Proof of the quantum null energy condition for fermions

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Null energy condition (NEC)

$$T_{kk} := T_{ab}k^a k^b$$

Condition on T_{ab} satisfied by most reasonable classical matter / fields

NEC ensures that light rays are never repelled by matter in Einstein's equation

NEC is crucial to many results in general relativity

Example: Hawking's area theorem

Quantum fields violate all local energy conditions

QNEC

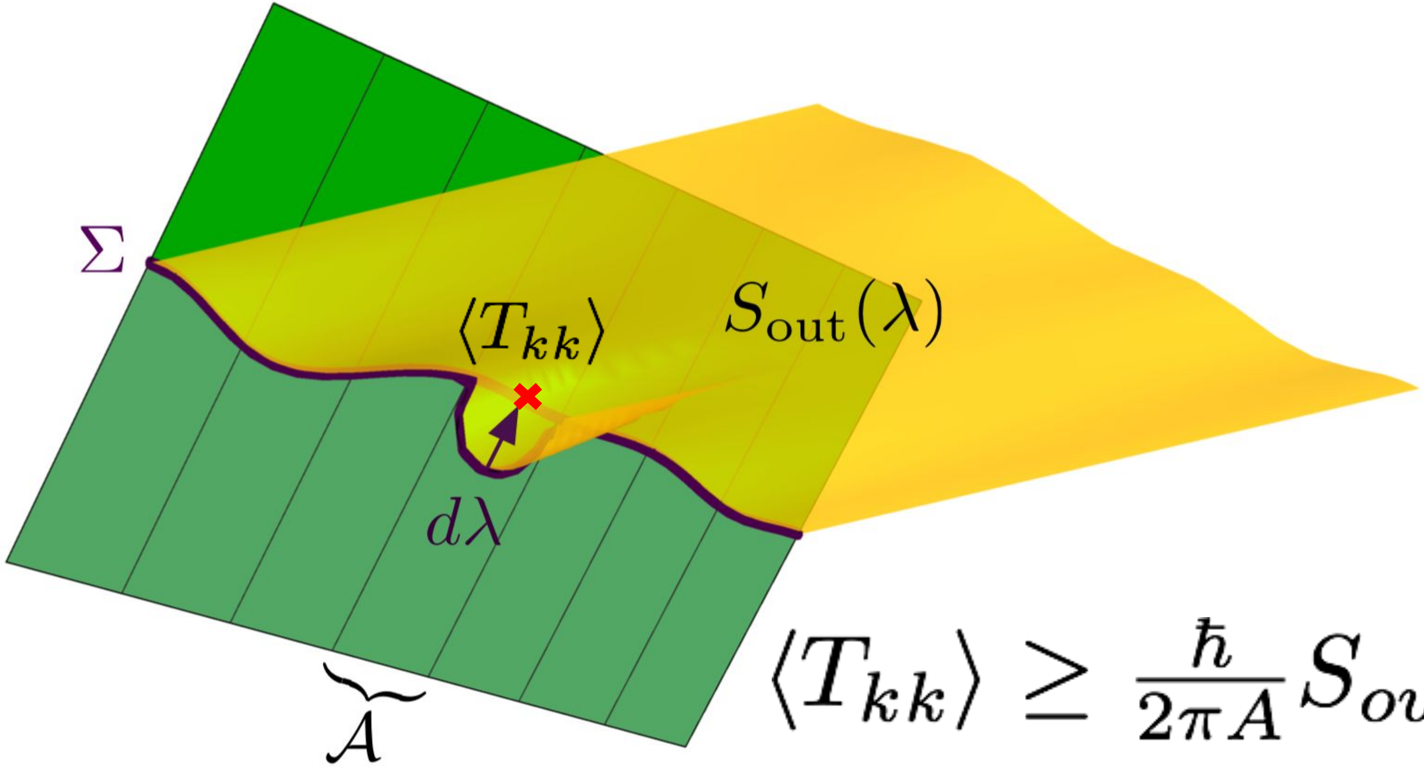
We consider a quantum generalization of NEC first proposed by R.Bousso, Z.Fisher S.Leichenauer and A.Wall (Phys. Rev. D **93**, 064044)

