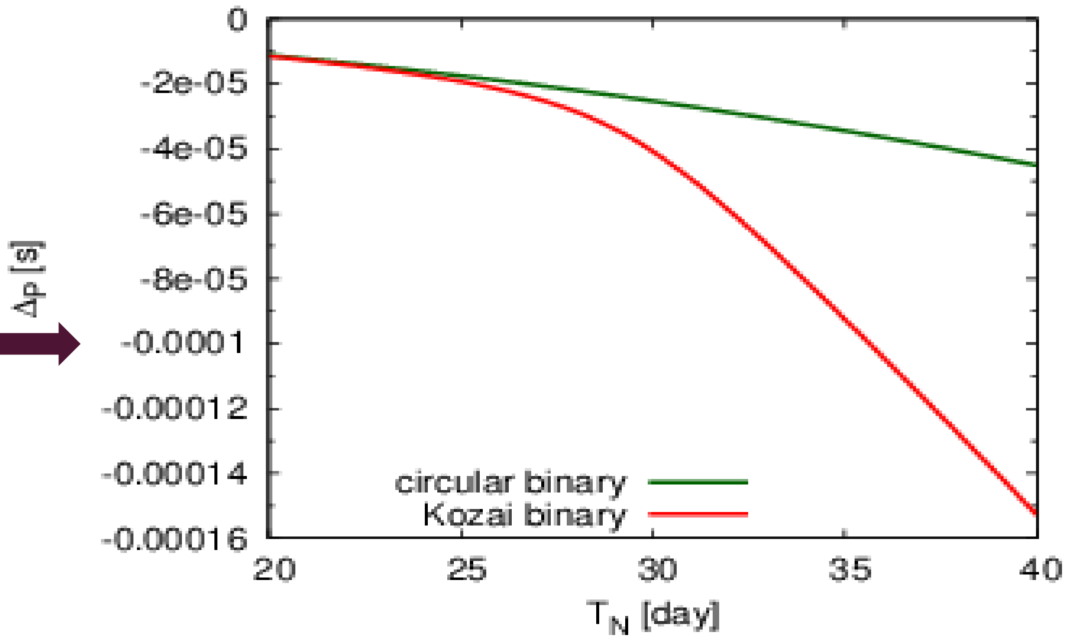
Results


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
eccentricity, Θ
Relative Inclination
Eccentricity
Θ
62
60
58
56
54
52
50
48
46
44
42
Relative Inclination[deg]
0
10
20
30
40
50
60
t [day]

