Wave propagation in force-free magnetosphere of a rotating black hole

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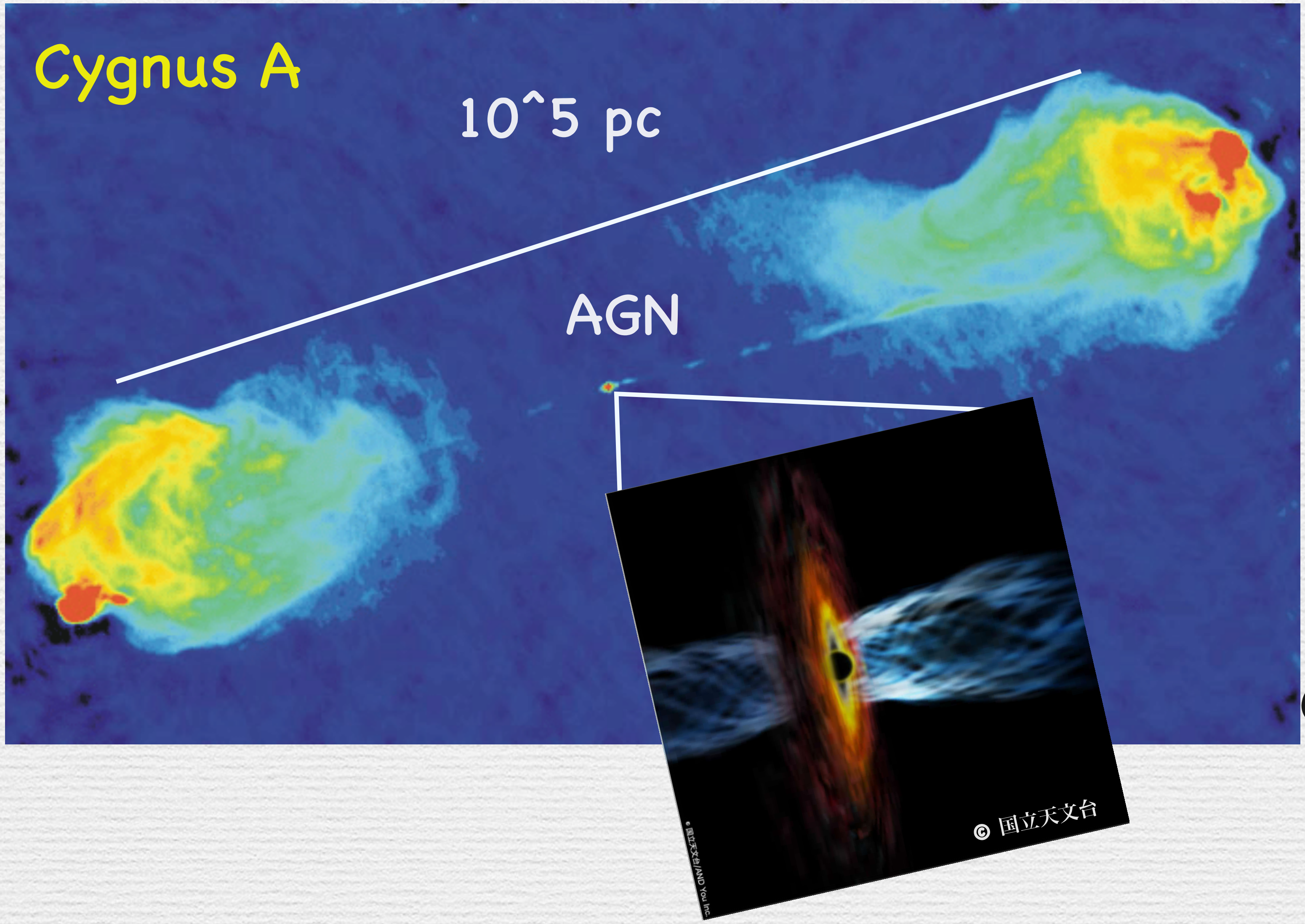
2019. Feb. 14 (YITP Asian pacific Winter school and Workshop on Gravitation and Cosmology)

Motivation



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Black Hole + Plasma, Electromagnetic fields

Engine: ⇓ rotation

Extraction of rotational energy (Blandford-Znajek process)

Blandford & Znajek (1977)

