2019 YITP Asian-Pacific Winter School and Workshop on Gravitation and Cosmology

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This is a pseudosphere

Based on arXiv:1902.03221

INFLATION ON A HYPERBOLIC PLANE

AND STRONGLY NON-GEODESIC MOTIONS

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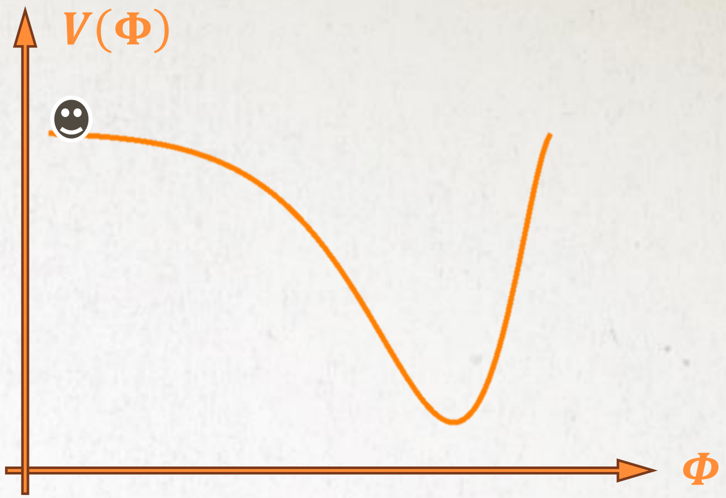
USUAL PICTURE OF INFLATION



$$\frac{M_p V'}{V} \ll 1\,, \qquad \frac{M_p^2 |V''|}{V} \ll 1$$

- Single-clock: only one scalar degree of freedom
- Slow-roll trajectory

How to embed the simple single-field dynamics (favored by data) in a theoretically motivated framework?

UV completion considerations:

	Examples
➢ Several scalar fields emerge as a low energy EFT	*dimensional compactification*
➢ Their kinetic terms are a priori non-canonical	*α– attractors*
➢ Small curvature of the potential is not natural	*eta problem*
➢ The slope of the potential is large in M_p units	*swampland de Sitter conjecture*

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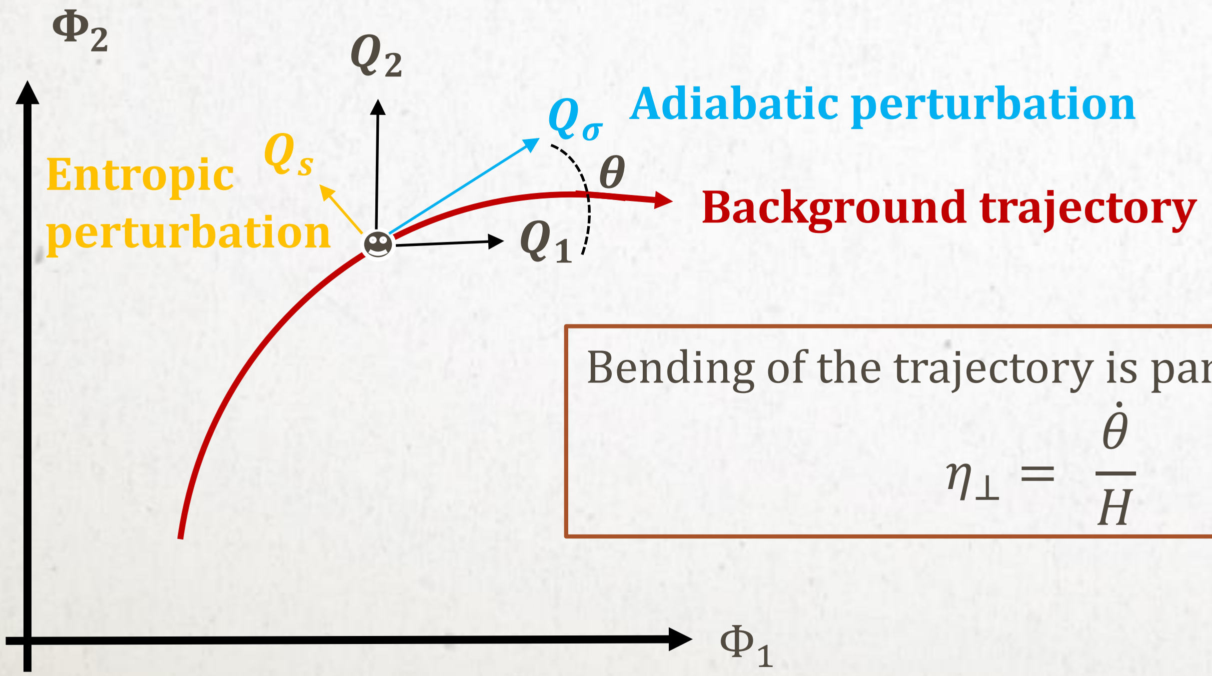
I. MULTI-FIELD MODELS OF INFLATION

