Gravitational Waves Induced by non-Gaussian Scalar Perturbations

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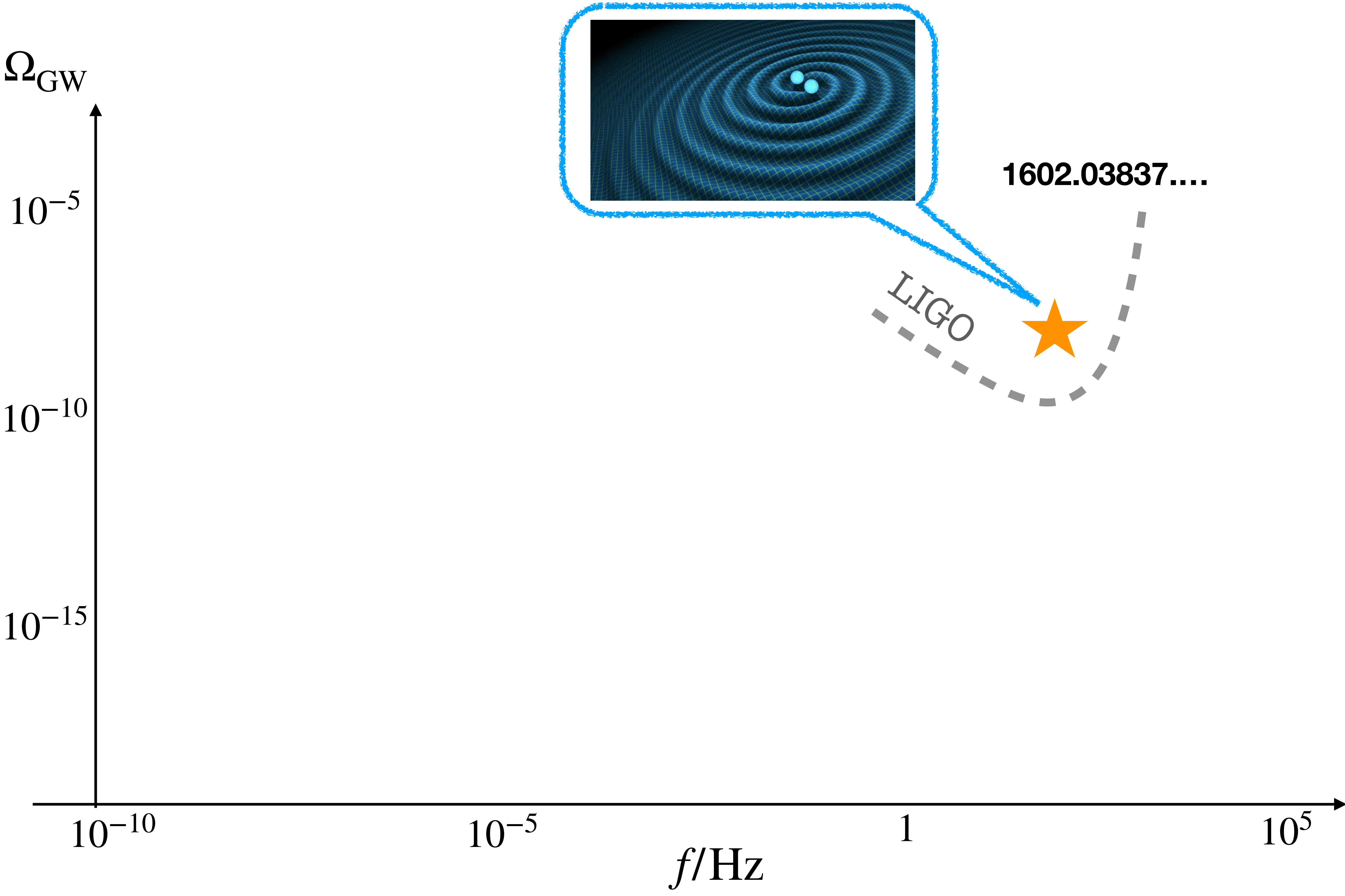
Based on arXiv:1810.11000, with Rong-Gen Cai and Misao Sasaki

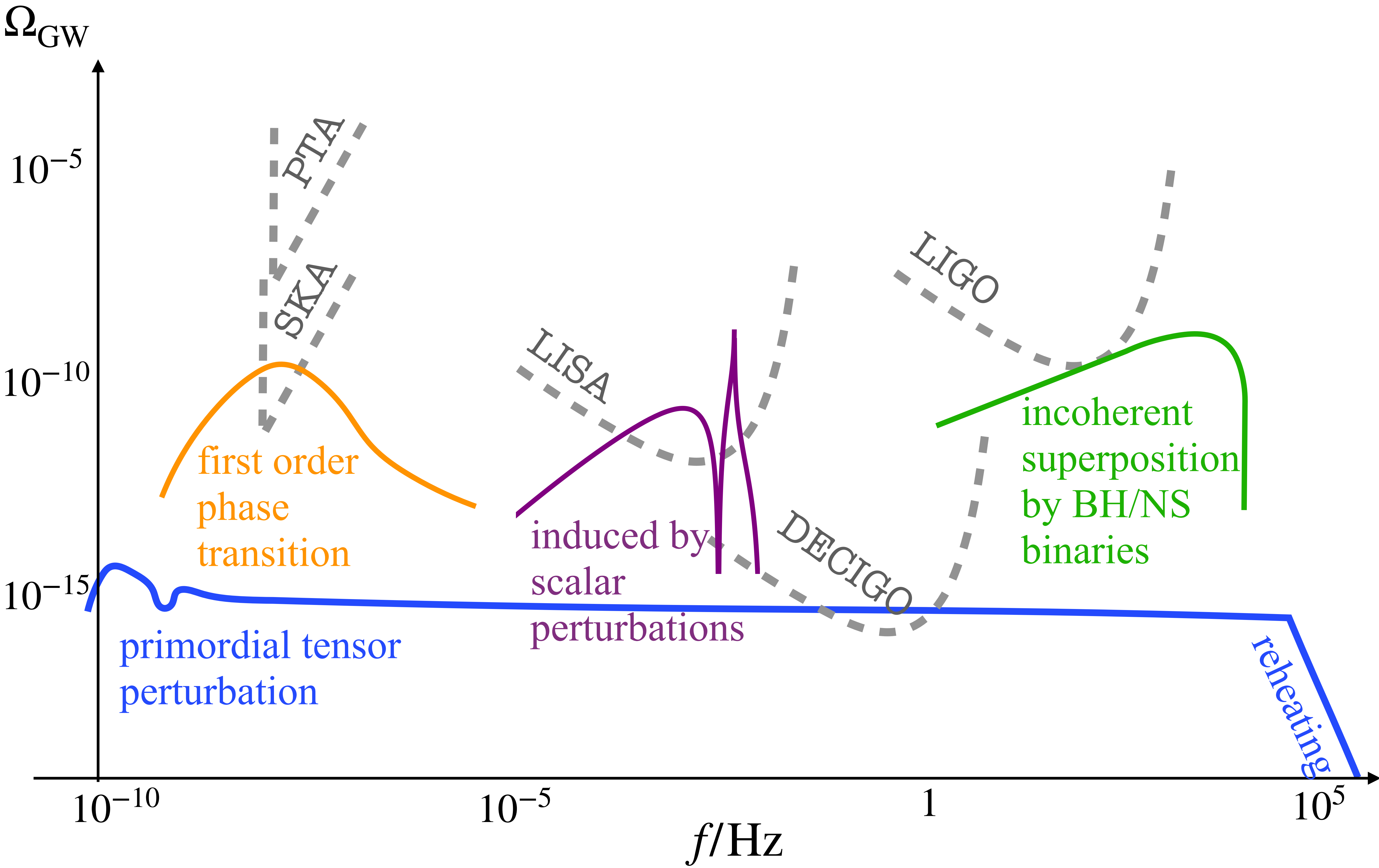
2019 YITP Asian-Pacific Winter School and Workshop on Gravitation and Cosmology

Content

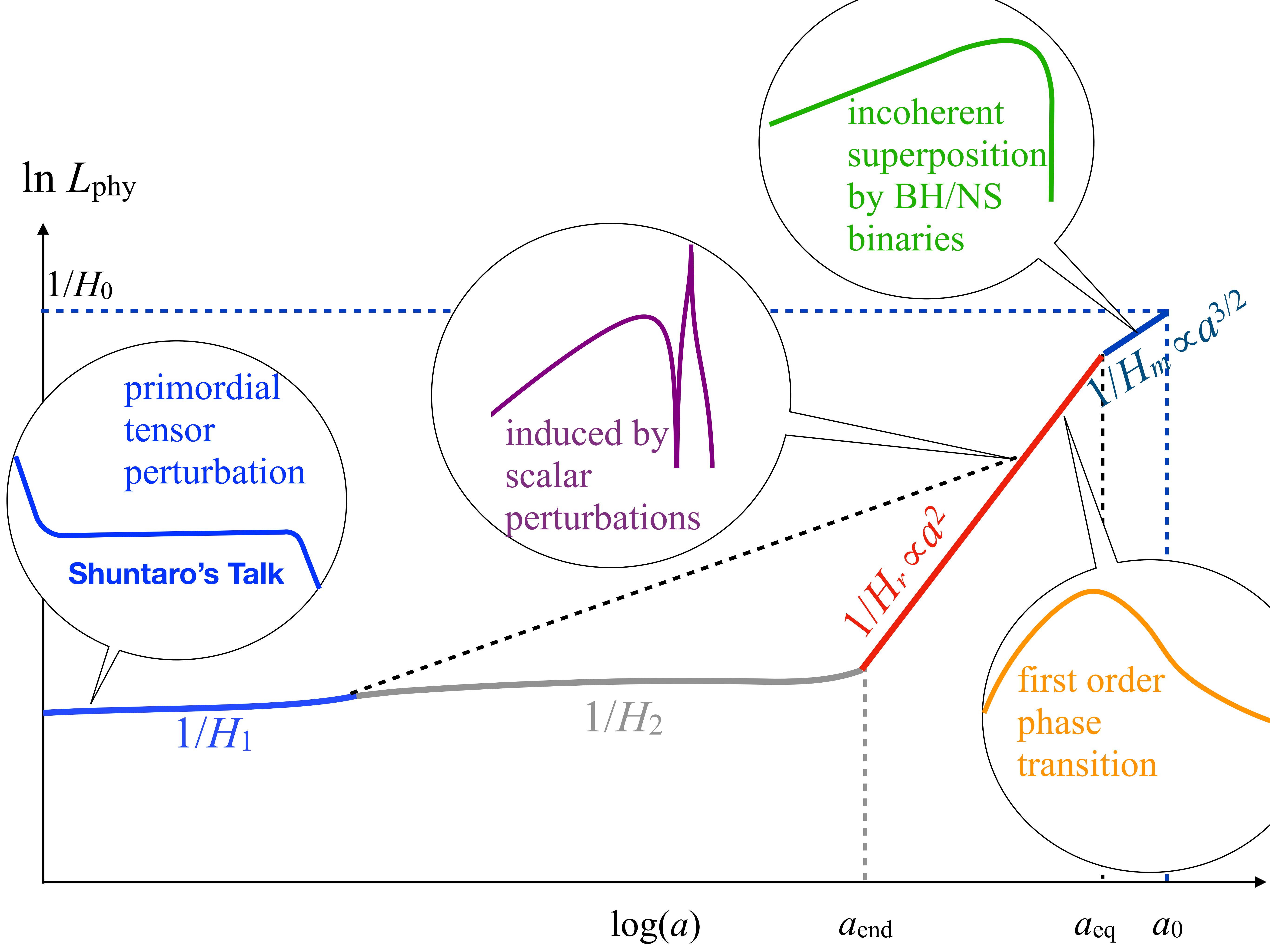
- Introduction to stochastic background of GWs
- PBH abundances
- Induced GWs: A probe for non-Gaussianity
- Conclusion



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
1602.03837....
LIGO
10^{-10}
10^{-5}
1
10^{5}
f/Hz



See for instance 1807.00786

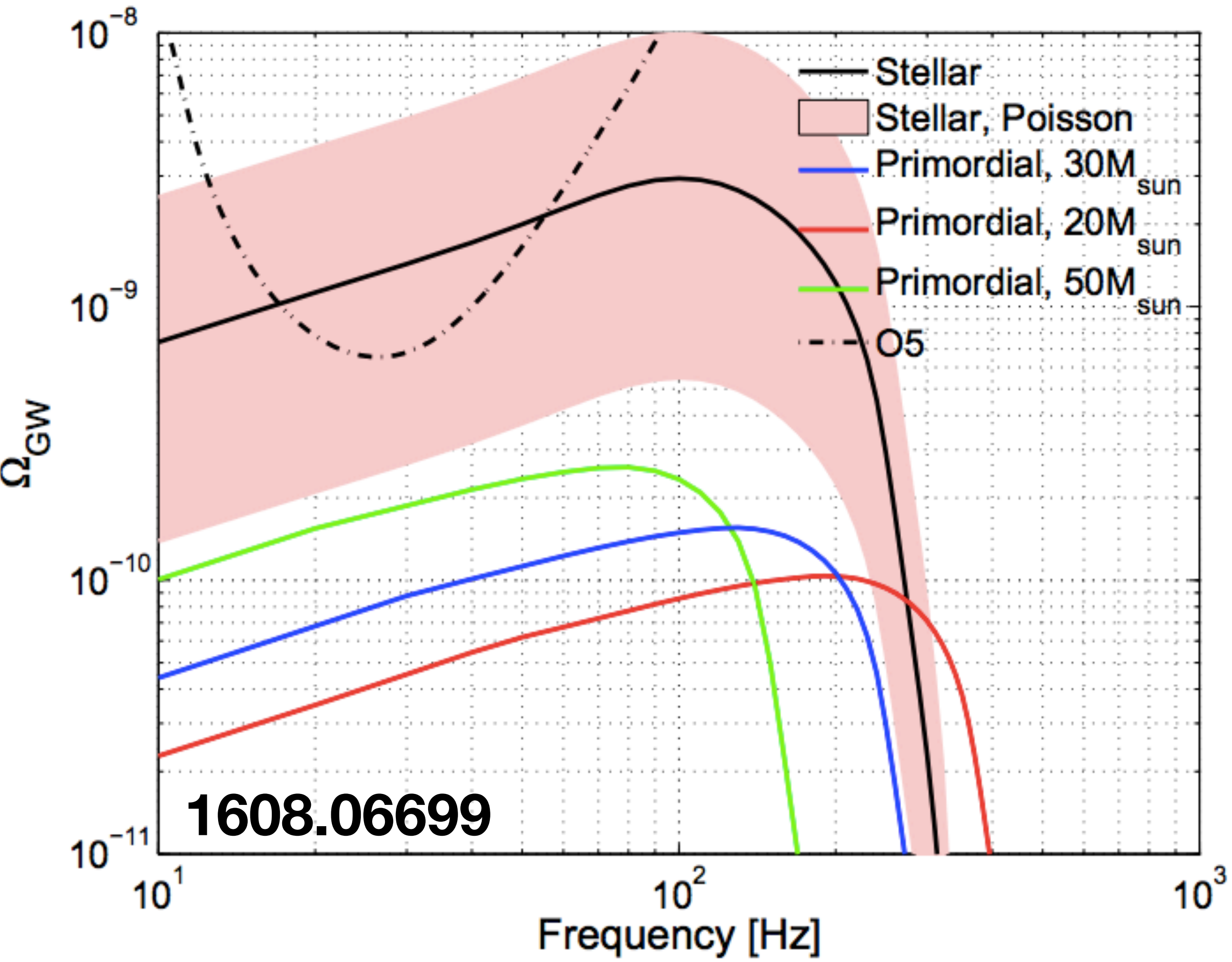


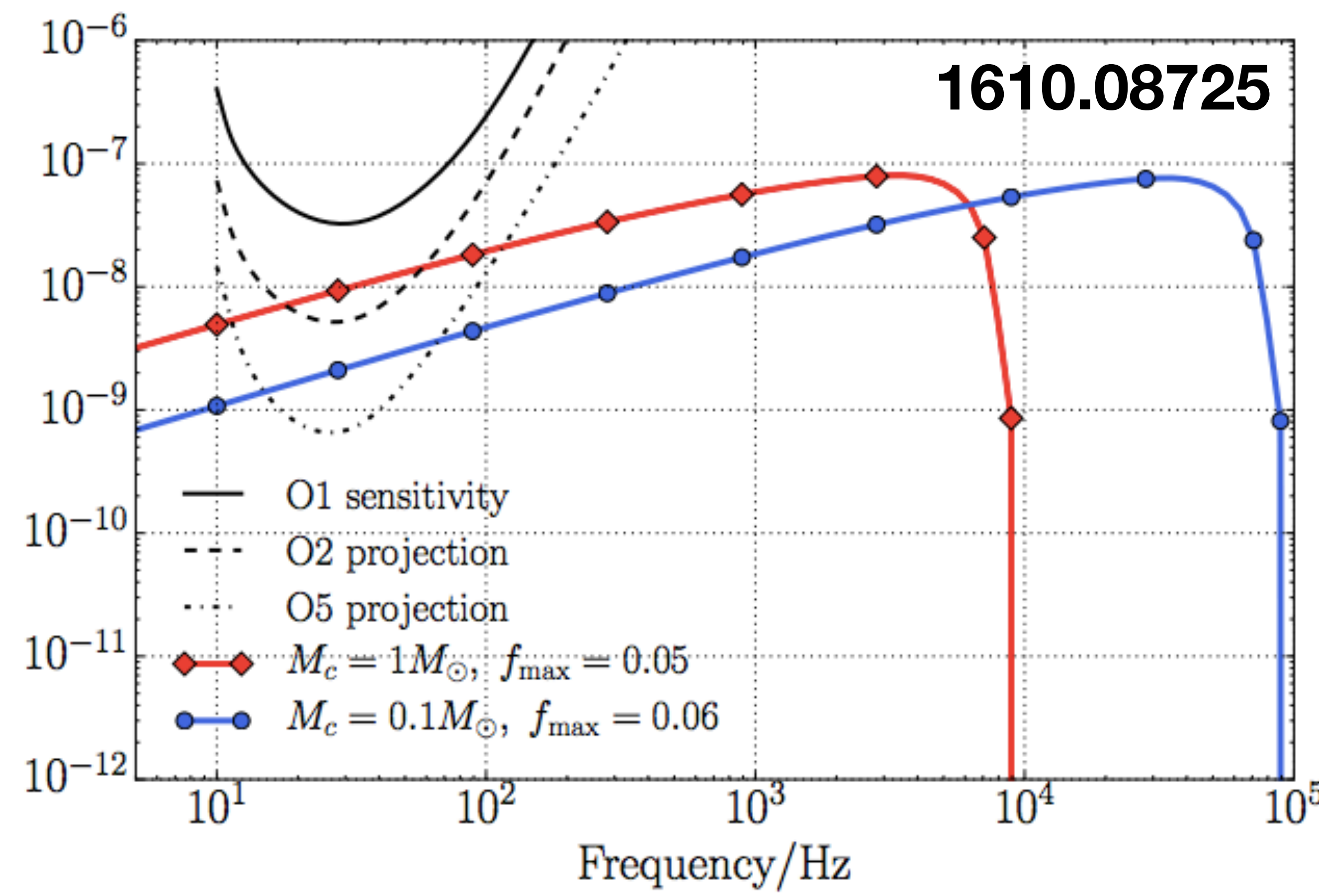
$\ln L_{\rm phy}$
$1/H_0$
primordial tensor perturbation
Shuntaro's Talk
induced by scalar perturbations
incoherent superposition by BH/NS binaries
first order phase transition
$1/H_m \propto a^{3/2}$
$1/H_r \propto a^2$
$1/H_1$
$1/H_2$
$\log(a)$
$a_{\rm end}$
$a_{\rm eq}$
a_0