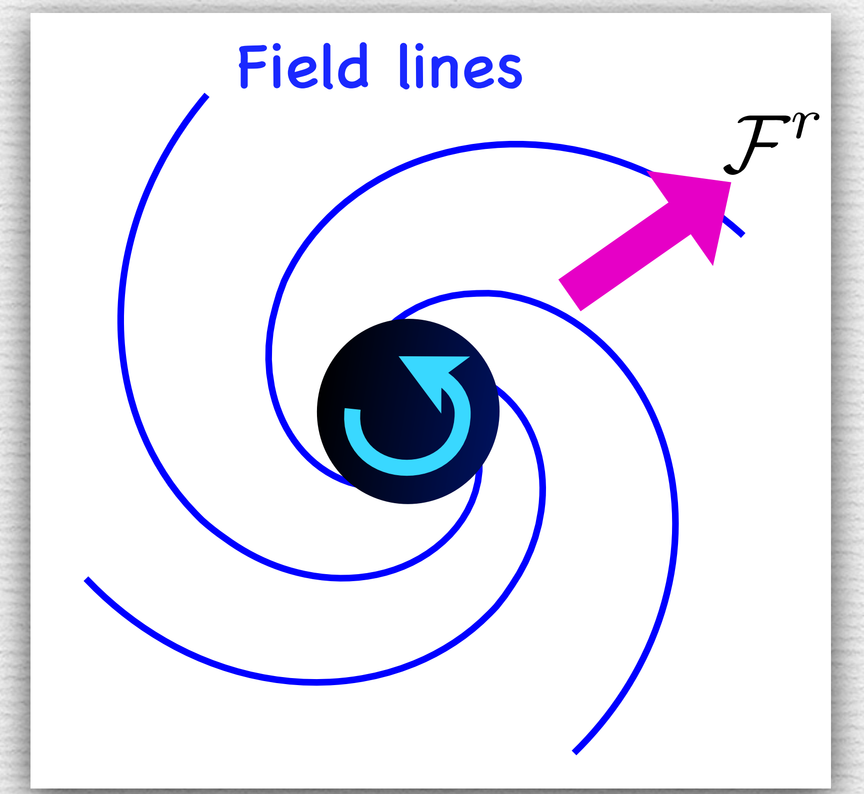
Blandford-Znajek process

Blandford & Znajek (1977)

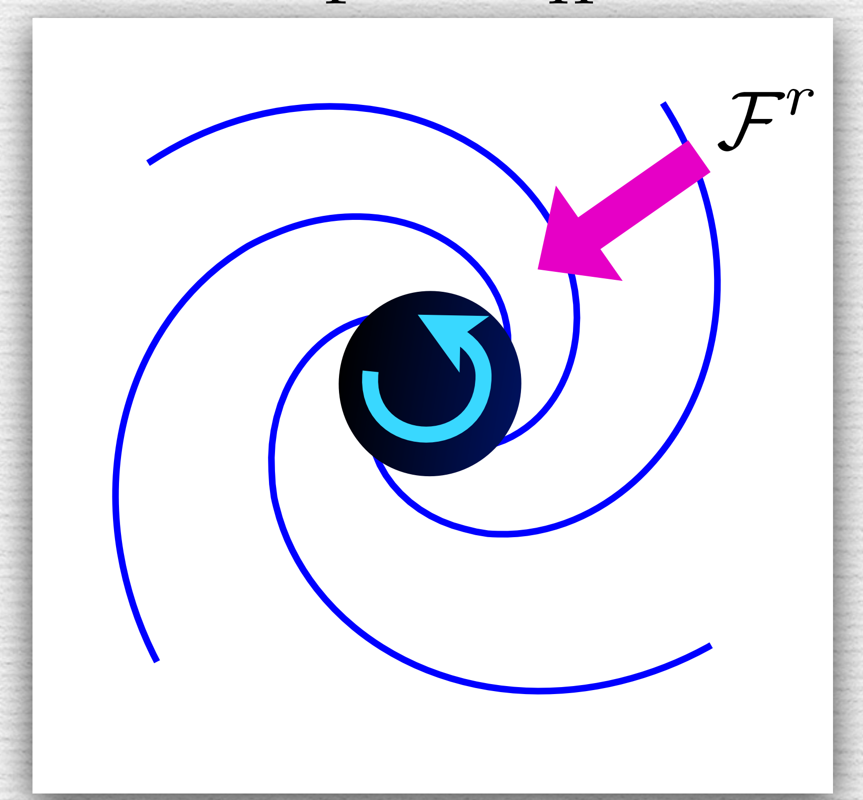
- Kerr metric , Stationary and axisymmetric magnetosphere
- Force-Free EM field (Magnetically dominated)
- Energy extraction by electromagnetic fields
- Rotational energy is transported as Poynting flux