BENDING OF THE TRAJECTORY

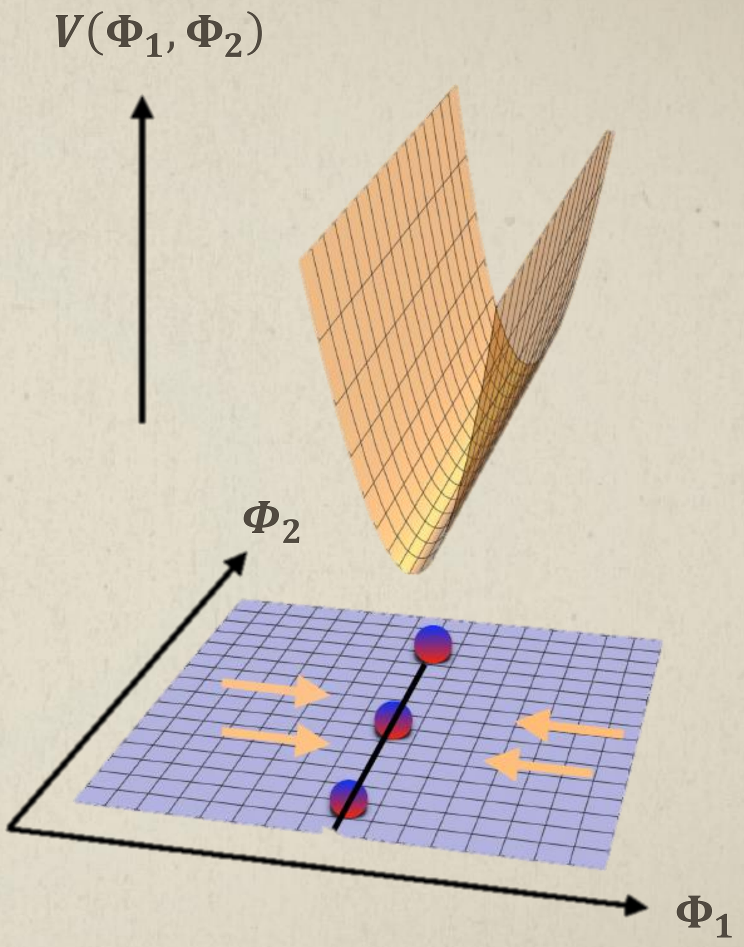
$$S = \int \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\delta_{AB}\partial_\mu\phi^A\partial_\nu\phi^B - V(\phi^I)\right)$$



Bending of the trajectory is parametrized by

$$\eta_\perp = \frac{\dot{\theta}}{H}$$

The Q's are the field perturbations $\delta\phi = \phi - \bar{\phi}$ in the flat gauge



Flat field space

I. MULTI-FIELD MODELS OF INFLATION

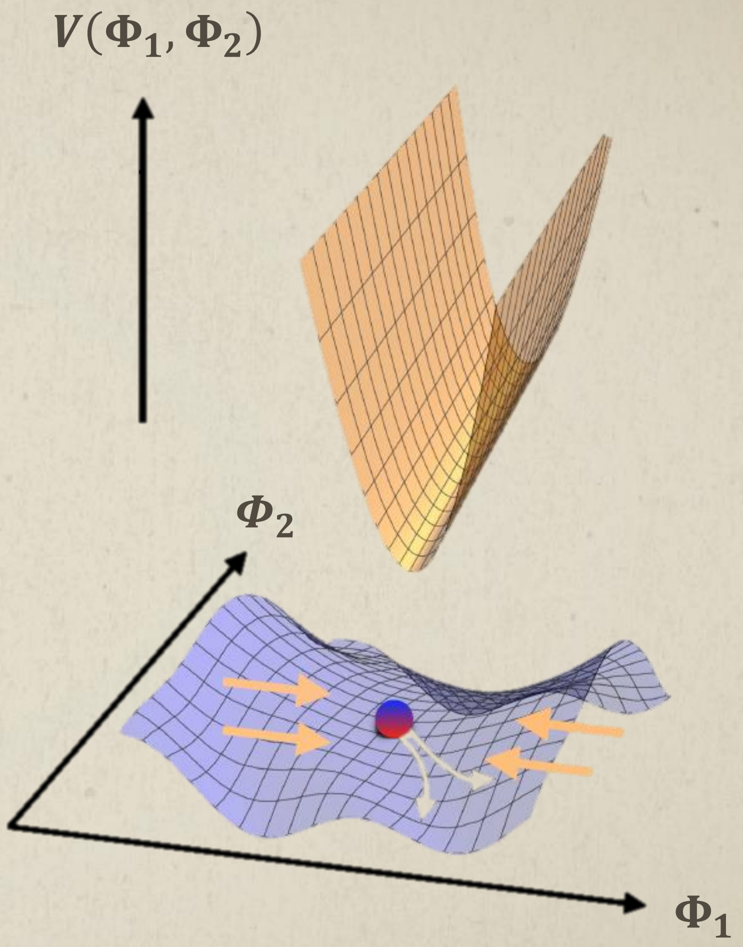
GEOMETRY AS A NEW DEGREE OF FREEDOM

$$S = \int \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} G_{AB} \left(\phi^I\right) \partial_\mu \phi^A \partial_\nu \phi^B - V\left(\phi^I\right) \right)$$