SGWB from binaries

- Origin: incoherent superposition of the GWs emitted by compact star binarys (BH, NS,…)

- Frequencies: 100 Hz(for 10M$_\odot$) , Amplitude: 10^{-9}





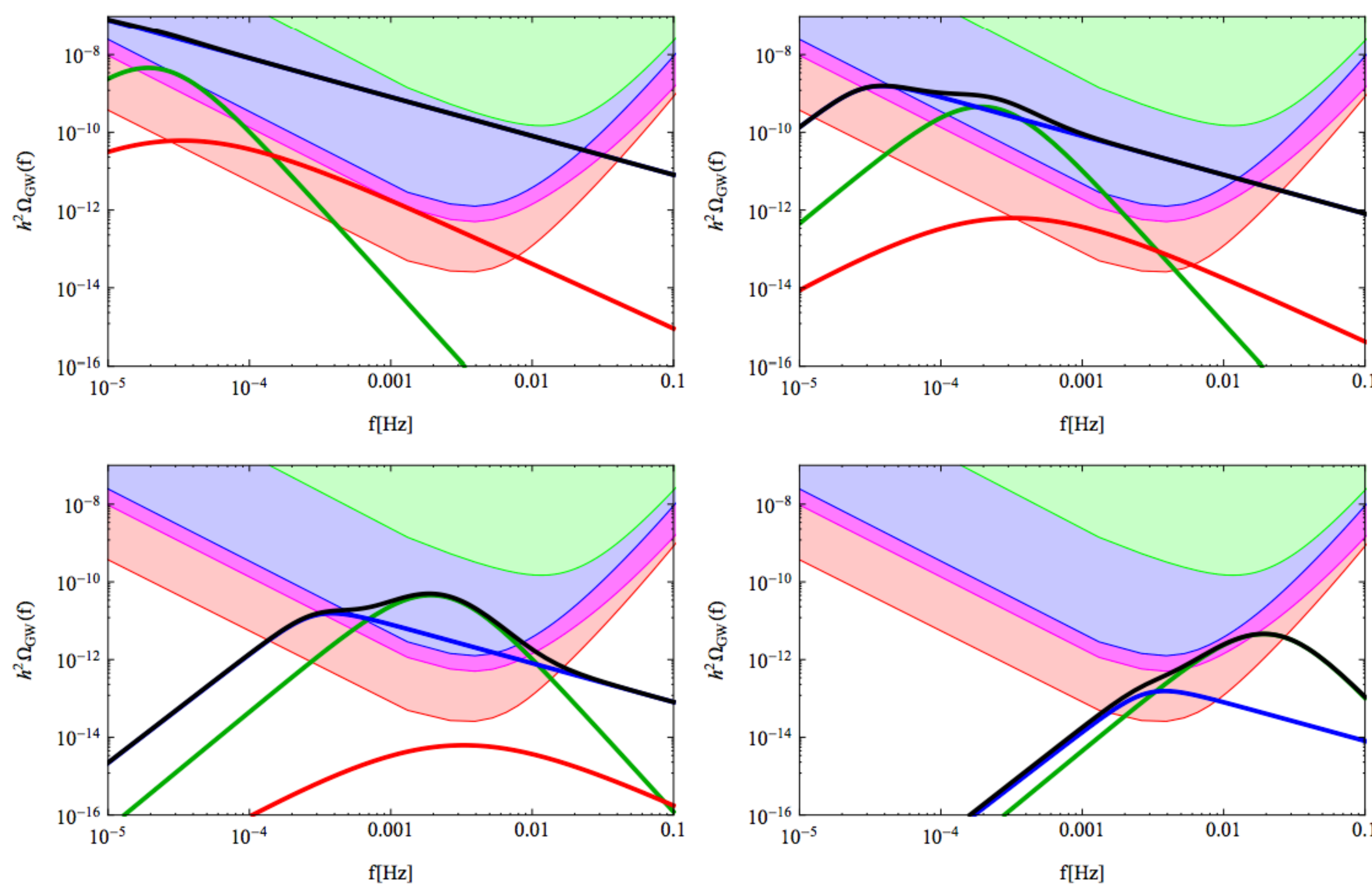
SGWB from 1OPT

$$f_{\rm peak} \simeq 10^{-6}{\rm Hz}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

1512.06239

$$\Omega_{\rm peak}h^2 \simeq 10^{-6}\left(\frac{\beta}{H_*}\right)^{-2}\left(\frac{g_*}{100}\right)^{-1/3}$$

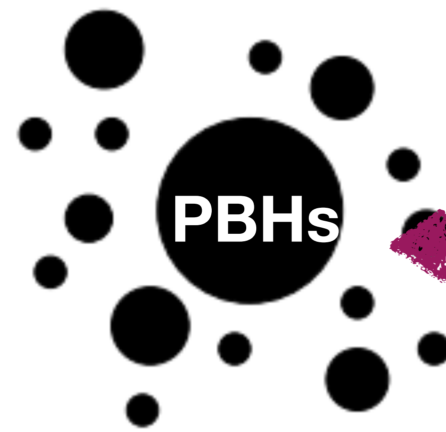
- For β/H*~100, frequency is 10^{-4}Hz, in LISA band, but the peak is only 10^{-10}.

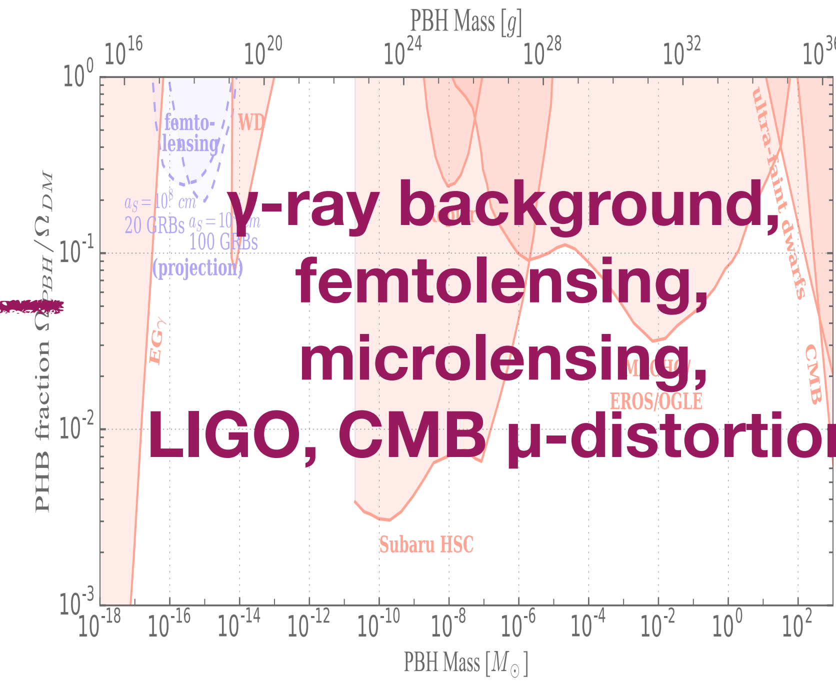


Induced GWs

Peak of scalar perturbation on small scales

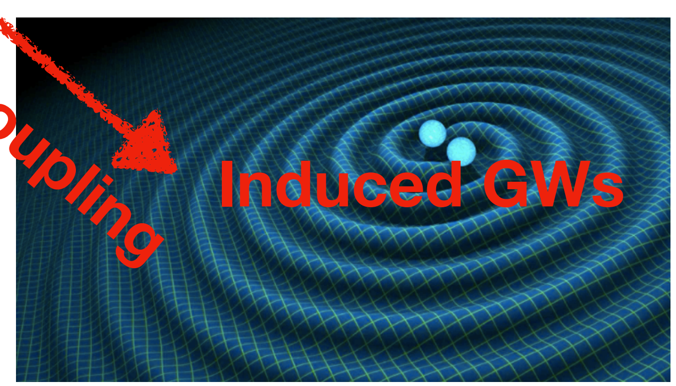
Peak Theory





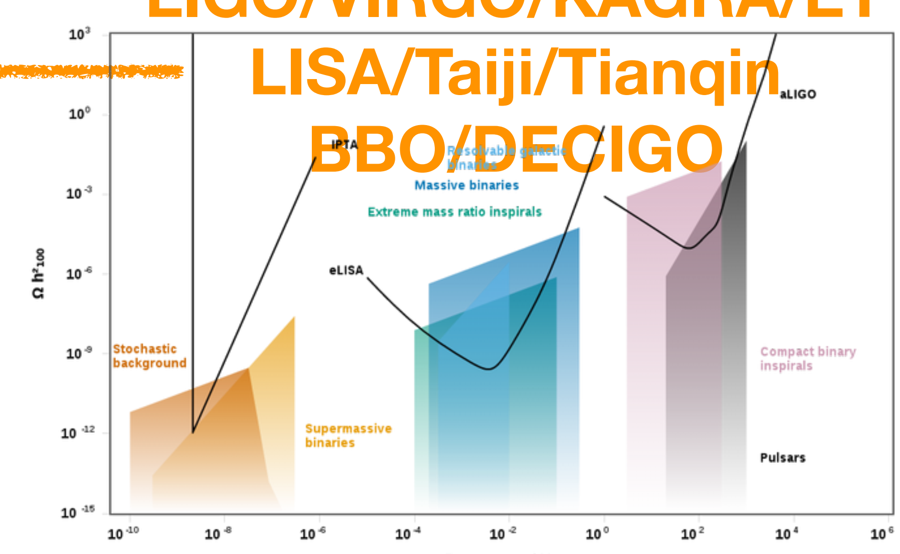
γ-ray background, femtolensing, microlensing, LIGO, CMB μ-distortion

secondary coupling



Induced GWs

LIGO/VIRGO/KAGRA/ET
LISA/Taiji/Tianqin
BBO/DECIGO

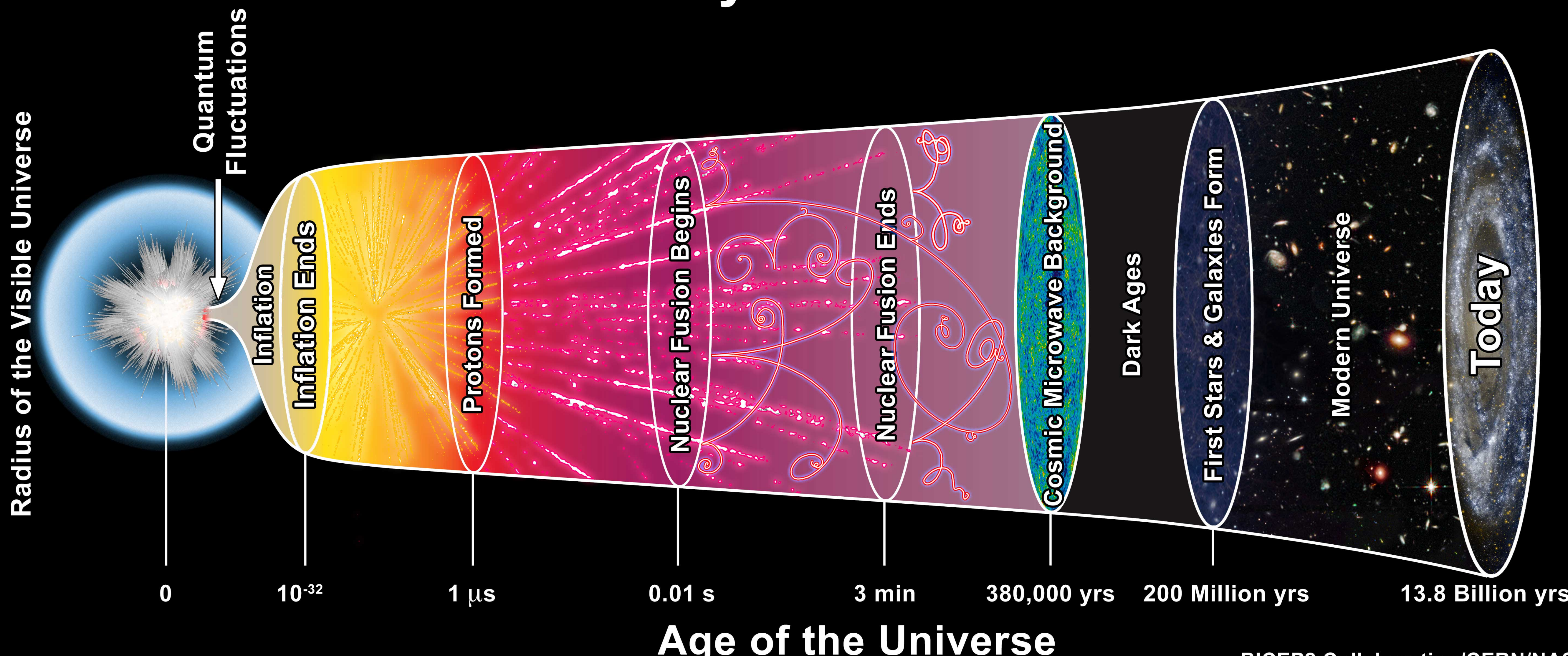


Saito & Yokoyama, 0812.4339

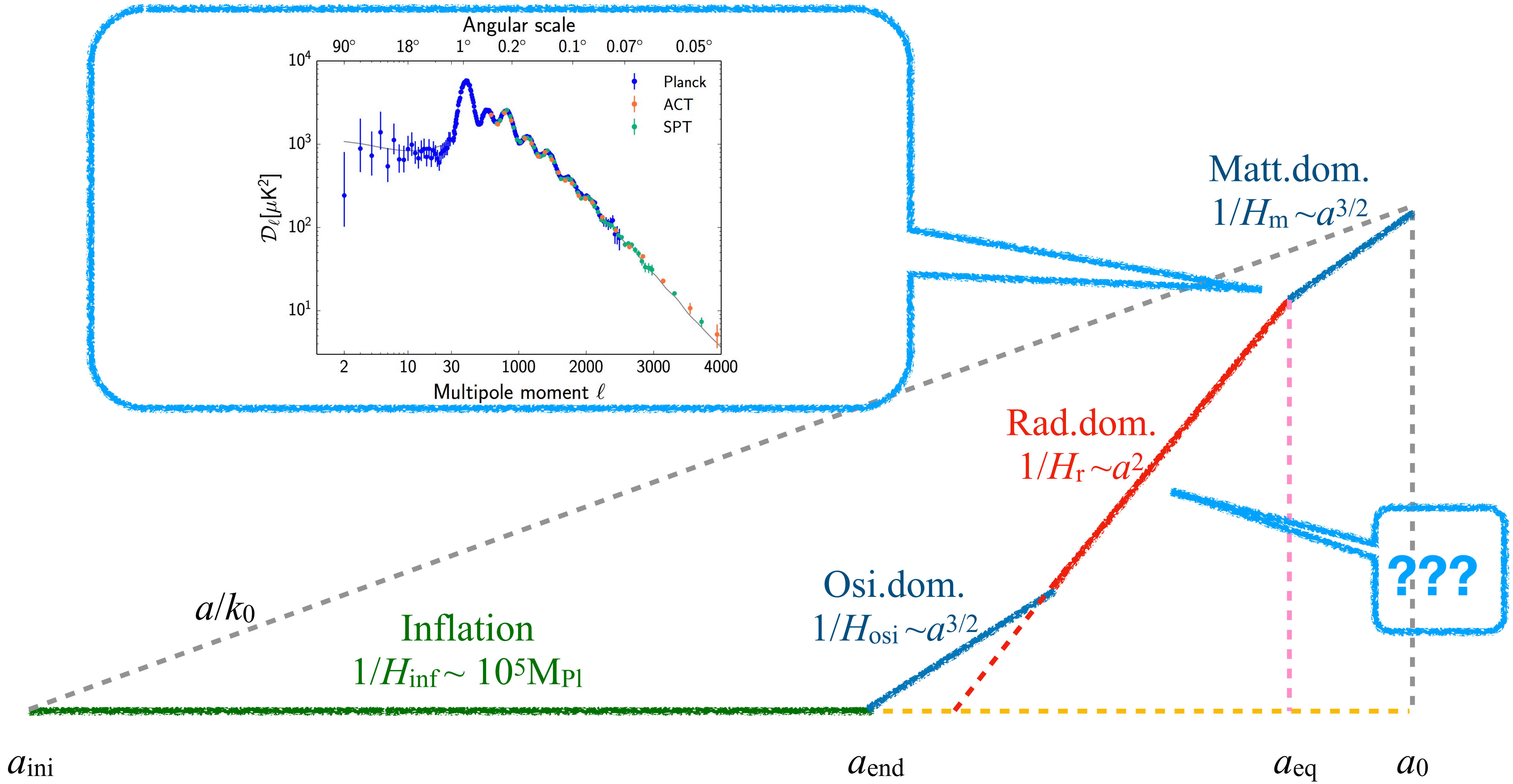
Content

- Mechanism of SGWB
- PBH abundances
- Induced GWs: A probe for non-Gaussianity
- Conclusion