$$\mathcal{F}^r \propto \Omega_{\rm F}(\Omega_{\rm H} - \Omega_{\rm F})$$

$\Omega_{\rm F} < \Omega_{\rm H}$

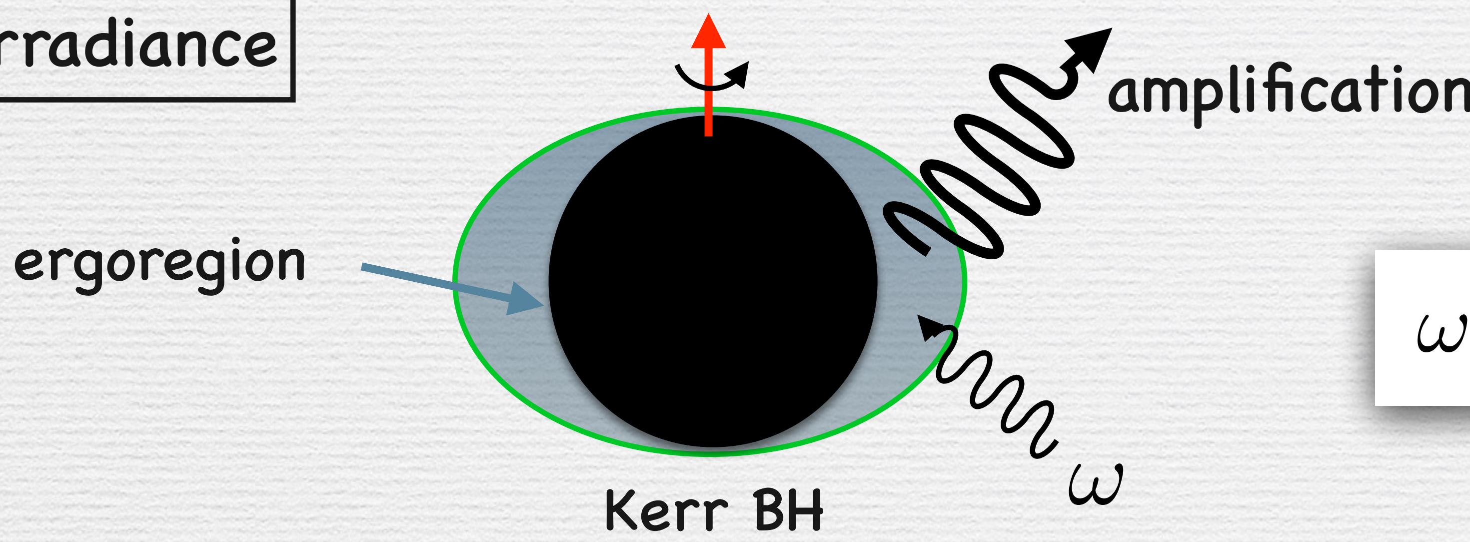


$\Omega_{\rm F} > \Omega_{\rm H}$



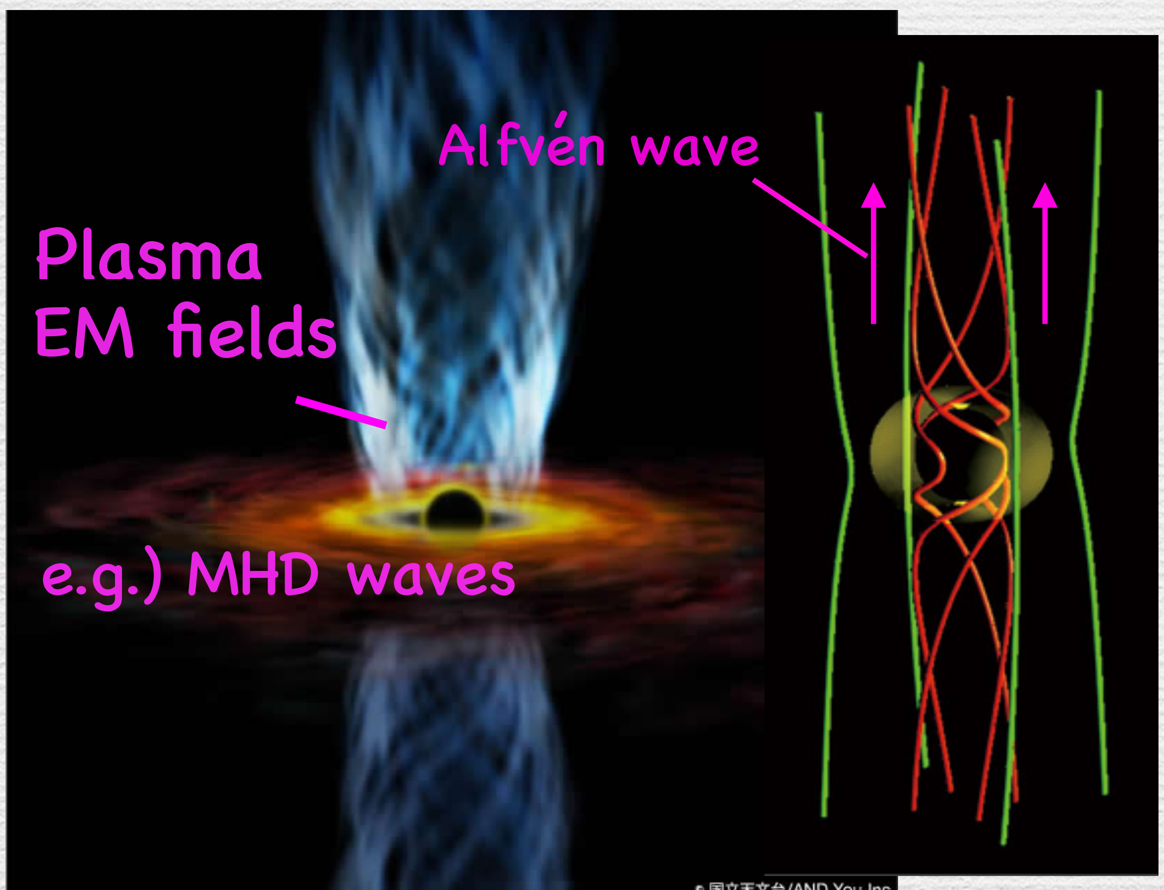
Energy Extraction by Waves

Superradiance



$$\omega < m\Omega_H \qquad m = 1, 2, \cdots$$

Waves in magnetosphere



Koide (2003)

BZ process

= energy extraction by background field

Does superradiance for waves

in magnetosphere contribute to the jet ?

