Primordial black holes in canonical single field inflation

Hayato Motohashi (YITP)

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YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

Gravitational Wave Physics and Astronomy Genesis

科研費
KAKENHI

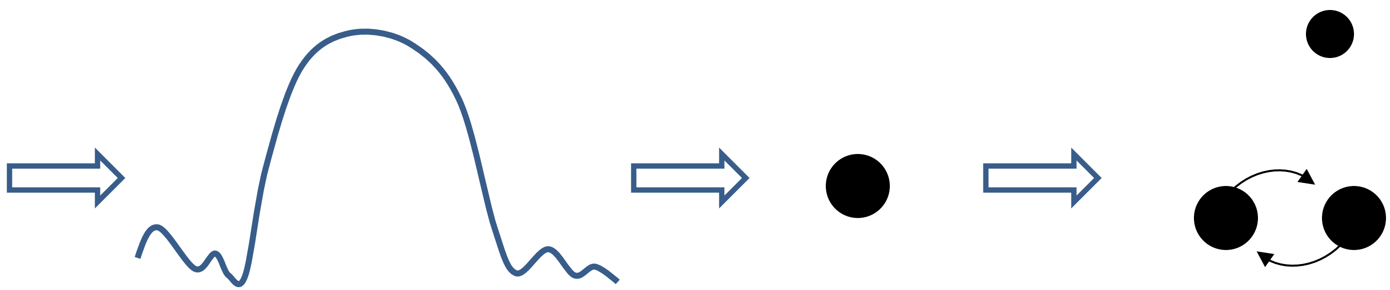
Primordial black holes (PBHs)

Zel'dovich, Novikov (1966)

Hawking (1971)

Carr (1975)