Local curvature in field space is given by the Ricci scalar R_{fs} constructed from the metric G

Geometry	Flat	Spherical	Hyperbolic
R_{fs}	0	> 0	< 0

Let us denote the total velocity $\dot{\sigma} = \sqrt{G_{AB}\dot{\phi}^A \dot{\phi}^B}$



Curved field space

Picture courtesy of S. Renaux-Petel

I. MULTI-FIELD MODELS OF INFLATION

EQUATIONS OF MOTION FOR THE LINEAR PERTURBATIONS

- Non-conservation equation for the comoving curvature perturbation $\zeta = \frac{H}{\dot{\sigma}} Q_\sigma$ (observable)

  on large scales: $\dot{\zeta} \simeq \frac{2H^2}{\dot{\sigma}} \eta_\perp Q_s$

- Dynamics of the entropic perturbations: $\ddot{Q}_s + 3H\dot{Q}_s + \left(\frac{k^2}{a^2} + m_s^2\right) Q_s = -2\dot{\sigma}\eta_\perp \dot{\zeta}$

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  with the entropic mass squared $m_s^2 = V_{;ss} - H^2\eta_\perp^2 + \epsilon R_{\rm fs} H^2 M_p^2$

  Hessian of the potential — Bending of the trajectory — Field-space curvature

[S. Renaux-Petel, K. Turzynski 2015]

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[S. Renaux-Petel, K. Turzynski 2015]

- Effective mass squared on large scales: $m_{s,\mathrm{eff}}^2 = m_s^2 + 4H^2\eta_\perp^2$

II. HYPERINFLATION

A CASE STUDY

[A. Brown 2018]
[S. Mukohyama, S. Mizuno 2018]

Setup:

radial ↘ ↙ angular

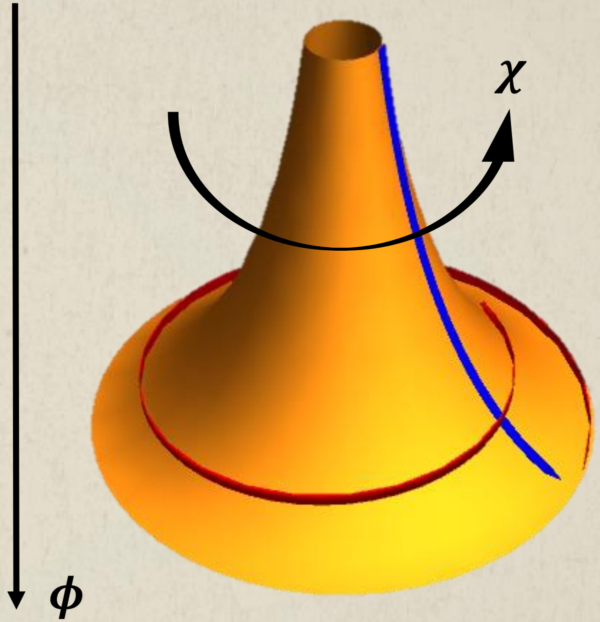
- The scalar fields ϕ , χ live on an internal hyperbolic plane
- The potential depends on ϕ only: $V(\phi, \cancel{\chi})$

Conservation of angular momentum $J = a^3 L^2 \sinh^2\left(\frac{\phi}{L}\right)\dot{\chi}$

Slow-roll

Hyper-trajectory $\cancel{\ddot{\phi}} + 3H\dot{\phi} - L\sinh\left(\frac{\phi}{L}\right)\cosh\left(\frac{\phi}{L}\right)\dot{\chi}^2 + V'(\phi) = 0$