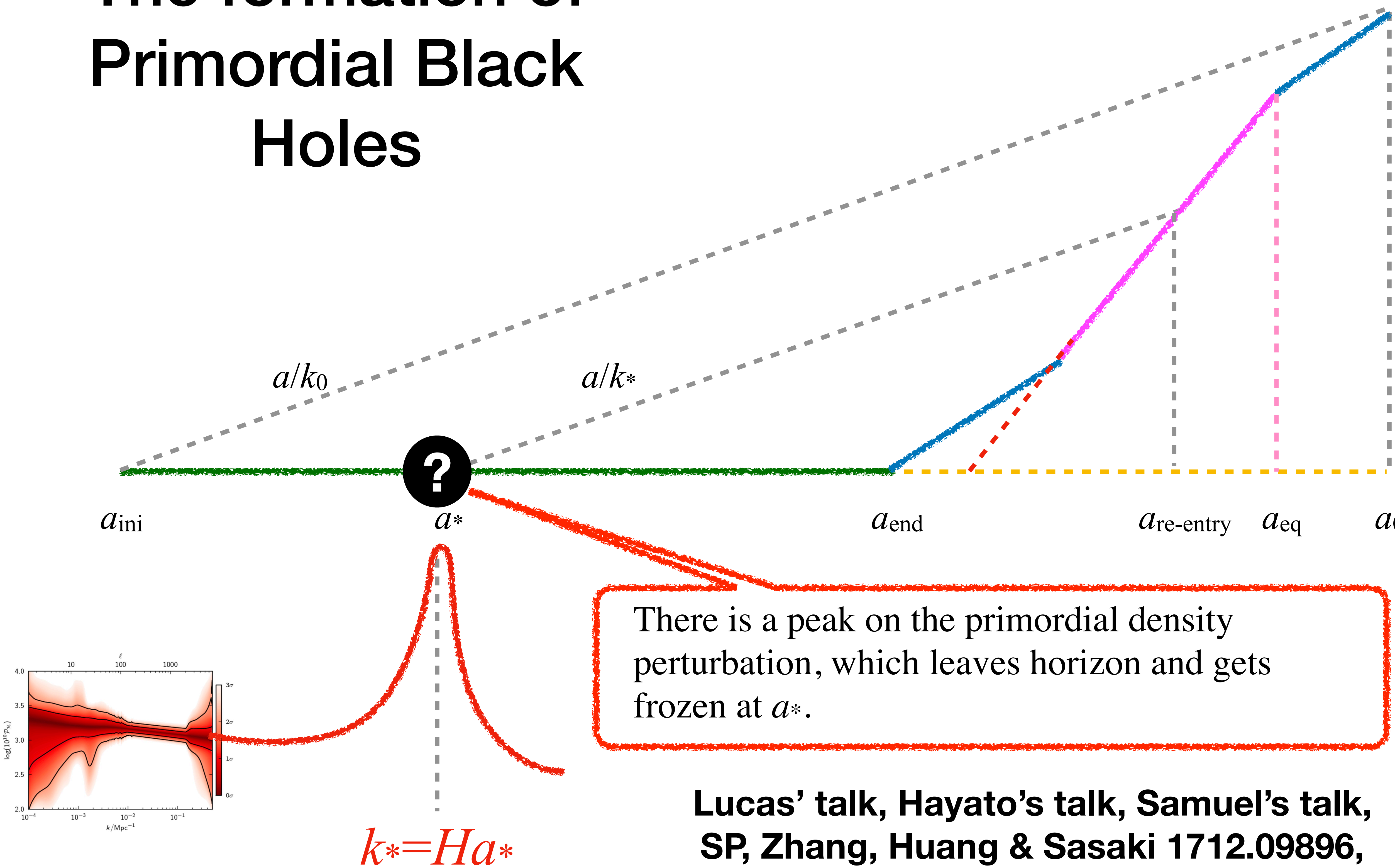
History of the Universe



BICEP2 Collaboration/CERN/NASA

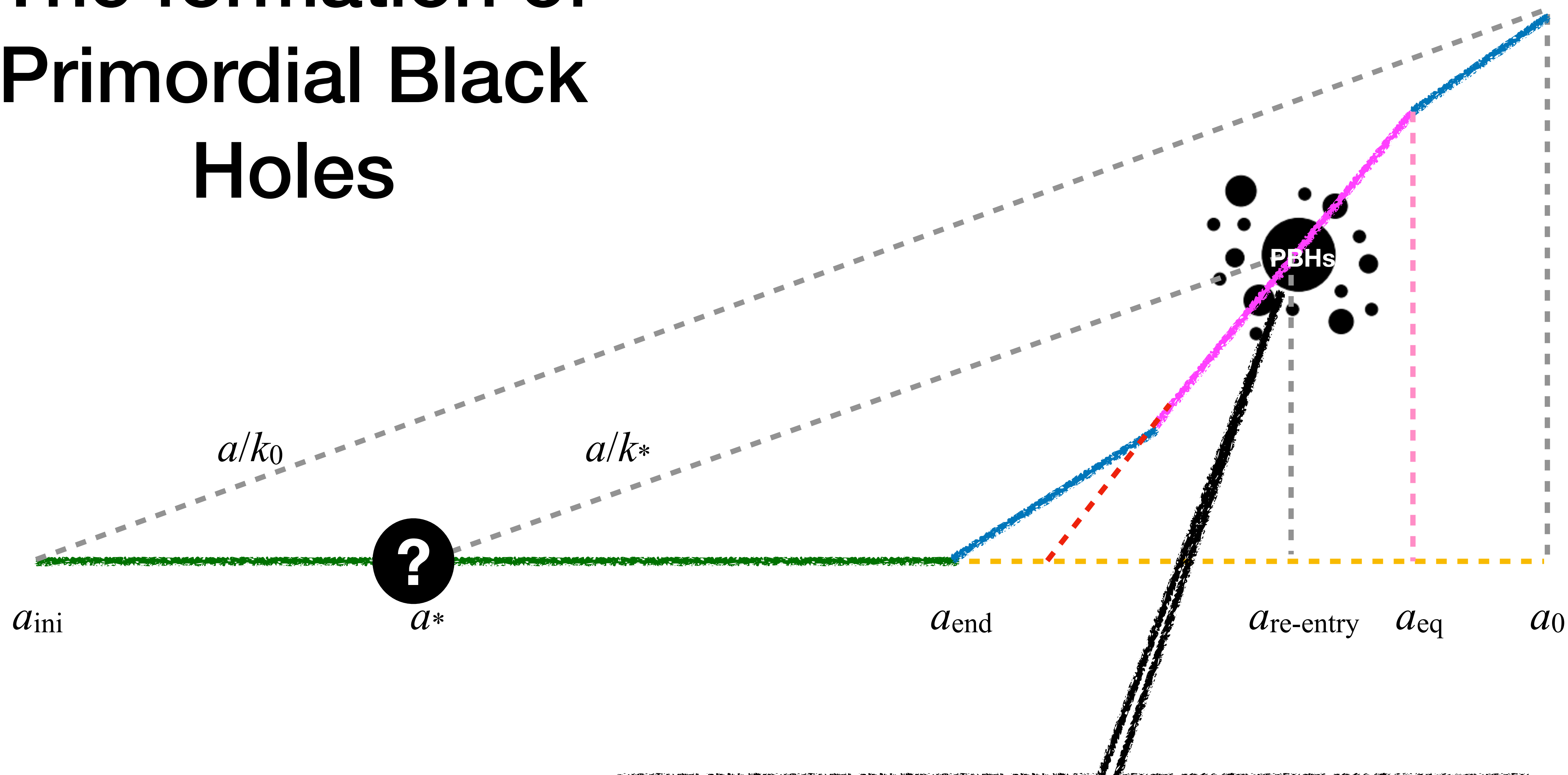

Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
10^4 10^3 10^2 10^1
$\mathcal{D}_\ell[\mu K^2]$
Planck
ACT
SPT
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Matt.dom.
$1/H_{\rm m} \sim a^{3/2}$
Rad.dom.
$1/H_{\rm r} \sim a^2$
???
Osi.dom.
$1/H_{\rm osi} \sim a^{3/2}$
a/k_0
Inflation
$1/H_{\rm inf} \sim 10^5 M_{\rm Pl}$
$a_{\rm ini}$
$a_{\rm end}$
$a_{\rm eq}$
a_0

The formation of Primordial Black Holes



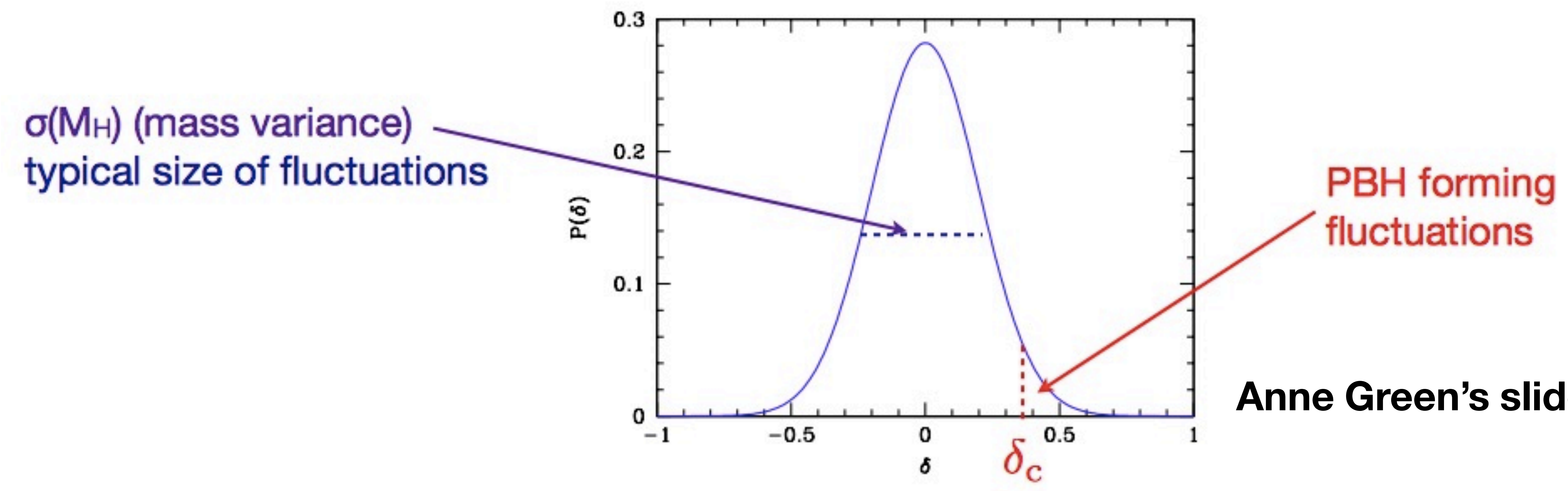
Lucas' talk, Hayato's talk, Samuel's talk, SP, Zhang, Huang & Sasaki 1712.09896, and many other models…

The formation of Primordial Black Holes



The peak scale re-enters the horizon at radiation dominated era. If it exceeded some critical value O(0.1), PBH will form. Its mass is O(M_H).

The Press-Schechter Mass Function

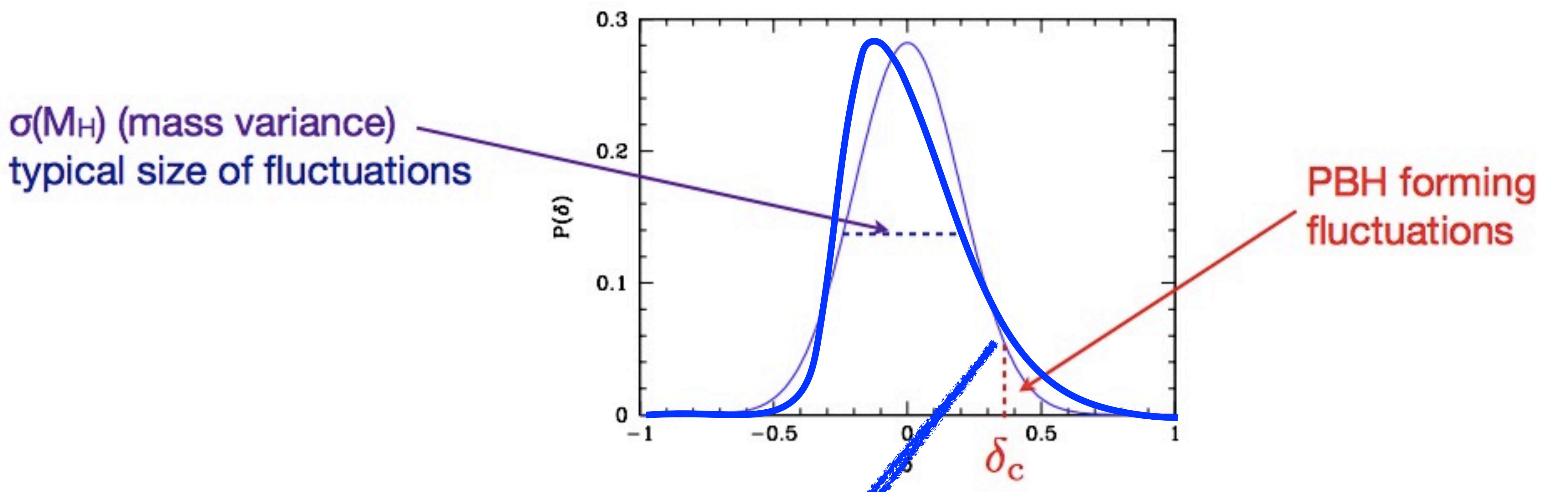


Anne Green's slides

- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}}\frac{\sigma_M}{\delta_c}\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

The Press-Schechter Mass Function



Non-Gaussianity can increase ($f_{NL}>0$) or decrease ($f_{NL}<0$) the PBH adundances.