Covariant version of spherical entropy bound

$$S \leq A/4$$

The bound on the null energy that a point *p* is computed from the von Neumann entropy of the quantum fields restricted to some spatial region **Σ.**

Boundary of **Σ** contains *p* and normal to *k*, and has vanishing null expansion at p



$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi \mathcal{A}} S_{out}(\Sigma(\lambda))''|_p$$

Consider deformations of spatial surface **Σ** about the point *p* in the null direction orthogonal **Σ**

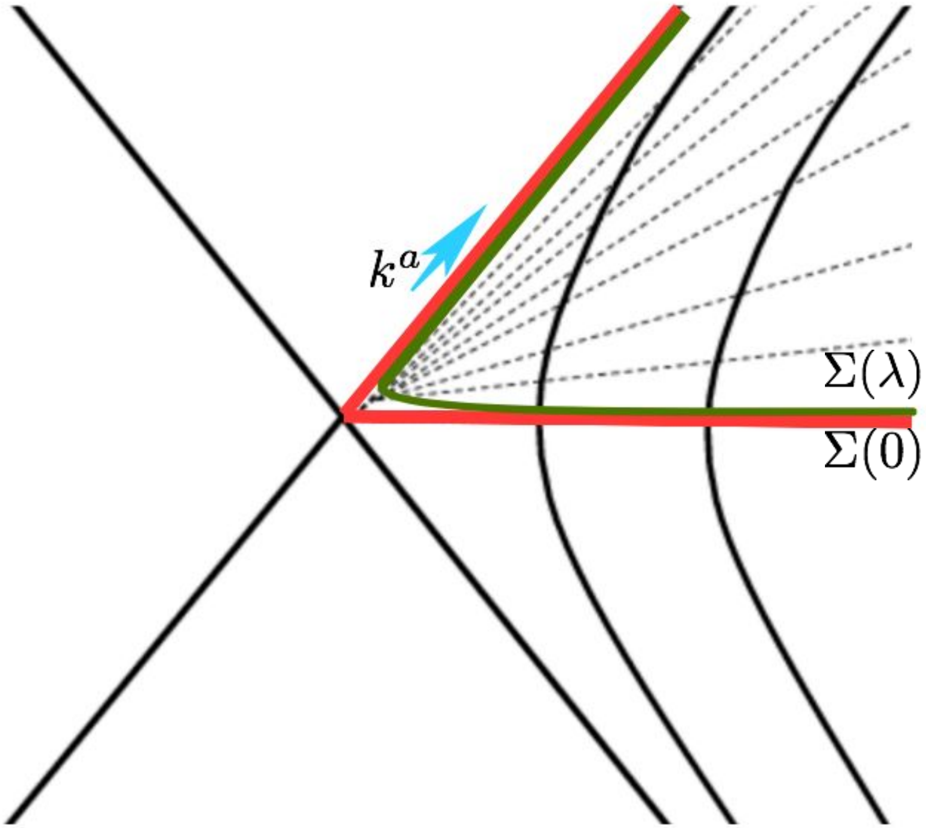
Label deformations by **λ**, affine parameter along null generator about *p*.

QNEC for fermions

Restrict spatial region **Σ** the half space.