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



$$ds^2_{fs} = d\phi^2 + L^2\sinh^2\left(\frac{\phi}{L}\right)d\chi^2$$

$$R_{fs} = -\frac{2}{L^2} \quad , L \ll M_p$$

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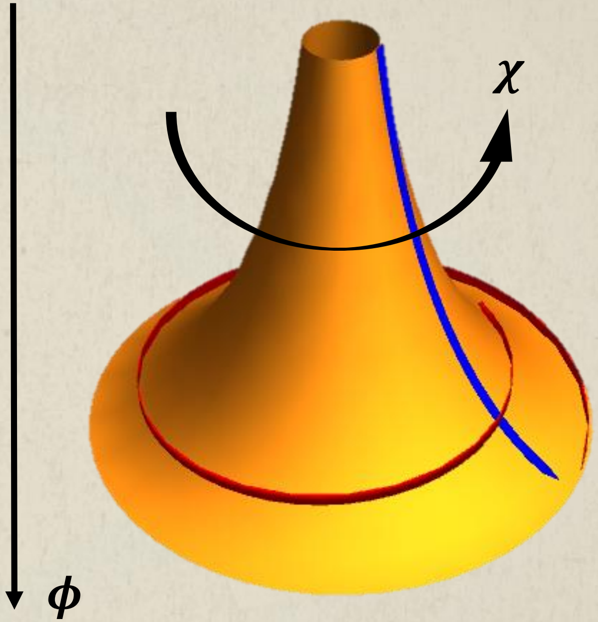
- Embedding of the hyperbolic plane in 3D
- Radial trajectory
- Hyperinflation trajectory



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$$\frac{LV'}{V} \ll 1 \quad \text{and} \quad \frac{L|V''|}{V'} \ll 1$$

$h^2 = \frac{V'}{LH^2} - 9 > 0$ necessary for hyperinflation

[T. Bjorkmo, D. Marsh 2019]

$\gg 1$ if strong bending (better for eta problem, swampland)

II. HYPERINFLATION

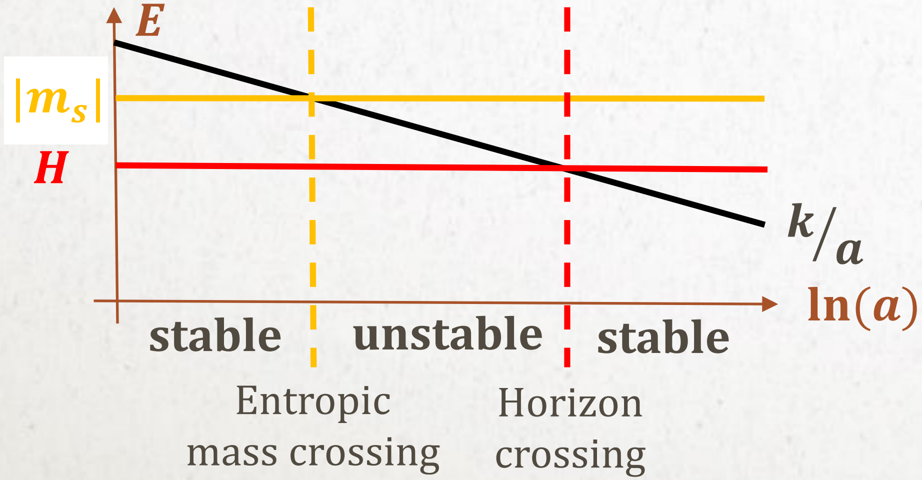
LINEAR PERTURBATIONS

We compute $\begin{cases} -H^2\eta_\perp^2 \approx -H^2h^2 \\ \epsilon R_{fs} H^2 M_p^2 \approx -H^2h^2 \end{cases} \Rightarrow \begin{cases} m_s^2 \approx -2H^2h^2 < 0 \\ m_{s,eff}^2 \approx 2H^2h^2 > 0 \end{cases}$

- $m_s^2 < 0$ → Unstable, growing sub-Hubble perturbations
- $m_{s,eff}^2 > 0$ → Stable, decaying super-Hubble perturbations

⚠ The tachyonic instability is only transient for each k-mode

Remember in the e.o.m. for Q_s , the mass term is $\left(\frac{k^2}{a^2} + m_s^2\right)$



⇒ Exponential enhancement of the power spectrum $\mathcal{P}_\zeta$ with respect to the standard one ⇒

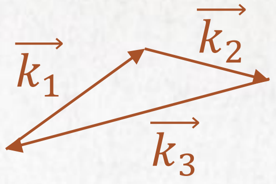
n_s	r
1.25	3.6×10^{-16}

II. HYPERINFLATION

BISPECTRUM USING PyTransport 2.0

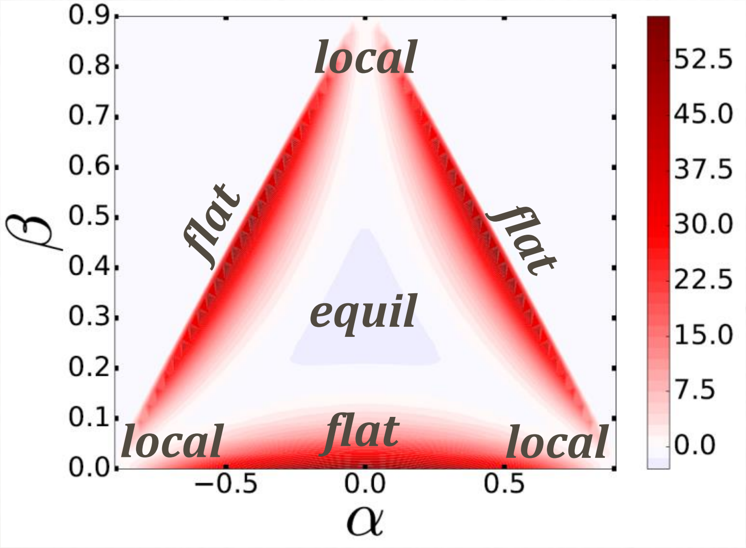
[D. Mulryne, J. Ronayne 2016]