Young & Byrnes, 1307.4995

The Press-Schechter Mass Function

- The current PBH mass measured in critical mass is

$$\Omega_{\rm PBH} = \beta \frac{a_{\rm eq}}{a_{\rm re}} = \beta \frac{a_{\rm eq}}{a_0} \frac{a_0}{a_{\rm re}} \simeq \beta \Omega_r (1 + z_{\rm re}(M))$$

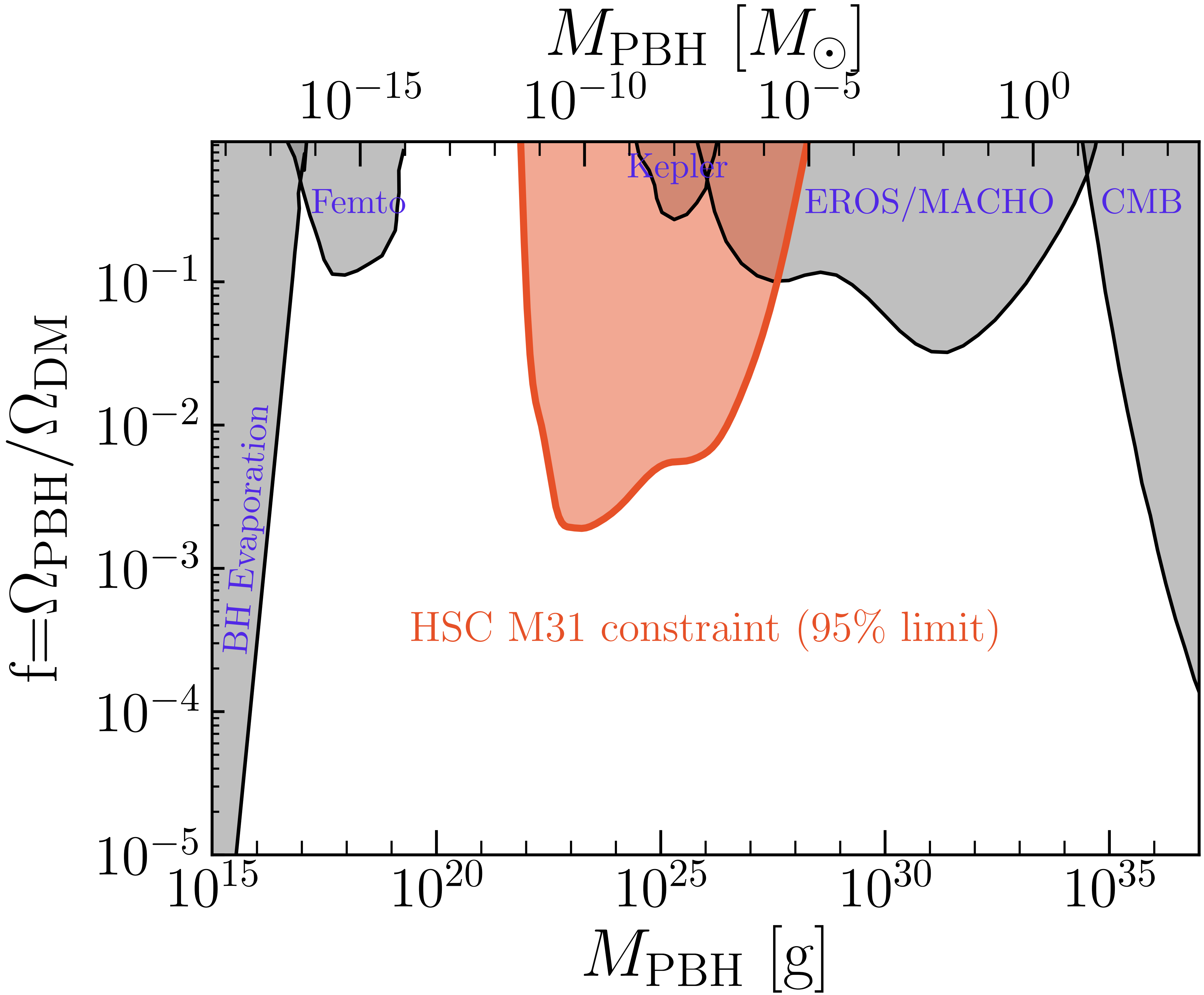
- where "eq" means equality and "re" means re-entry for the peak of the variance of the density perturbation at mass M.

- It is easy to estimate $z(M)$ relation at horizon reentry

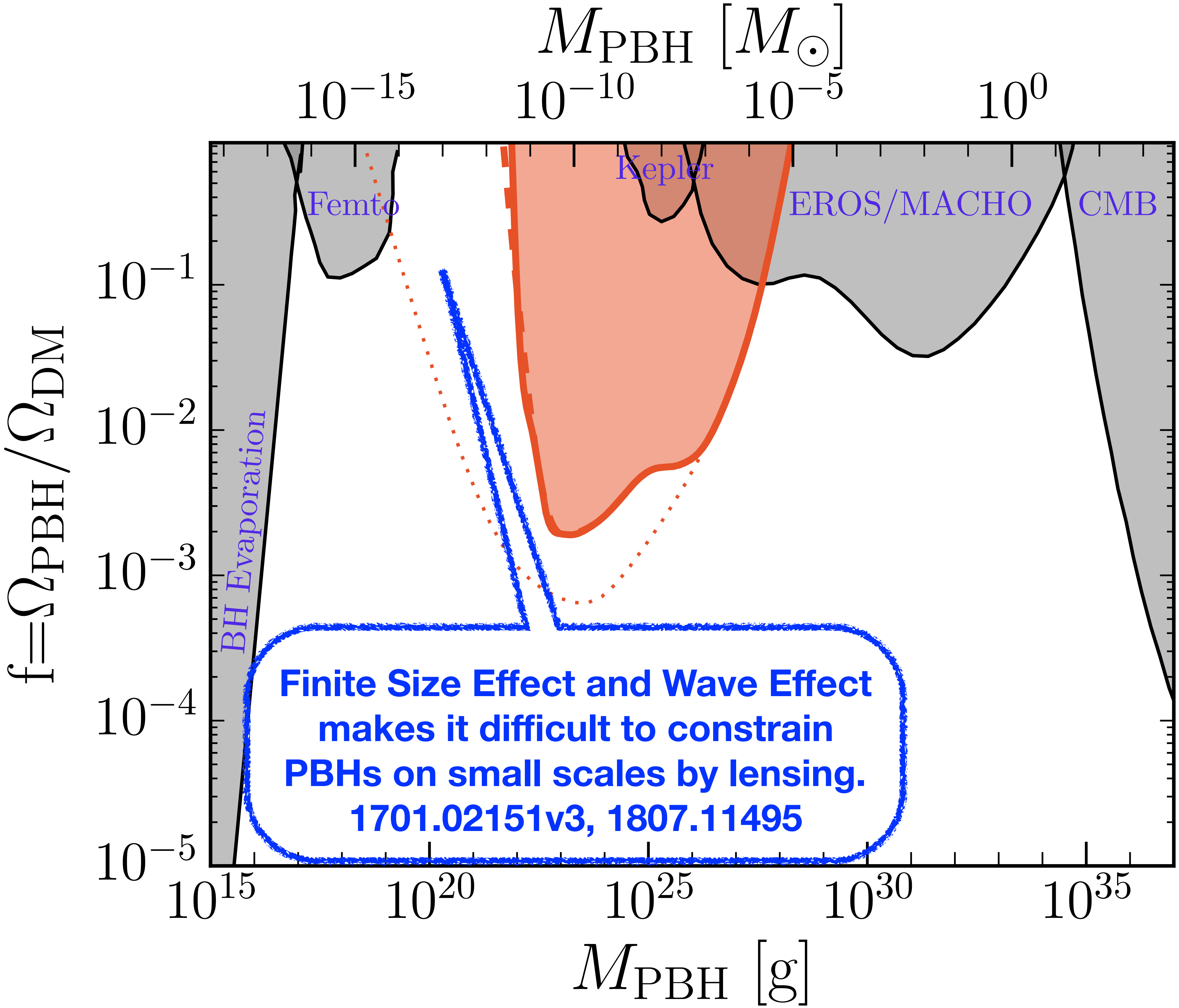
$$M = \frac{c^3}{G H_{\rm re}} = \frac{c^3}{G \Omega_r^{1/2} (1+z)^2 H_0}$$

- Therefore we have

$$f \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm CDM}} \approx 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot} \right)^{-1/2}$$



Niikura et al., 1701.02151v3



Niikura et al., 1701.02151v3

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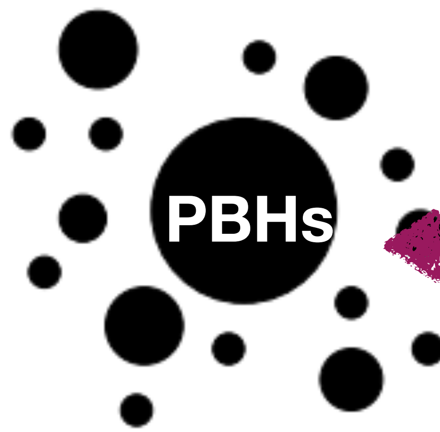
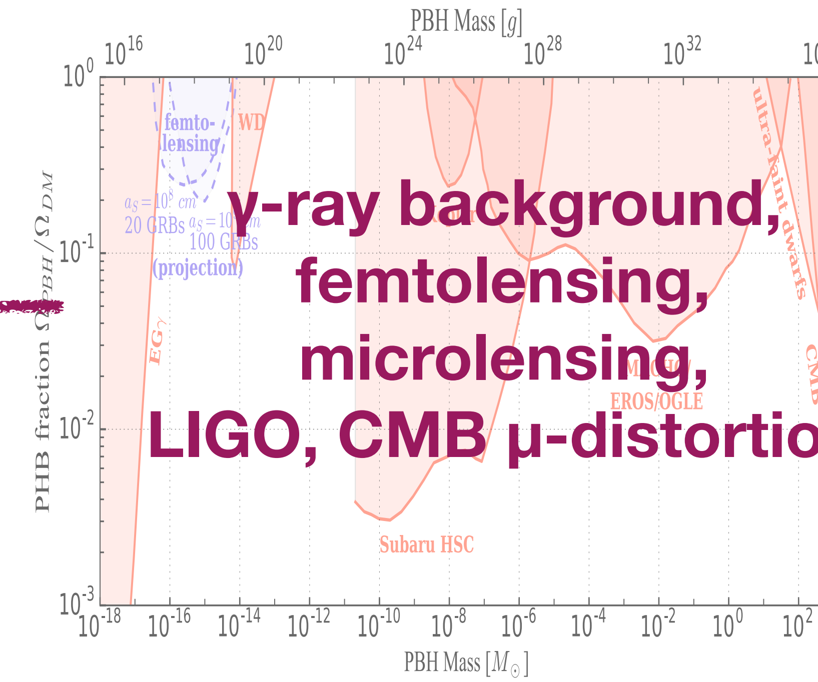
Induced GWs



Peak Theory



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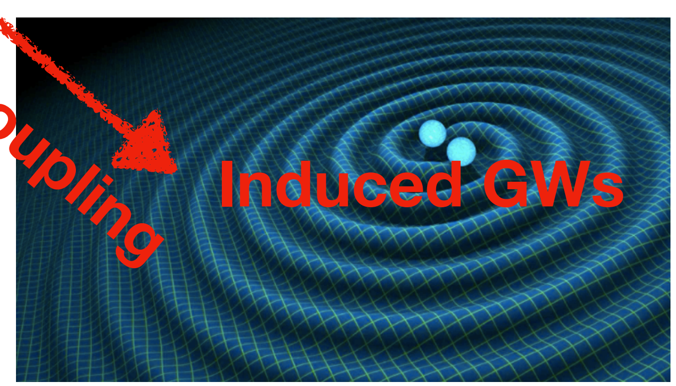
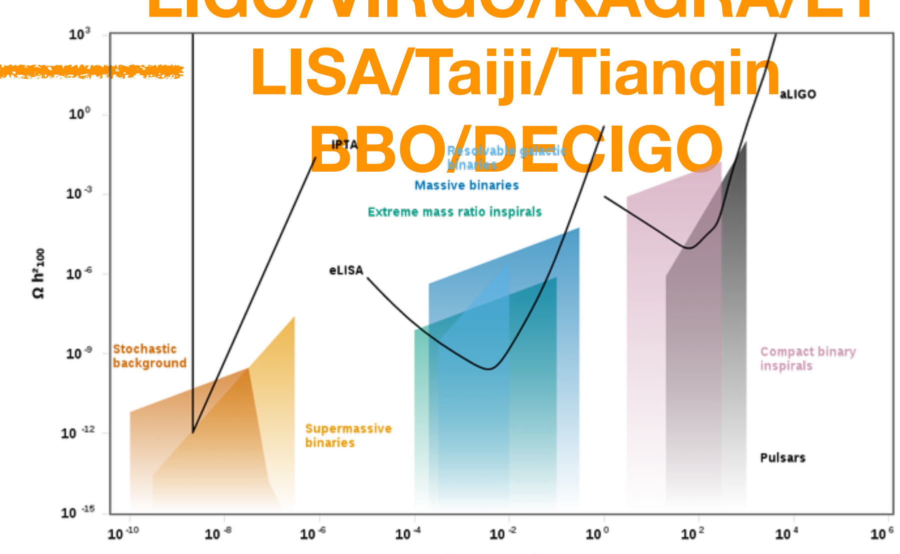


Induced GWs



LIGO/VIRGO/KAGRA/ET
LISA/Taiji/Tianqin
BBO/DECIGO

Saito & Yokoyama, 0812.4339

Induced GWs

- From the nonlinear equation of motion for the tensor perturbation

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k},\eta) \sim \int d^3l \; l_i l_j \Phi_{\mathbf{l}}(\eta)\Phi_{\mathbf{k}-\mathbf{l}}(\eta)$$

- where the source term is (Ananda et al. gr-qc/0612013)

$$\mathcal{S}(\mathbf{k},\eta) = 36 \int \frac{d^3l}{(2\pi)^{3/2}} \frac{l^2}{\sqrt{2}} \sin^2\theta \begin{pmatrix} \cos 2\varphi \\ \sin 2\varphi \end{pmatrix} \Phi_{\mathbf{l}}\Phi_{\mathbf{k}-\mathbf{l}} \times \left[j_0(ux)j_0(vx) - 2\frac{j_1(ux)j_0(vx)}{ux} - 2\frac{j_0(ux)j_1(vx)}{vx} + 3\frac{j_1(ux)j_1(vx)}{uvx^2} \right].$$

- This equation can be solved by the Green function method.

Induced GWs

- The quantity we want to calculate is

$$\Omega_{\rm GW}(k) \equiv \frac{1}{12}\left(\frac{k}{Ha}\right)^2 \frac{k^3}{\pi^2}\overline{\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}}(\eta)\rangle}.$$

- Then we know that $\Omega_{\rm GW} \sim \langle hh \rangle \sim \langle \mathcal{S}\mathcal{S} \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_\Phi^2$

- It is naive to believe that Φ stays Gaussian when it becomes very large on small scales. (Hayato's and Samuel's talks)

- Therefore we want to consider the non-Gaussian scalar induced GWs (Komatsu & Spergel astro-ph/0005036)

$$\mathscr{R}(\mathbf{x}) = \mathscr{R}_g(\mathbf{x}) + F_{\rm NL}\left[\mathscr{R}_g^2(\mathbf{x}) - \langle \mathscr{R}_g^2(\mathbf{x})\rangle\right].$$

Induced GWs

- Then the 2pt of Φ is

$$\langle \Phi_{\mathbf{k}} \Phi_{\mathbf{p}} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{p}) \frac{4}{9} \left(P_{\mathscr{R}}(k) + 2F_{\mathrm{NL}}^2 \int d^3 l \; P_{\mathscr{R}}(|\mathbf{k}-\mathbf{l}|) P_{\mathscr{R}}(l) \right).$$

- And we have to specify the power spectrum of the primordial curvature perturbation. As we mentioned, we suppose there is a narrow peak at around k*.

$$P_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{(2\pi)^{3/2} 2\sigma k_*^2} \exp\left(-\frac{(k-k_*)^2}{2\sigma^2} \right).$$

- Narrow means σ<<k*. This is for simplicity.