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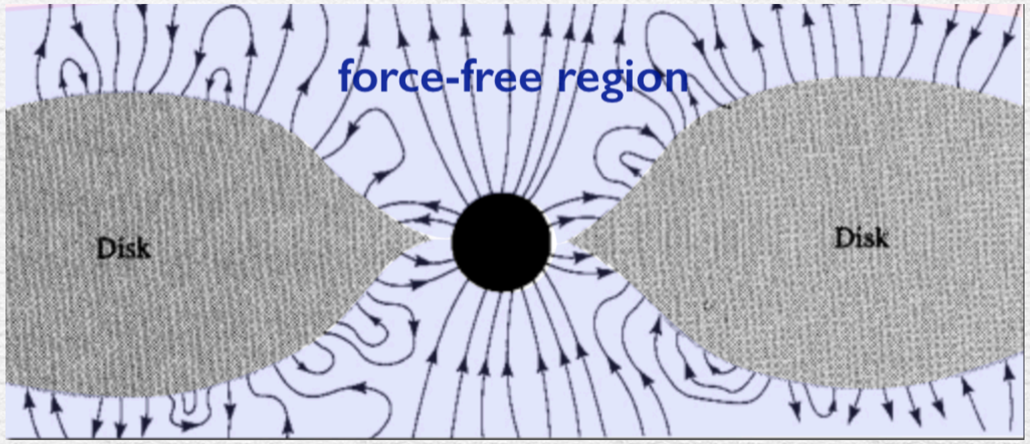
Force-free electromagnetic field

Maxwell eq.

$$\nabla_{[a}F_{bc]} = 0 \qquad \nabla_b T^{ab}_{(\mathrm{EM})} = -F^{ab}j_b$$

$$\nabla_b F^{ab} = 4\pi j^a \quad \nabla_b \left(T^{ab}_{(\mathrm{EM})} + T^{ab}_{\mathrm{plasma}}\right) = 0$$



If the EM fields are dominant , $\nabla_b T^{ab}_{(\mathrm{EM})} \approx 0 \quad \rightarrow \quad F^{ab}j_b \approx 0.$ (Force-free approximation)

$$\nabla_{[a}F_{bc]} = 0 \ , \quad F_{ab}\nabla_c F^{bc} = 0 \ , \quad j^a \propto \nabla_b F^{ab} \neq 0$$

Carter (1979)
Uchida (1997)
Gralla & Jacobson (2014)

$$\Rightarrow \quad \boxed{F = d\phi_1 \wedge d\phi_2}$$

ϕ_1 , ϕ_2 : Euler potential

Force-free EM field eq.

$$\partial_a \phi_1 \partial_b \left[\sqrt{-g}\left(\partial^a\phi_1\partial^b\phi_2 - \partial^b\phi_1\partial^a\phi_2\right)\right] = 0$$

$$\partial_a \phi_2 \partial_b \left[\sqrt{-g}\left(\partial^a\phi_1\partial^b\phi_2 - \partial^b\phi_1\partial^a\phi_2\right)\right] = 0$$

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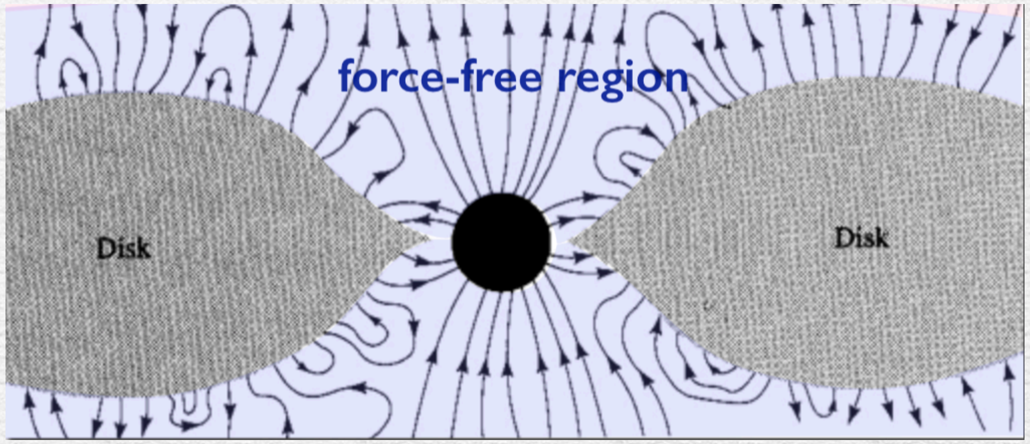