3-point function $< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^3\delta^{(3)}\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)B_\zeta(\vec{k}_1,\vec{k}_2,\vec{k}_3)$



$$\frac{k_{55}}{3} = k_1 + k_2 + k_3$$

$$k_1 = \frac{3k_{55}}{4}(1+\alpha+\beta), k_2 = \frac{3k_{55}}{4}(1-\alpha+\beta) \text{ and } k_3 = \frac{3k_{55}}{2}(1-\beta)$$



Characteristic flattened non-Gaussianities

f_{NL}^{equil}	f_{NL}^{flat}
-2	53.8

In the range of detectability of future experiments

Single-clock inflation with Bunch-Davies initial states predicts equilateral non-Gaussianities

III. EFFECTIVE FIELD THEORY FOR THE PERTURBATIONS

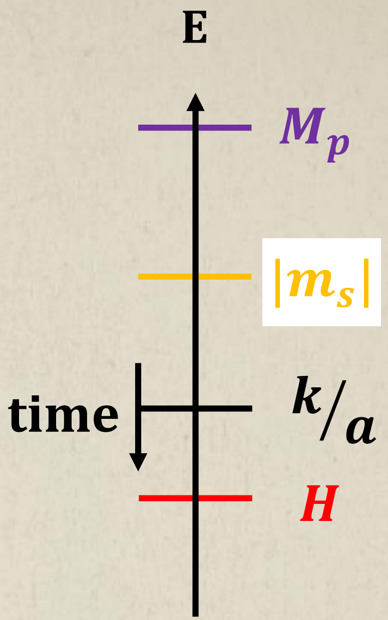
AN IMAGINARY SPEED OF SOUND

When this hierarchy is satisfied, one finds

$$Q_s^{\rm EFT} = -2\frac{\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}$$

from the e.o.m. of Q_s

A hierarchy of scales



Energy of the "experiment"

$$E \sim \frac{k}{a} \ll |m_s|$$

Integrate out the heavy field

Like in the Fermi theory:
Integrate out the heavy W boson and consider contact interactions for fermions

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When this hierarchy is satisfied, one finds $$Q_s^{\mathrm{EFT}} = -2\frac{\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}$$

$\Downarrow$ *from the e.o.m. of Q_s*

Effective single-field action for the curvature perturbation

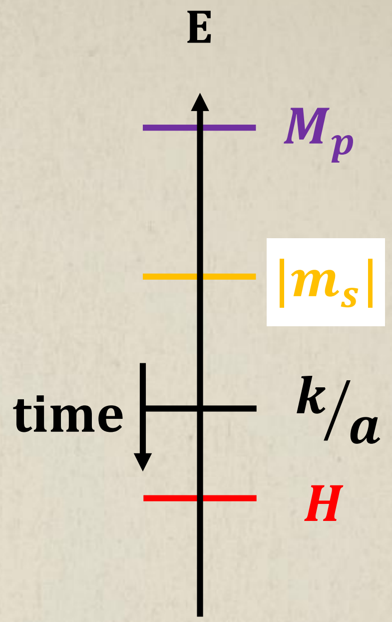
$$S_{(2)}^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x\, a^2\epsilon^2\left[\frac{\zeta'^2}{c_s^2} - (\vec{\nabla}\zeta)^2\right]$$

in conformal time $d\tau = {}^{dt}/_{a}$

With a speed of sound c_s such that $$\frac{1}{c_s^2} \equiv 1 + \frac{4H^2\eta_\perp^2}{m_s^2} = \frac{m_{s(\mathrm{eff})}^2}{m_s^2}$$

