Initial Conditions for Inflation in an FRW Universe
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Inflationary Dynamics of a Scalar Field

Action of a scalar field minimally coupled to gravity

$$S[\phi] = \int \mathrm{d}^4 x \sqrt{-g}\, \mathcal{L}(F, \phi)$$

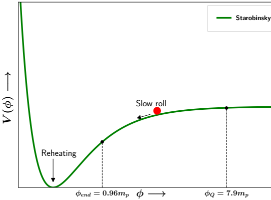
with $F = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi$ which for a canonical kinetic term leads to

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right),$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)\left(\rho_\phi + 3\,p_\phi\right) = -\frac{1}{3m_p^2}\left(\dot{\phi}^2 - V(\phi)\right)$$



The dynamics of the scalar field is governed by

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

while the equation of state is

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

Inflationary Dynamics

For an extended period of inflation,

$$\epsilon_{H} = -\frac{\dot{H}}{H^2} < 1 \ , \quad \eta_{H} = -\frac{\ddot{\phi}}{H\dot{\phi}} < 1$$

Primordial Power-spectra

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{K}{K_*}\right)^{n_S - 1}$$

$$\Delta^2_t = A_t \left(\frac{K}{K_*}\right)^{n_t - 1}$$

with

$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_{H}}, \ A_t = \frac{2}{\pi^2}\left(\frac{H_*}{m_p}\right)^2$$

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Scalar Spectral Index

$$n_S = 1 + 2\eta_{_H} - 4\epsilon_{_H}$$

Tensor to Scalar Ratio

$$r = 16\epsilon_{_H}$$

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Potential slow-roll parameters

$$\epsilon_V(\phi) = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V(\phi) = m_p^2\left(\frac{V''}{V}\right)$$

Slow-roll conditions corresponds to $\epsilon_H, \eta_H \ll 1$, where
$\epsilon_H \simeq \epsilon_V$, $\eta_H \simeq \eta_V - \epsilon_V$

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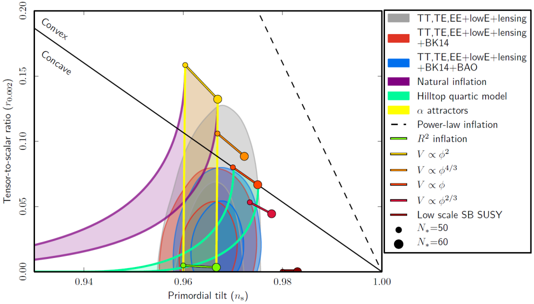
What do we know from CMB Observations



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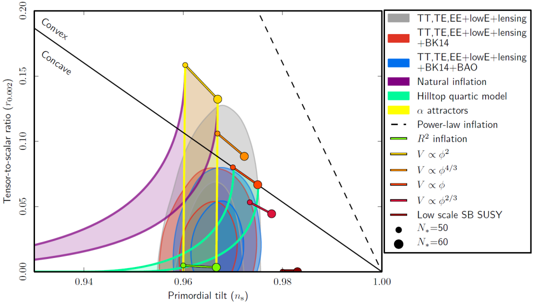

$$n_S \in [0.953, 0.977]$$

$$r \leq 0.06$$

$$\Rightarrow$$

$$\epsilon_H \leq 0.00375$$

$$|\eta_H| < 0.02$$

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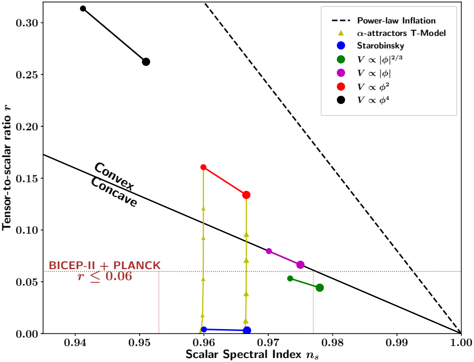