Inflation



- DM
- GW source

Reviews

Carr et al, 0912.5297

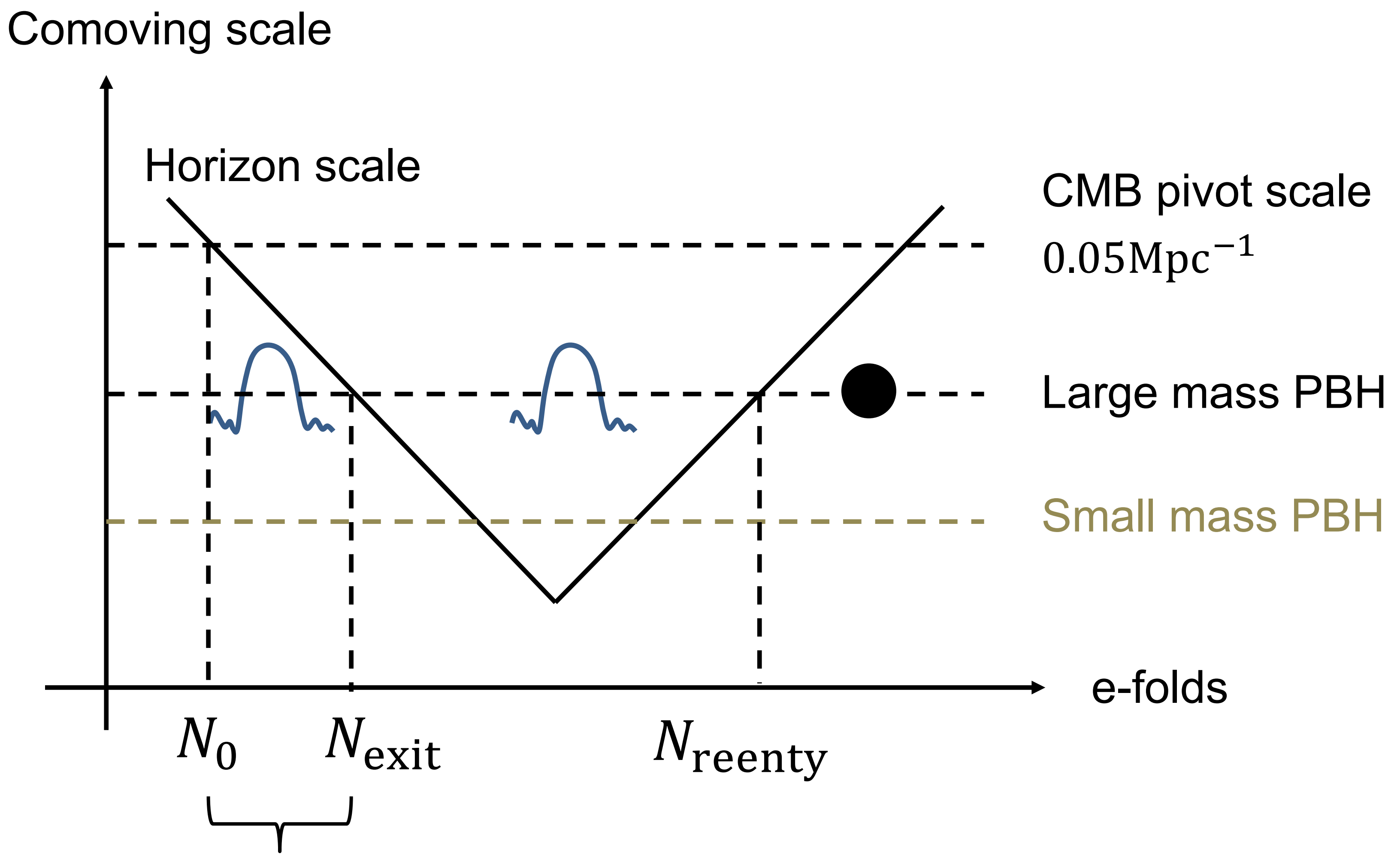
Carr et al, 1607.06077

Sasaki et al, 1801.05235

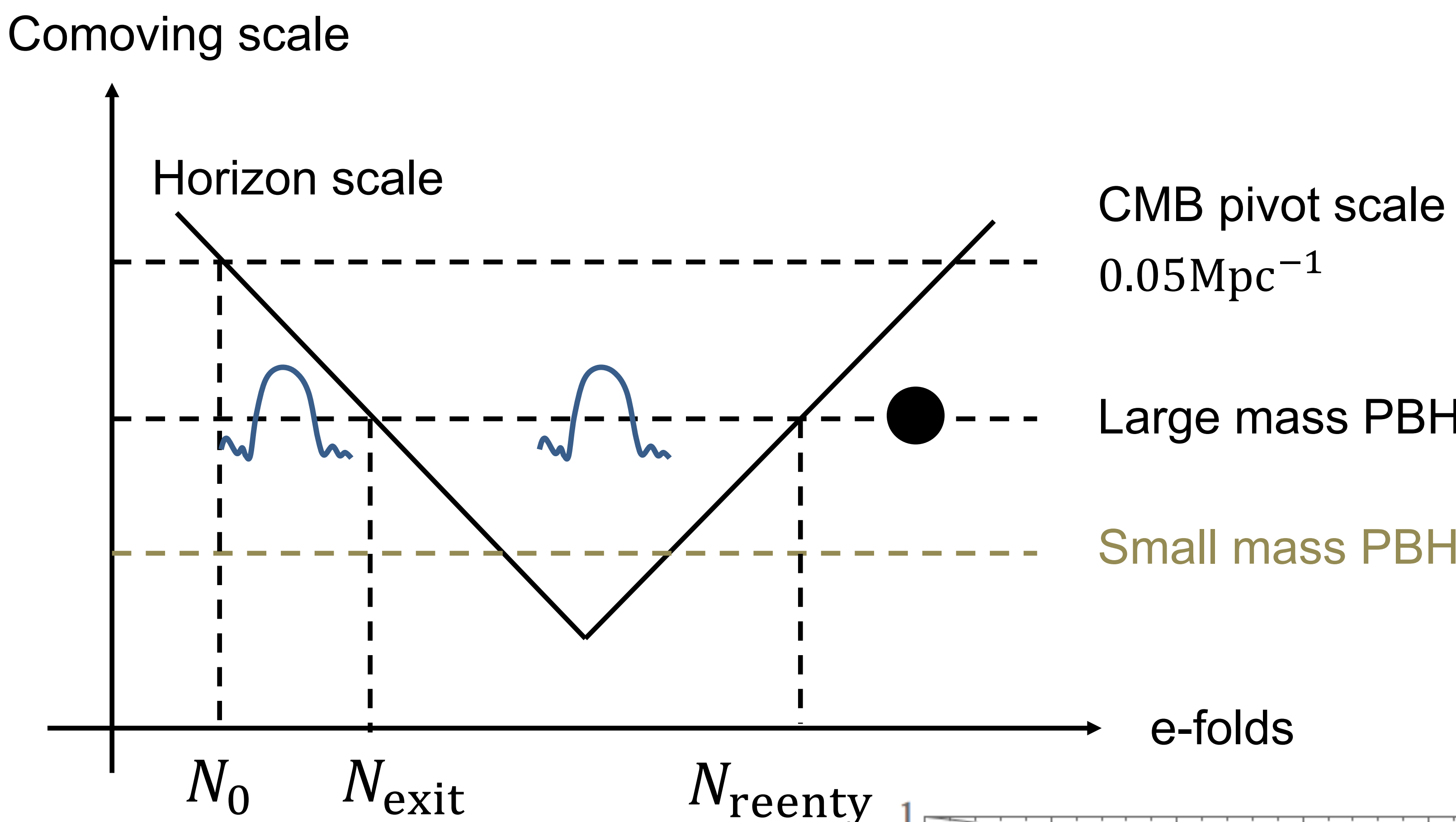
Bird et al, 1603.00464

Clesse et al, 1603.05234

Sasaki et al, 1603.08338



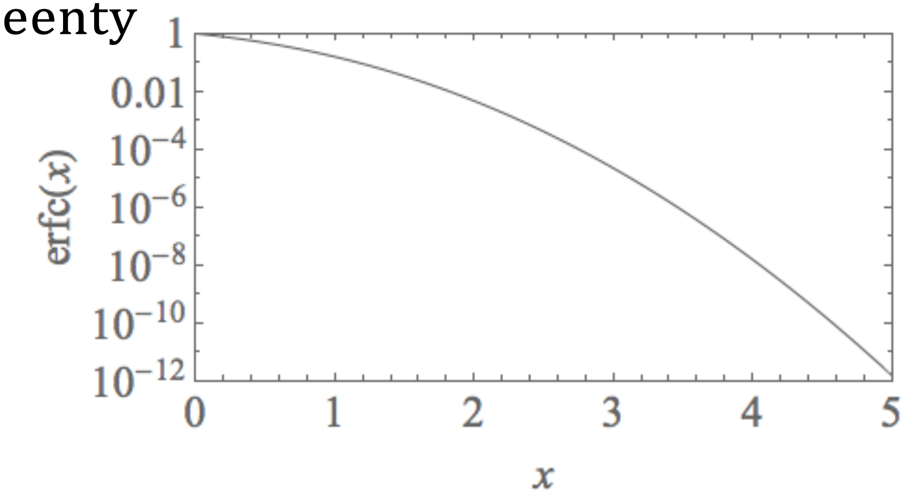
$$\Delta N \approx 18 - \frac{1}{2} \ln \frac{M}{M_{\odot}}$$

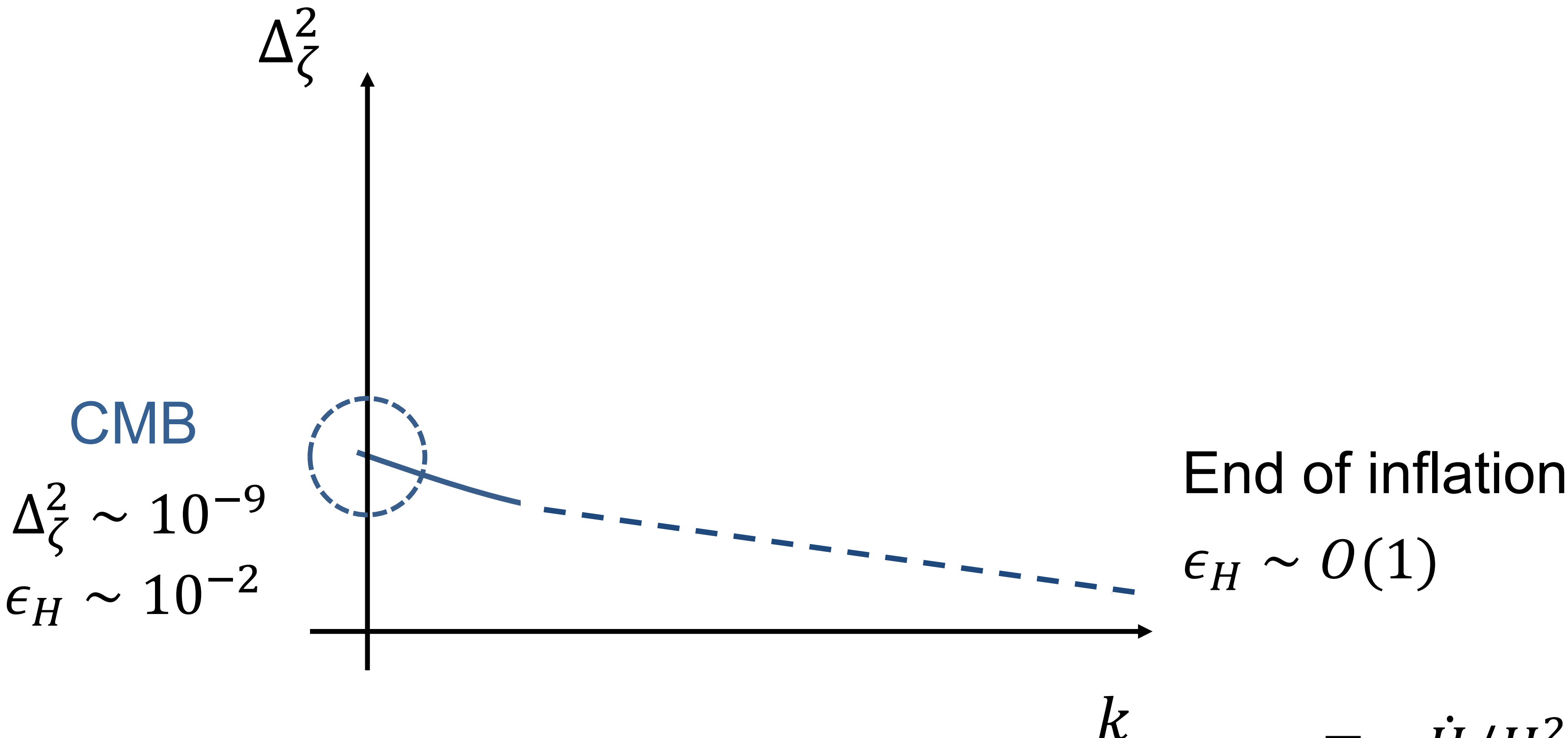


PBH abundance

$$\beta \equiv \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{tot}}}\Big|_{\mathrm{reenty}} \approx \mathrm{erfc}\left(\frac{\zeta_c}{\sqrt{2}\Delta_\zeta}\right)$$

A large peak of Δ_ζ^2 is required.

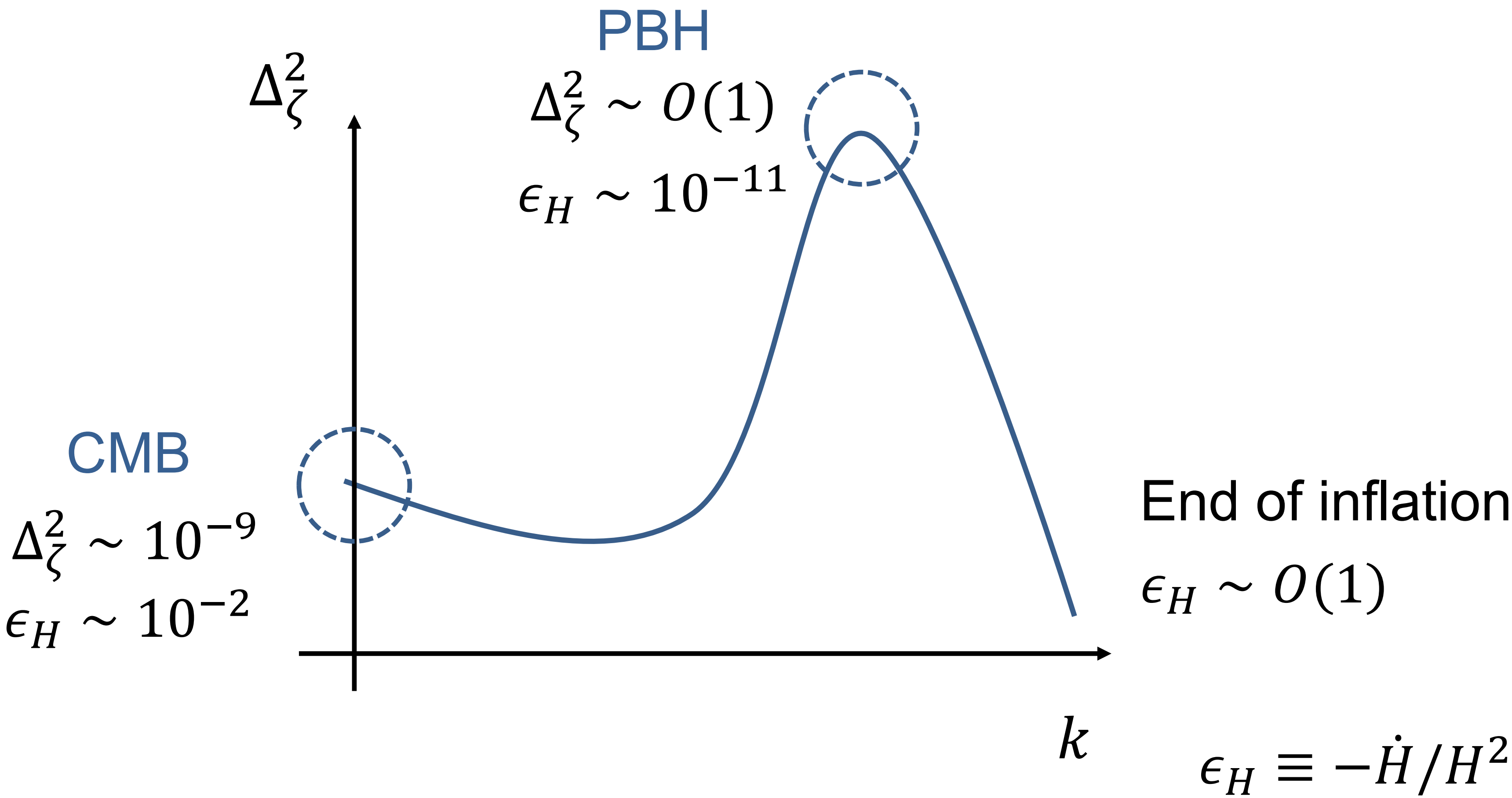




$$\epsilon_H \equiv -\dot{H}/H^2$$

Leading-order slow-roll approx.