$$< 0 \text{ if } m_s^2 < 0 < m_{s(\mathrm{eff})}^2$$

A hierarchy of scales



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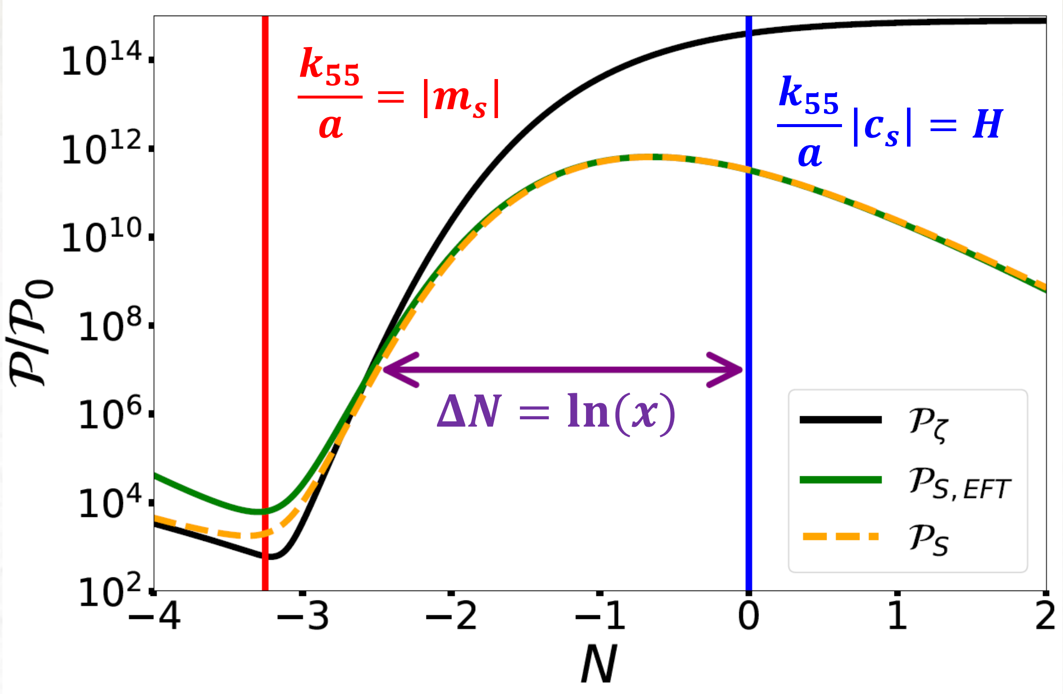
III. EFFECTIVE FIELD THEORY FOR THE PERTURBATIONS

AN EXPONENTIAL GROWTH

In hyperinflation, we find $c_s^2 \approx -1$

$$x^2 \approx h^2 \gg 1$$

The EFT for ζ is valid for $\frac{k}{a}|c_s| < x \times H$



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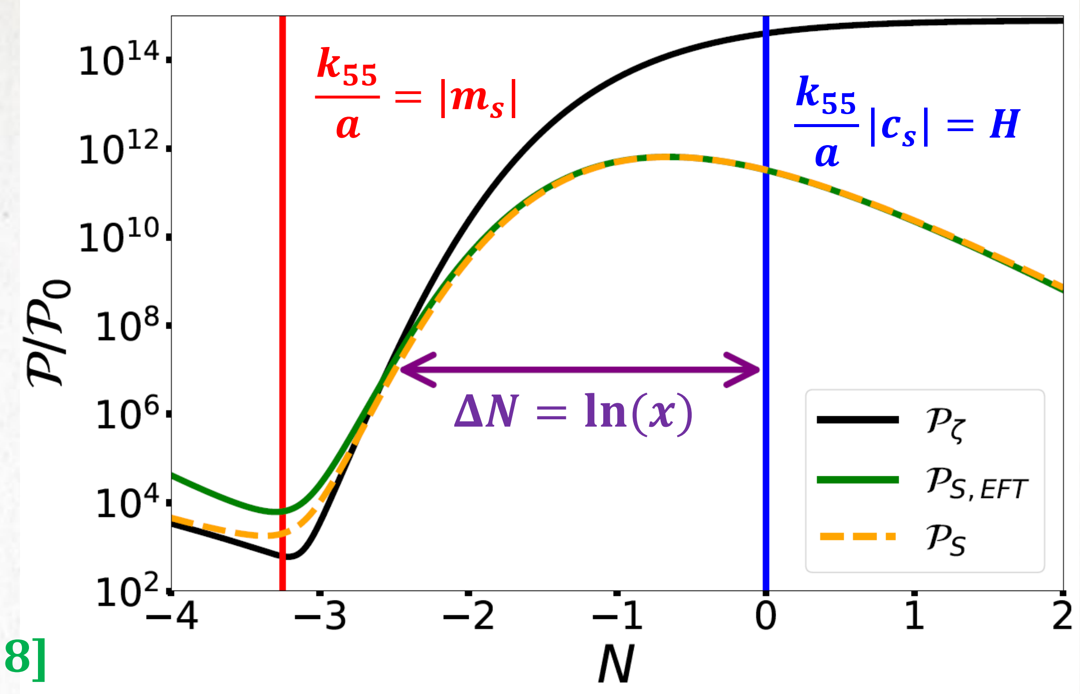