By unitary invariance of entropy, we can shift **Σ** along the null hypersurface

Deformation of surface equivalent to tracing out degrees of freedom along null hypersurface



R.Bousso, Z.Fisher, J.Koeller, S.Leichenauer and A.Wall (Phys. Rev. D **93**, 024017)

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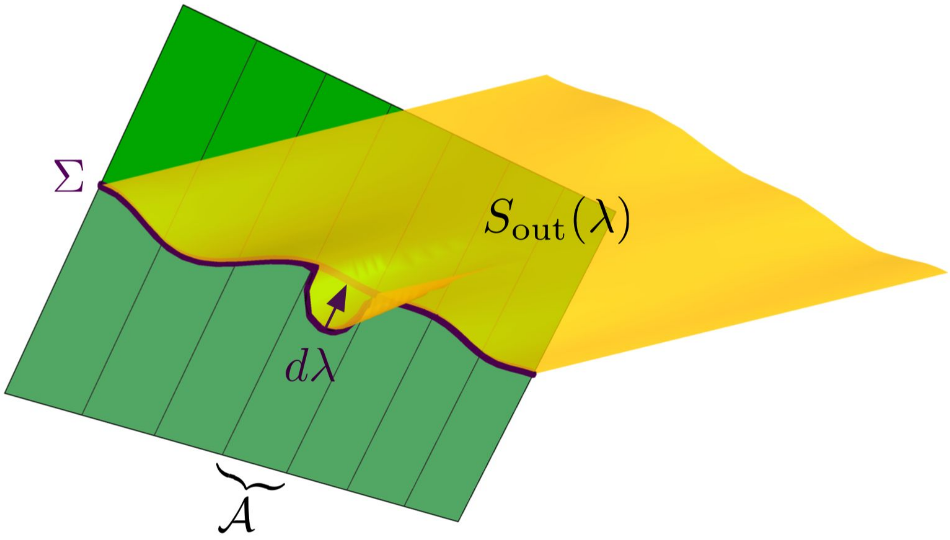

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Null quantization

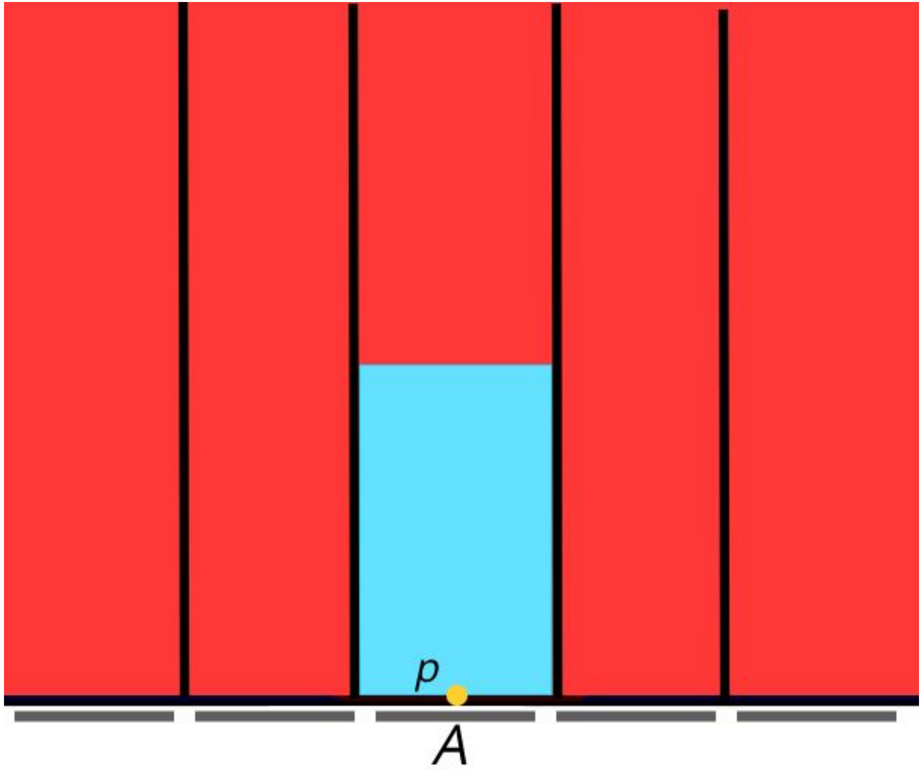
D>2

Discretize and quantize on the null hypersurface.