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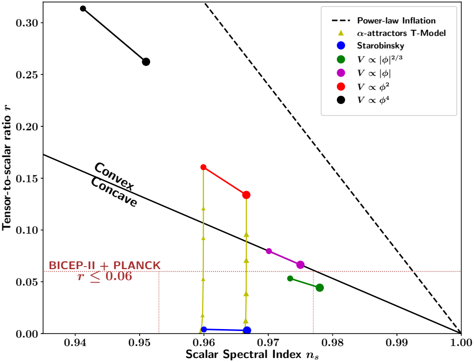

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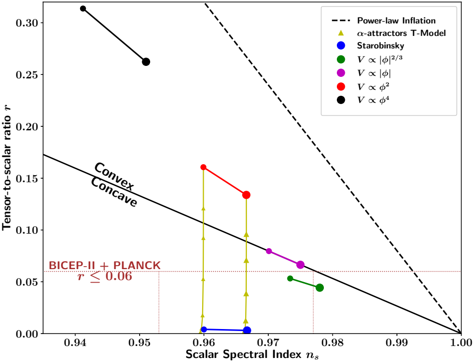

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concave potentials over convex potentials. Tight constraints on the upper bound of r imply asymptotically flat plateau like potentials are favoured. Issue of initial conditions for Inflation becomes more important!!

Initial Conditions for Inflation in an FRW Universe

Inflation $\Rightarrow$ Accelerated Expansion $\Rightarrow \epsilon_H < 1$. When the energy budget of the universe is dominated by the inflaton field, this implies,

$$w_\phi < -\frac{1}{3} \quad \text{or} \quad \dot{\phi}^2 < V(\phi)$$

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(In fact recent observations of $r \leq 0.06$ imply $\frac{1}{2}\dot{\phi}^2 \leq 0.001 \;\, V(\phi)$), Even though this condition seems restrictive, it is actually quite generic and can be quantified, at least for monotonically increasing potentials.

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Role of Friction term $H\dot{\phi}$ and existence of inflationary separatrix (indicating inflationary trajectories are local attractors at least for large field inflation) [Albretch and Brandenberger (1985)], [Brandenberger and Kung (1990)], [Remmen and Carroll (2013)]

Hamilton-Jacobi Formalism: $H_{,\phi}^2 - \frac{3}{2}H^2 = -\frac{1}{2}V(\phi)$, $\Delta N = \frac{1}{\sqrt{2\epsilon_H}}\frac{|\Delta\phi|}{m_p}$

$$\delta H(\phi) = \delta H(\phi_i)e^{-3(N-N_i)}$$

Methodology

- We explicitly calculate the set of initial conditions that yield **adequate inflation** i.e $N_e \geq 60$, where $a(t_e) = a(t_i)e^{N_e}$ so $N_e = \int_{t_i}^{t_e} H dt$
- For a given fixed initial energy scale $\rho(\phi_i)$ of inflation (or equivalently H_i), the first Friedmann equation becomes the equation of a circle

$$R^2 = X^2 + Y^2$$

where

$$R = \sqrt{6}\frac{H}{m_p}, \quad X = \hat{\phi}\frac{\sqrt{2V(\phi)}}{m_p^2}, \quad Y = \frac{1}{m_p^2}\frac{d\phi}{dt}$$

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with $\hat{\phi} = \frac{\phi}{|\phi|}$ (this ensures that X and ϕ have same sign). So fixing initial energy scale of inflation is equivalent to fixing the radius of the circle R while varying X and Y **uniformly**. Our analysis [Mishra, Sahni and Toporensky PRD 2018] is a natural and generalized extension of the original work by [Belinsky et. al 1985].