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$$\partial_a \phi_2 \partial_b \left[\sqrt{-g} \left(\partial^a \phi_1 \partial^b \phi_2 - \partial^b \phi_1 \partial^a \phi_2 \right) \right] = 0$$

Wave propagation in force-free fields

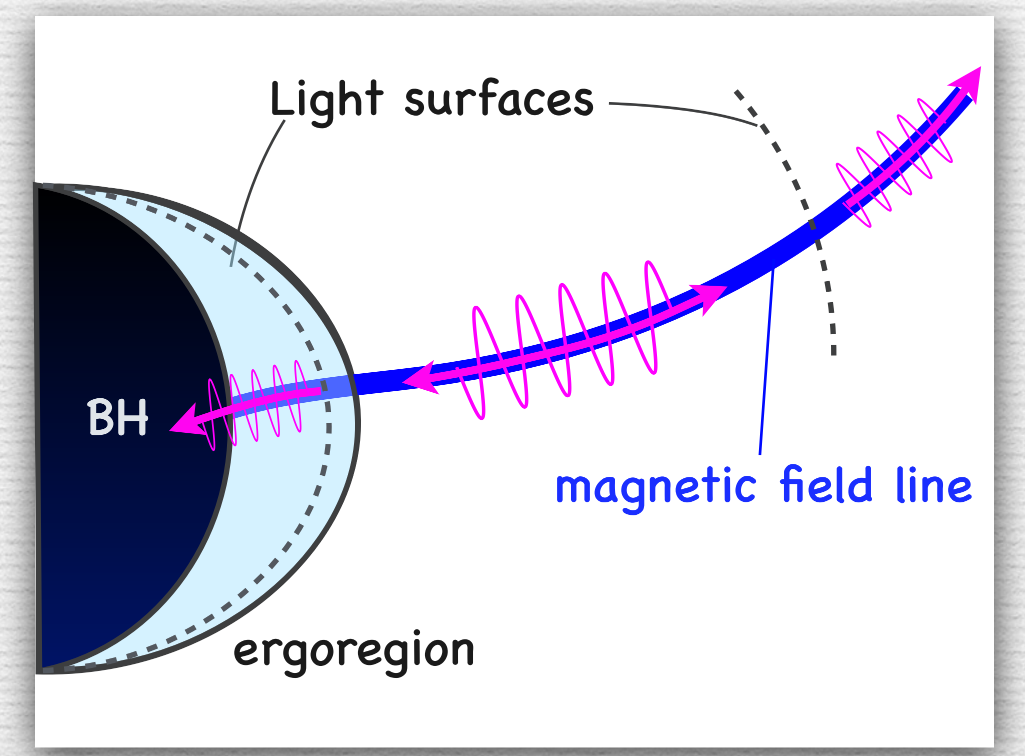
$\phi_i \to \phi_i + \delta\phi_i$

$$\partial_a \underline{\delta\phi_1} \partial_b \left[\sqrt{-g}\,\partial^{[a}\phi_1 \partial^{b]}\phi_2\right] + \partial_a\phi_1\partial_b\left[\sqrt{-g}\left(\partial^{[a}\underline{\delta\phi_1}\partial^{b]}\phi_2 + \partial^{[a}\phi_1\partial^{b]}\underline{\delta\phi_2}\right)\right] = 0$$

$$\partial_a \underline{\delta\phi_2} \partial_b \left[\sqrt{-g}\,\partial^{[a}\phi_1 \partial^{b]}\phi_2\right] + \partial_a\phi_2\partial_b\left[\sqrt{-g}\left(\partial^{[a}\underline{\delta\phi_1}\partial^{b]}\phi_2 + \partial^{[a}\phi_1\partial^{b]}\underline{\delta\phi_2}\right)\right] = 0$$

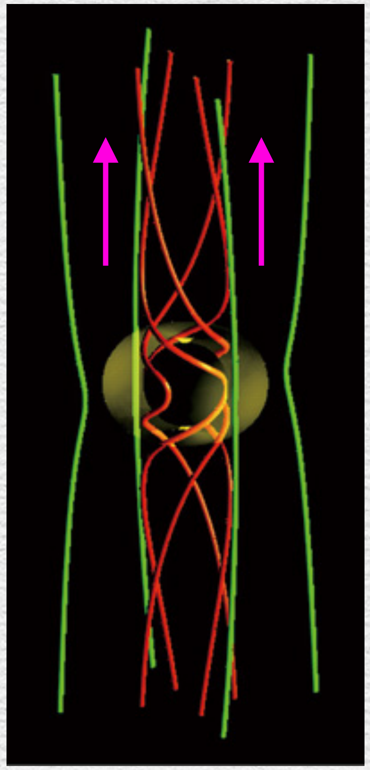
Fast mode ··· sound wave like mode (Longitudinal mode)

Alfvén mode ··· propagating along magnetic field lines (Transverse mode)



Questions

- Superradiance occur for Alfvén wave ?
- Relation to BZ process ?



