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Blue-tilted primordial gravitational waves from massive gravity

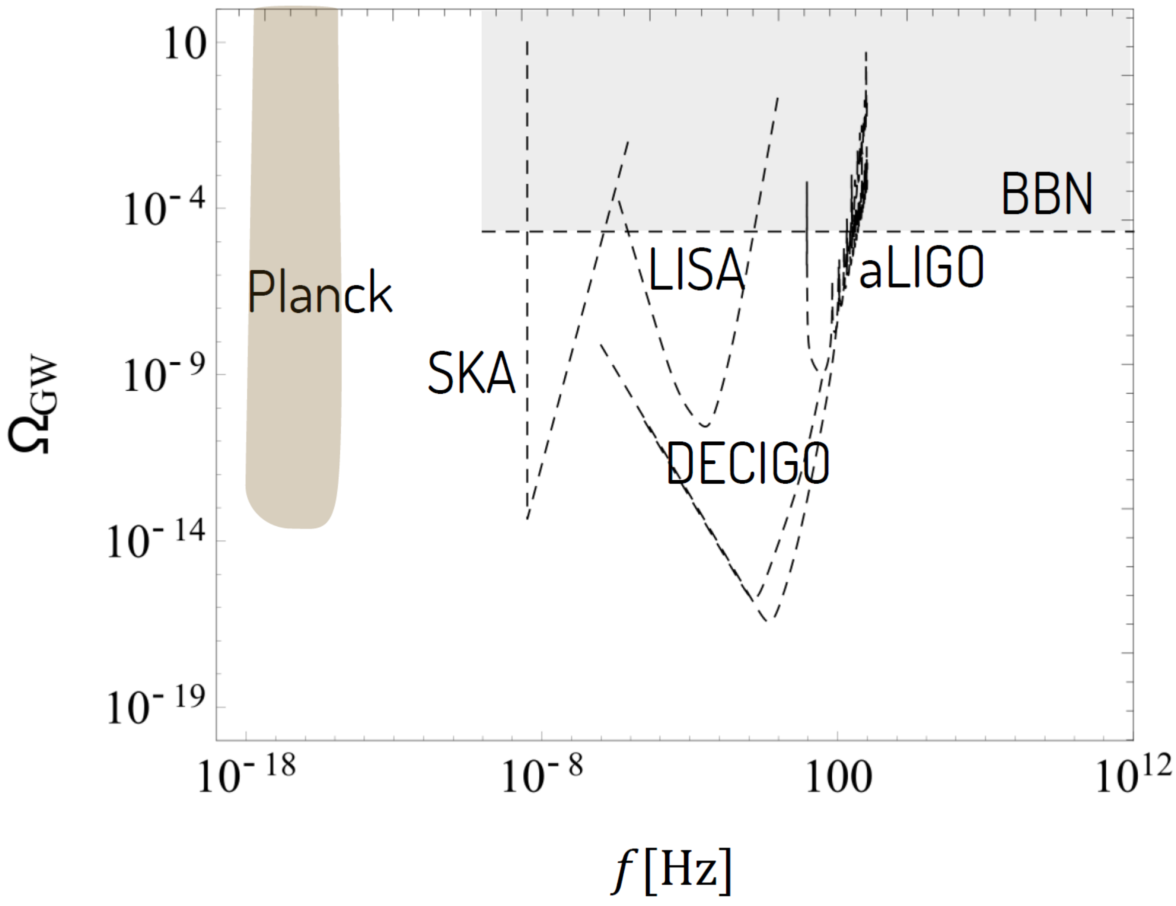
Shuntaro Mizuno (YITP, Kyoto)

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Fujita, Kuroyanagi, SM, Mukohyama Phys. Lett. B789, 215

Fujita, SM, Mukohyama in preparation

Primordial Gravitational Waves (PGWs)



Density parameter

$$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_{\mathrm{tot}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k}$$