1+1 CFT free chiral fermion

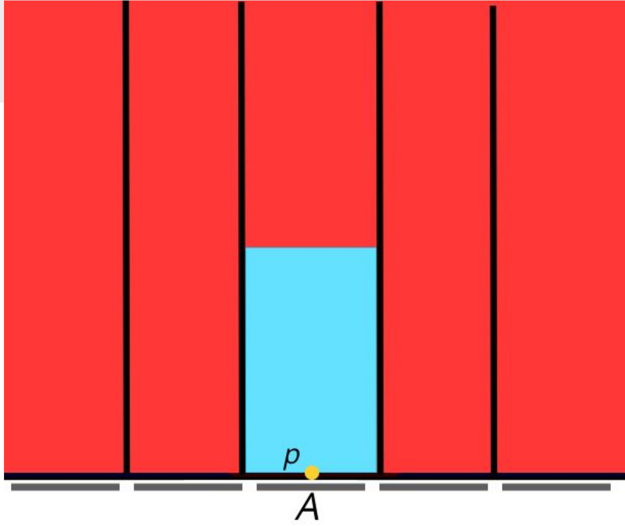
Take limit as *A* goes to *0*

Distinguished pencil



$$\mathcal{H} = \mathcal{H}_{pen} \otimes \mathcal{H}_{aux}$$

State



$$\mathcal{H} = \mathcal{H}_{pen} \otimes \mathcal{H}_{aux}$$

$$\rho(\lambda) = \rho^{(0)}_{pen}(\lambda) \otimes \rho^{(0)}_{aux} + \sigma(\lambda)$$

$\rho^{(0)}_{pen}(\lambda)$ is the vacuum density matrix on the pencil with affine parameter greater than **λ**

$\rho^{(0)}_{aux}$ is some arbitrary state on the auxiliary system

$\sigma(\lambda)$ is small perturbation of order $A^{\frac{1}{2}}$ (higher order terms are irrelevant)
obtained by taking partial trace of **|0><1|** and **|1><0|**

Expansion of entropy

Expanding in $\sigma(\lambda)$

$$S_{out}(\lambda) = S^{(0)}(\lambda) + S^{(1)}(\lambda) + S^{(2)}(\lambda)$$

Expansion on entropy

$$\left(S^{(0)} + S^{(1)}\right)'' = \frac{2\pi A}{\hbar} \langle T_{kk} \rangle$$

$$\frac{\hbar}{2\pi A} S''_{out} - \langle T_{kk} \rangle = \frac{\hbar}{2\pi A} S^{(2)''} + \ldots$$

Expansion on entropy

$$\left(S^{(0)} + S^{(1)}\right)'' = \frac{2\pi A}{\hbar} \langle T_{kk} \rangle$$

$$\frac{\hbar}{2\pi A} S''_{out} - \langle T_{kk} \rangle = \frac{\hbar}{2\pi A} S^{(2)''} + \dots$$

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi A} S''_{out} \quad \Leftrightarrow \quad S^{(2)''} \leq 0$$

Replica trick

$$S_{out} = (1 - n\partial_n)\log\left(\mathrm{Tr}[\rho^n]\right)|_{n=1}$$

$$\tilde{Z}_n = \mathrm{Tr}[\rho^n] = \mathrm{Tr}[\left(\rho^{(0)} + \sigma\right)^n]$$