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$$x^2 \approx h^2 \gg 1$$

The EFT for ζ is valid for $\frac{k}{a}|c_s| < x \times H$

With $c_s^2 < 0$, the mode function verifies: **[S. Garcia-Saenz, S. Renaux-Petel 2018]**

$$\zeta_k(\tau) = \left(\frac{2\pi^2}{k^3}\right)^{1/2} \alpha \left(e^{k|c_s|\tau + x}(k|c_s|\tau - 1) - \rho e^{i\theta} e^{-(k|c_s|\tau + x)}(k|c_s|\tau + 1)\right)$$

$$Q_s^{\mathrm{EFT}} = -2\frac{\dot\sigma\eta_\perp}{m_s^2}\dot\zeta$$

exponentially growing mode (real) — exponentially decaying mode

$$\mathcal{P}_\zeta = \alpha^2 e^{2x}$$

! The overall amplitude needs to be matched

- EFT not predictive for the power spectrum
- Powerful for non-Gaussianities

III. EFFECTIVE FIELD THEORY FOR THE PERTURBATIONS

BISPECTRUM

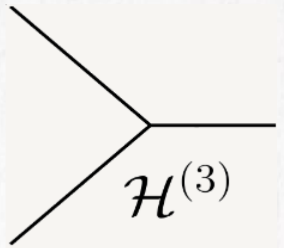
[Cheung et al. 2007]

Effective single-field cubic action

$$S^{\mathrm{EFT}}_{(3)}[\zeta] = \int d\tau d^3x \, \frac{a\,\epsilon^2}{H}\left(\frac{1}{c_s^2}-1\right)\left[\zeta'(\vec{\nabla}\zeta)^2 + \frac{A}{c_s^2}\zeta'^3\right]$$

A: $O(1)$

Tree-level 3-point function



→ No exponential enhancement of $f_{NL} \sim \dfrac{<\zeta^3>}{<\zeta^2>^2}$

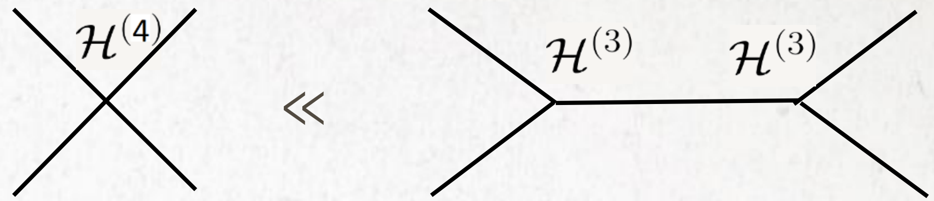
$$f_{NL}^{equil} \sim O(1) \quad \text{and} \quad f_{NL}^{flat} \sim O(0.05) \times P_3(x)$$

Cubic polynomial in x with $x \sim 10$ in hyperinflation $\Rightarrow f_{NL}^{flat} \sim O(50)$

III. EFFECTIVE FIELD THEORY FOR THE PERTURBATIONS

HIGHER-ORDER CORRELATION FUNCTIONS

- Trispectrum: contact and scalar-exchange diagrams



→ Exponential enhancement of $g_{NL} \sim \frac{< \zeta^4 >}{< \zeta^2 >^3} \sim e^{4x} \sim 10^{30}$ in hyperinflation

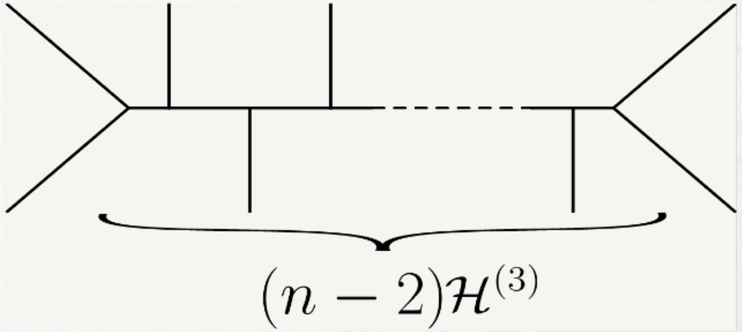
- Higher-order non-Gaussianities dominated by scalar-exchange diagrams with cubic interactions

Gaussian field

$$\zeta = \zeta_g \left(1 + f_{NL}^{(1)} \zeta_g + f_{NL}^{(2)} \zeta_g^2 + \ldots + f_{NL}^{(n)} \zeta_g^n + \ldots\right)$$