$$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2 \epsilon_H}$$



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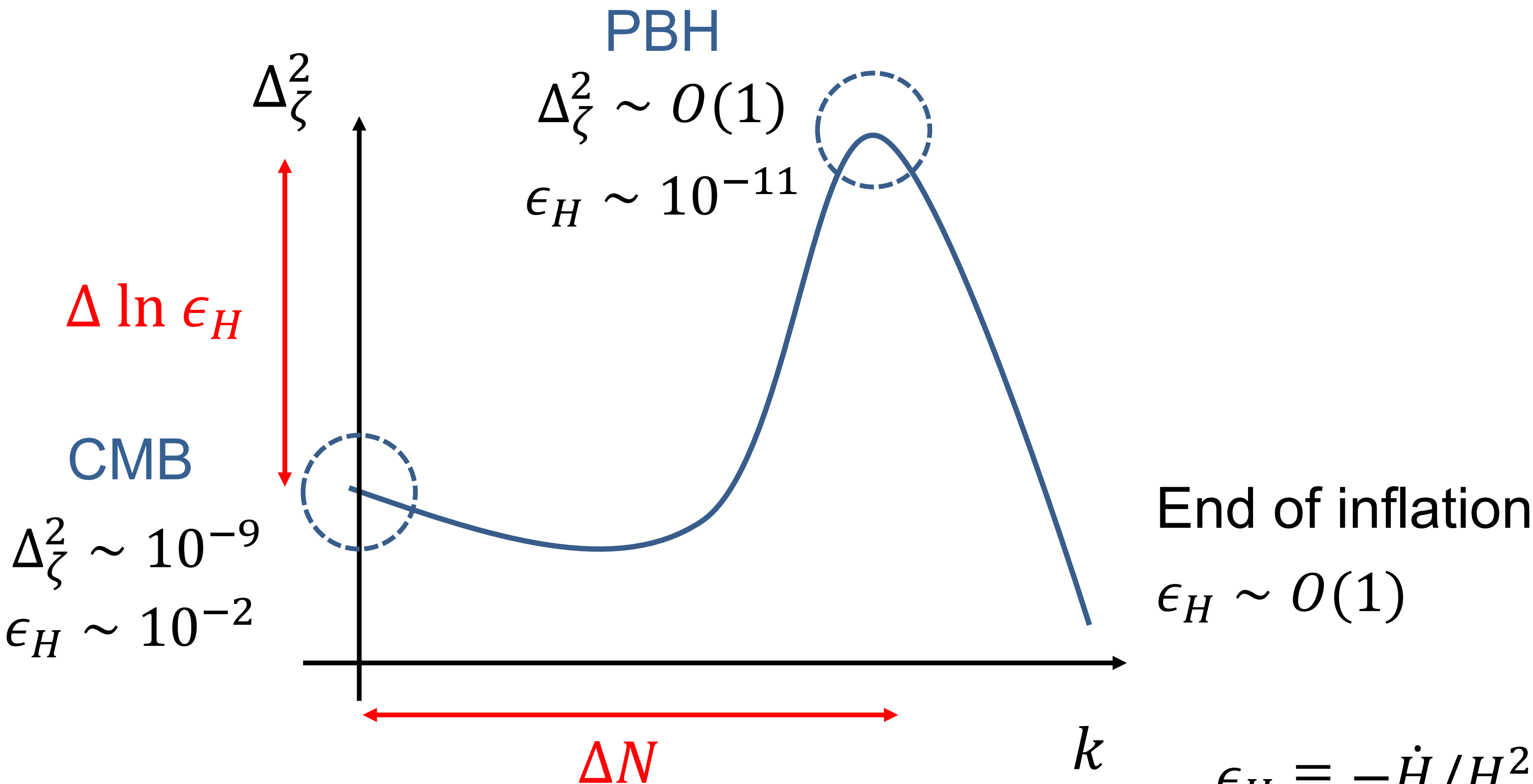


$$\epsilon_H \equiv -\dot{H}/H^2$$

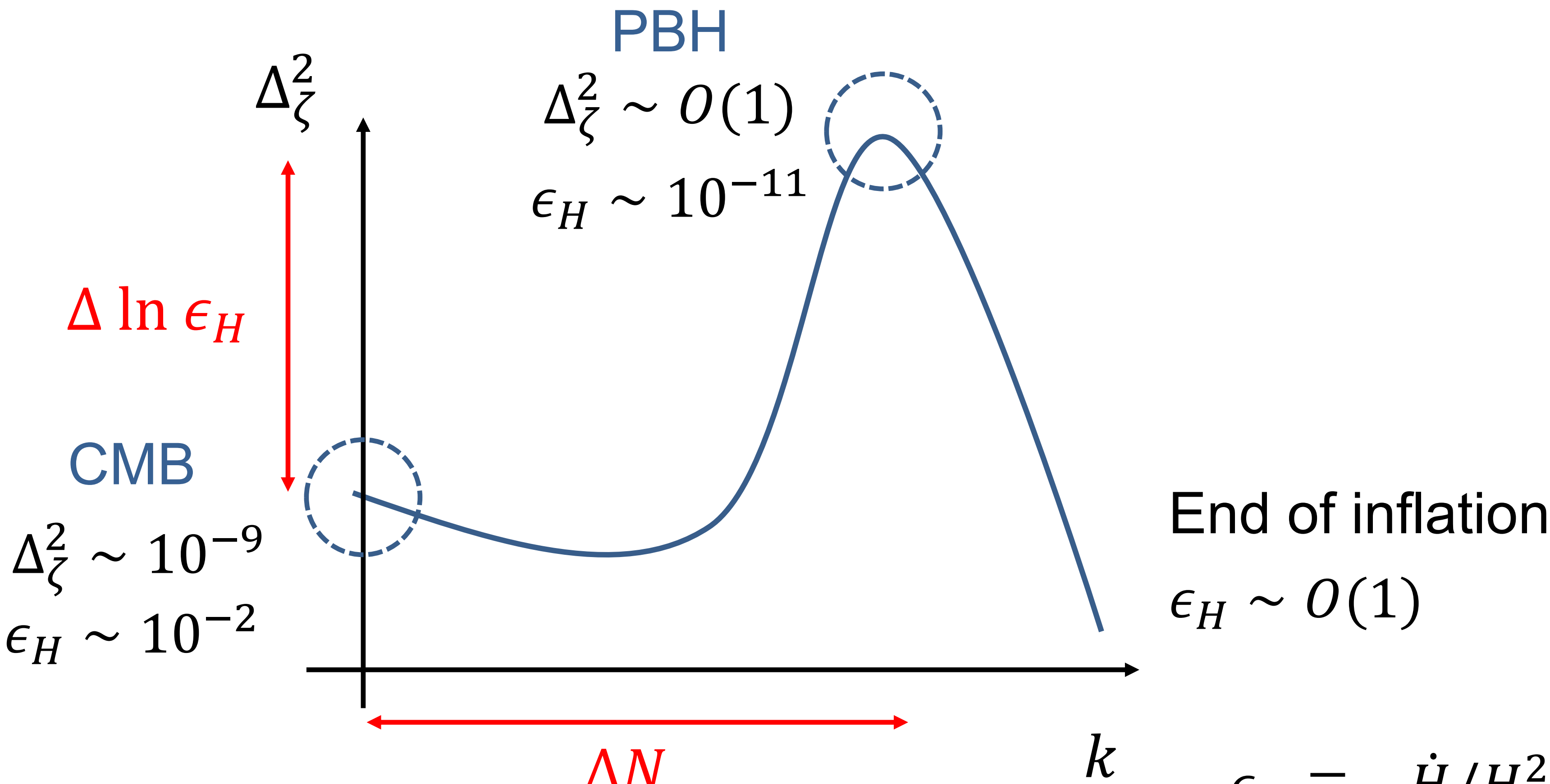
Leading-order slow-roll approx.

