Holographic heat engine in Horndeski model with the $k-\mathrm{essence}$ sector

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Outline

- Background: Extended black hole thermodynamics
- Motivation
- Holographic heat engine in a Horndeski model
- Prospects

1.Background: Extended Black Hole Thermodynamics

For asymptotically AdS black holes, negative cosmological constant provides positive pressure to the whole spacetime thus yields the definition of the pressure for thermodynamics as

$$\mathrm{P} = -\frac{\Lambda}{8\pi}$$

- The extension shifts the identification of the mass M from being the energy U to being the enthalpy， to wit

$$M = H \equiv U + PV$$

- The first law now becomes $dM = TdS + VdP + \Phi dQ + \Omega dJ$
- The thermodynamic volume is the conjugate of pressure

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,Q,J}$$

With pressure and volume in play alongside temperature and entropy, extracting mechanical useful work from heat energy becomes natural.

In this engine, the input/exhaust of heat and the mechanical work can all be determined from the gravitational system, so the engine efficiency can be evaluated directly.

[1] C. V. Johnson, ‘Holographic Heat Engines’, Class. Quant. Grav. 31(2014)205002

[2] A. Karch and B. Robinson, ‘Holographic Black Hole Chemistry’, JHEP 12(2015)073

[3] Shi-Qian Hu and Xiao-Mei Kuang, ‘Holographic heat engine in Horndeski model with the k-essence sector’, Arxiv:1808.00176

[4] Shi-Qian Hu,Xiao-Mei Kuang, and Yen Chin Ong, ‘A note on Smarr Relation and Coupling Constants’, Arxiv:1810.06073

Thermodynamics and extended thermodynamics

Phenomena in the extended thermodynamics of black hole including Van der Waals like phase transition, triple points and reentrant phase transition and so on have been investigated. The possibility of extracting mechanical useful work from heat energy naturally materializes.

Thermodynamics		Black Hole Mechanics	
Enthalpy	$H = E + PV$	Mass	M
Temperature	T	Surface Gravity	$\frac{\kappa}{2\pi}$
Entropy	S	Horizon Area	$\frac{A}{4}$
Pressure	P	Cosmological Constant	$-\frac{\Lambda}{8\pi}$
First Law	$\delta H = T\delta S + V\delta P + \ldots$	First Law	$\delta M = \frac{\kappa}{8\pi}\delta A + V\delta P + \ldots$

[1] D. Kubizňák and R. B. Mann, JHEP 1207(2012)033

[2] S. Gunasekaran, R. B. Mann and D. Kubiznak, JHEP 1211(2012)110

[3] R. A. Hennigar and R. B. Mann, Entropy 17(2015)8056-8072

[4] N. Altamirano, D. Kubizňák, R. Mann and Z. Sherkatghanad, Class. Quant. Grav. 31(2014) 042001

[5] S. W. Wei, Y. X. Liu, Phys. Rev. D 90 (2014)044057

Thermodynamic Cycles and Heat Engines

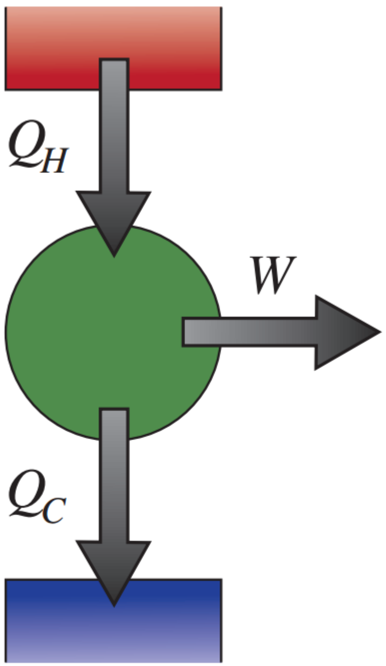


Figure 1: The heat engine flows.