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We begin explaining our methodology for power-law potentials through the example of simple Quadratic Chaotic Inflation

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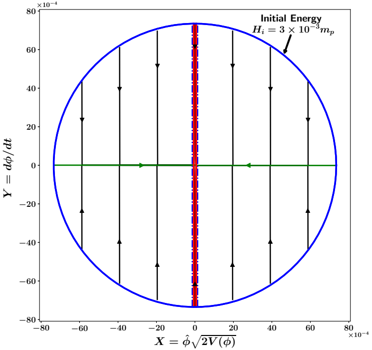

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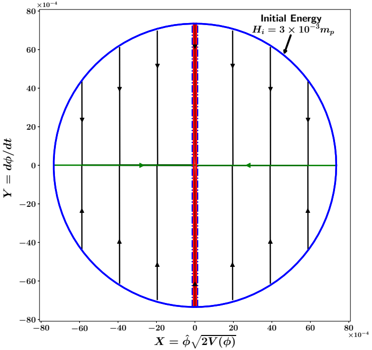

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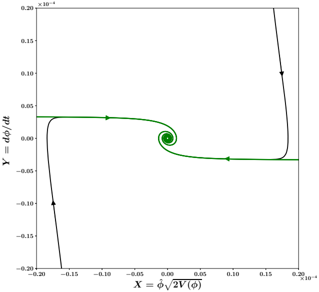

Zoomed View: Attractor Slow-roll Trajectories

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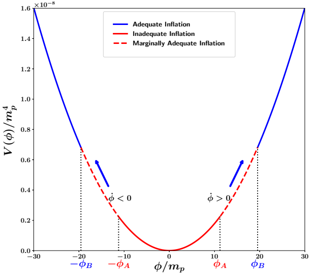
Range of Initial Conditions in the Field Space



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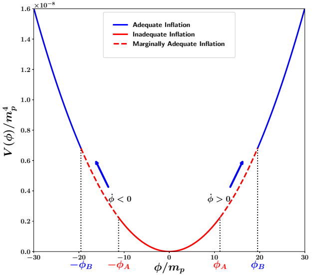
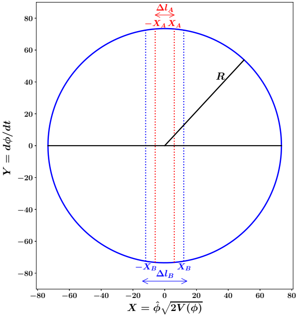


Defining the 'Measure'



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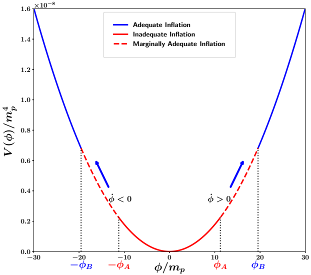
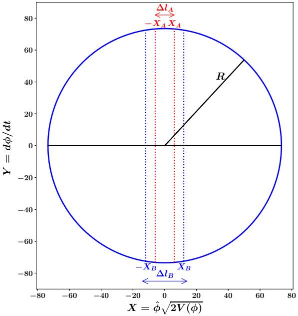


Defining the 'Measure'



H_i (in m_p)	ϕ_A (in m_p)	ϕ_B (in m_p)	$f_A = 2\frac{\Delta l_A}{l}$	$f_B = 2\frac{\Delta l_B}{l}$
3×10^{-3}	11.22	19.55	5.80×10^{-3}	1.01×10^{-2}
3×10^{-2}	9.33	21.38	4.83×10^{-4}	1.11×10^{-3}
3×10^{-1}	7.47	23.27	3.86×10^{-5}	1.20×10^{-4}

Initial Conditions for Power-law Potentials: Axion-Monodromy

Axion-Monodromy potential is given by [Silverstein, Westphal, McAllister 2008-09]

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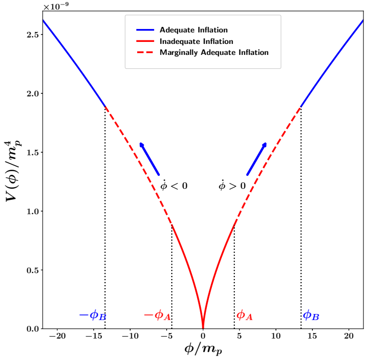
Fractional Monodromy Inflation $p = 2/3$