$$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2 \epsilon_H}$$

$\Leftarrow$ $\epsilon_H \ll 1$ and

$$\left|\frac{d \ln \epsilon_H}{dN}\right| \ll 1\ ?$$

HM, Hu, 1706.06784



$$\epsilon_H \equiv -\dot{H}/H^2$$

$$\beta \approx \mathrm{erfc}\left(\frac{\zeta_c}{\sqrt{2}\Delta_\zeta}\right) \approx 10^{-9}\left(\frac{\Omega_{\mathrm{PBH}}h^2}{0.12}\right)\left(\frac{M}{M_\odot}\right)^{1/2}$$

$$\Delta N \approx 18 - \frac{1}{2}\ln\frac{M}{M_\odot}$$

$\epsilon_H \ll 1$ and

$$\left|\frac{d\ln\epsilon_H}{dN}\right| \ll 1\,?$$

HM, Hu, 1706.06784

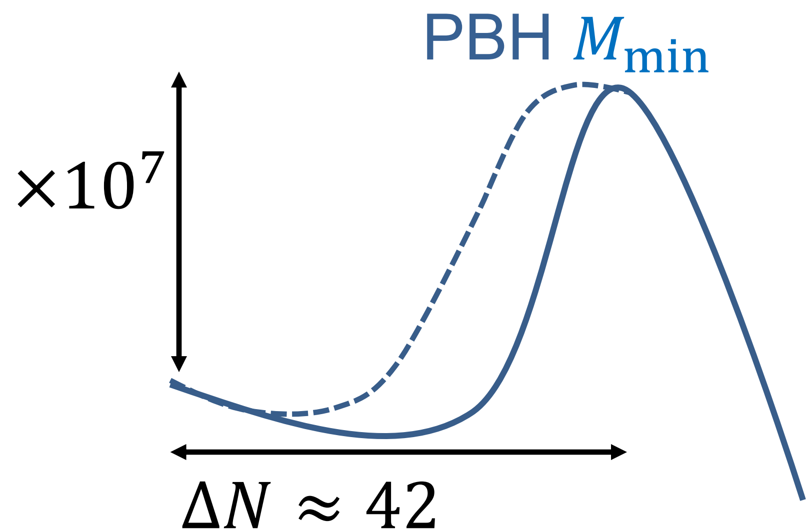
No go for slow roll

$$(\Omega_{\mathrm{PBH}}, M) = \begin{cases} (\Omega_{\mathrm{DM}}, M_{\mathrm{min}}) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38 \\ \\ \end{cases}$$

Slow-roll violation

$$\left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38$$

for any single-field canonical inflation



$M_{\mathrm{min}} \approx 10^{-21} M_\odot$: Smallest PBH mass that does not evaporate by matter-radiation equality barring merging and accretion $\Longrightarrow$ Lower bound on SR violation

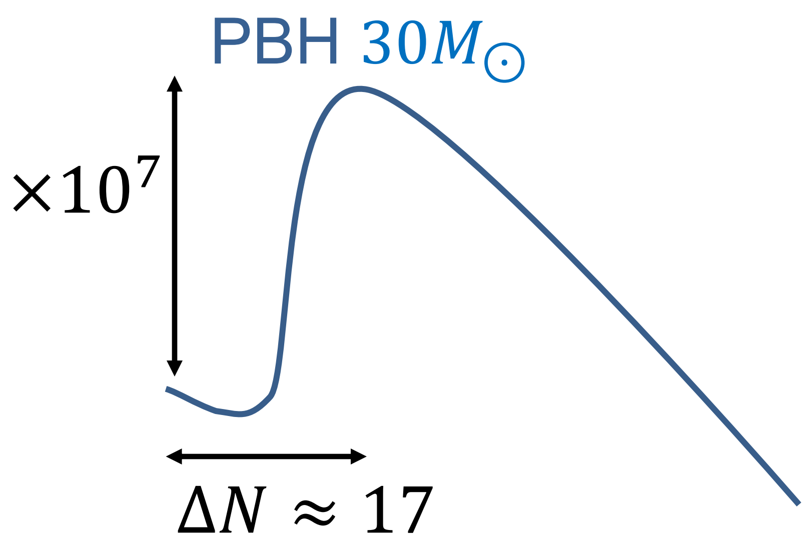
HM, Hu, 1706.06784

No go for slow roll

$$(\Omega_{\rm PBH}, M) = \begin{cases} (\Omega_{\rm DM}, M_{\rm min}) & \Rightarrow \left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38 \\ (10^{-3}\Omega_{\rm DM}, M_{\rm min}) & \Rightarrow \left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.37 \\ (10^{-3}\Omega_{\rm DM}, 30M_\odot) & \Rightarrow \left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.99 \end{cases}$$

PBH = GW events scenario

Sasaki, Suyama,
Tanaka, Yokoyama, 1603.08338



Various approximations

- Standard SR : $\Delta_\zeta^2 \approx \left.\frac{H^2}{8\pi^2\epsilon_H}\right|_{k\eta=1}$ $\Longrightarrow$ Not good

Various approximations

- SR-V : $$\Delta_\zeta^2 \approx \frac{V}{24\pi^2\epsilon_V}\bigg|_{k\eta=1}$$ $\Longrightarrow$ Particularly bad

$$\epsilon_H \approx \epsilon_V, \ \frac{d\phi}{dN} = -\frac{V\prime}{V}$$

- Standard SR : $$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2\epsilon_H}\bigg|_{k\eta=1}$$ $\Longrightarrow$ Not good

Exact relation

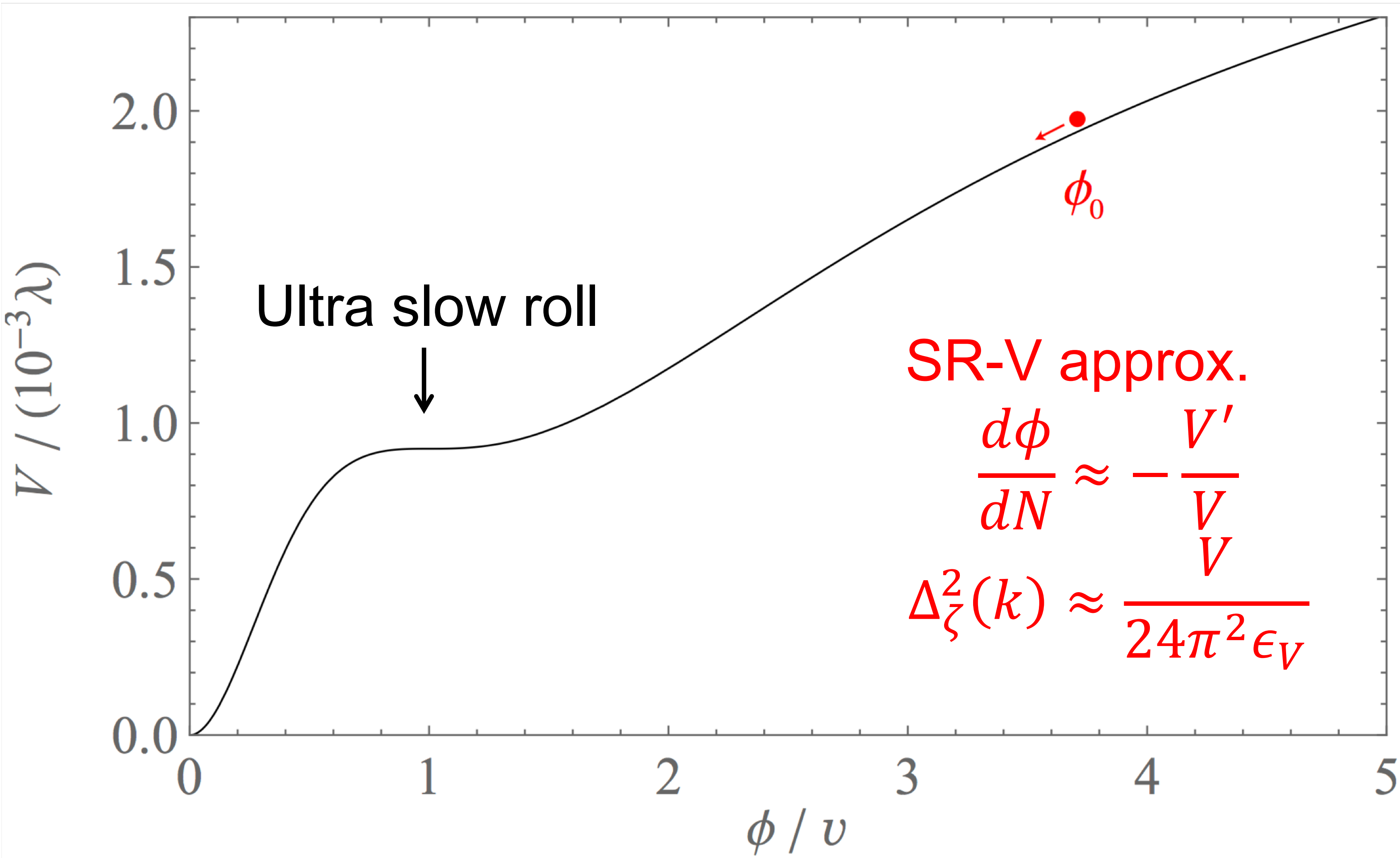
$$\frac{\epsilon_V}{\epsilon_H} = \left(1 + \frac{1}{2(3-\epsilon_H)}\frac{d\ln\epsilon_H}{dN}\right)^2$$