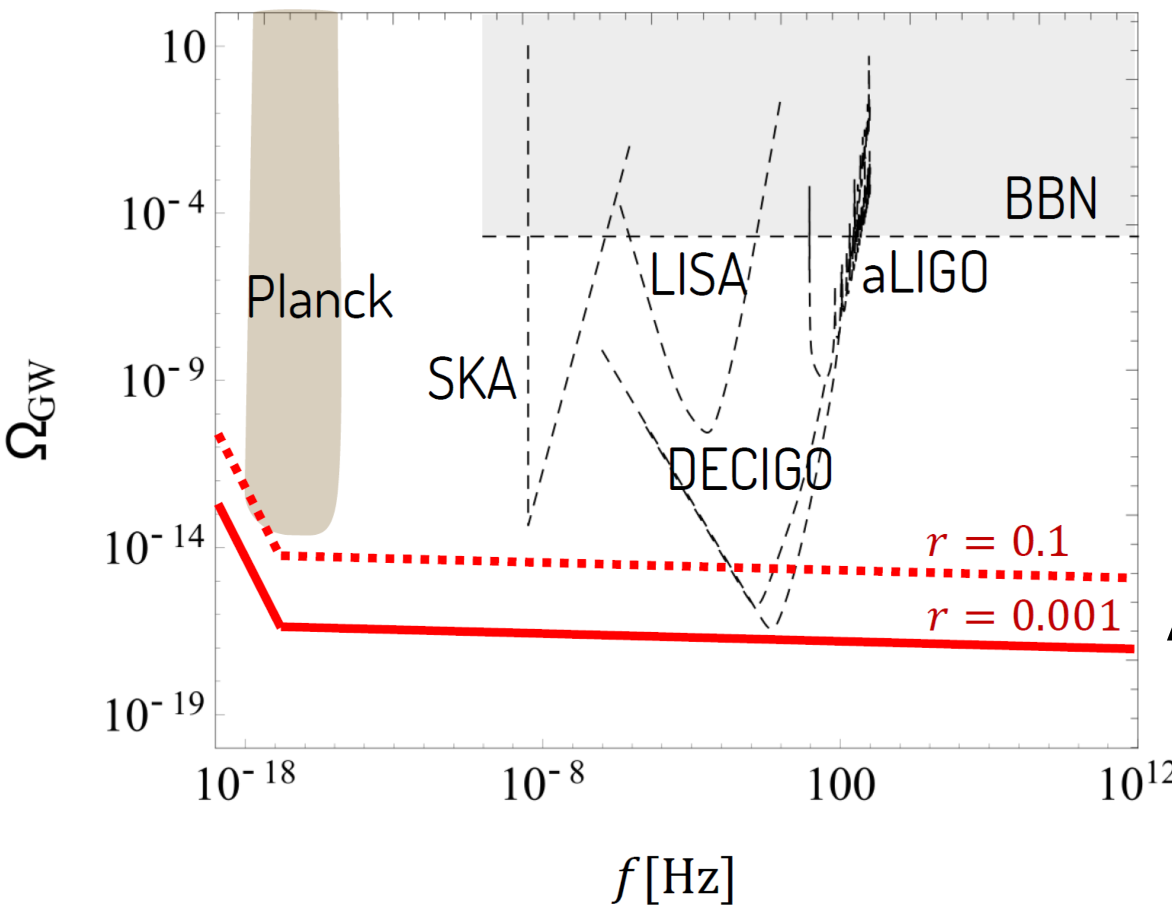
Frequency

$$f \equiv k/2\pi$$

We have already constraints on Ω_{GW} from BBN and Planck

Interferometers can get information of PGWs on various scales !!