The input of an amount of heat

$$Q_H$$

and the exhaust of an amount

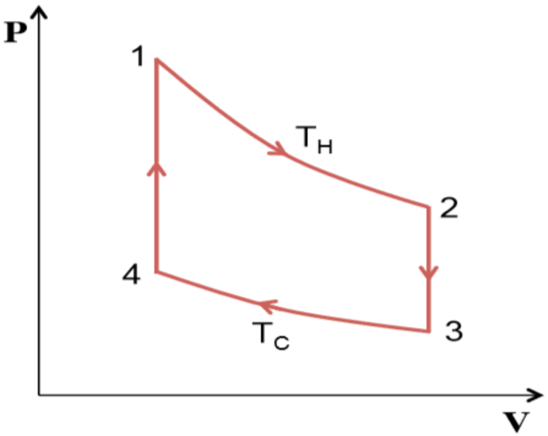
$$Q_C$$

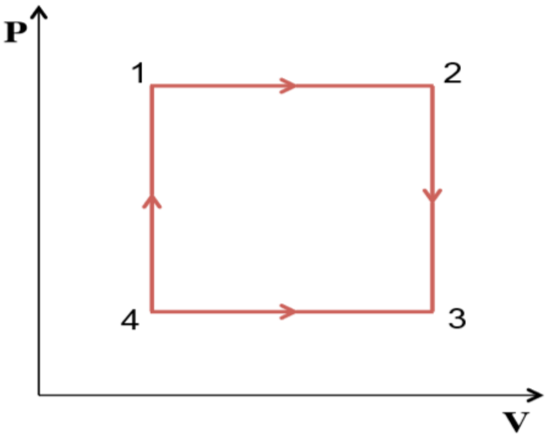
By the first law $W = Q_H - Q_C$

The efficiency of heat engine

$$\eta = W/Q_H = 1 - Q_C/Q_H$$

This is the maximum efficiency any heat engine can have.





Consider black holes undergoing the Carnot cycle. The efficiency of the Carnot cycle does not depend on the equation of state and hence should be the same for all black hole systems. It is given by

$$\eta_{Carnot} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

The rectangular paths in the $P - V\ plane$. The heat flow only occurs for the horizontal paths which are at constant pressure, using the first law $\delta M = \delta H = T\delta S + V\delta P$ with $\delta P = 0$,we get the efficiency

$$\eta = 1 - \frac{M_3 - M_4}{M_2 - M_1}$$

In our work, we investigate a charged black hole with k-essence, as a simple holographic realization of momentum dissipation.

Motivation

- In systems with prefect translational symmetry, the particles cannot dissipate their momentum. For heat engine, the implement of momentum dissipation is necessary.

- Previously in the context of holographic optical conductivity, translational symmetry was broken by either introducing a graviton mass term, or a lattice. Both methods are rather complicated.

David Vegh, CERN-PH-TH/2013-357,arXiv:1302.6586

Gary T. Horowitz and Jorge E. Santos, 10.1007/JHEP06(2013)087,arXiv:1301.0537

A simpler method to achieve this is via k-essence.

Tomas Andrade and Benjamin Withers, 10.1007/JhEp05(2014)101, arXiv:1311.5157

A Horndeski black hole model

- The four-dimensional Horndeski gravity with the $k-\mathrm{essence}$ sector:

$$S=\frac{1}{16\pi}\int\sqrt{-g}\,(R-2\Lambda-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\sum_{i=1}^{2}(\mathrm{X_i}+\gamma X_i^k))d^4x, \qquad \mathrm{X_i}=\frac{1}{2}\nabla^{\mu}\Phi_i\nabla_{\mu}\Phi_i$$

- From this, we can get the following exact black hole solution

$$ds^2=-f(r)dt^2+\frac{dr^2}{f(r)}+r^2\left(dx_1^2+dx_2^2\right)$$

With $$f(r)=\frac{r^2}{l^2}-\frac{2m}{r}-\frac{\lambda^2}{2}+\gamma\frac{\lambda^{2k}}{2^k(2k-3)}r^{2(1-k)}+\frac{q^2}{4r^2}$$

and the matter fields $\phi_1=\lambda\, x_1,\ \phi_2=\lambda\, x_2,$

$$A=\left(\mu-\frac{q}{r}\right)\mathrm{dt}$$

From this, we get the state equation

$$P=\frac{T}{2r_h}+\frac{q^2}{32\pi r_h^4}+\frac{\lambda^2}{16\pi r_h^2}+\frac{\gamma 2^{-k-3}\lambda^{2k}r_h^{-2k}}{\pi}$$

r_h : black hole horizon

3.Holographic heat engine in Horndeski

- The general specific heat is defined as $C = T(\partial S/\partial T)$,
- From the expression of temperature, we can get

$$\frac{\partial T}{\partial r_h} = \frac{2P + \frac{3q^2}{16\pi r_h^4} + \frac{\lambda^2}{8\pi r_h^2} + \frac{\gamma 2^{-k-2}(2k-1)\lambda^{2k} r_h^{-2k}}{\pi}}{1 - 2r_h \frac{\partial P}{\partial T}}$$

So the formula of the specific heat is

$$C = T\frac{\partial S}{\partial T} = T\frac{\frac{\partial S}{\partial r_h}}{\frac{\partial T}{\partial r_h}} = \left(1 - 2r_h\frac{\partial P}{\partial T}\right)\frac{2\pi P r_h^6 - \frac{1}{16}q^2 r_h^2 - \frac{1}{8}\lambda^2 r_h^4 - \gamma 2^{-k-2}\lambda^{2k} r_h^{6-2k}}{4\pi P r_h^4 + \frac{1}{4}\lambda^2 r_h^2 + \frac{3}{8}q^2 + \gamma 2^{-k-1}(2k-1)\lambda^{2k} r_h^{4-2k}}$$

The state equation gives us $(\frac{\partial P}{\partial T})_V = \frac{1}{2r_h}$. Thus, the specific heat at constant volume is reduced to $Cv = T(\frac{\partial P}{\partial T})_V$=0, which implies that adiabats and isochores are equivalent.