$$\epsilon_H \equiv -\dot{H}/H^2$$
$$\epsilon_V \equiv (V'/V)^2/2$$

Case study 1: Inflection model

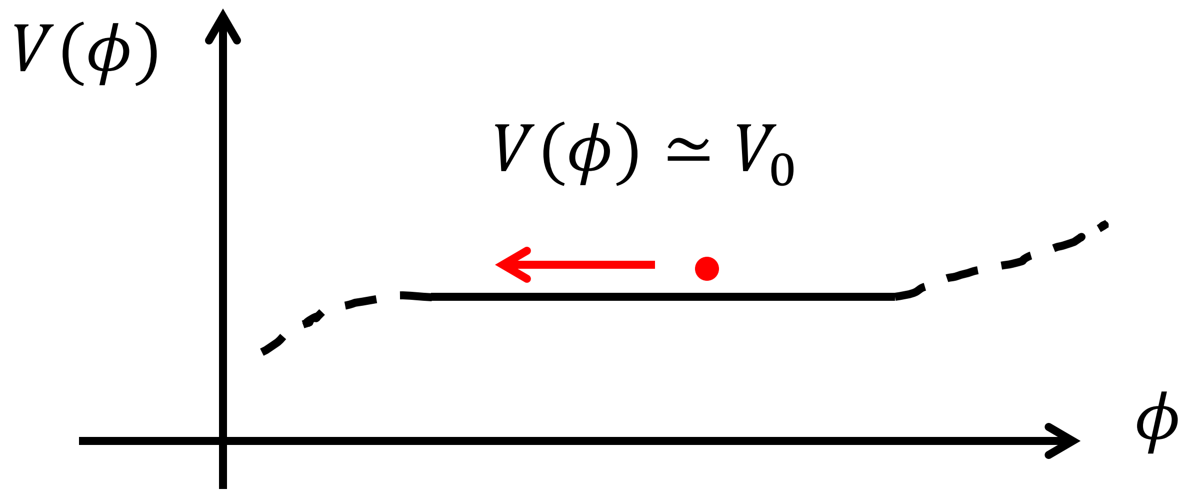
Garcia-Bellido, Morales, 1702.03901

Ezquiaga, Garcia-Bellido, Morales, 1705.04861



Ultra slow-roll inflation

Kinney, gr-qc/0503017



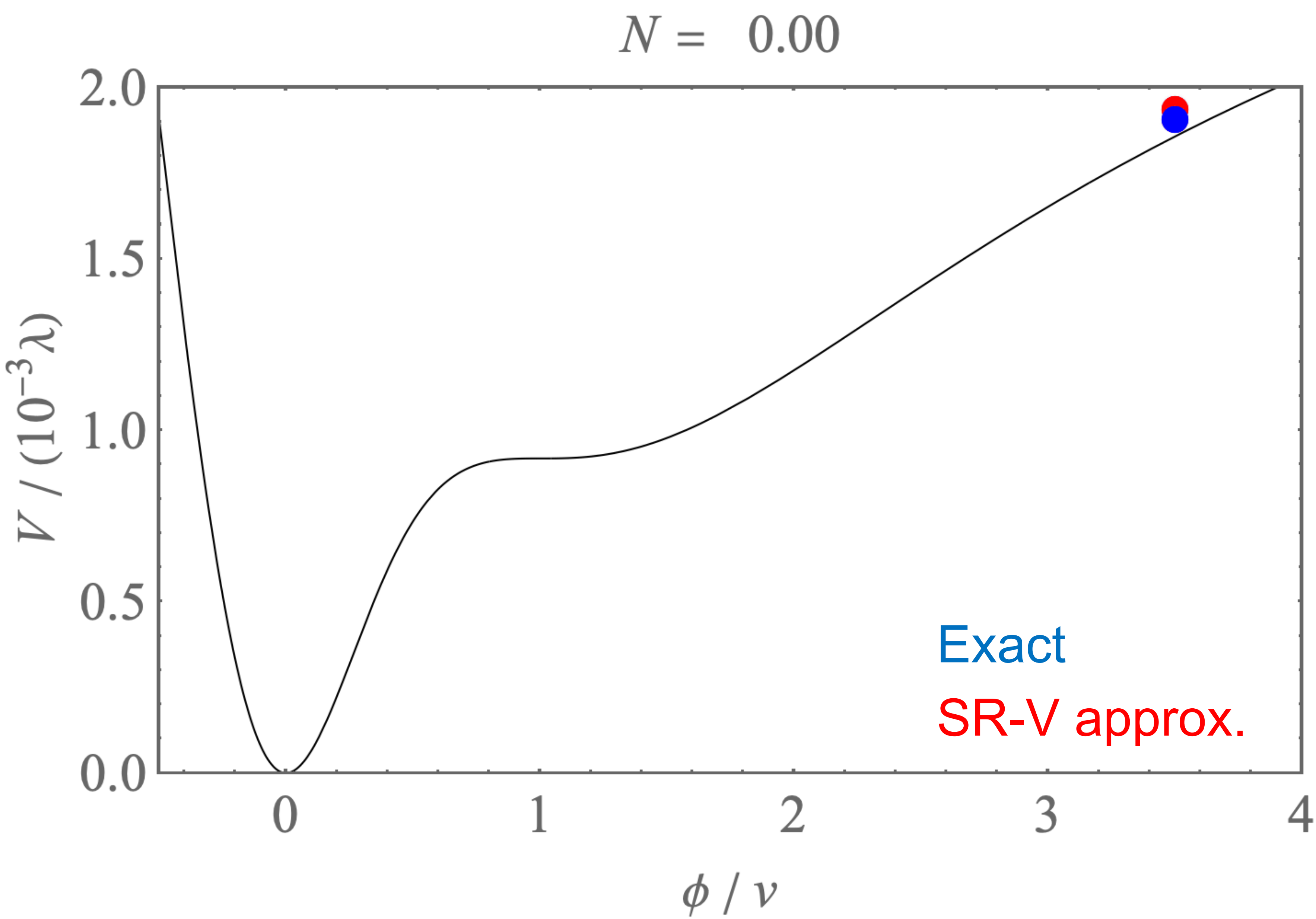
$$\ddot{\phi} = -3H\dot{\phi} \quad \Rightarrow \quad \dot{\phi} \propto a^{-3}$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-6}$$

SR Violation: $\frac{d\ln\epsilon_H}{dN} = -6$

Namjoo, Hassan, Sasaki, 1210.3692
Martin, HM, Suyama, 1211.0083

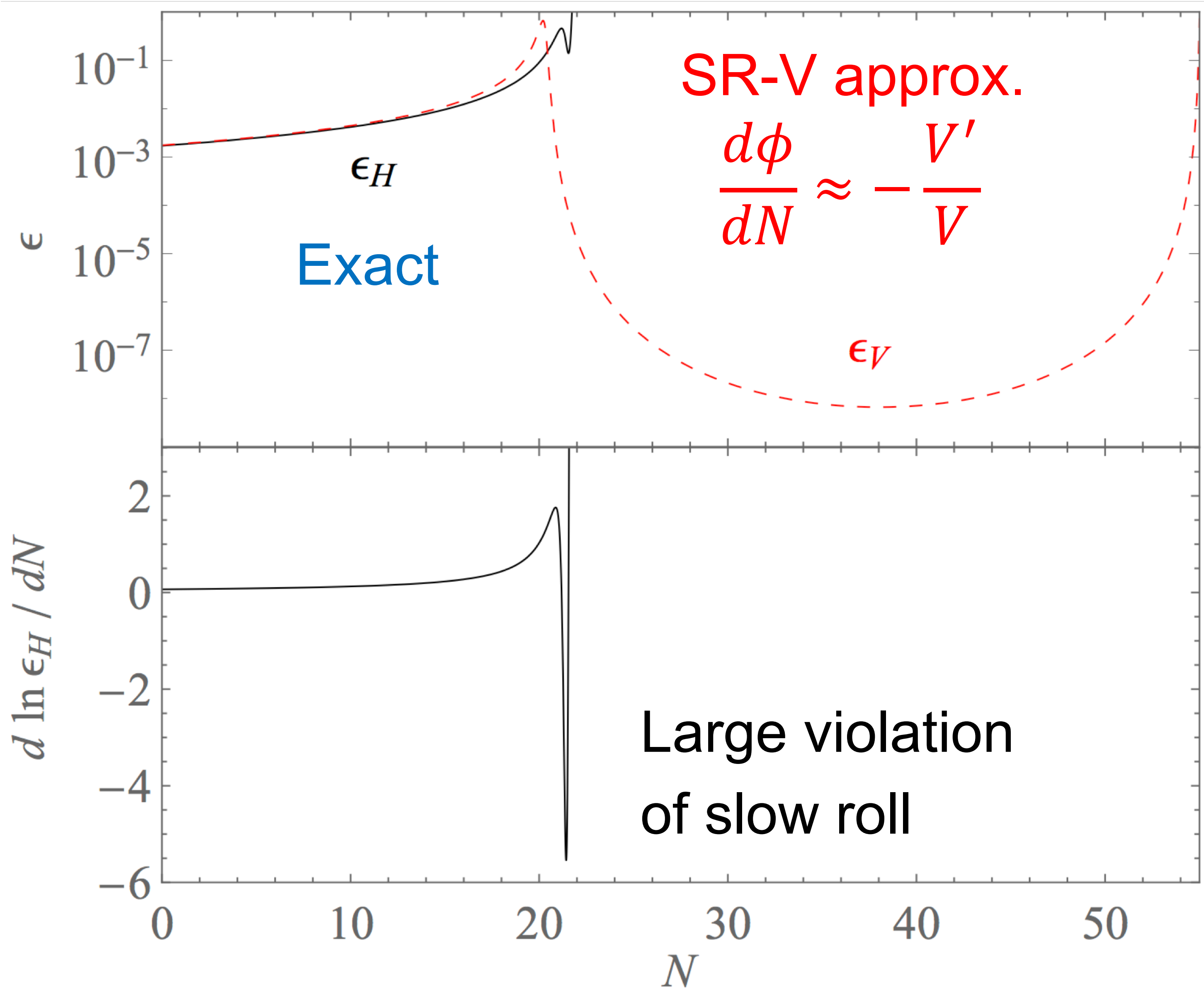
- Violation of non-Gaussianity consistency relation
- Growing mode of ζ_k on superhorizon scales



N = 0.00
V / (10^−3 λ)
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
ϕ / v
Exact
SR-V approx.