Induced GWs

- The result is the integral (Cai, SP & Sasaki, 1810.11000):

$$\Omega_{\rm GW} = 6\mathcal{A}_{\mathcal{R}}^2 \frac{k^2}{2\pi\sigma^2}\left(\frac{k}{k_*}\right)^4 \int_0^\infty dv \int_{|1-v|}^{1+v} du\, uv\, \mathcal{T}(u,v)$$

$$\times \left[e^{-\frac{(vk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}} F_{\rm NL}^2 \frac{\sigma}{vk}\sqrt{\frac{\pi}{2}}\,{\rm erf}\left(\frac{vk}{2\sigma}\right)\right]$$

$$\times \left[e^{-\frac{(uk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}} F_{\rm NL}^2 \frac{\sigma}{uk}\sqrt{\frac{\pi}{2}}\,{\rm erf}\left(\frac{uk}{2\sigma}\right)\right].$$

$$\mathcal{T}(u,v) = \frac{1}{4}\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 \left(\frac{u^2+v^2-3}{2uv}\right)^2$$

$$\times \left\{\left(-2+\frac{u^2+v^2-3}{2uv}\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2\right.$$

$$\left. + \pi^2\left(\frac{u^2+v^2-3}{2uv}\right)^2 \Theta\left(u+v-\sqrt{3}\right)\right\}.$$

Induced GWs

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Saito & Yokoyama, 0812.4339

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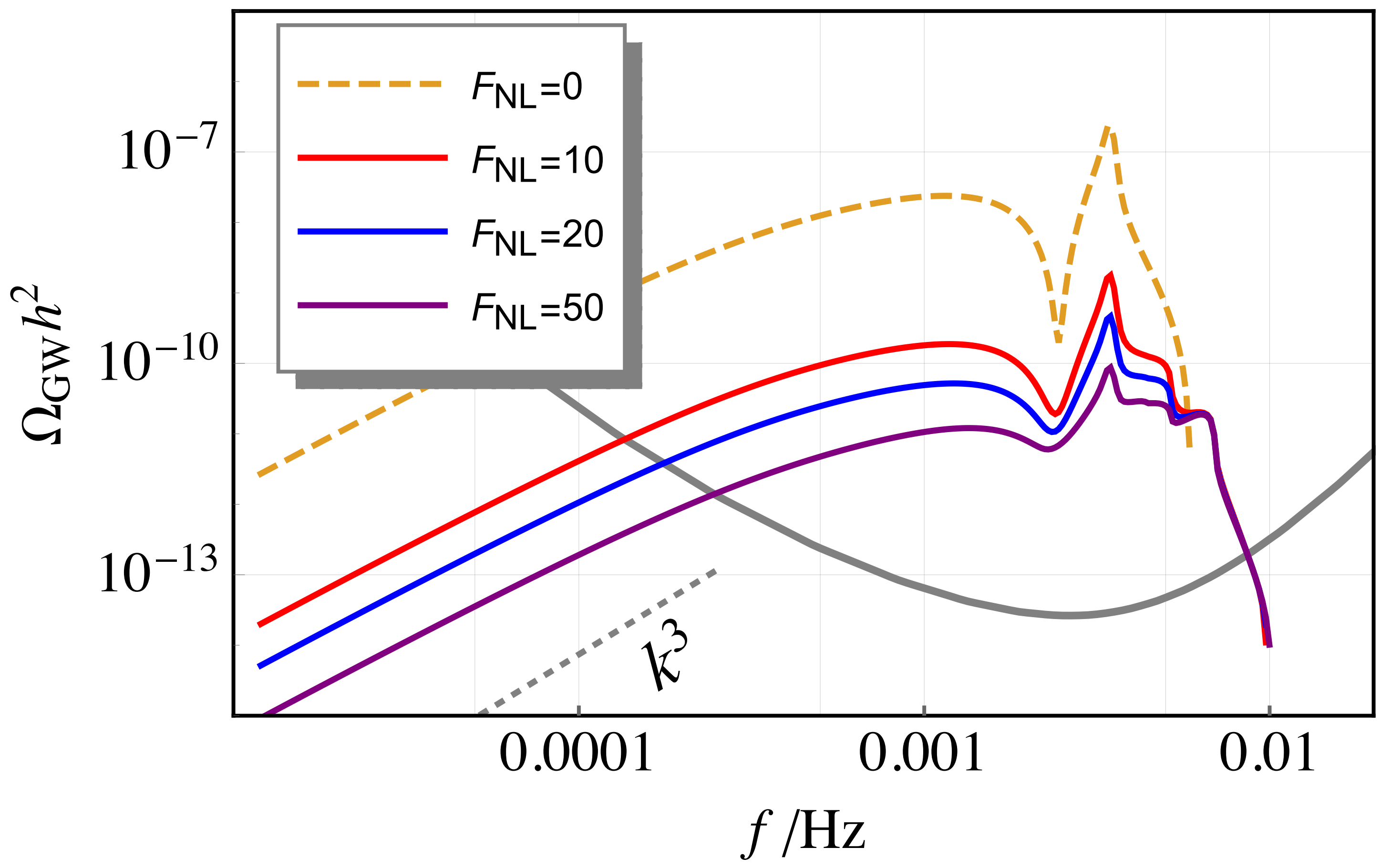
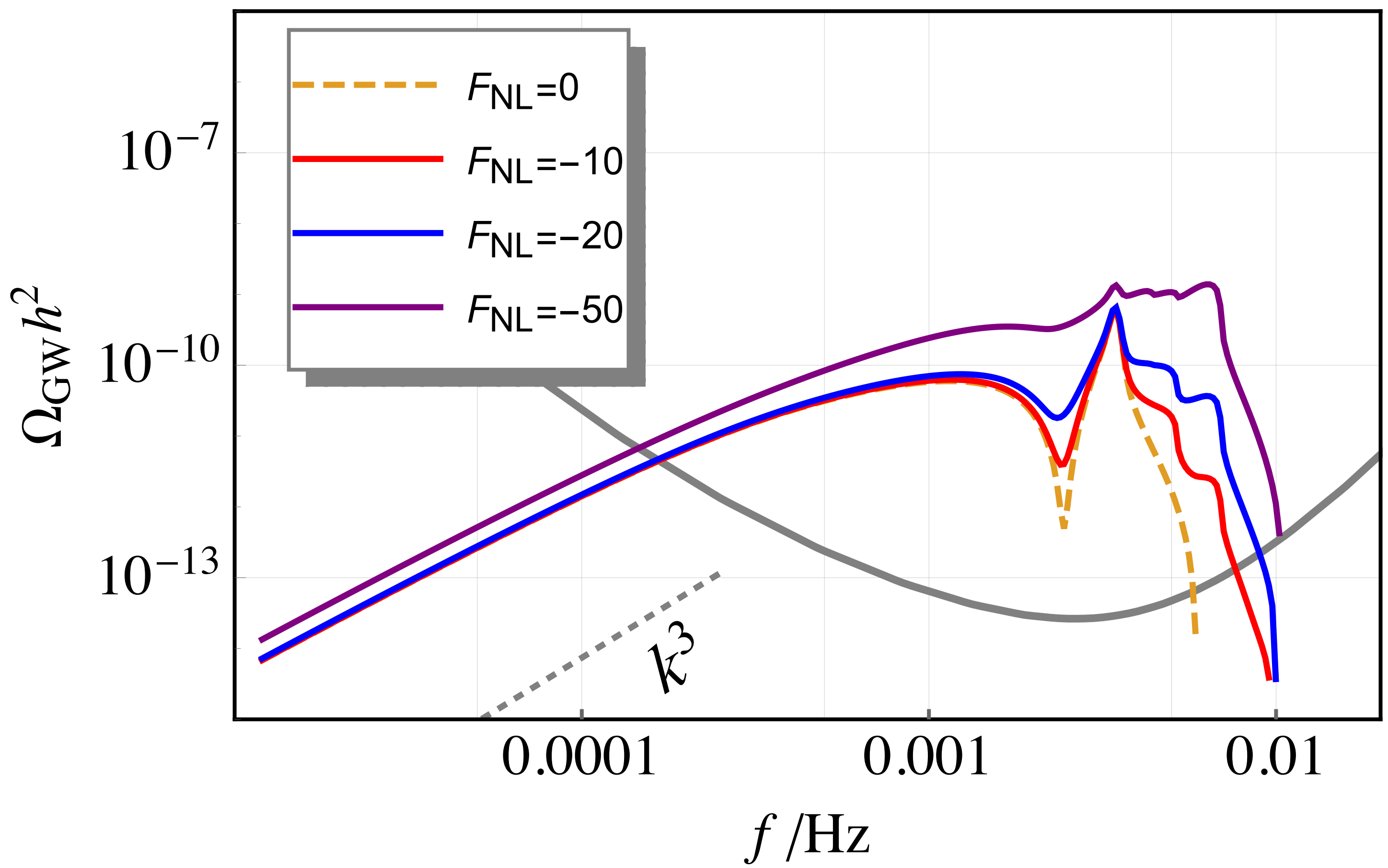
non-Gaussian contributions

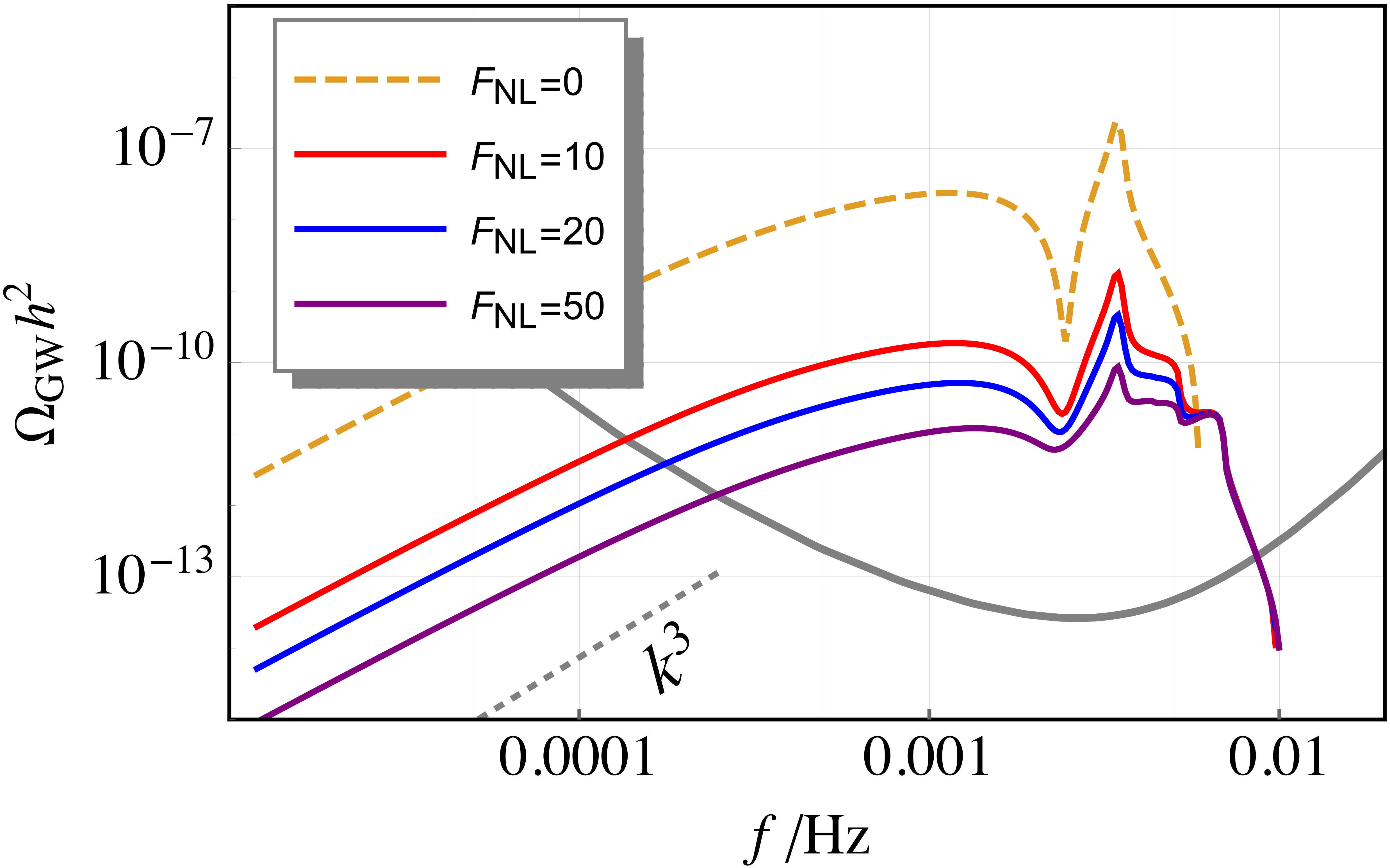
Saito & Yokoyama, 0812.4339

Kohri & Tareda, 1804.08577

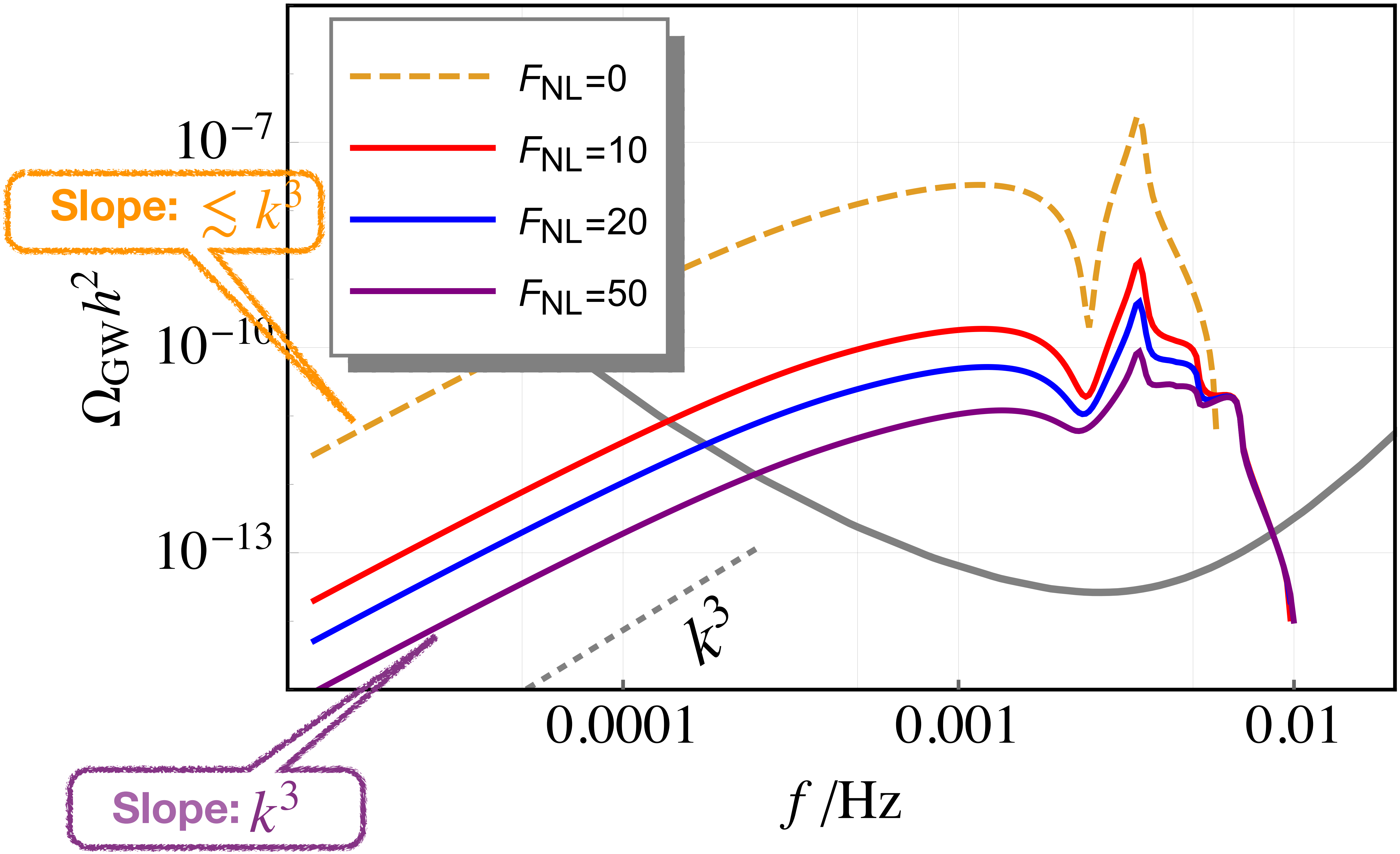
- Up: $F_{NL} > 0$, fix the PBH abundance to be 1.
- Down: $F_{NL} < 0$, fix the peak amplitude to be $\mathscr{A}_{\mathscr{R}} = 10^{-2}$
- Gray curve: LISA
- Frequency: PBH window <—> LISA band
- Coincidence, but fortunate for our universe.