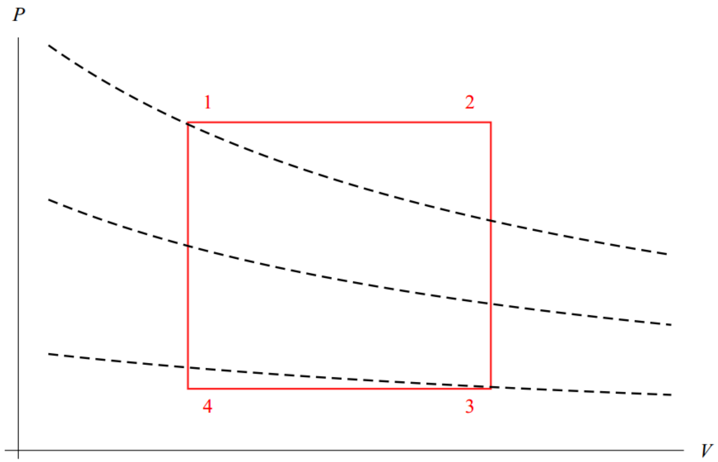


FIG. 1: Cartoon of the engine.

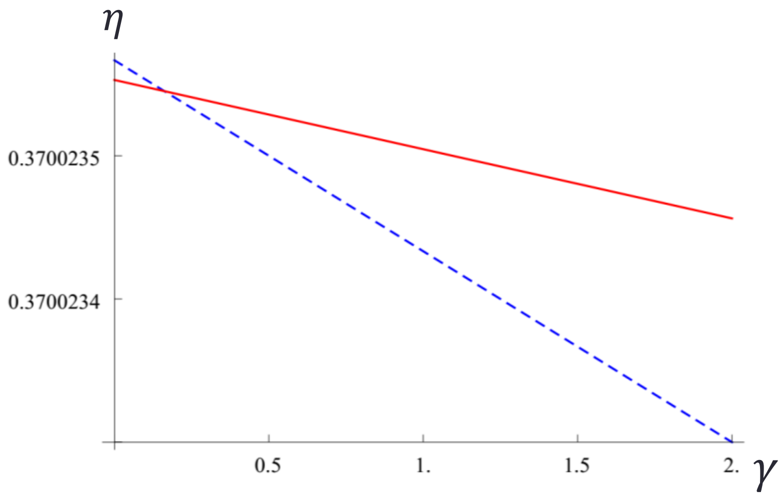
Now, we are ready to define a heat engine dual to the black hole. we consider a rectangular cycle in the P - V plane. The cycle consists of two isobars and two isochores as shown in the left, where 1;2;3;4 are four corners in the thermal flow cycle.

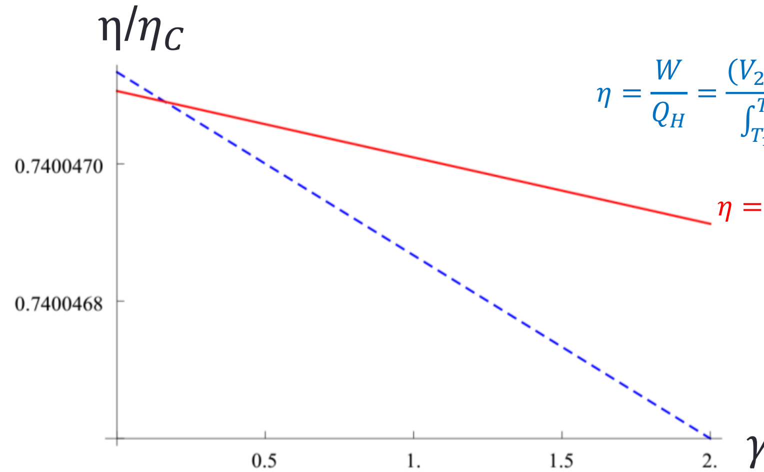
The engine efficiency is computed by

$$\eta = \frac{W}{Q_H} = \frac{(V_2 - V_1)(P_1 - P_4)}{\int_{T_1}^{T_2} C_p(P_1, T)\,dT}$$

In P – V plane, the isotherms at temperatures T_h and T_l with $T_h > T_l$ give the Carnot efficiency ηc and for our engine, Carnot efficiency $\eta_c = 1 - \frac{T_l}{T_h} = 1 - \frac{T_4}{T_2}$

Results





$$\eta = \frac{W}{Q_H} = \frac{(V_2 - V_1)(P_1 - P_4)}{\int_{T_1}^{T_2} C_p(P_1, T)\, dT}$$

$$\eta = 1 - \frac{M_3 - M_4}{M_2 - M_1}$$

$$\eta_C = 1 - \frac{T_C}{T_H}$$

η and η/η_C are both suppressed by increasing the coupling parameter γ and η is always lower than the Carnot efficiency.