HM, Hu, 1706.06784

Case study 1: Inflection model



HM, Hu, 1706.06784

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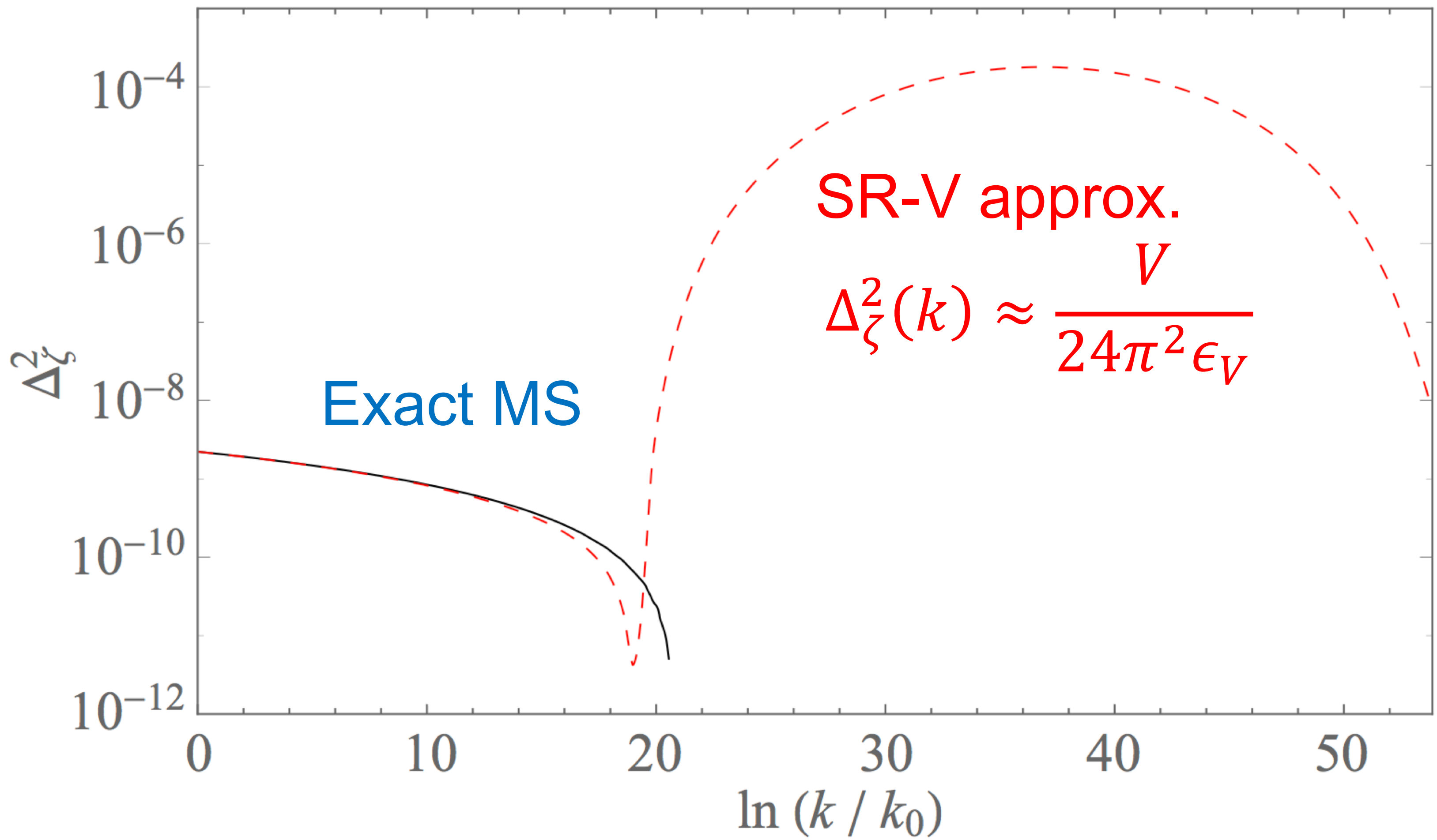
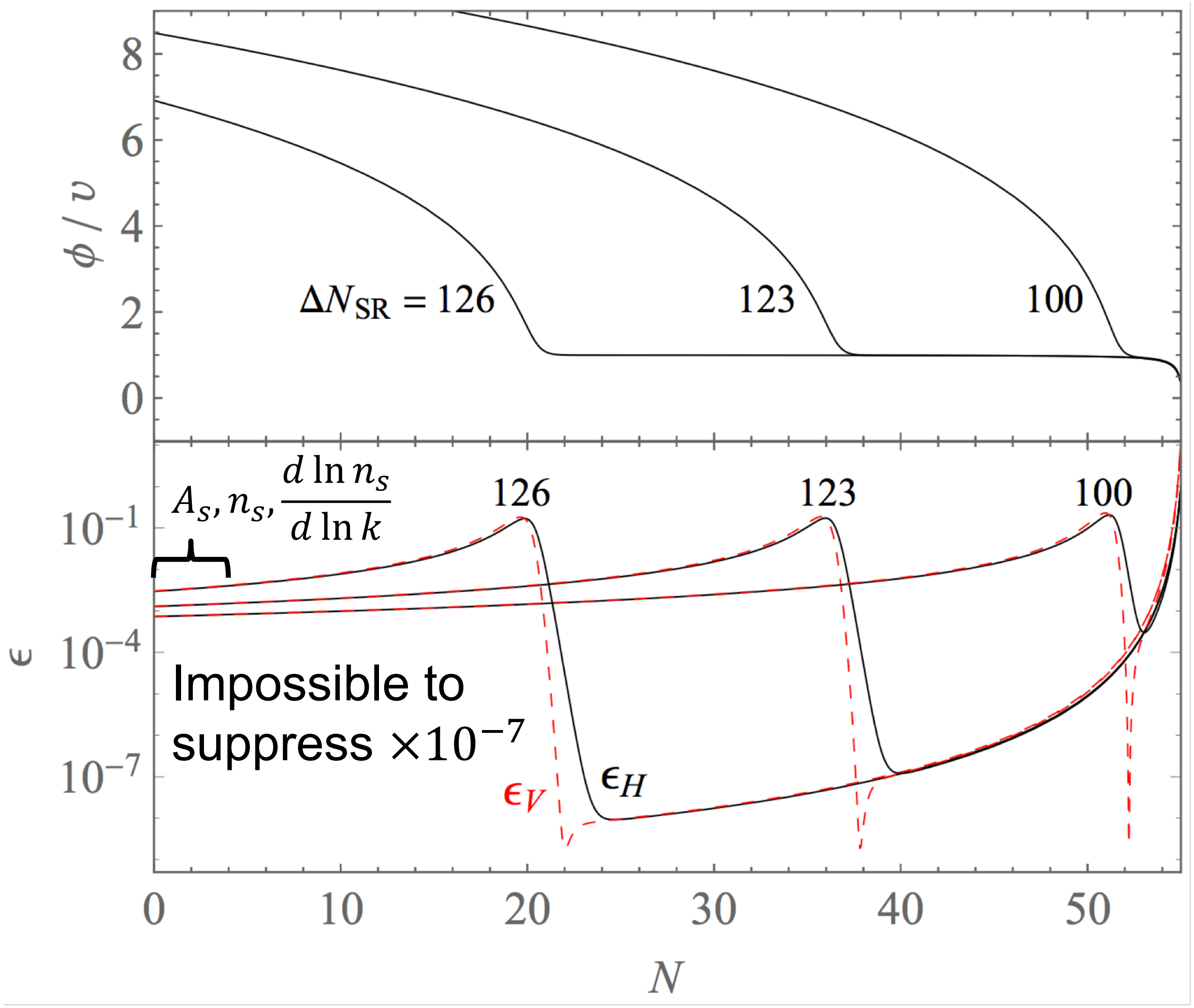


HM, Hu, 1706.06784

Case study 1: Inflection model



For sufficient suppression in inflection model, a fine-tuned shape of the potential is required.