Range of Initial field values

Comparison of Power-law Potentials

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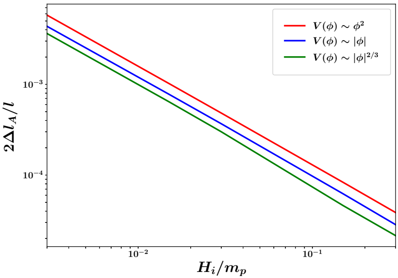

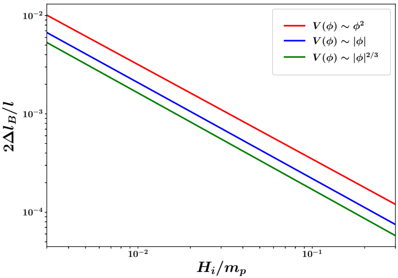

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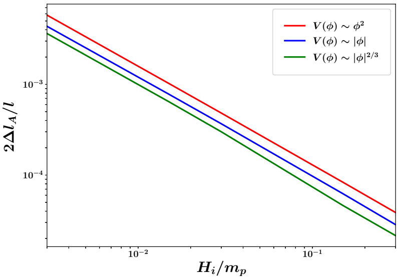

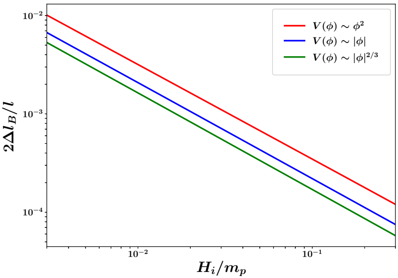

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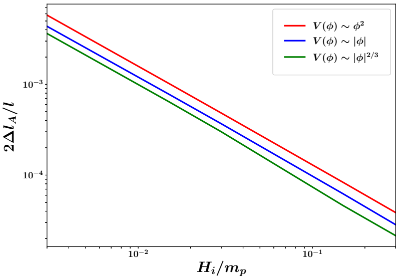

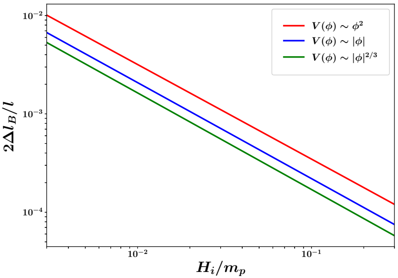

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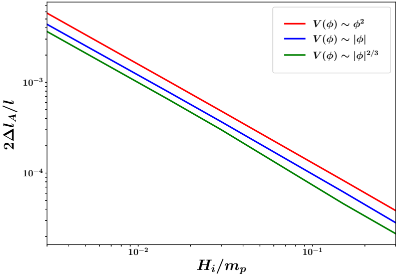

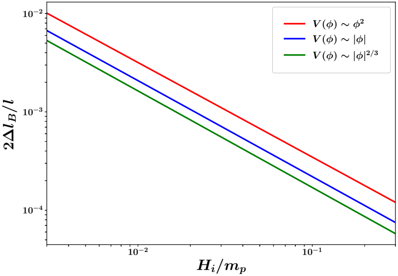

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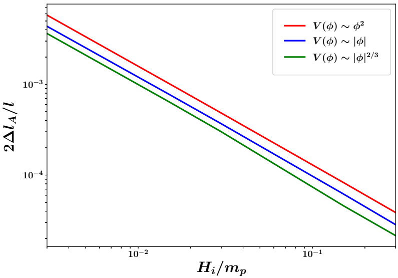