A simple example of the force-free field

Alfvén wave propagation and superradiance

Alfvén wave and Blandford-Znajek mechanism

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Model — Black cylinder spacetime

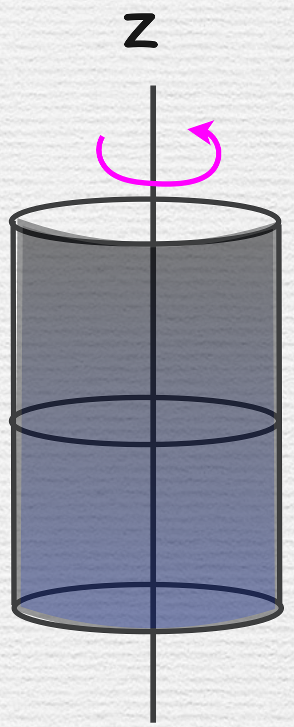
Axisymmetric spacetime

$$ds^2 = -\alpha^2 dt^2 + \frac{dr^2}{\alpha^2} + r^2(d\varphi - \Omega dt)^2 + dz^2$$

$$\Omega = \frac{r_H r_-}{r^2 \ell}$$

$$\alpha^2 = \frac{(r^2 - r_H^2)(r^2 - r_-^2)}{r^2 \ell^2}$$

BTZ black hole + 1dim



Axisymmetric force-free field and BZ mechanism

Jacobson & Rodriguez (arXiv 1709.10090)

$$\phi_1 = z$$

$$\phi_2 = C \int \frac{dr}{r\alpha^2} + \varphi - \Omega_F t$$

$$C = r_H(\Omega_H - \Omega_F)$$

Ω_F : angular velocity of the magnetic field lines

C : const. determined by a regularity condition at horizon (Znajek condition)

Light surfaces (corotating vector = null)

$$\xi_F^\mu = (1, 0, 0, \Omega_F) \; , \quad \Gamma \equiv -g_{\mu\nu}\xi_F^\mu \xi_F^\nu = 0$$

BZ condition

$$0 < \Omega_F < \Omega_H$$

A simple example of the force-free field

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Alfvén wave propagation and superradiance

Alfvén wave equation

$$\partial_a \phi_1 \partial_b \left(\sqrt{-g} \partial^{[a} \phi_1 \partial^{b]} \delta\phi \right) = 0$$

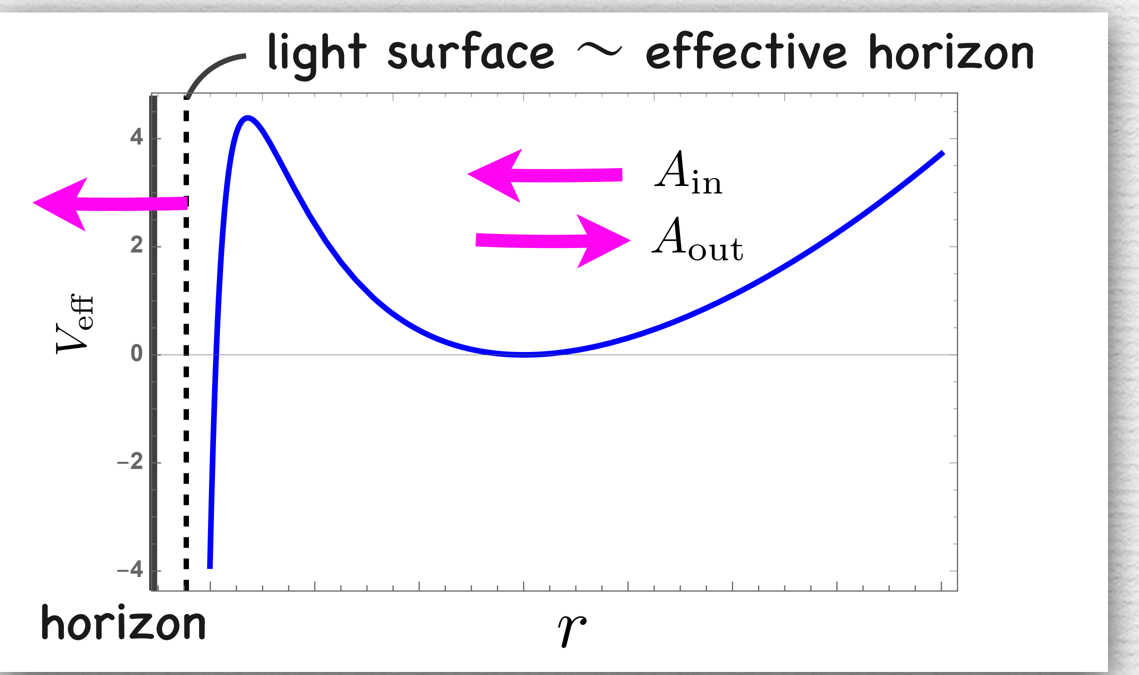
$$\delta\phi = e^{-1/(2r^2)} \psi(r) e^{-i\omega t_*} e^{im\varphi_*}$$

$$dt_* = dt + \frac{Cr}{\Gamma}(g^{t\varphi} - \Omega_F g^{tt}) dr \ , \quad d\varphi_* = d\varphi + \frac{Cr}{\Gamma}(g^{\varphi\varphi} - \Omega_F g^{t\varphi}) dr$$