Interferometers and ``Standard" PGWs



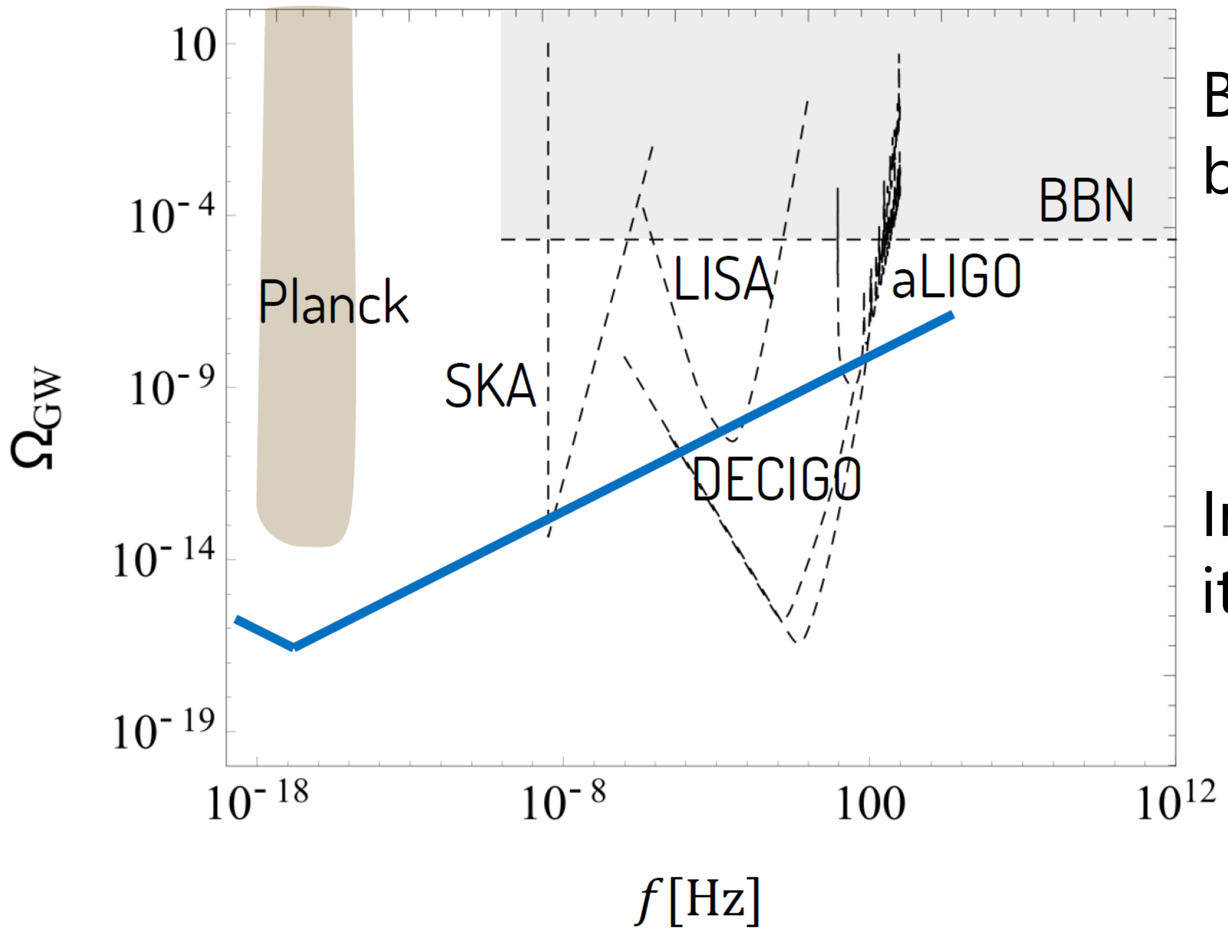
PGWs from inflation

$$\mathcal{P}_h^{\rm standard} = \frac{2H^2}{\pi^2 M_{\rm Pl}^2}$$

Almost flat (Red-tilted) PGWs spectrum

Planck constrains $\Omega_{\rm GW} \lesssim 10^{-15}$ on interferometers' scales !!

Interferometers and Blue-tilted PGWs



Blue-tilted PGWs are better for detection

In standard inflation, it is difficult to realize

Can we obtain consistent and detectable blue-tilted PGWs ?

Minimal theory of massive gravity (MTMG)

・Properties of MTMG De Felice, Mukohyama `15

Having only 2 propagating DOFs (No scalar & vector gravitons)

Other points are same as dRGT de Rham, Gabadadze, Tolley `11

ex.) FRW background, tensor perturbations around it,…

・Construction of MTMG

Method to remove extra d.o.f s is based on ADM vielbein

⇒ Lorentz violating massive gravity

cf.) Solid inflation, super solid inflation Endlich, Nicolis, Wang `12

Effective Field Theory (EFT) of inflation with ~~space diffs~~

Set-up

- Decomposition and quantization of h_{ij} with $g_{ij} = a^2 \left[e^h \right]_{ij}$

$$h_{ij}(\tau, \boldsymbol{x}) = \frac{2}{a M_{\rm Pl}} \sum_{\lambda=+,-} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i \boldsymbol{k} \cdot \boldsymbol{x}} e^{\lambda}_{ij} \left[v^{\lambda}_k(\tau) \hat{a}^{\lambda}_{\boldsymbol{k}} + \mathrm{h.c.} \right]$$