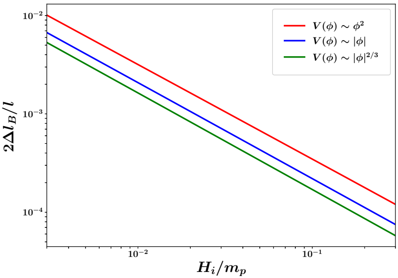

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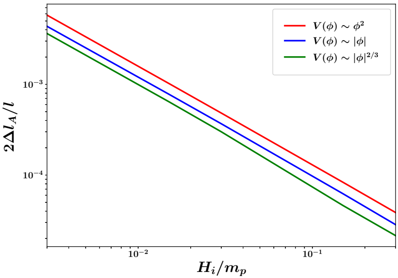

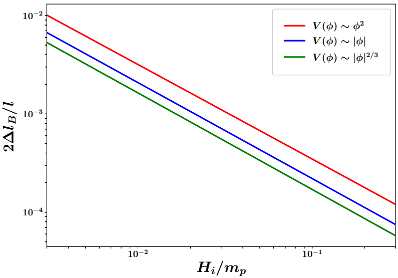

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But [Planck2018] strongly favours **Asymptotically Flat (Plateau-like)** potentials.

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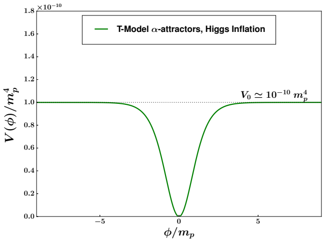

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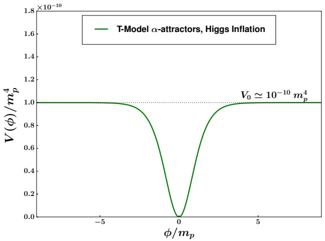

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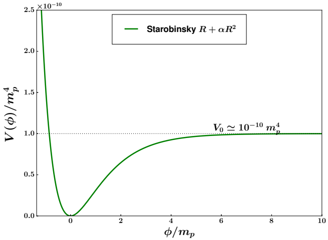
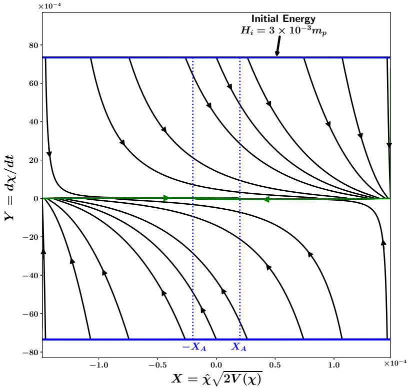


[Starobinsky 1980, Phys. Lett 91B]

Asymptotically Flat Potentials: Initial Conditions



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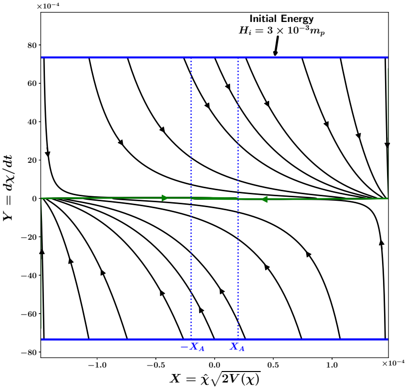

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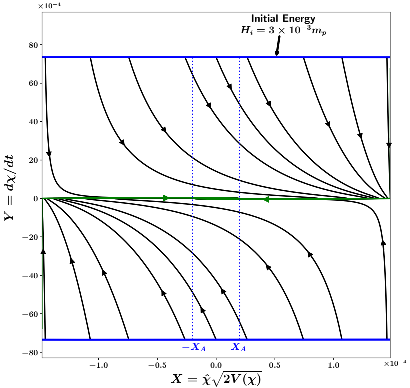