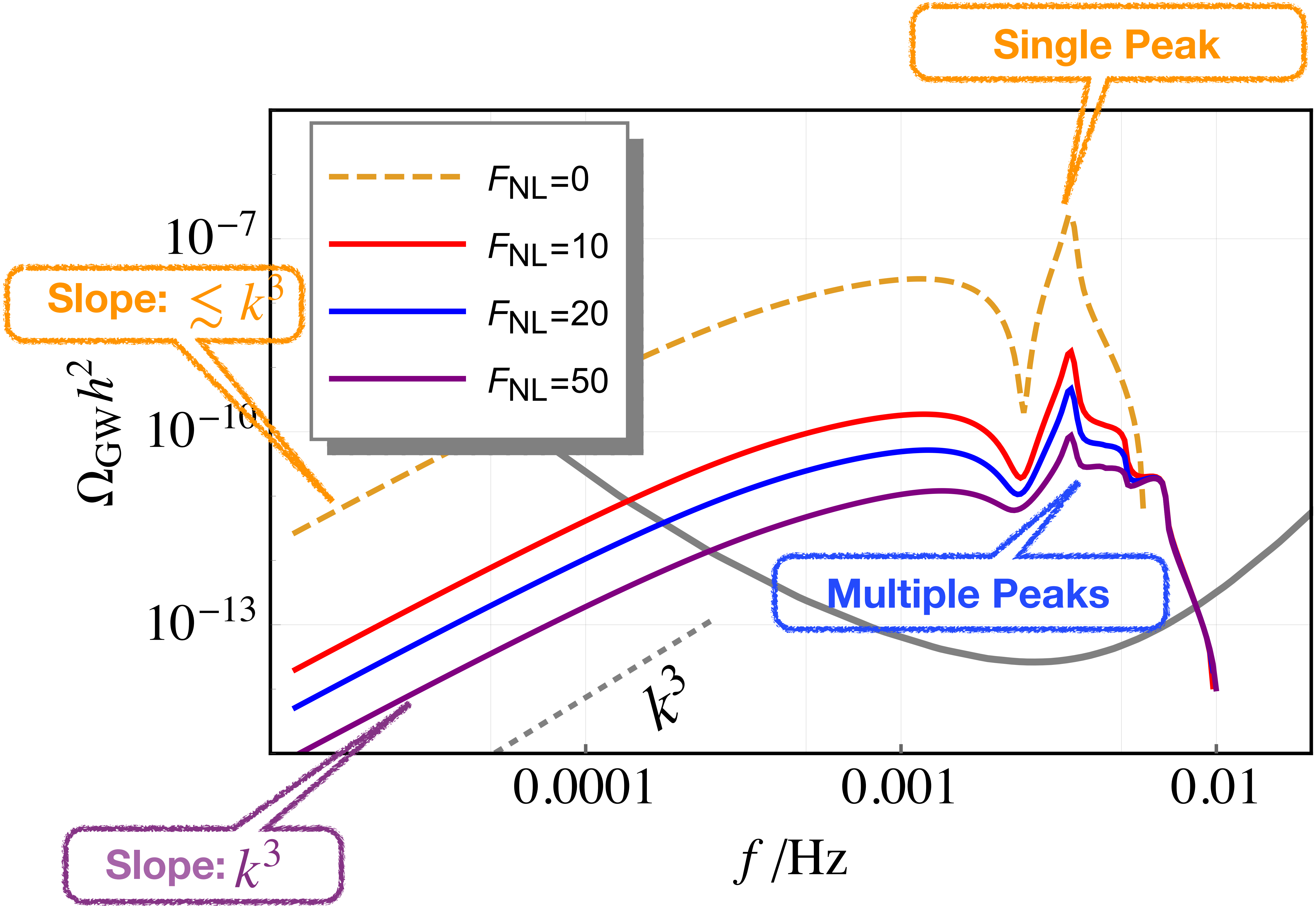




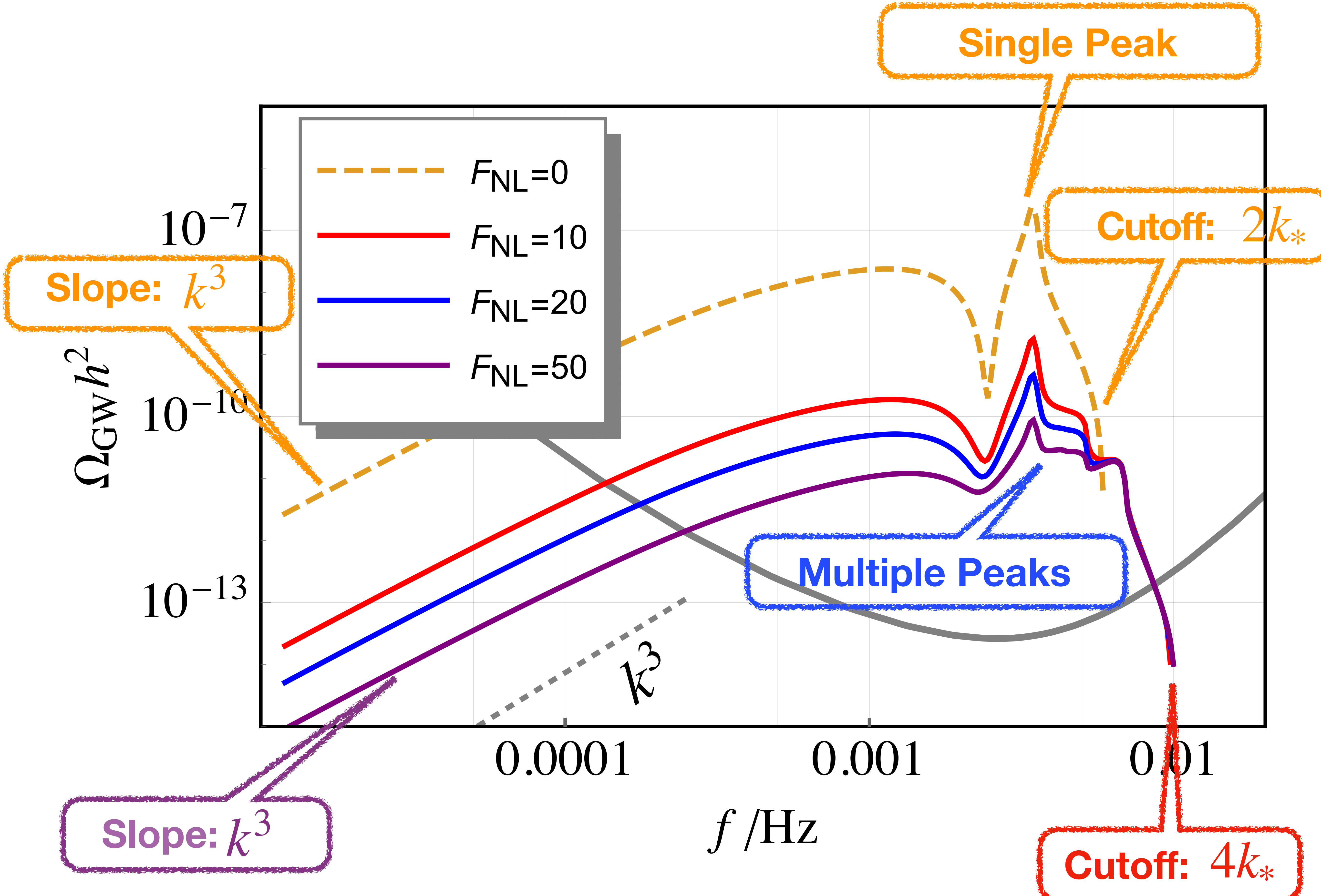


$F_{\rm NL}$=0
$F_{\rm NL}$=10
$F_{\rm NL}$=20
$F_{\rm NL}$=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{\rm GW} h^2$
k^3
0.0001
0.001
0.01
f /Hz

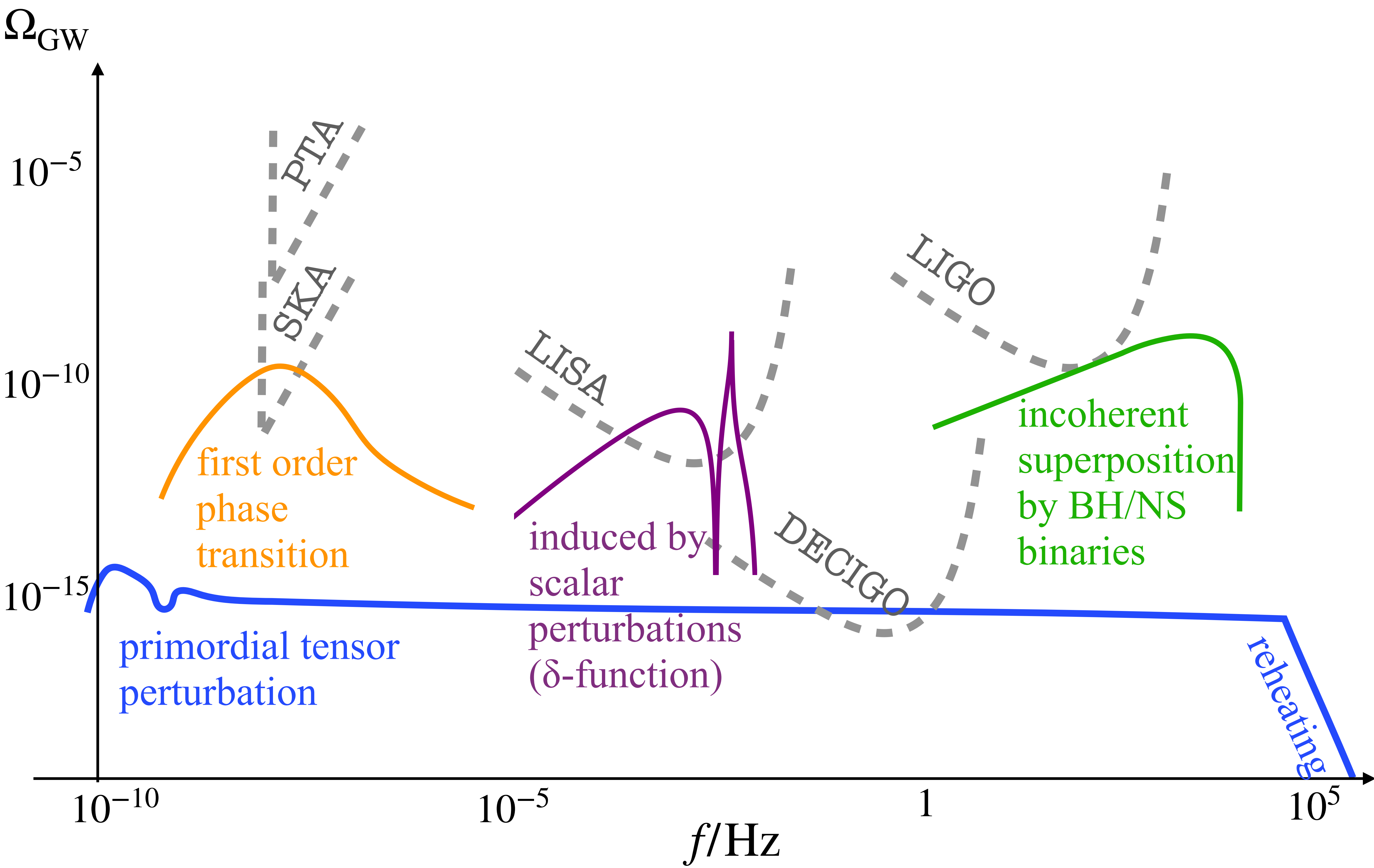

F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
Slope: $\lesssim k^3$
Slope: k^3
k^3
10^{-7}
10^{-10}
10^{-13}
0.0001
0.001
0.01
$\Omega_{GW}h^2$
f /Hz



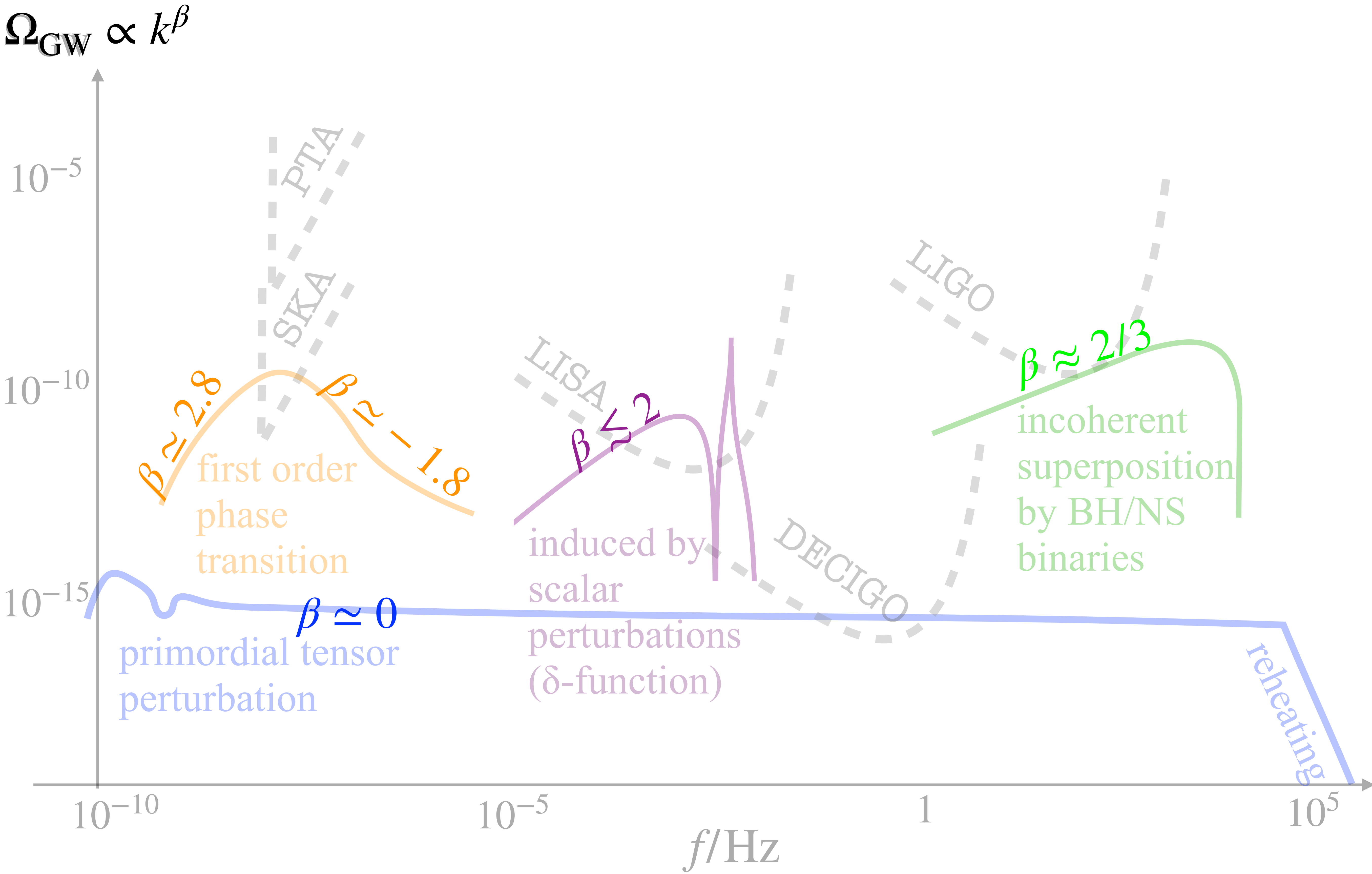
Single Peak
Slope: $\lesssim k^3$
F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{GW}h^2$
Multiple Peaks
k^3
0.0001
0.001
0.01
f /Hz
Slope: k^3



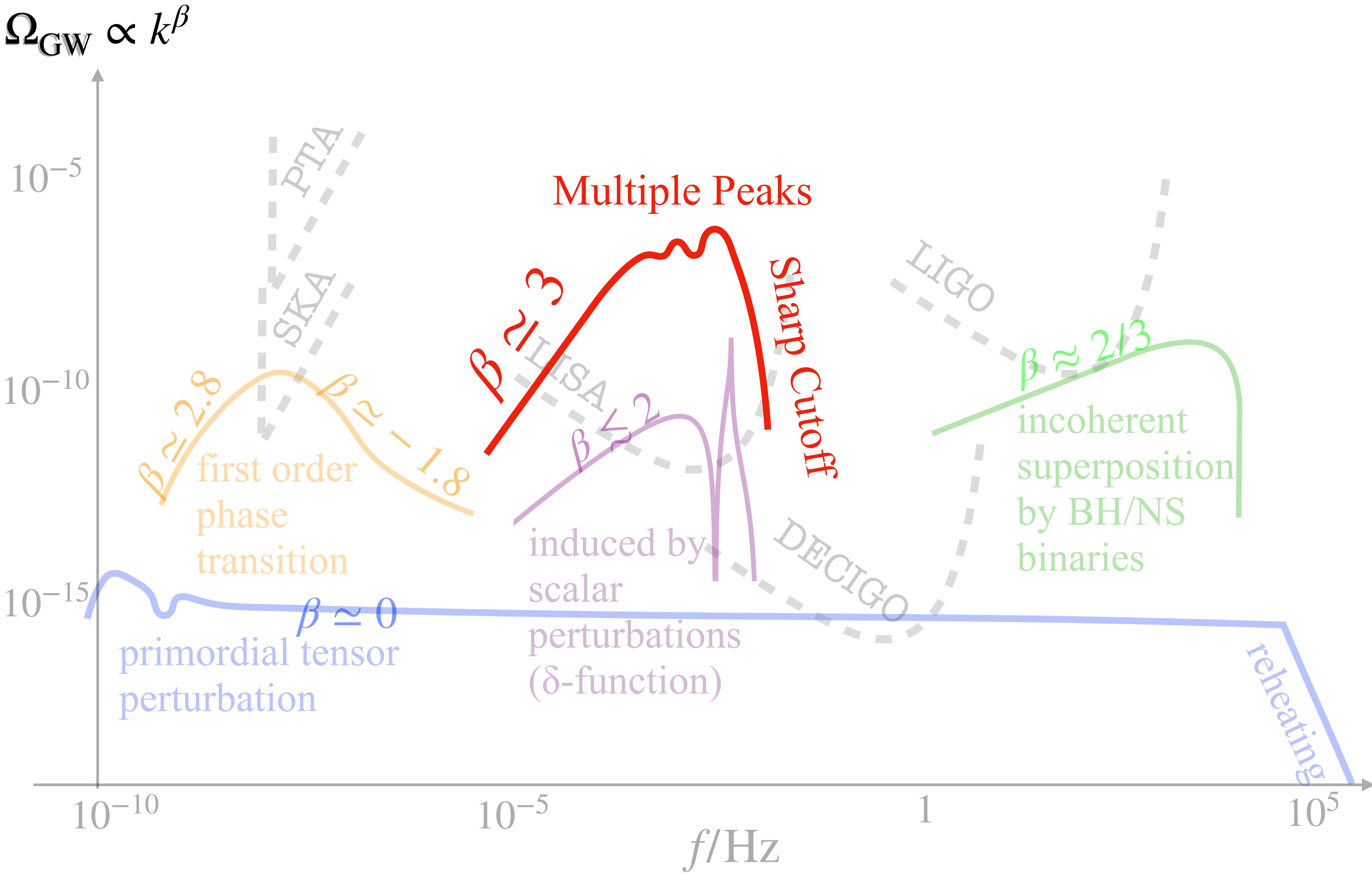
Single Peak
Cutoff: $2k_*$
Slope: k^3
F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{GW}h^2$
k^3
Multiple Peaks
0.0001
0.001
0.01
f /Hz
Slope: k^3
Cutoff: $4k_*$