Radial equation

$$\frac{d^2\psi}{dr_*^2} + \left(\omega^2 - V_{\rm eff}\right)\psi = 0$$

$$\frac{dr_*}{dr} = \frac{1}{\Gamma} \qquad \text{tortoise coodinate}$$



Conservation of the Wronskian

$$W = \psi' \bar{\psi} - \psi \bar{\psi}' = \text{const}$$

$$\left| \frac{A_{\rm out}}{A_{\rm in}} \right|^2 = 1 - \mathcal{P}(\Omega_H - \Omega_F)(\omega - m\Omega_F)$$

reflection rate

Superradiant condition

$$(\Omega_H - \Omega_F)(\omega - m\Omega_F) < 0$$

Superradiance & Blandford-Znajek

Alfvén wave

$$(\Omega_{\rm H} - \Omega_{\rm F})(\omega - m\Omega_{\rm F}) < 0$$

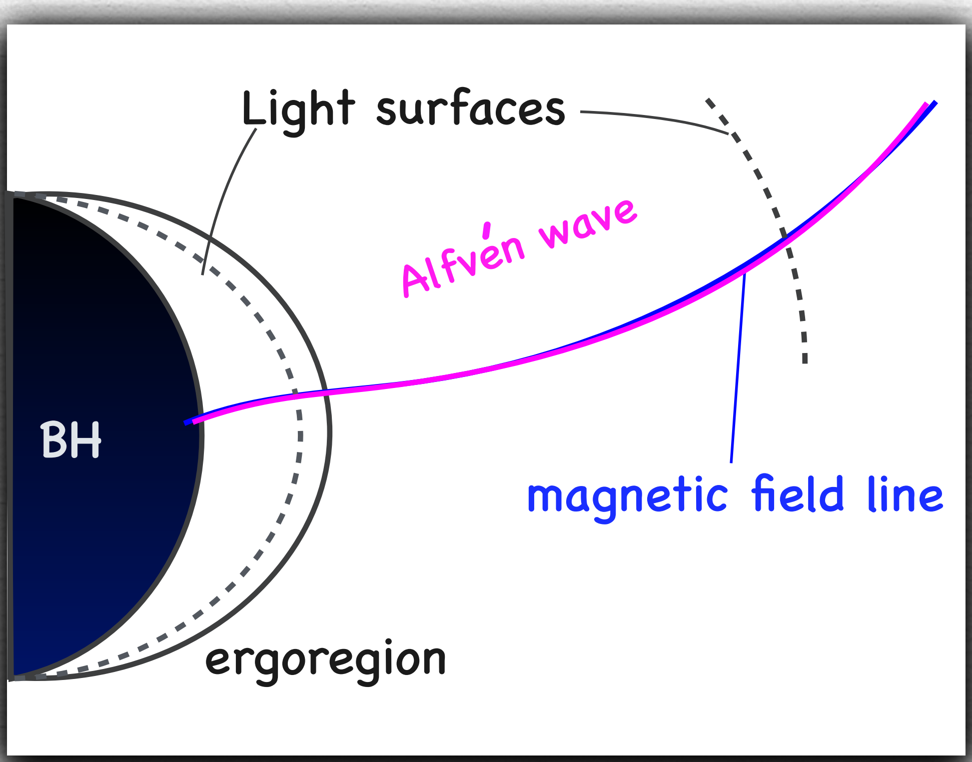
Reflection rate of Alfvén wave

$\omega \to 0$

Blandford-Znajek process

$$\mathcal{F}^r \propto \Omega_{\rm F}(\Omega_{\rm H} - \Omega_{\rm F}) > 0$$

Poynting flux (Background field)



Besides Blandford-Znajek mechanism,
Alfvén wave can also extract the rotational energy of a rotating BH (superrdiance) as long as the wave length is finite,