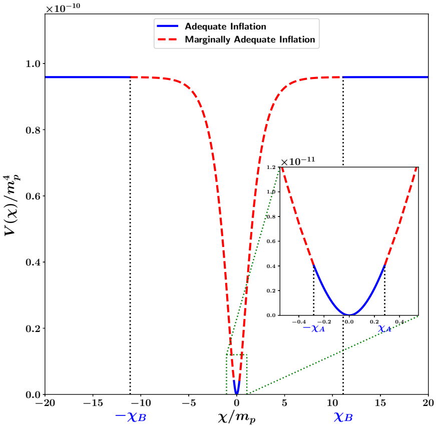

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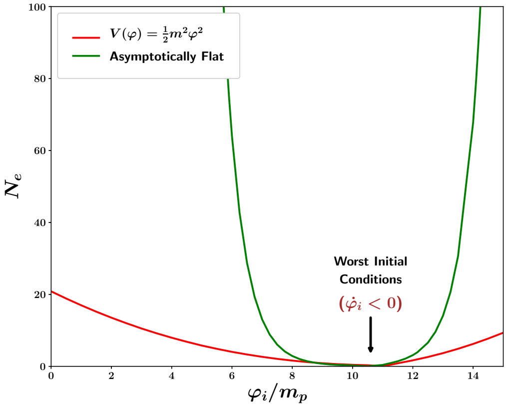

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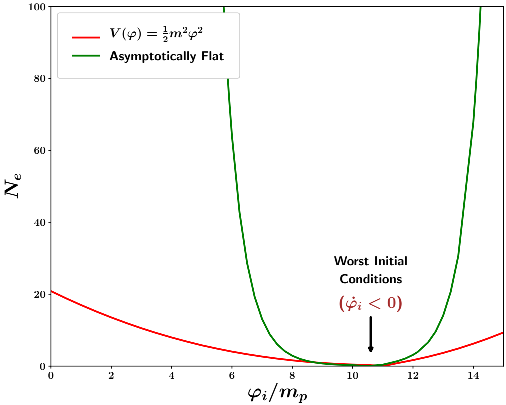

However Positive Spatial Curvature is a serious problem for Asymptotically Flat potentials [Steinhardt, Ijjas and Loeb 2014]

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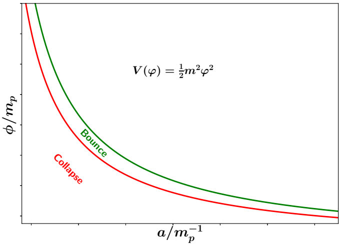
So starting from $H_i = 0$ at some fixed initial energy scale, we have
Collapse: $\frac{V(\varphi)}{m_p^4} < \frac{2}{a^2 m_p^2}$ **Bounce**: $\frac{2}{a^2 m_p^2} < \frac{V(\varphi)}{m_p^4} \leq \frac{3}{a^2 m_p^2}$

Trouble with asymptotically flat potentials for $K = +1$



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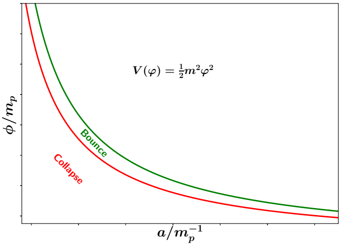

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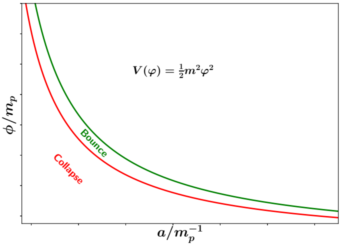

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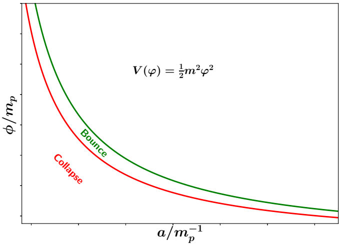