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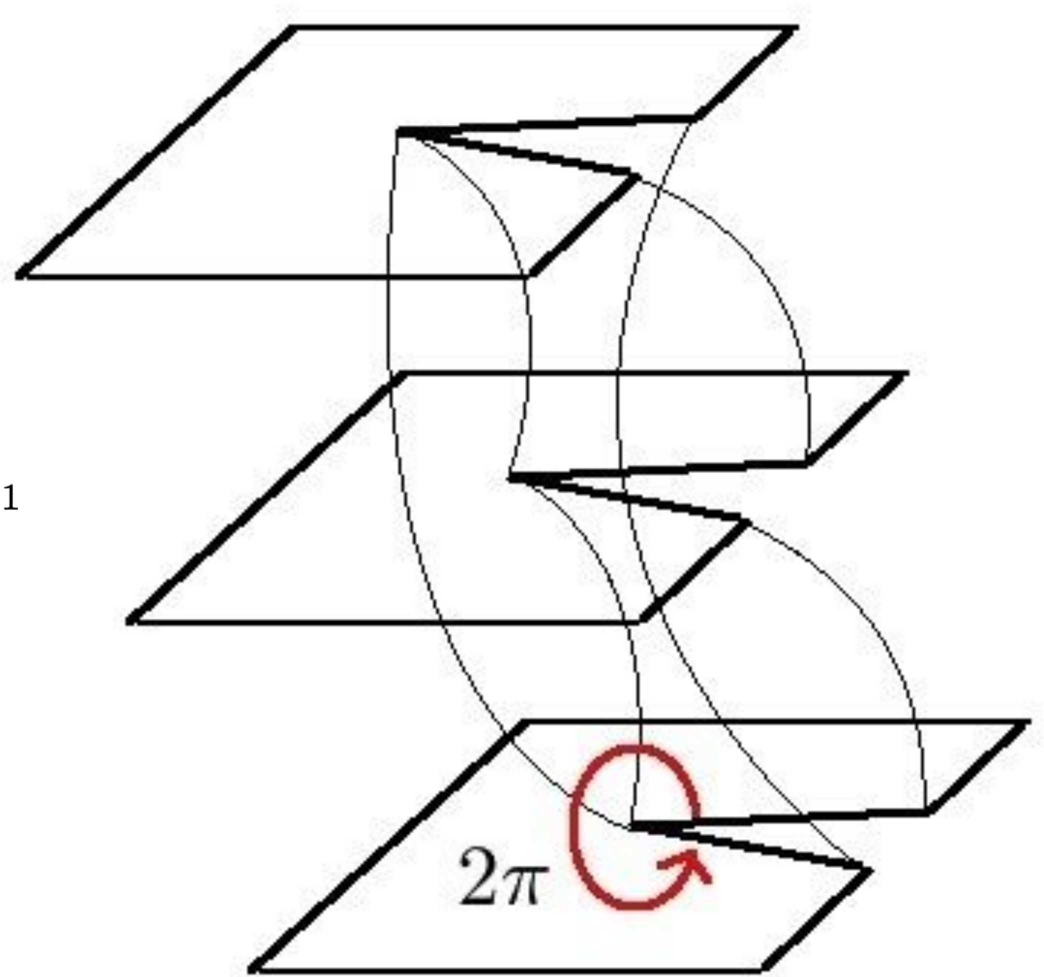

Replica trick + analytical continuation

$$S^{(2)''} \leq 0$$

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi A} S''_{out}$$

Summary

Shown that QNEC holds for free fermionic field theories, and to any points that lie on a stationary null surface.

Used methods similar to R.Bousso, Z.Fisher, J.Koeller, S.Leichenauer and A.Wall (Phys. Rev. D **93**, 024017)

Null quantization - 1+1 CFT free chiral fermions

Replica trick

Analytical continuation

Acknowledgement

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Questions?