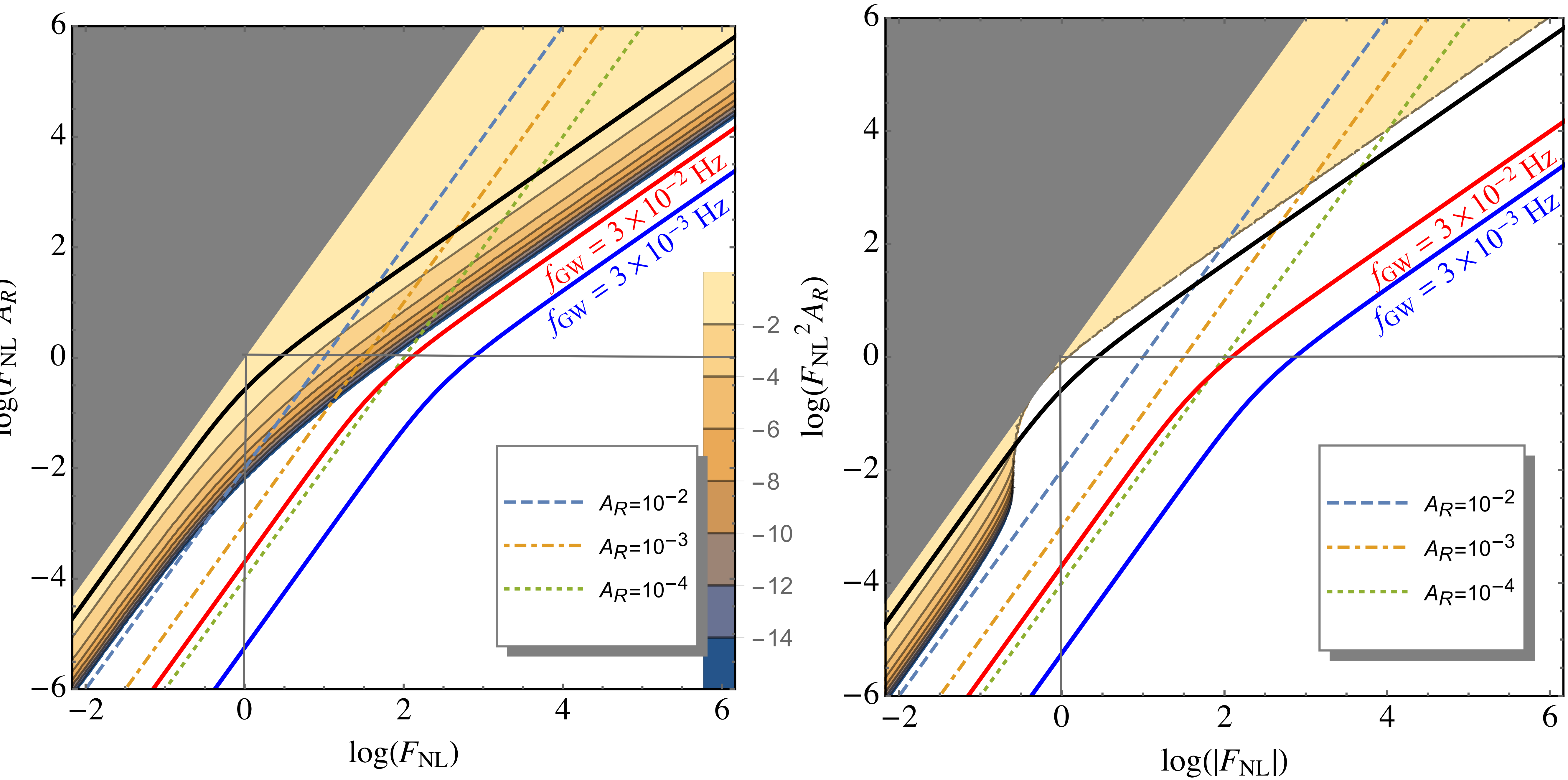
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
first order phase transition
induced by scalar perturbations (δ-function)
incoherent superposition by BH/NS binaries
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



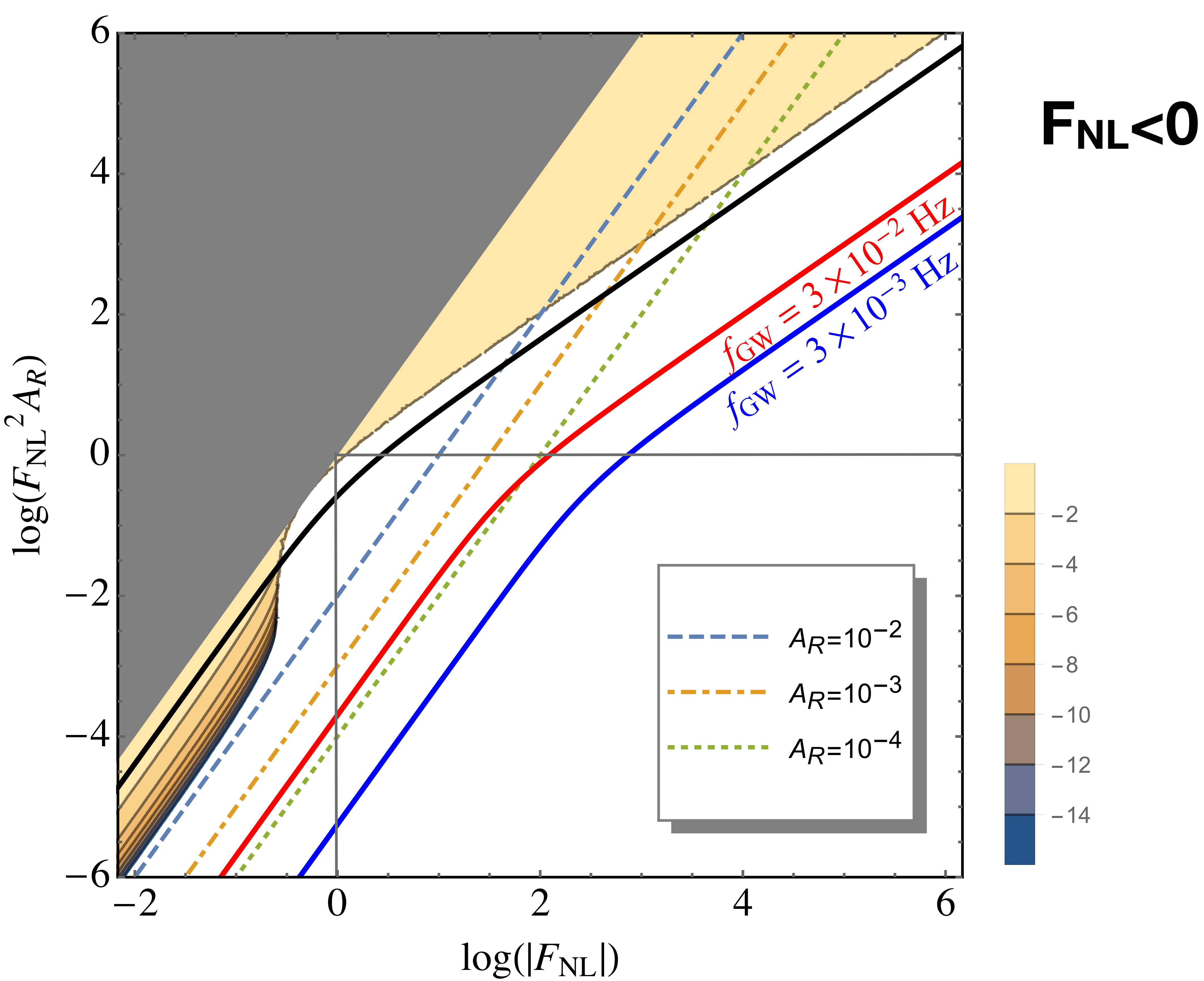
$\Omega_{GW} \propto k^{\beta}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
$\beta \simeq 2.8$
$\beta \simeq -1.8$
first order
phase
transition
$\beta \lesssim 2$
induced by
scalar
perturbations
(δ-function)
$\beta \approx 2/3$
incoherent
superposition
by BH/NS
binaries
$\beta \simeq 0$
primordial tensor
perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



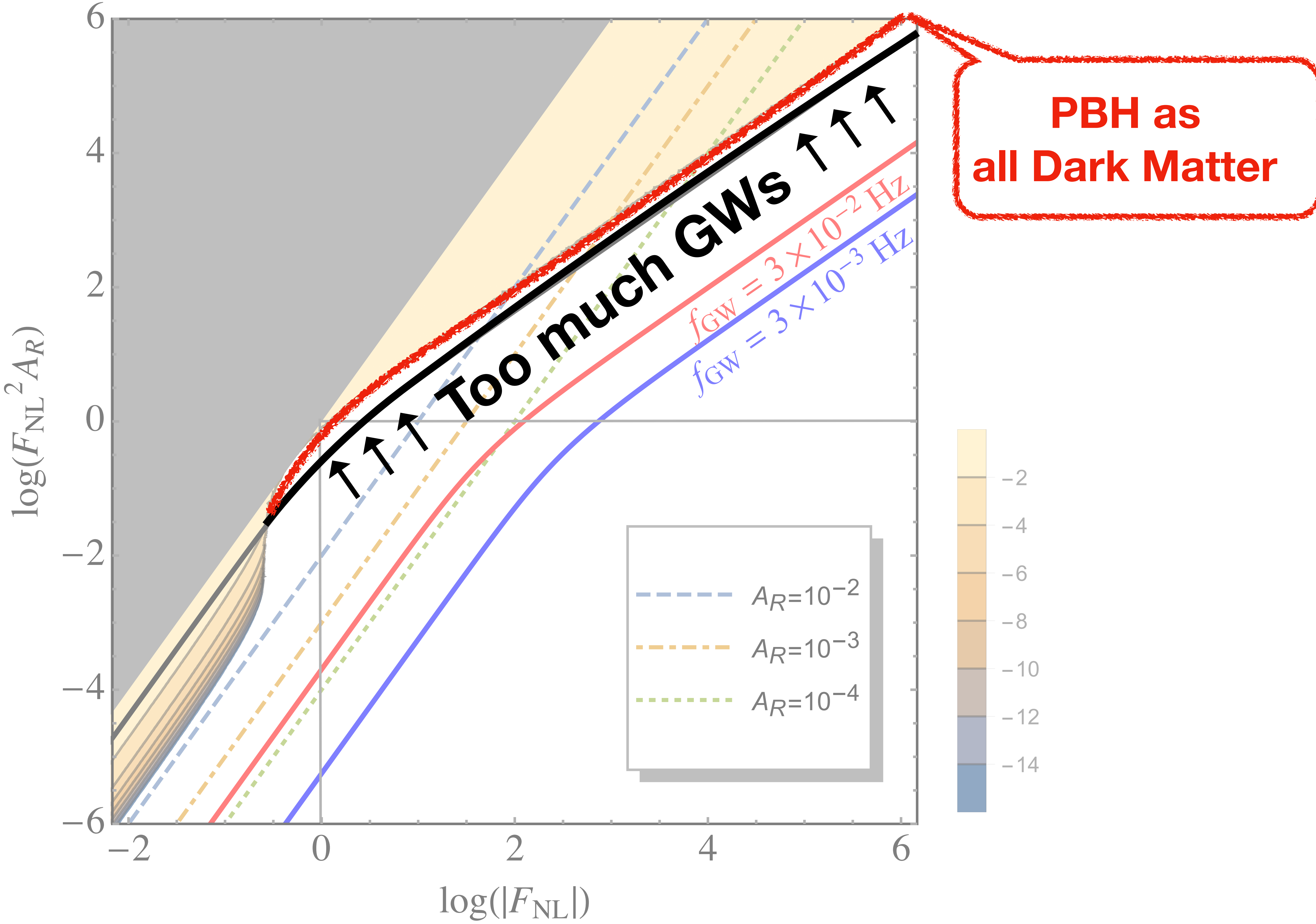
$\Omega_{GW} \propto k^{\beta}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
Multiple Peaks
$\beta \simeq 3$
Sharp Cutoff
$\beta \simeq 2.8$
$\beta \simeq -1.8$
first order phase transition
$\beta \leq 2$
induced by scalar perturbations (δ-function)
$\beta \approx 2/3$
incoherent superposition by BH/NS binaries
$\beta \simeq 0$
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



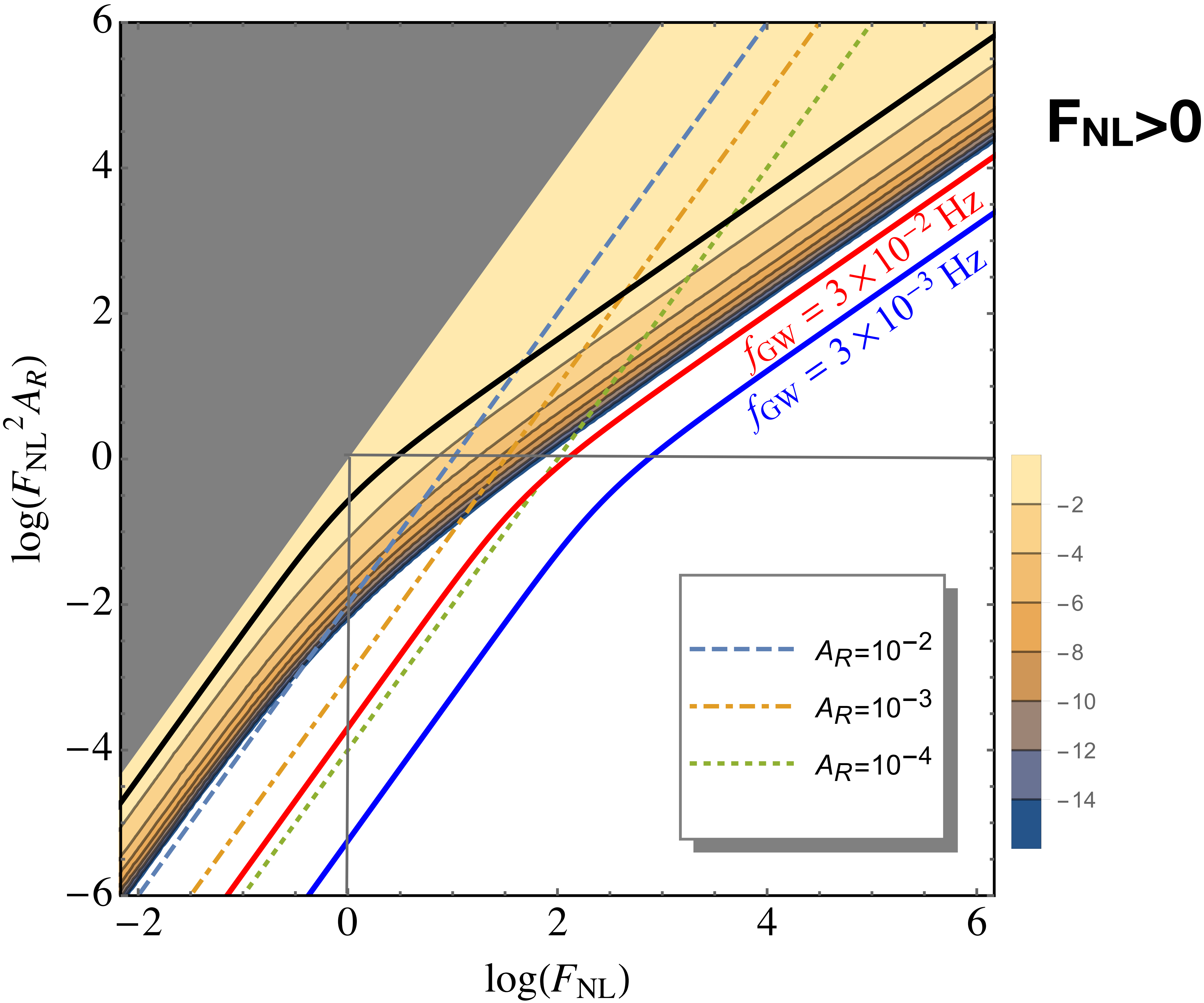
PBH abundance at the formation: β(M)