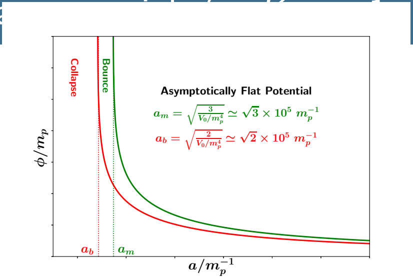

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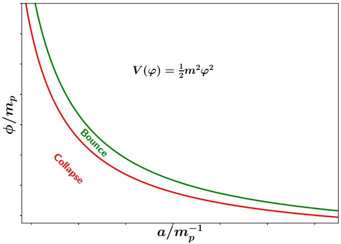

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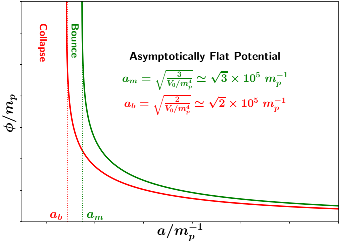

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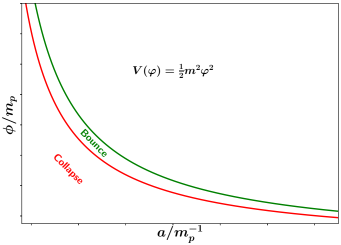

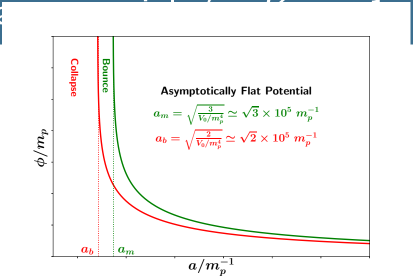

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$$\left|H_i - H_i|_{K=0}\right| \leq 4 \times 10^{-7}$$

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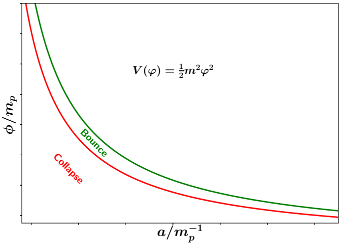
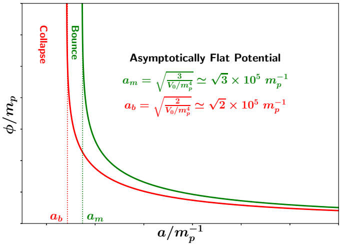




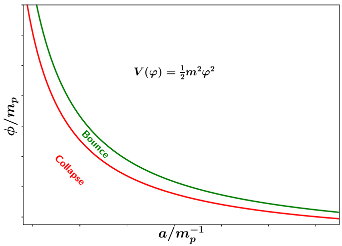
At $H_i = 0$, for any given initial value of ρ_ϕ (hence a), there is a range of values of ϕ for which universe bounces back and inflation is possible. For higher $H_i > 0$, probability of bounce increases (quantifiable).

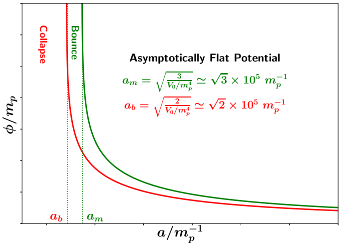
For $H_i = 0$, **Collapse is inevitable**. While for $H_i > 0$, bounce becomes possible only if $a_i > 1600\ m_p^{-1}$ which puts H_i close to its flat universe value upto 7 decimal places

$$|H_i - H_i|_{K=0}| \leq 4 \times 10^{-7}$$

Not very good for Inflation!!