- Equation of motion for the mode function

$$v_k'' + \left[k^2 + a^2 \mu^2 - \frac{a''}{a} \right] v_k = 0,$$

	inflation	radiation dom.	radiation dom.
$a(\tau)$	$-1/(H_{\rm inf}\tau)$	$a_r \tau / \tau_r$	$a_r \tau / \tau_r$
	massive	massive	massless
$\mu(\tau)$	m	m	0

Inflation era

$$v_k'' + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right] v_k = 0 \qquad \nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H_{\rm inf}^2}}$$

- Power spectrum of PGWs

$$\mathcal{P}_h \equiv \frac{4k^3|v_k(\tau)|^2}{\pi^2 M_{\rm Pl}^2 a(\tau)^2} \simeq \frac{2H_{\rm inf}^2}{\pi^2 M_{\rm Pl}^2}\left(\frac{k}{k_{\rm UV}}\right)^{3-2\nu} \quad \text{for} \quad k < k_{\rm UV}$$

(at the end of inflation) $k_{\rm UV} \equiv a_r H_{\rm inf}$

Usually, $m/H_{\rm inf}$ cannot be $\mathcal{O}(1)$ (Higuchi bound)

In MTMG, it is possible (only 2 propagating DOFs)

⇨ PGWs are highly blue-tilted (suppressed on large scales)

Evolution of PGWs after inflation

- Graviton energy density

$$T^{(\mathrm{GW})}_{\mu\nu} = \frac{M_{\mathrm{Pl}}^2}{4}\langle \partial_\mu h_{ij} \partial_\nu h_{ij} \rangle \Longrightarrow \rho^{(\mathrm{GW})} \propto \frac{1}{2a^2}(h'_{ij})^2 \quad \text{(massless)}$$

(analogy with scalar field) $\Longrightarrow$ $\rho^{(\mathrm{GW})} \propto \frac{1}{2a^2}(h'_{ij})^2 + \frac{1}{2}m^2 h_{ij}^2$

- Massive phase

$$\rho_k^{\mathrm{GW}} \propto m^2 h_k^2 \propto a^{-2} v_k^2 \propto a^{-3}$$

decays like non-relativistic matter!!

- Massless phase

$$\rho_k^{\mathrm{GW}} \propto a^{-2} {h'_k}^2 \propto a^{-2}[(a^{-1}v_k)']^2 \propto a^{-4}$$

decays like relativistic matter (as usual)

Power spectrum of PGWs at late time

1. Inflation → **2. Mass-dominant** → **3. Massless**

From BD-vacuum, GWs are produced and decay on super-horizon scales in same way as $\delta\phi_k$

After instant re-heating, $k \ll am$ and gravitons behave as matter.

At some point in RD era, gravitons lose the mass to avoid some obs. bounds.

Blue-tilt

$$\frac{\mathcal{P}_h}{\mathcal{P}_h^{\rm standard}} \sim \left(\frac{k}{k_{\rm UV}}\right)^{3-2\nu}$$

Slow decay

$$\rho_k^{\rm GW} \propto a^{-3}$$

Detection

$$\rho_k^{\rm GW} \propto a^{-4}$$

(at late time)