Various approximations

- SR-V : $$\Delta_\zeta^2 \approx \frac{V}{24\pi^2\epsilon_V}\Big|_{k\eta=1}$$ $\Longrightarrow$ Particularly bad

$$\epsilon_H \approx \epsilon_V, \ \frac{d\phi}{dN} = -\frac{V'}{V}$$

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- Standard SR : $$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2\epsilon_H}\Big|_{k\eta=1}$$ $\Longrightarrow$ Not good

- Optimized SR : $$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2\epsilon_H}\Big|_{k\eta=x_1}$$ $\Longrightarrow$ Works well

HM, Hu, 1503.04810, 1704.01128

- Minimize truncation error by optimization $k\eta = x_1$
- $x_1 = \frac{7}{3} - \ln 2 - \gamma_E \approx 2.89$: model independent
- Apply to general class of single-field inflation
  e.g. Horndeski, GLPV, subclass of DHOST

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

1. Formal solution of Mukhanov-Sasaki equation by using Green function (Generalized SR) Stewart, astro-ph/0110322
2. First order iteration

$x \equiv ks$

Sound horizon

$$\ln \Delta^2 (k) = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

Model-indep window function

Source function (model)

HM, Hu, 1503.04810, 1704.01128

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$x \equiv ks$ ← Sound horizon

$$\ln \Delta^2(k) = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

$W'(x)$: Model-indep window function; $G(\ln x)$: Source function (model)

3. Taylor expand $G(\ln x)$ around the evaluation point $\ln x_f$

$$\ln \Delta^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

q_p: Model-indep. polynomial

SR: $\ln x_f = 0$ horizon exit

HM, Hu, 1503.04810, 1704.01128

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Model-indep. polynomial

SR: $\ln x_f = 0$ horizon exit

4. Truncate at p and optimize $\ln x_f$ so that $q_{p+1}(\ln x_f) = 0$

OSR: $\ln x_f \neq 0$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

Consider general inflation model with feature $\sim \Delta N$

$$\ln \Delta^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

$\uparrow$ $\sim 1/\Delta N^p$

$x \equiv ks$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

Consider general inflation model with feature $\sim \Delta N$

$$\ln \Delta^2 (k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

$\uparrow \sim 1/\Delta N^p$ $\quad x \equiv ks$

- Standard SR

$$\ln \Delta^2 = G(0) \text{ : correction} = O(q_1(0) G'(0))$$

$\ln x_f = 0$: horizon exit

$q_1(0) \approx 1.06$ $\quad 1/\Delta N$

$\simeq 0.35$ for $\Delta N \sim 3$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

Consider general inflation model with feature $\sim \Delta N$

$$\ln \Delta^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

$G^{(p)} \sim 1/\Delta N^p$ $\qquad x \equiv ks$

- Standard SR

$$\ln \Delta^2 = G(0) : \text{correction} = O(q_1(0) G'(0))$$

$\ln x_f = 0$: horizon exit $\qquad q_1(0) \approx 1.06 \qquad G'(0) \sim 1/\Delta N$

$\simeq 0.35$ for $\Delta N \sim 3$

- Optimized SR

$$\ln \Delta^2 = G(\ln x_1) : \text{correction} = O(q_2(\ln x_1) G''(\ln x_1))$$

$\ln x_f = \ln x_1 \approx 1.06$ with $q_1(\ln x_1) = 0$

$\sim$1 efold before horizon exit

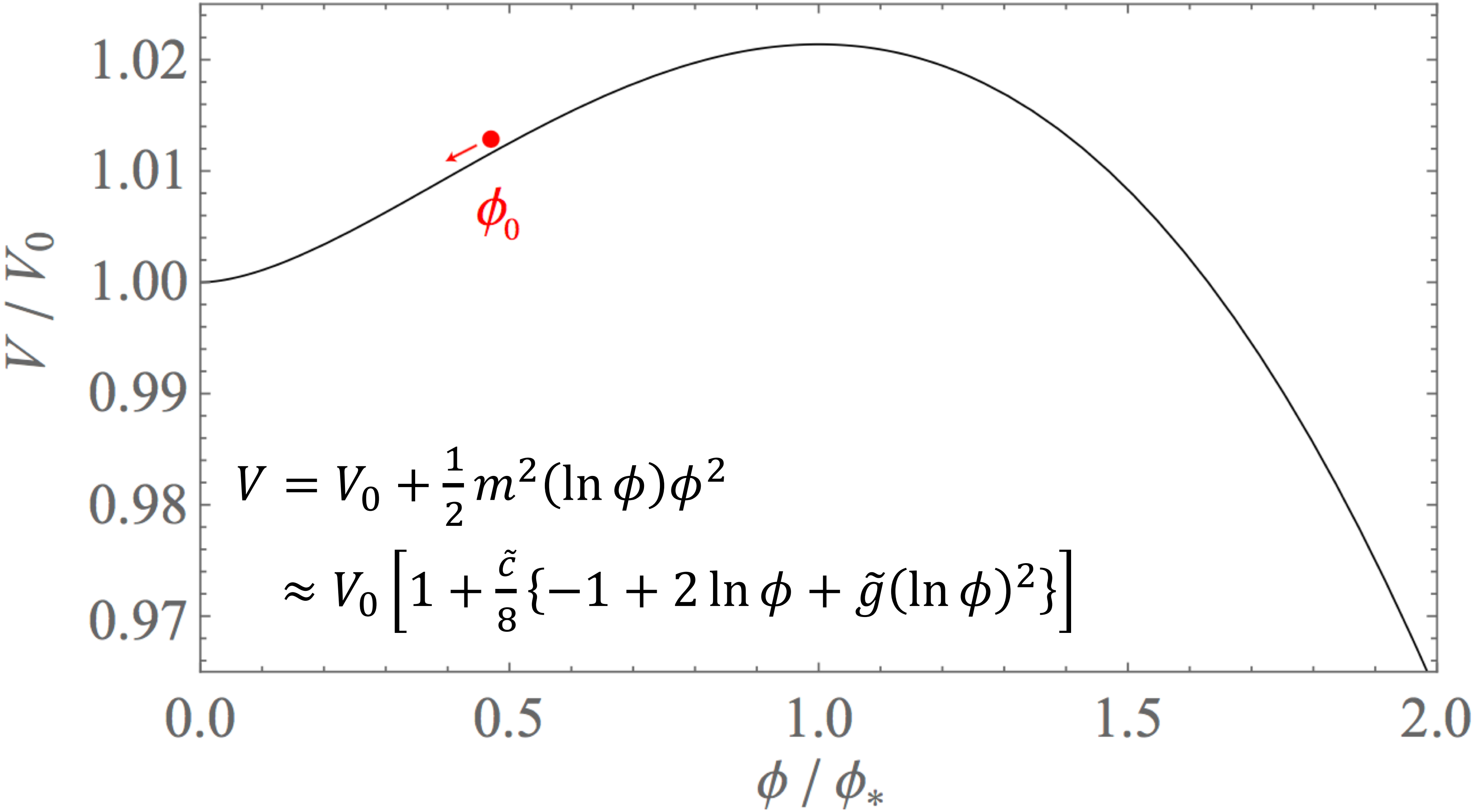
$q_2(\ln x_1) \approx -0.36 \qquad G''(\ln x_1) \sim 1/\Delta N^2$

$\simeq 0.04$ for $\Delta N \sim 3$

HM, Hu, 1706.06784

Case study 2: Running mass model

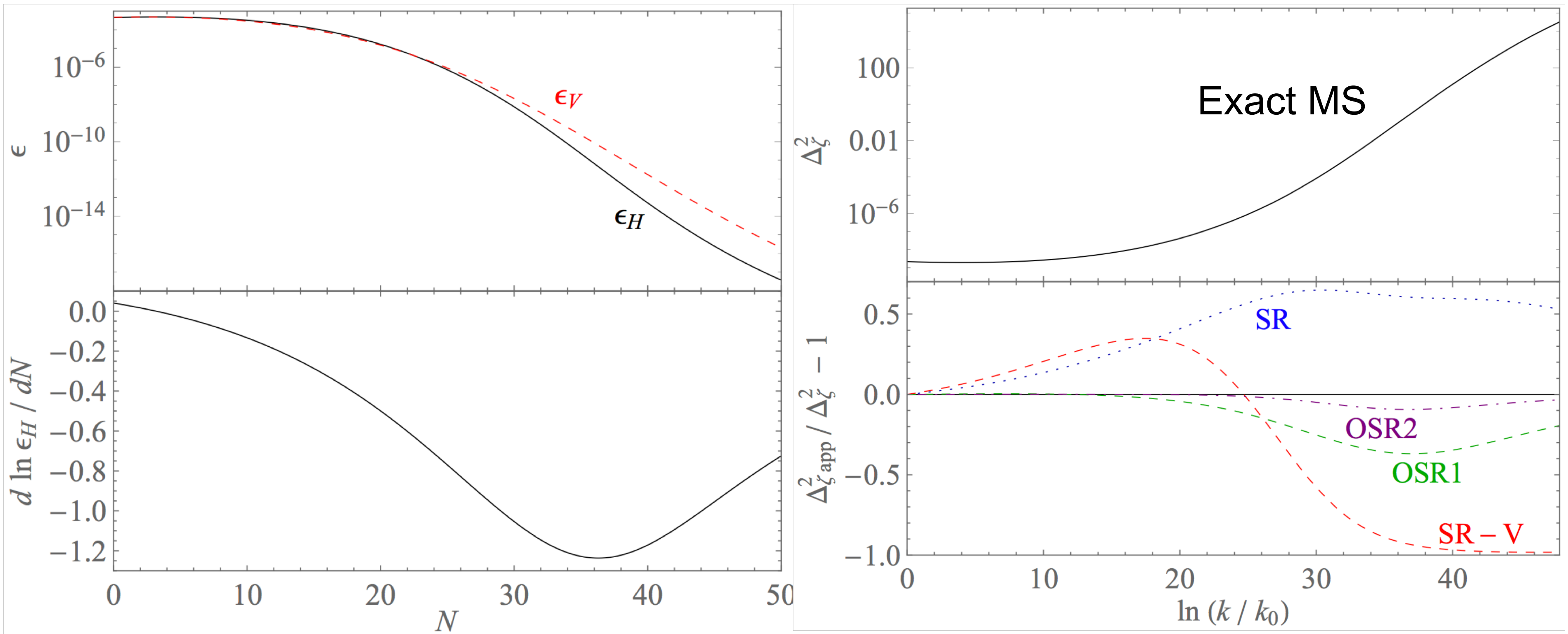
Drees, Erfani,1102.2340



$$V = V_0 + \frac{1}{2} m^2 (\ln\phi)\phi^2$$
$$\approx V_0\left[1 + \frac{\tilde{c}}{8}\{-1 + 2\ln\phi + \tilde{g}(\ln\phi)^2\}\right]$$