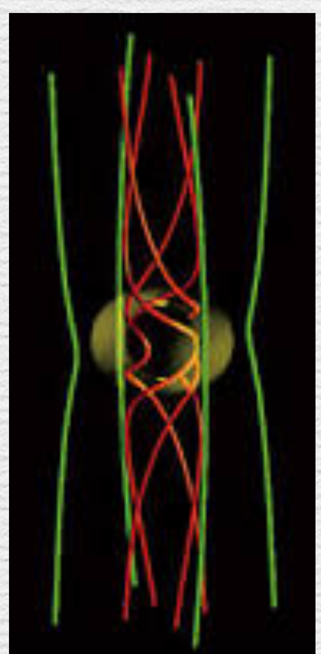
Summary

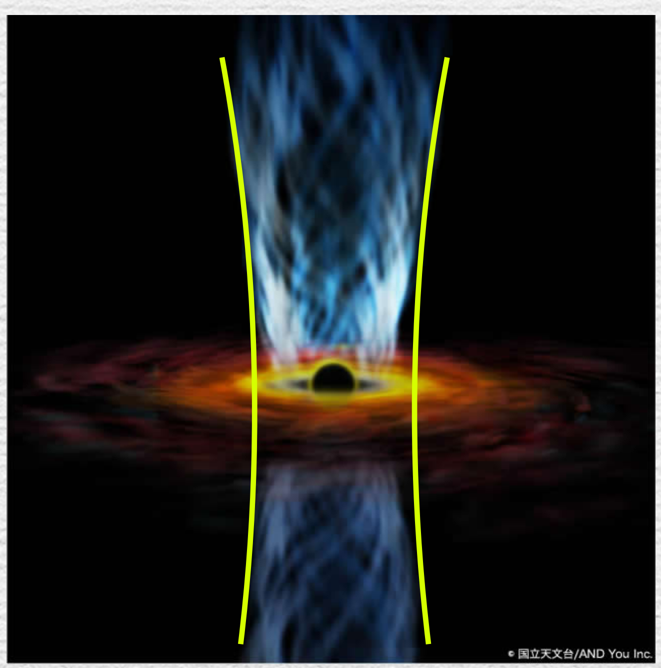
- Wave propagation in axisymmetric force-free EM field
- Alfvén wave can extract the rotational energy
- Long wavelength limit of Alfvén wave → BZ process

What's next?

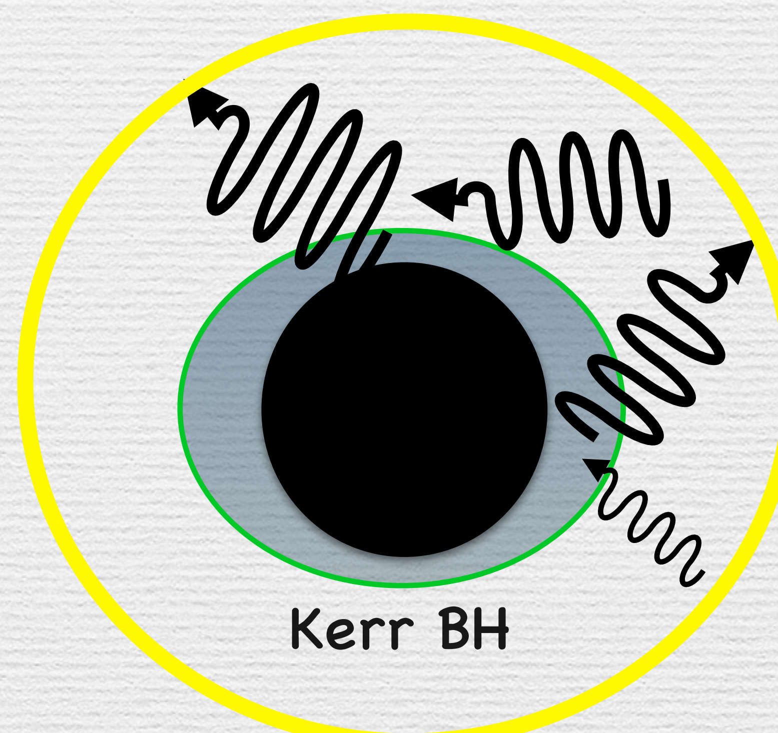
More realistic situation
(Kerr BH case)

Superradiant instability?
(Black Hole Bomb)





Koide (2003)

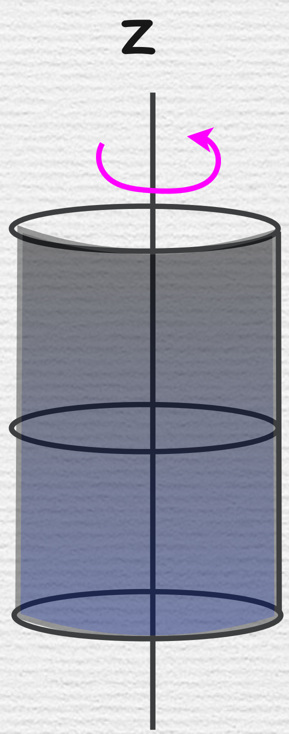


Backup

Fast wave in black cylinder

$$\Box\delta\phi_{\text{fast}} = 0 \qquad \delta\phi_{\text{fast}} = \frac{\psi(r)}{r}e^{-i\omega t}e^{im\phi}$$

$$\frac{d^2\psi}{dr_*^2} + (\omega^2 - V_{\text{eff}})\psi = 0$$



Same as the scalar perturbation to this spacetime

⟶ Superradiance $\boxed{\omega < m\Omega_H}$ $m = 1, 2, \cdots$