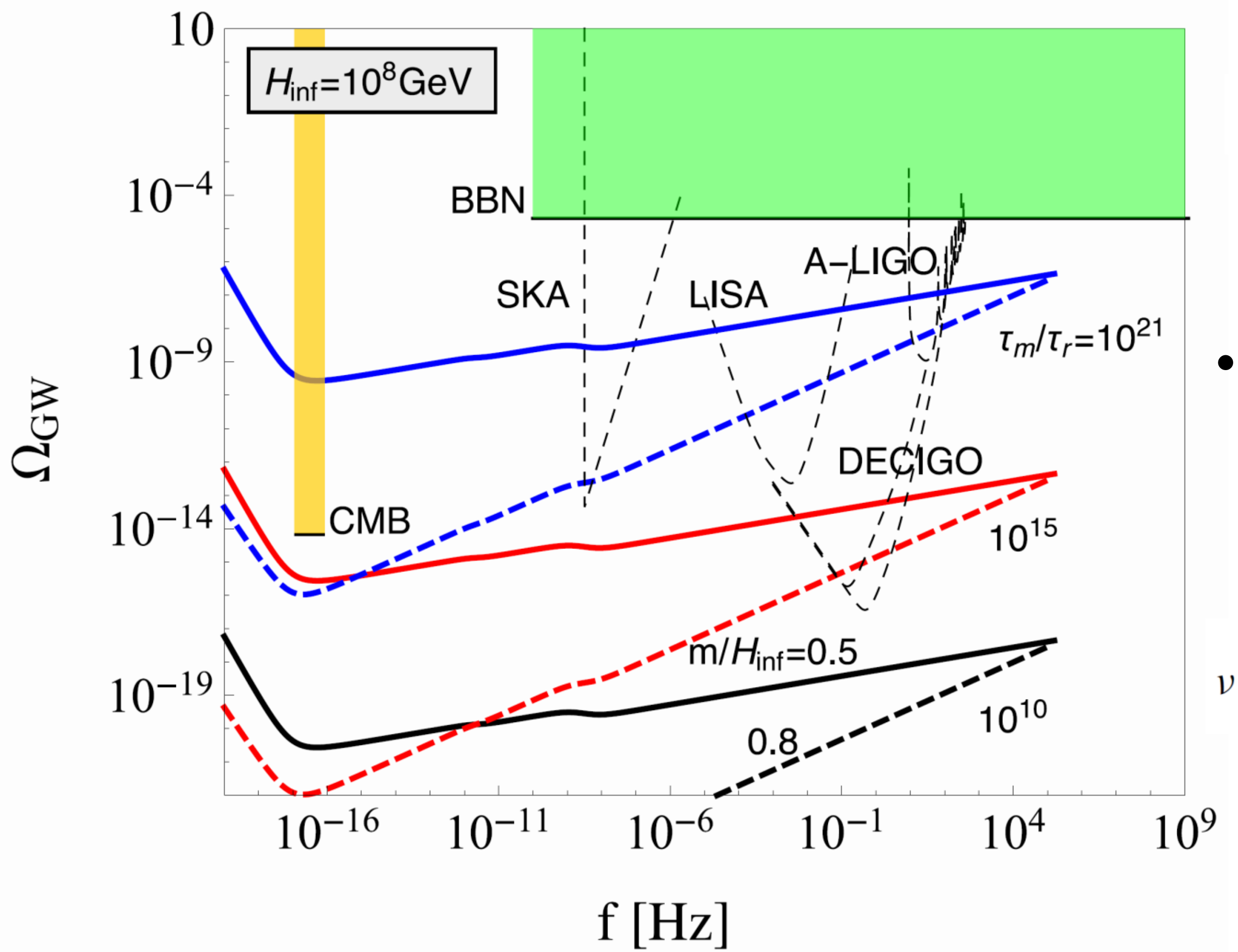
$$\mathcal{P}_h^{\rm massive} \sim \frac{\tau_m}{\tau_r}\left(\frac{k}{k_{\rm UV}}\right)^{3-2\nu} \mathcal{P}_h^{\rm standard}$$

$$\nu \equiv \sqrt{9/4 - m^2/H_{\rm inf}^2} \qquad k_{\rm UV} \equiv a_r H_{\rm inf}$$

Theoretical prediction for $\Omega_{\rm GW}$

$$\Omega_{\rm GW}(f) \simeq 10^{-15}\frac{\tau_m}{\tau_r}\left[\frac{H_{\rm inf}}{10^{14}{\rm GeV}}\right]^{\nu+\frac{1}{2}}\left[\frac{f}{2\times 10^8{\rm Hz}}\right]^{3-2\nu}$$

$$f < f_{\rm UV}$$



$$f_{\rm UV} = 2\times 10^8 H_{14}^{1/2}{\rm Hz}$$

$$H_{14} \equiv H_{\rm inf}/(10^{14}{\rm GeV})$$

- Constraints

$$\frac{\tau_m}{\tau_r} \lesssim 10^{10}H_{14}^{-2}, \quad \text{(BBN)}$$

$$\nu \lesssim \frac{75-\log_{10}(H_{14}^{1/2}\tau_m/\tau_r)}{50+\log_{10}(H_{14})}. \quad \text{(CMB)}$$

Primordial tensor non-Gaussianity

- How to distinguish scenarios with detectable PGWs ?