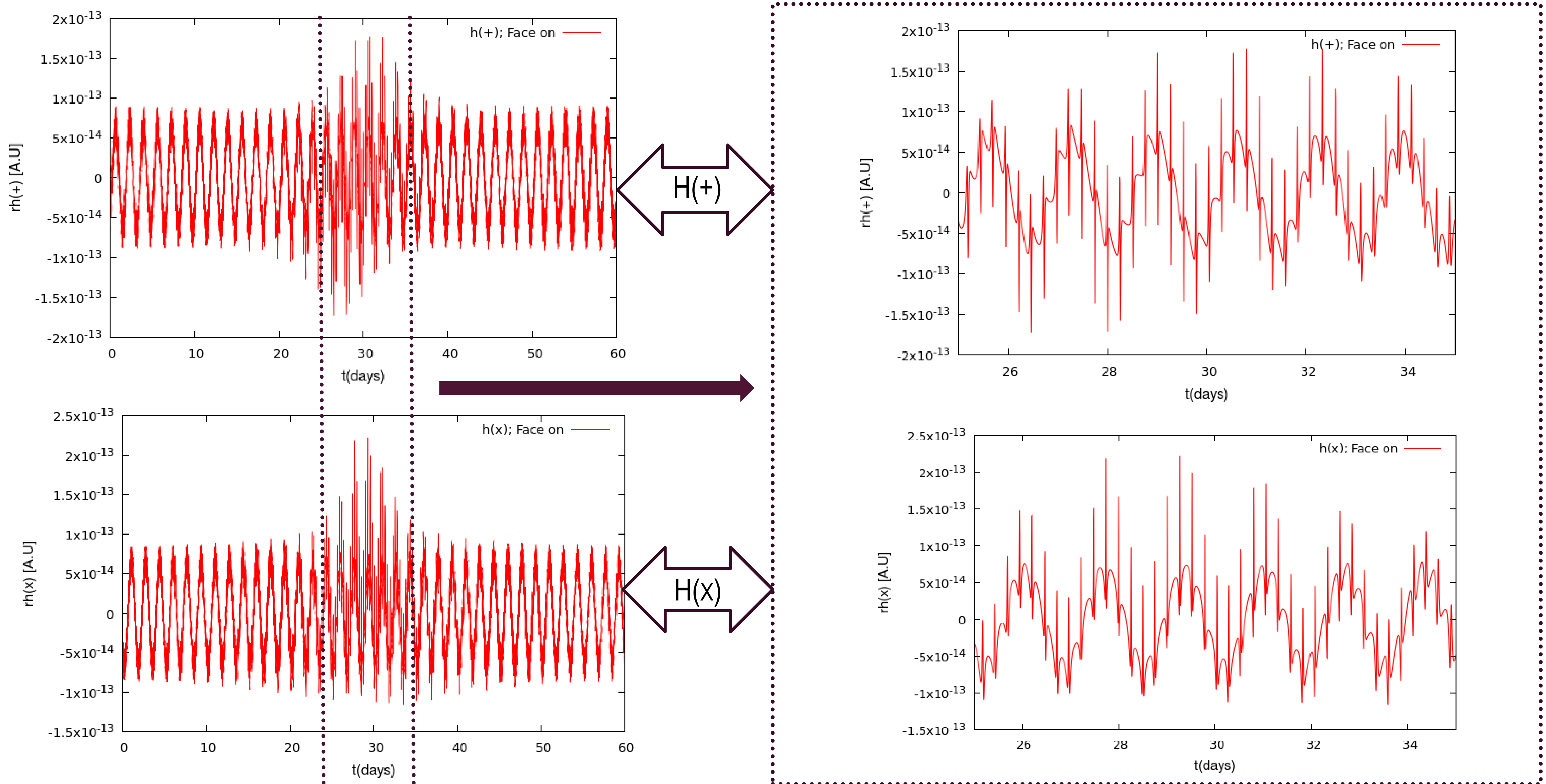

I large
e_{in} small
I small
e_{in} large

Result – Periastron Shift

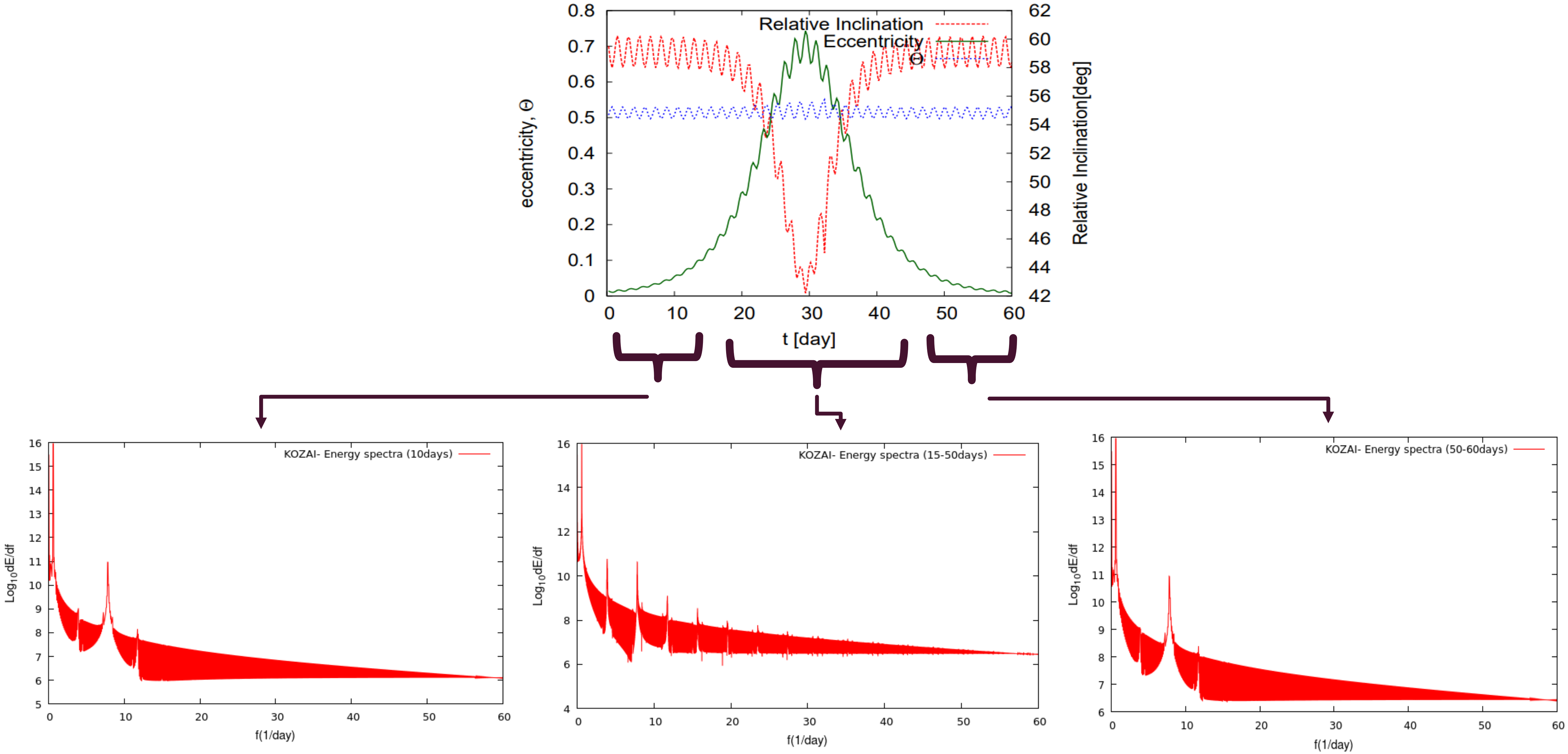




Result – Gravitational Waveform



Result – Energy Spectra



SUMMARY AND FUTURE WORK

- ✓ Kozai-Lidov effect can be seen in the time evolution of the cumulative shift of periastron time and the waveform of the inner binary in a hierarchical triplet.
- ✓ Due to high eccentricity, we can see many harmonics in the energy spectra of the waveform.
- ✓ The timescale of this effect corresponds to the Kozai-Lidov timescale.

- ➢ Now we are doing parameter search (for detectable range) and consider more realistic models.
- ➢ Future Task : Including back reaction effect in orbital evolution.