HM, Hu, 1706.06784

Case study 2: Running mass model



Slow-roll violation

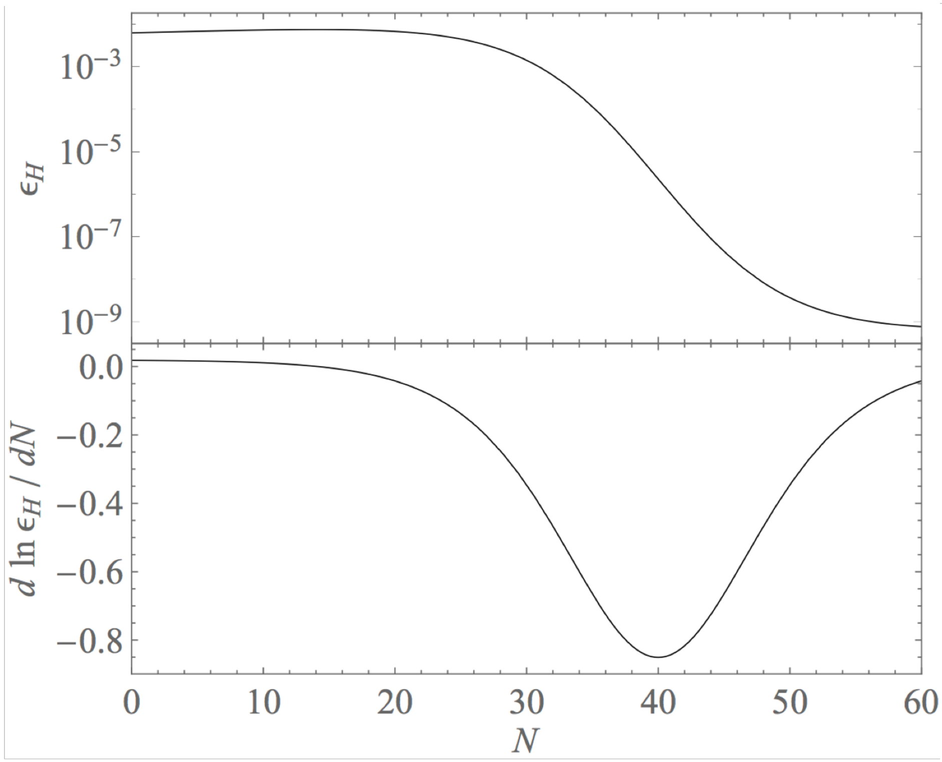
OSR still works well

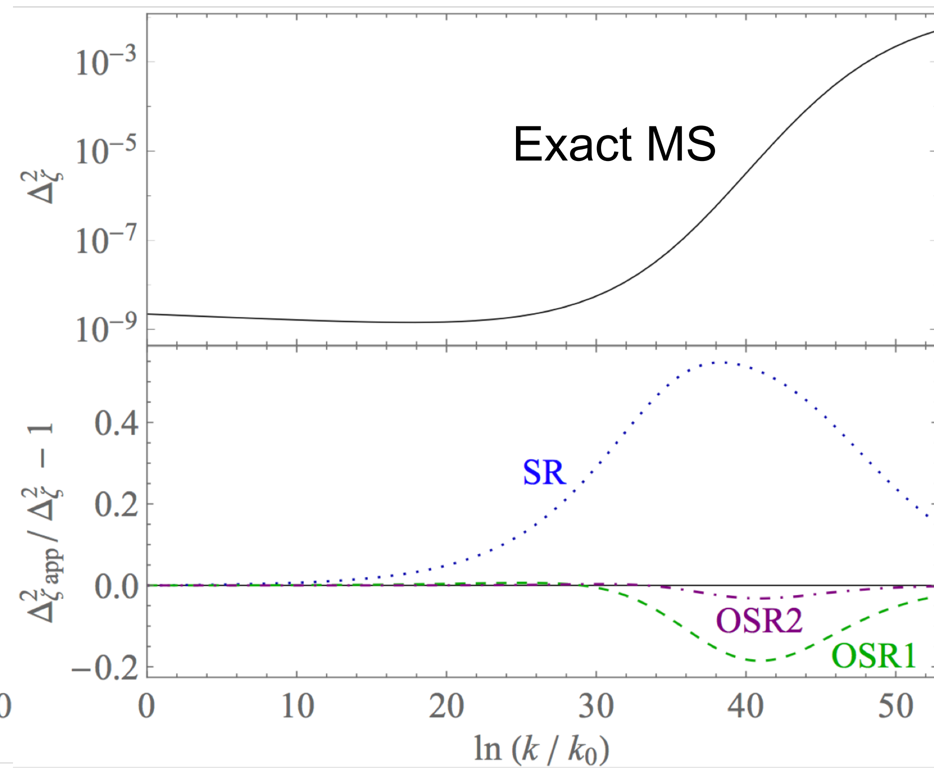
HM, Hu, 1706.06784

Case study 3: Slow roll step model

Parametrize $\ln \epsilon_H$ directly

$$\ln \epsilon_H = C_1 + C_2 N - C_3 \left[1 + \tanh\left(\frac{N - N_s}{d}\right)\right]$$





HM, Hu, 1706.06784

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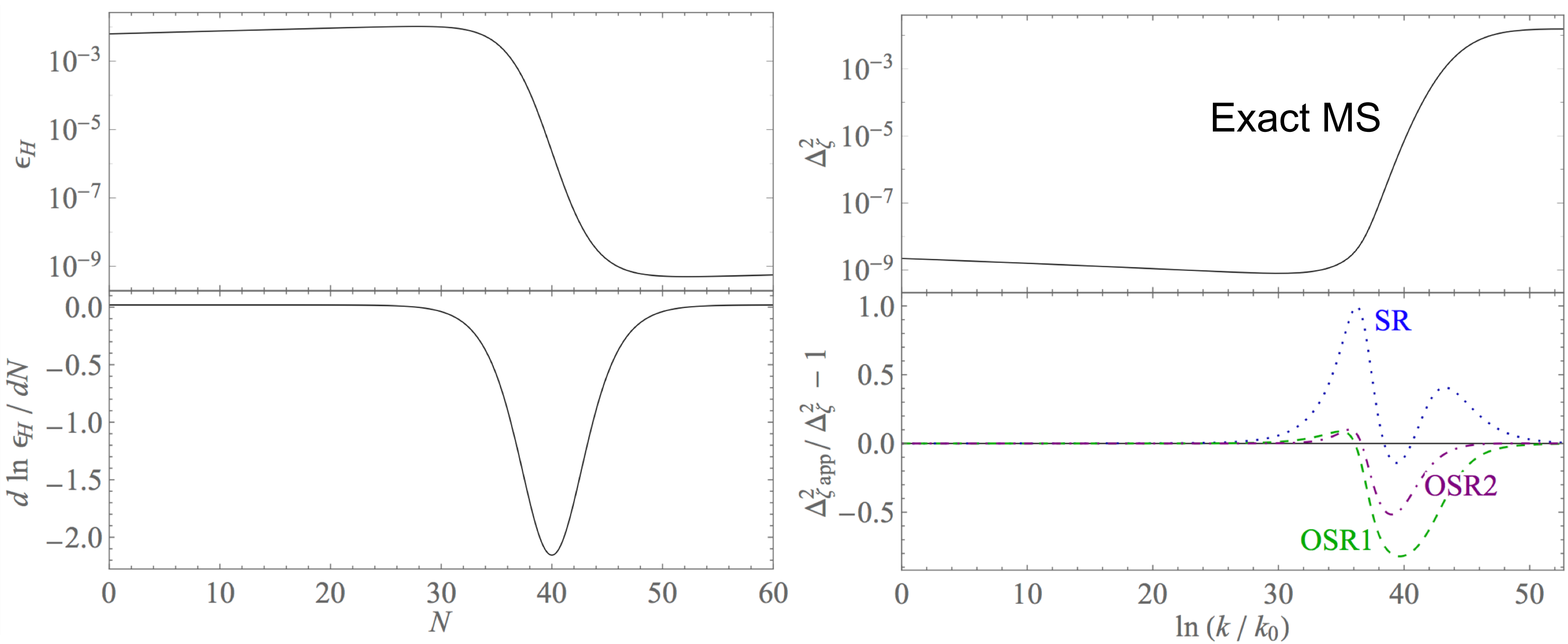

For $\Delta N < 10$, all approximations do not work.

Summary

- No go for slow roll: HM, Hu, 1706.06784

  PBH production requires $\mathcal{O}(1)$ slow-roll violation

  $\Rightarrow$ Previous analyses based on slow-roll approx. need reconsideration.

- Improved approximation: Optimized slow roll HM, Hu, 1503.04810 1704.01128

  $\Rightarrow$ Remains a good description for models with 10^7 amplification of Δ_ζ^2 in $\Delta N > 10$.

  $\supset$ PBH = (part of) DM or GW events scenarios

- No go is robust even with non-Gaussianity Passaglia, Hu, HM, 1812.08243

  $\Leftarrow$ *Next talk by **Samuel Passaglia***