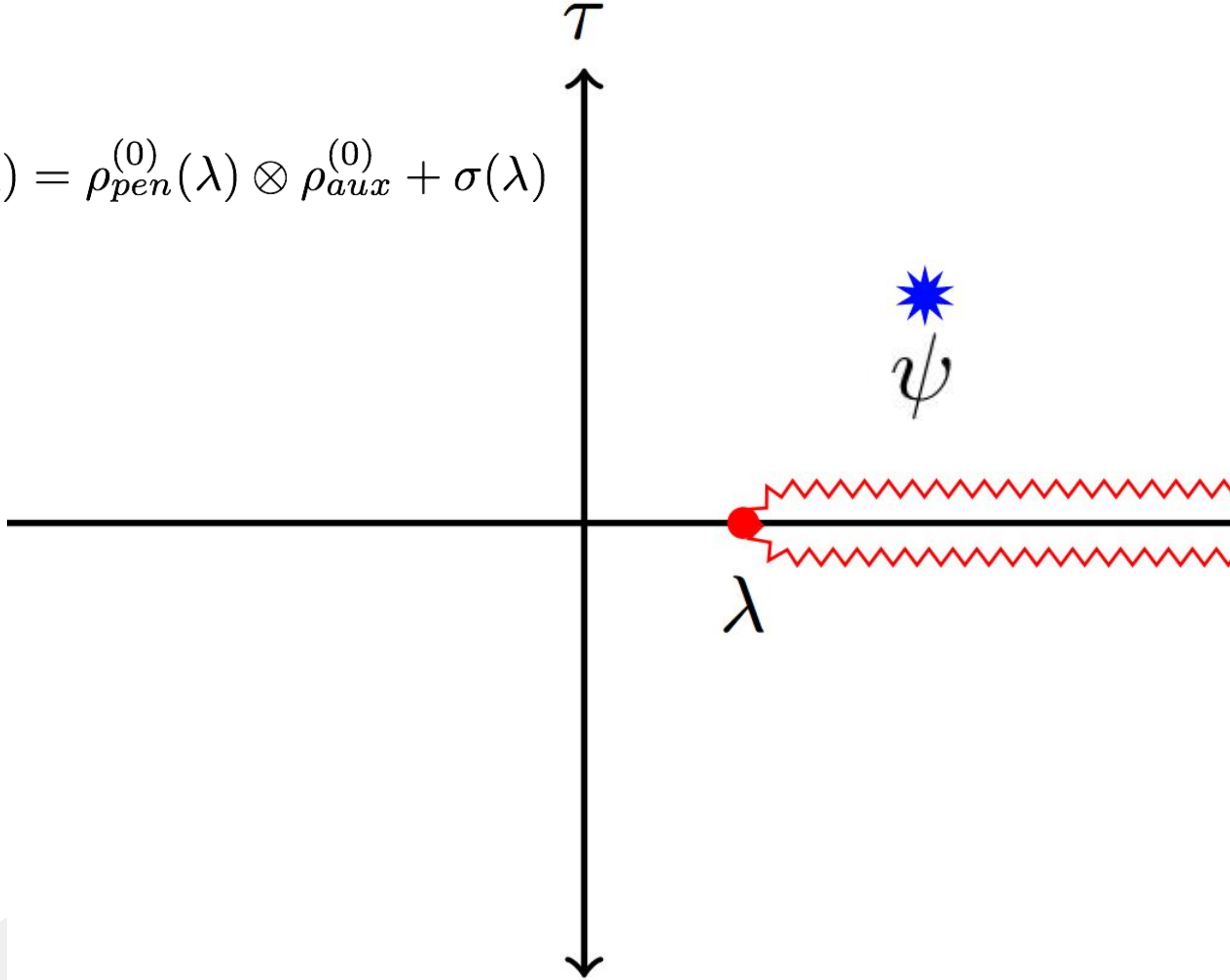
Operator insertions - Extra

Quantization on a surface of constant Euclidean time *it=0* equivalent to quantization on the Rindler horizon

In chiral theory, translations in *z* is equivalent to translation in *x*

z=x+t

$$\rho(\lambda) = \rho^{(0)}_{pen}(\lambda) \otimes \rho^{(0)}_{aux} + \sigma(\lambda)$$