PGWs from vacuum fluctuations of metric are almost Gaussian
Maldacena `02

Stochastic GWs by uncorrelated astrophysical sources are also almost Gaussian (Central limit theorem)

⇨ Tensor non-Gaussianity is powerful discriminator

- Primordial tensor bispectrum

$$\langle h_{i_1j_1}(\tau,\boldsymbol{k}_1)h_{i_2j_2}(\tau,\boldsymbol{k}_2)h_{i_3j_3}(\tau,\boldsymbol{k}_3)\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B^h_{i_1j_1i_2j_2i_3j_3}(k_1,k_2,k_3)$$

Depending on amplitude, shape of triangle, and chiralities

$$\langle h_{i_1j_1}(\tau,\boldsymbol{k}_1)h_{i_2j_2}(\tau,\boldsymbol{k}_2)h_{i_3j_3}(\tau,\boldsymbol{k}_3)\rangle = i\int_{-\infty}^{\tau}\mathrm{d}\eta\langle[H_{\mathrm{int}}(\eta),h_{i_1j_1}(\tau,\boldsymbol{k}_1)h_{i_2j_2}(\tau,\boldsymbol{k}_2)h_{i_3j_3}(\tau,\boldsymbol{k}_3)]\rangle$$

Shape of tensor bispectrum

- Interaction Hamiltonian at third order

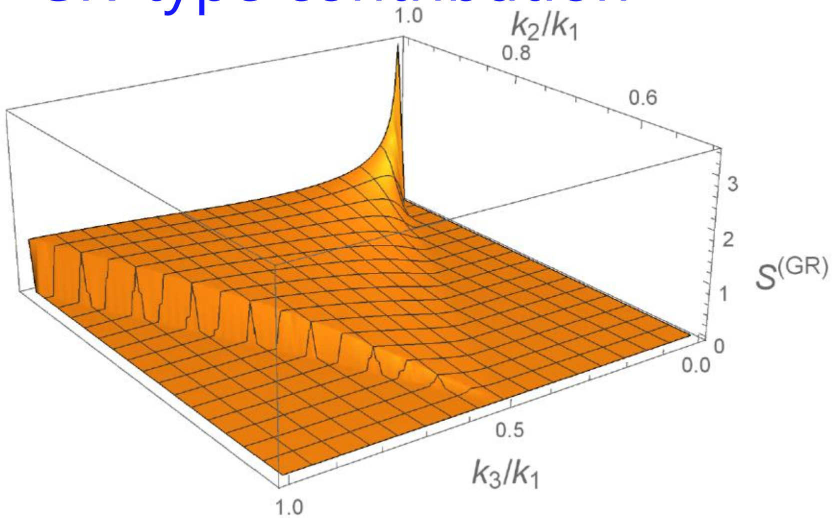
$$H_{\rm int} = H_{\rm int}^{\rm (GR)} + H_{\rm int}^{\rm (mass)}$$

$$H_{\rm int}^{\rm (GR)} = -\frac{M_{\rm Pl}^2}{4} a^2 \int {\rm d}^3x \, h_{ij} h_{kl} \left(\partial_j \partial_l h_{ik} - \partial_i \partial_j \frac{1}{2} h_{kl} \right) \qquad H_{\rm int}^{\rm (mass)} = -g \frac{M_{\rm Pl}^2}{4} a^4 \int {\rm d}^3x \, h_{ij} h_{jk} h_{ki}$$