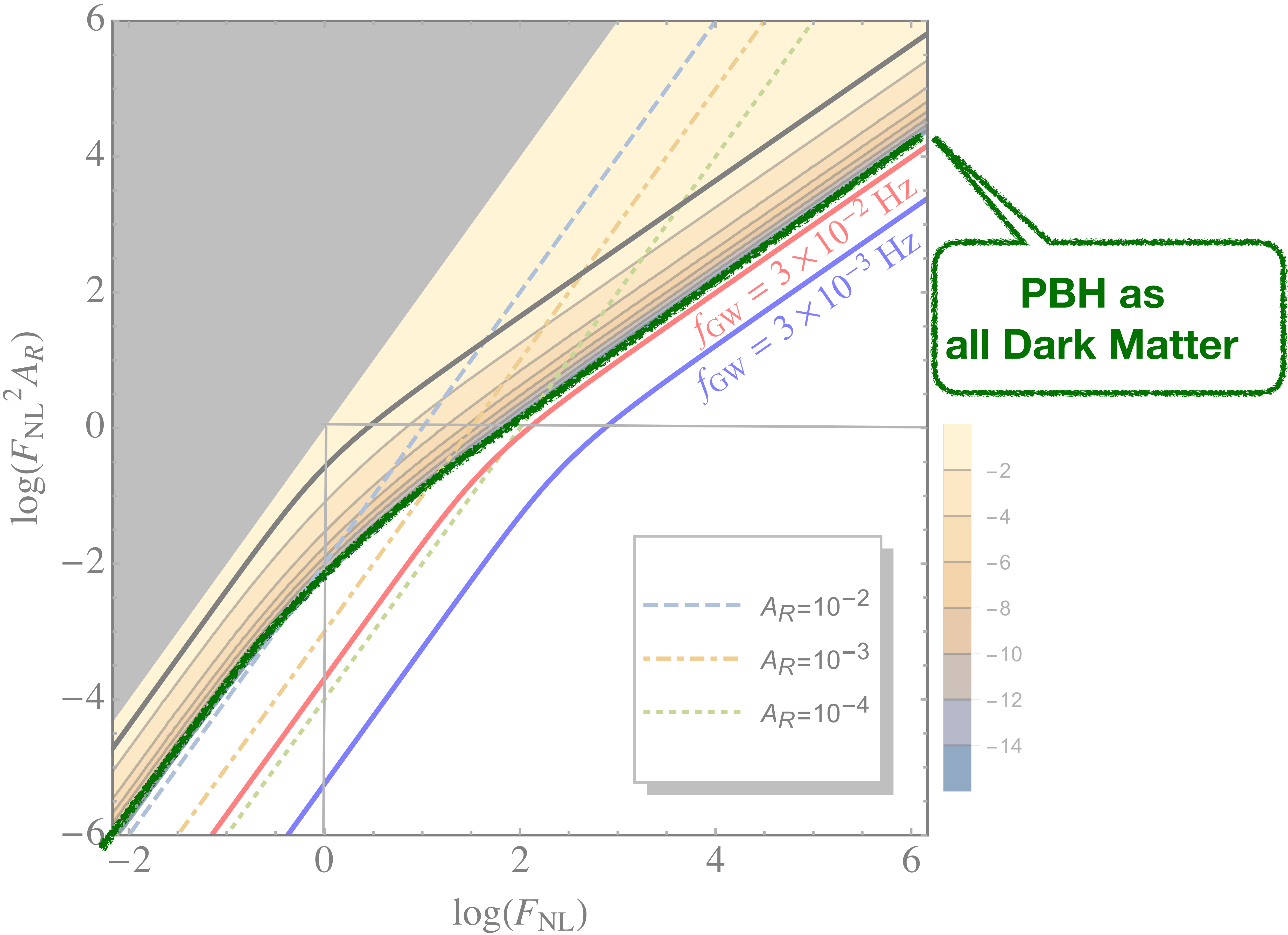
FNL<0
log(FNL^2 A_R)
log(|FNL|)
f_GW = 3×10^-2 Hz
f_GW = 3×10^-3 Hz
A_R=10^-2
A_R=10^-3
A_R=10^-4
-2
-4
-6
-8
-10
-12
-14



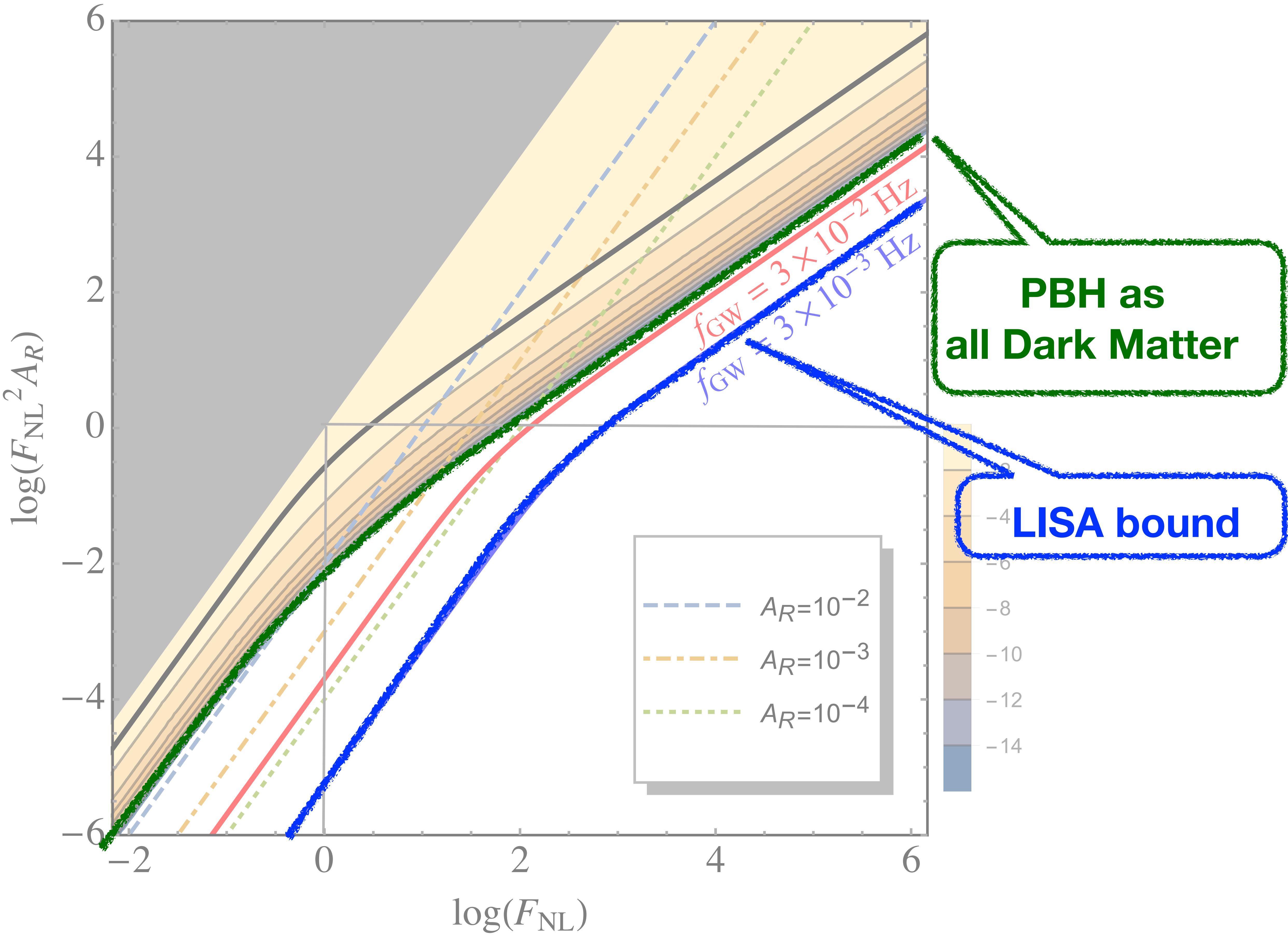
PBH as
all Dark Matter
Too much GWs
$f_{\rm GW} = 3\times10^{-2}$ Hz
$f_{\rm GW} = 3\times10^{-3}$ Hz
$A_R=10^{-2}$
$A_R=10^{-3}$
$A_R=10^{-4}$
$\log(F_{\rm NL}{}^2 A_R)$
$\log(|F_{\rm NL}|)$
−2
−4
−6
−8
−10
−12
−14



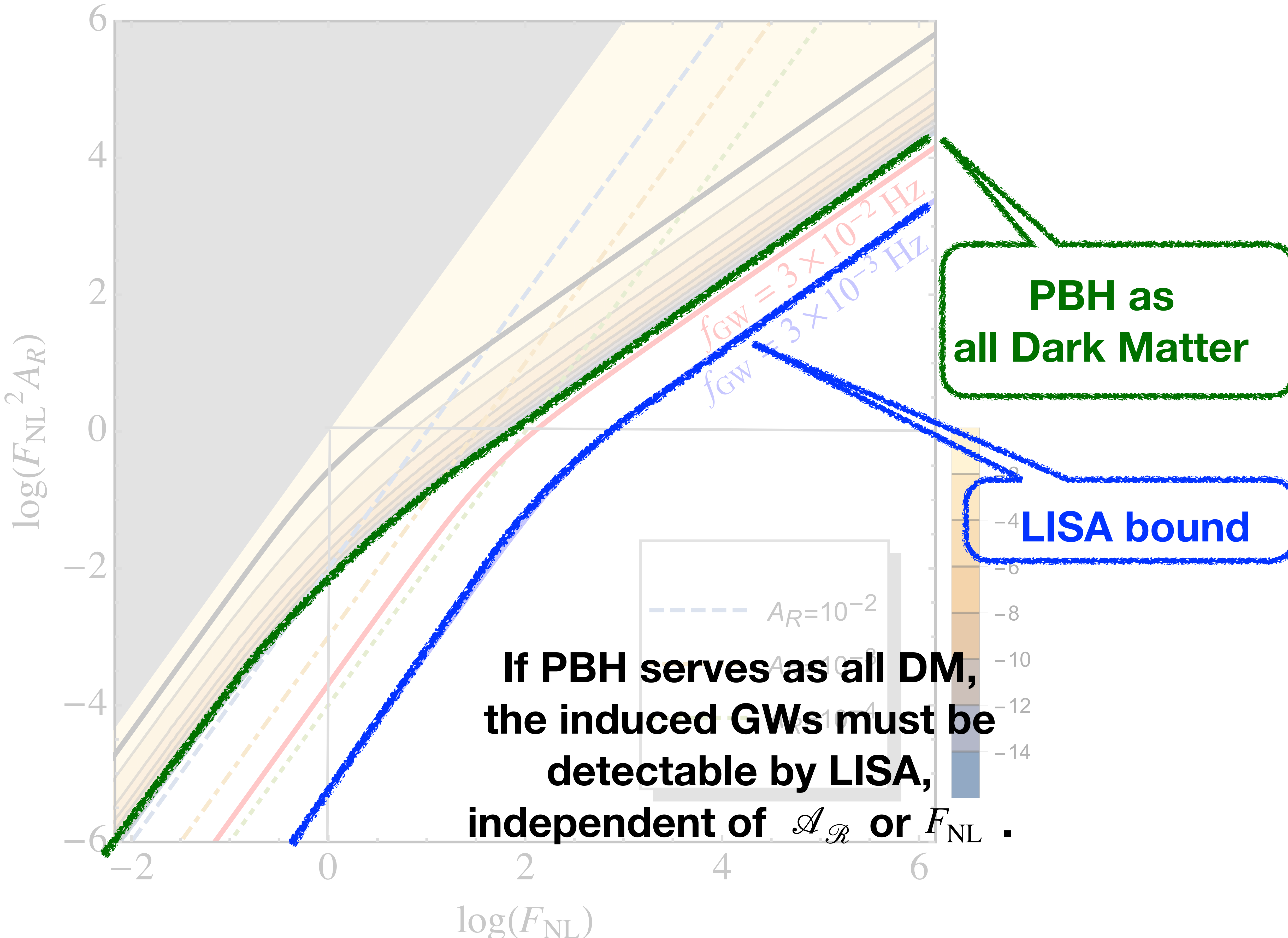
$\mathbf{F_{NL}>0}$
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
$\log(F_{NL}{}^2 A_R)$
$\log(F_{NL})$
6
4
2
0
−2
−4
−6
−2
0
2
4
6
−2
−4
−6
−8
−10
−12
−14



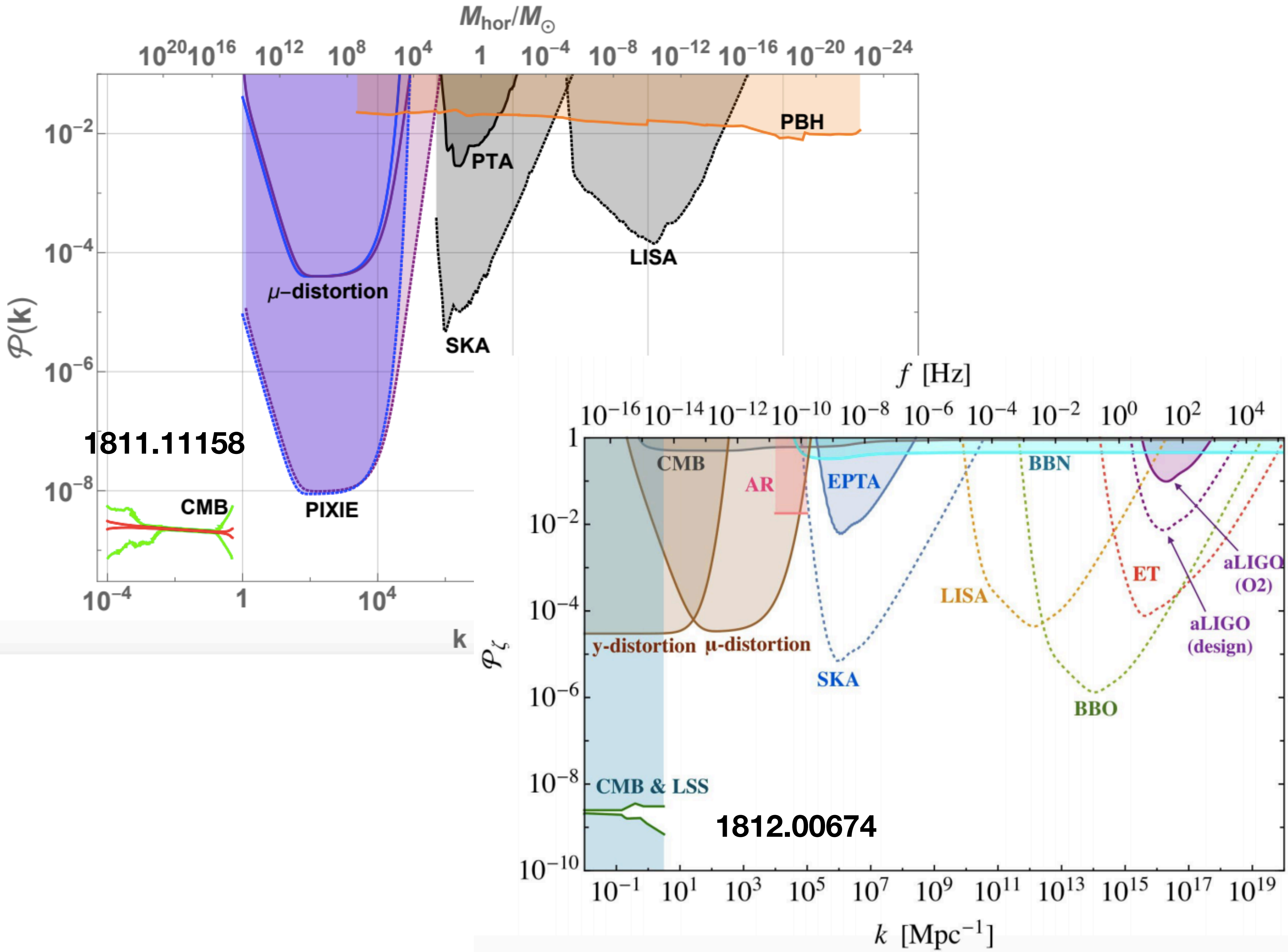
PBH as
all Dark Matter
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
$\log(F_{NL}{}^2 A_R)$
$\log(F_{NL})$
−2
−4
−6
−8
−10
−12
−14



PBH as all Dark Matter
LISA bound
$f_{GW} = 3\times10^{-2}$ Hz
$f_{GW} = 3\times10^{-3}$ Hz
A_R=10^{-2}
A_R=10^{-3}
A_R=10^{-4}
log($F_{NL}{}^2 A_R$)
log(F_{NL})
−4
−6
−8
−10
−12
−14



PBH as
all Dark Matter
LISA bound
If PBH serves as all DM,
the induced GWs must be
detectable by LISA,
independent of $\mathcal{A}_{\mathcal{R}}$ or $F_{\rm NL}$.
$\log(F_{\rm NL})$
$\log(F_{\rm NL}{}^2 A_R)$
$f_{\rm GW} = 3\times10^{-2}$ Hz
$f_{\rm GW} = 3\times10^{-3}$ Hz
$A_R=10^{-2}$



$M_{hor}/M_\odot$
10^{20} 10^{16} 10^{12} 10^{8} 10^{4} 1 10^{-4} 10^{-8} 10^{-12} 10^{-16} 10^{-20} 10^{-24}
$\mathcal{P}(\mathbf{k})$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
PBH
PTA
LISA
μ–distortion
SKA
1811.11158
CMB
PIXIE
10^{-4} 1 10^{4}
k
f [Hz]
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4}
$\mathcal{P}_\zeta$
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
CMB
AR
EPTA
BBN
ET
LISA
aLIGO (O2)
aLIGO (design)
y-distortion
μ-distortion
SKA
BBO
CMB & LSS
1812.00674
10^{-1} 10^{1} 10^{3} 10^{5} 10^{7} 10^{9} 10^{11} 10^{13} 10^{15} 10^{17} 10^{19}
k [Mpc^{-1}]

Summary

- GWs induced by non-Gaussian scalar perturbations: k^3-slope, multiple peaks, cutoff.
- If PBHs can serve as all the DM on mass range 10^{19}g to 10^{22}g , induced GWs must be detectable by LISA, no matter how small $\mathcal{A}_{\mathcal{R}}$ or $f_{\rm NL}$ is.
- Conversely if LISA can not detect the induced GWs, we can put an independent constraint on the PBH abundances on mass range 10^{19}g to 10^{22}g where no current experiment can explore.

Thank you!