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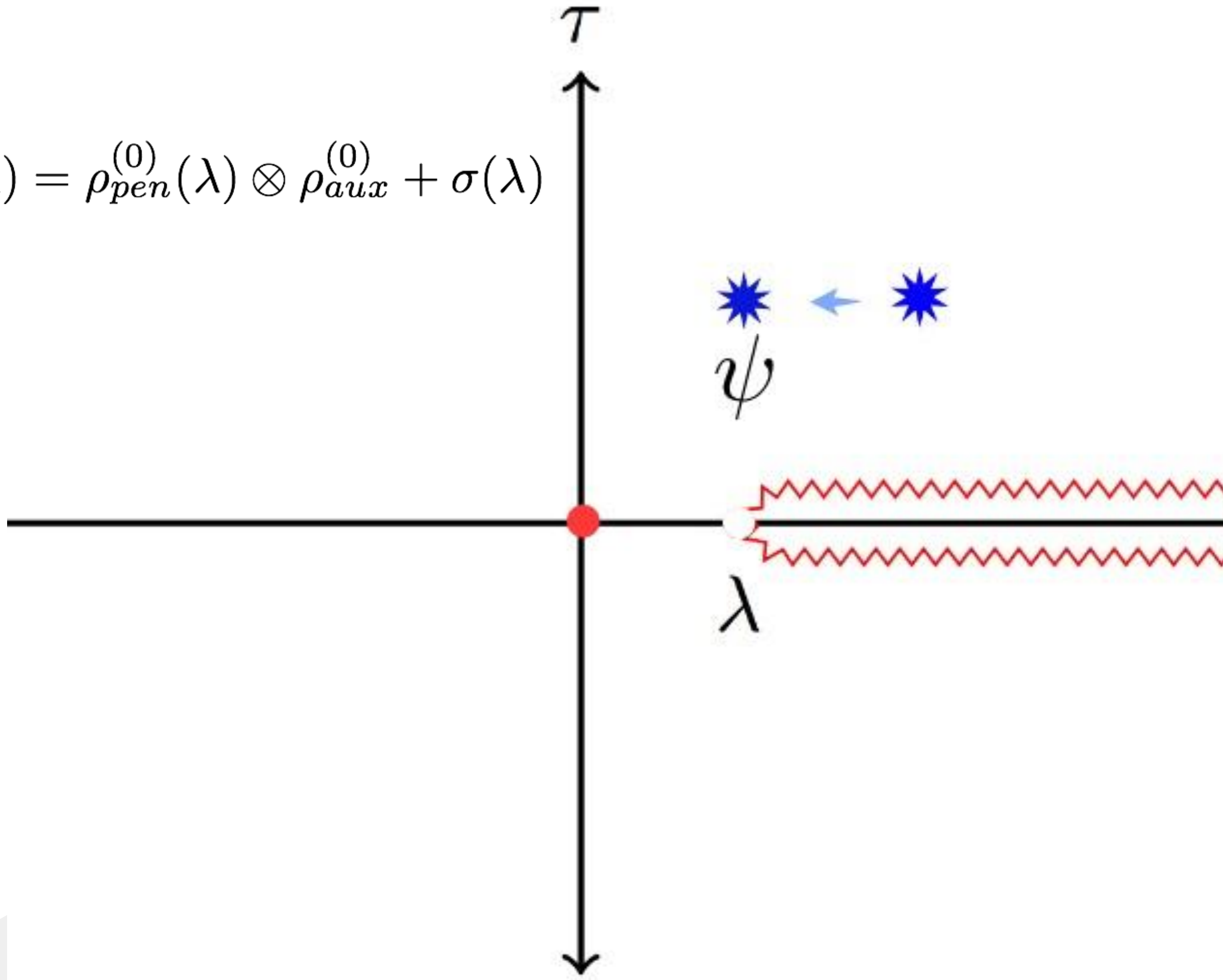

Extra

$$D(f) := (1 - n\partial_n) f|_{n=1}$$

$f(1)=0$

$D(fg)=g(1)D(f)$

Extra

$Tr(p^n) = O(n)A(n)+E(n)B(n)$

O is an odd switch function, E is an even switch function

$D(O(n)A(n)+E(n)B(n))=D(A)$

Extra