Trouble with asymptotically fla





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This is a serious issue and needs to be resolved! [Mishra, Sahni and Toporensky (in preparation)]

Possible Resolution

Consider the Margarita Potential
[Bag, Mishra and Sahni JCAP 2017]

$$V(\phi) = V_0 \tanh\left(\frac{\lambda_1 \phi}{m_p}\right) \cosh\left(\frac{\lambda_2 \phi}{m_p}\right), \;\; \lambda_1 > \lambda_2$$

which has three asymptotes

$$\begin{aligned} \text{Exponential wing} \quad V(\varphi) &\simeq \frac{V_0}{2} \exp\left(\lambda_2 |\varphi|/m_p\right) , \quad \frac{|\varphi|}{m_p} \gg \frac{1}{\lambda_2} , \\ \text{Flat wing:} \quad V(\varphi) &\simeq V_0 + \frac{1}{2} m_2^2 \varphi^2 , \quad \frac{1}{\lambda_1} \ll \frac{|\varphi|}{m_p} \ll \frac{1}{\lambda_2} , \\ \text{Oscillatory region:} \quad V(\varphi) &\simeq \frac{1}{2} m_1^2 \varphi^2 , \quad \frac{|\varphi|}{m_p} \ll \frac{1}{\lambda_1} , \end{aligned}$$

where $m_1^2 = \frac{2V_0\lambda_1^2}{m_p^2}$ and $m_2^2 = \frac{V_0\lambda_2^2}{m_p^2}$
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Exponential wing $V(\varphi) \simeq \frac{V_0}{2}$ ex

Flat wing: $V(\varphi) \simeq V_0 +$

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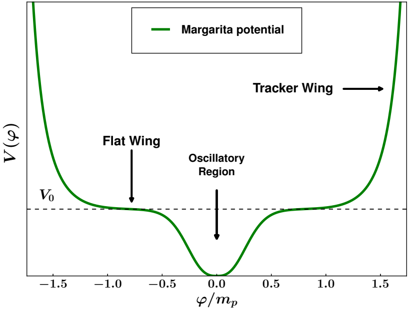

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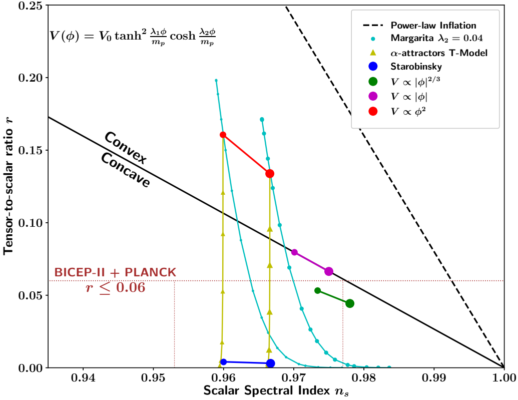
Margarita Potential r vs n_s

[Mishra, Sahni and Toporensky (in preparation)]



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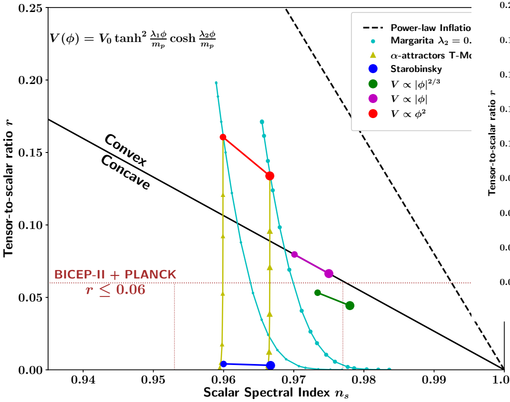

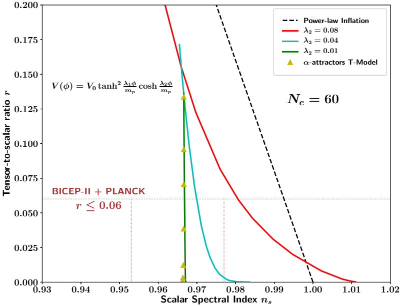

Concluding Remarks: Key Points

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- Initial conditions should also be thoroughly analysed for alternatives to Inflation like String Gas Cosmology, Matter Bounce Models, Ekpyrotic Models, Emergent Scenarios etc.

References

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