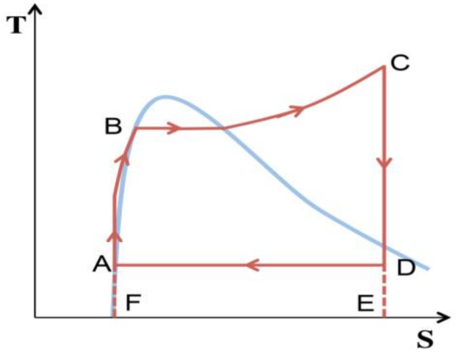
k	η	η/η_c
2	0.37002343	0.74004687
5/2	0.37002355	0.74004712
3	0.37002357	0.74004713

TABLE I: Engine efficiency effected by k with $\gamma = 1$.

Table 1 list the related efficiency for samples of exponent k. As k increases, η increases slowly, so does η/η_C.

Prospects

- Study the efficiency with other heat flow cycles such as Rankine circle.



The Rankine cycle, which is used in for example, steam turbines, and makes use of the liquid/gas phase transition. The efficiency for this cycle is given by

$$\eta = 1 - \frac{Area(ADEFA)}{Area(FABCDEF)}$$

- Physics of heat flow from CFT viewpoint?

The stress tensor's properties are consistent with that of a conformally invariant fluid with density ρ(proportional to pressure). Both of them are set by the energy. So that fluid pressure is not the p of the AdS thermodynamics. Consequently, it is interested to figure out the meaning of p and mechanical work done over the circle in the field theory.

Thank you!

Shi-Qian Hu and Xiao-Mei Kuang,
'Holographic heat engine in Horndeski model with the k-essence sector',
Sci. China-Phys. Mech. Astron. 62 (2019) 060411, arxiv:1808.00176

In order to study the efficiency directly from the definition. We consider the large temperature limit , in term of large T ,we obtain

$$
\begin{aligned}
r_h &= \frac{T}{2P} + \frac{\lambda^2}{8\pi T} + \frac{8\pi P^2 q^2 - \lambda^4 P}{32\pi^2 T^3} + \frac{\lambda^6 P^2 - 16\pi\lambda^2 P^3 q^2}{64\pi^3 T^5} + \cdots \\
&+ \gamma\left(\frac{2^{k-4}\lambda^{2k}P^{2k-2}}{\pi T^{2k-1}} - \frac{2^{k-5}k\lambda^{2k+2}P^{2k-1}}{\pi^2 T^{2k+1}} + \cdots\right).
\end{aligned}
$$

$$
\begin{aligned}
V &= \frac{T^3}{24P^3} + \frac{\lambda^2 T}{32\pi P^2} + \frac{q^2}{16\pi T} + \frac{\lambda^6 - 48\pi\lambda^2 P q^2}{1536\pi^3 T^3} - \frac{128\pi^2 P^3 q^4 - 48\pi\lambda^4 P^2 q^2 + \lambda^8 P}{2048\pi^4 T^5} + \cdots \\
&+ \frac{\gamma}{4\pi}\left(\frac{2^{k-5}(1-k)\lambda^{2k+2}P^{2k-3}}{T^{2k-1}} + \frac{2^{k-4}\lambda^{2k}P^{2k-4}}{T^{2k-3}} + \cdots\right),
\end{aligned}
$$

$$
\begin{aligned}
C_P &= \frac{T^2}{8P^2} + \frac{\lambda^4 - 16\pi P q^2}{128\pi^2 T^2} + \frac{24\pi\lambda^2 P^2 q^2 - \lambda^6 P}{128\pi^3 T^4} + \frac{960\pi^2 P^4 q^4 - 480\pi\lambda^4 P^3 q^2 + 15\lambda^8 P^2}{2048\pi^4 T^6}. \\
&+ \gamma\left(\frac{2^{k-5}(k-1)\lambda^{2k}P^{2k-3}}{\pi T^{2k-2}} + \frac{2^{k-7}k(2k-1)\lambda^{2k+2}P^{2k-2}}{\pi^2 T^{2k}} + \cdots\right),
\end{aligned}
$$