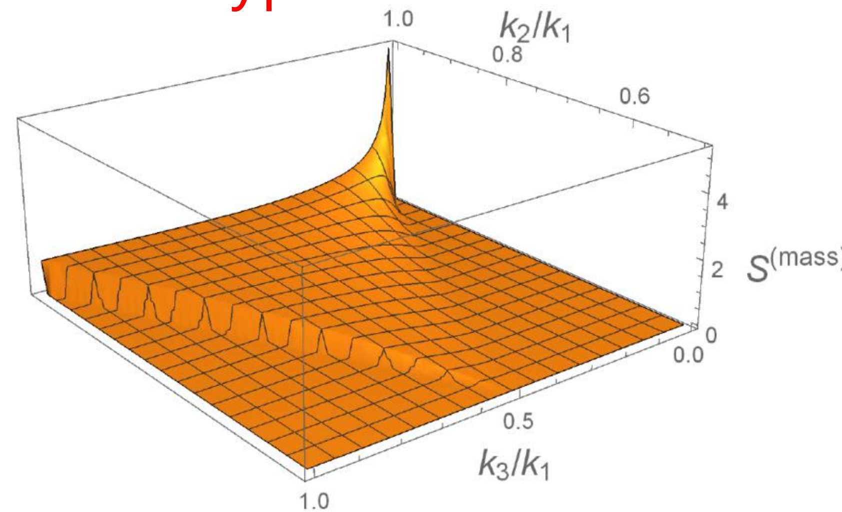
appears in GR — peculiar to MTMG

- ``Shape function" of tensor bispectrum ($m/H_{\rm inf} = 0.8$)

GR-type contribution



MTMG-type contribution



Both are maximized in the squeezed limit

Detectability of tensor bispectrum by LISA

- Amplitude of tensor bispectrum

$$\frac{(k_1k_2k_3)^2 B_h^{\rm equil}}{[\mathcal{P}_h(k_1)\mathcal{P}_h(k_2)\mathcal{P}_h(k_3)]^{1/2}} \simeq \left(\frac{g_{\rm inf}}{H_{\rm inf}^2}\right)\left(\frac{H_{\rm inf}}{10^{-3}M_{\rm Pl}}\right)$$

(``test of non-Gaussianity" for LISA) $B_h = \delta_{j_1 i_2}\delta_{j_2 i_3}\delta_{j_3 i_1} B^h_{i_1 j_1 i_2 j_2 i_3 j_3}$

Bartolo et al `18

⇒ chance for LISA to detect for $g_{\rm inf} \geq 10^{-3} H_{\rm inf} M_{\rm Pl}$

If curvature perturbation is generated by single-field inflation

$$\frac{H_{\rm inf}}{M_{\rm Pl}} = \sqrt{8\pi\epsilon\mathcal{P}_\zeta} \approx 10^{-4}\left(\frac{\epsilon}{0.1}\right)^{\frac{1}{2}}\left(\frac{\mathcal{P}_\zeta}{2\times 10^{-9}}\right)^{\frac{1}{2}} \qquad g_{\rm inf} \geq 10 H_{\rm inf}^2$$

But for models with suppressed curvature perturbation,

$g_{\rm inf}/H_{\rm inf}^2 \sim \mathcal{O}(1)$ is possible

Conclusions

- PGWs gives information of scales different from CMB, which is very helpful to distinguish and/or constrain inflation models

- Highly blue-tilted PGWs can be detected by interferometers, even if their signal is not observed on the CMB scales

- We construct a consistent model producing highly blue-tilted and largely amplified PGWs based on MTMG

- We also calculate the non-Gaussianity of PGWs for the model and discuss the detactability by LISA

Discussions

- Squeezed limit of tensor bispectrum

  Consistency Relation (CR) for adiabatic tensor perturbations

  Maldacena `02

$$\lim_{\boldsymbol{q}\to 0}\langle h^{s_1}_{\boldsymbol{q}} h^{s_2}_{\boldsymbol{k}}\ h^{s_3}_{-\boldsymbol{k}}\ \rangle' = \frac{3}{2}\delta^{s_2 s_3}\mathcal{P}_h(q)\mathcal{P}_h(k)e^{s_1}_{ij}(\boldsymbol{q})\frac{k^i k^j}{k^2}$$

  If CR holds, effect of superhorizon mode is unobservable

  Pajer, Schmidt, Zaldarriaga `13

  In solid inflation, CR breaks and there are observable effects

  Bordin, Creminelli, Mirbabayi, Norea `16

$$\mathcal{P}_{h,\bar{h}}(k) = \mathcal{P}_{h,0}(k)\left(1 + \mathcal{Q}_{ij}\frac{k^i k^j}{k^2}\right) \qquad \mathcal{Q}_{ij} \propto f_{\rm NL}^{h,\rm squeezed}$$

- Relation between curvature perturbation and PGWs

Thank you very much !!