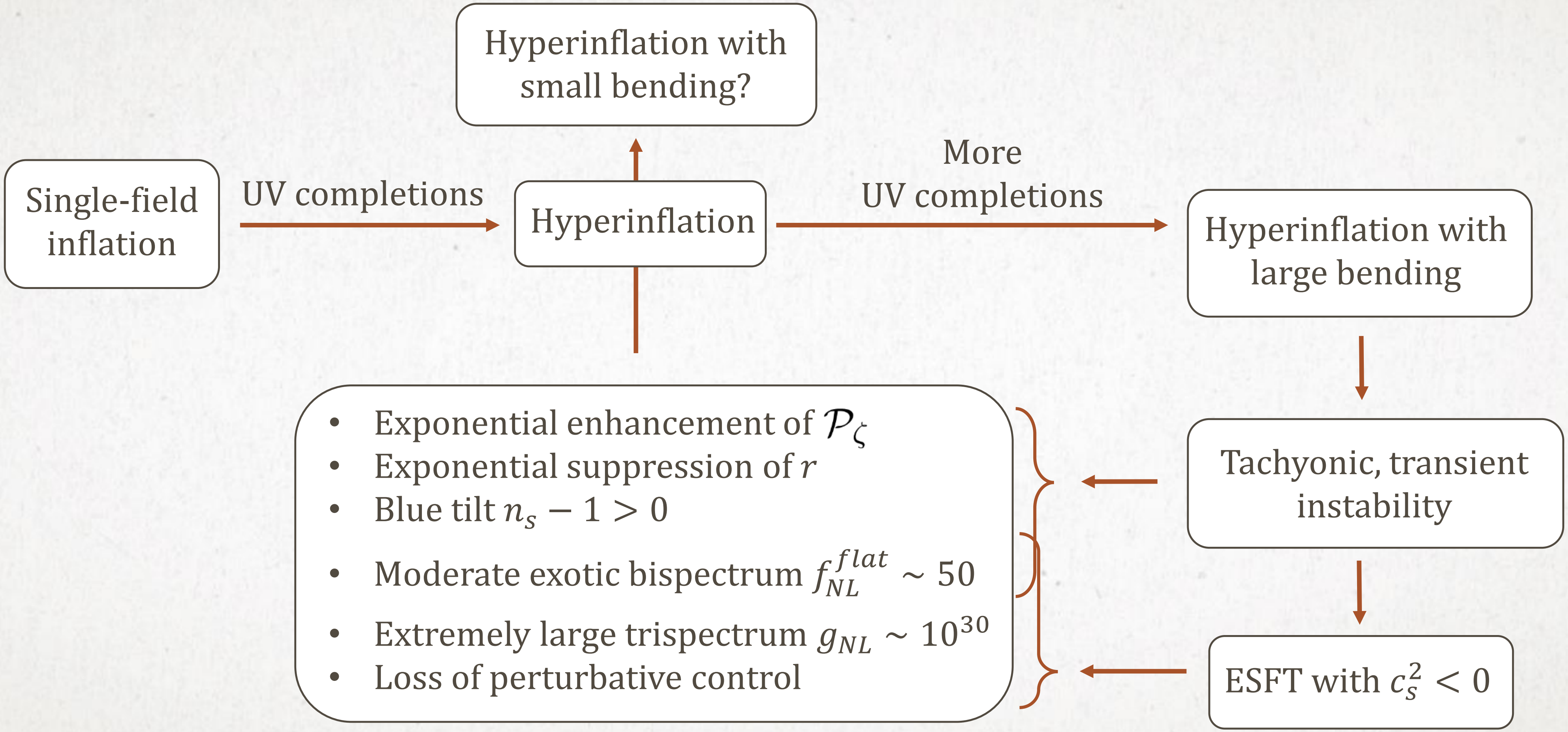
$f_{NL}^{(1)} = f_{NL} \ll f_{NL}^{(2)} = g_{NL} \ll \ldots$

Perturbative expansion



→ **Loss of perturbative control**

CONCLUSION


Hyperinflation with small bending?
Single-field inflation
UV completions
Hyperinflation
More UV completions
Hyperinflation with large bending
Tachyonic, transient instability
ESFT with $c_s^2 < 0$
• Exponential enhancement of $\mathcal{P}_\zeta$
• Exponential suppression of r
• Blue tilt $n_s - 1 > 0$
• Moderate exotic bispectrum $f_{NL}^{flat} \sim 50$
• Extremely large trispectrum $g_{NL} \sim 10^{30}$
• Loss of perturbative control

THANKS FOR YOU ATTENTION!

REQUIREMENTS ON THE POTENTIAL

Upper bounds:

- Single-field slow-roll $\epsilon \ll 1, \quad \eta \ll 1$

$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \ , \quad \eta_V = M_p^2 \frac{V''}{V} \ll 1$$

- Hyperinflation $\epsilon \ll 1, \quad \eta \ll 1$

$$\frac{LV'}{V} \ll 1 \ , \qquad \frac{L|V''|}{V'} \ll 1$$

Lower bounds:

- Swampland conjecture, eta problem $\epsilon_V \geq 1 \ or \ -\eta_V \geq 1, \quad |\eta_V| \geq 1$

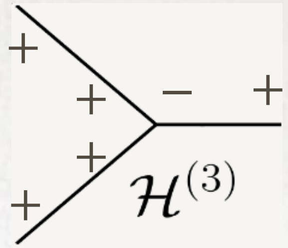
$$\frac{M_p V'}{V} \gtrsim 1 \ , \qquad \frac{M_p^2 |V''|}{V} \gtrsim 1$$

- Hyperinflation (with strong bending)

$h^2 \geq 0$ $(h^2 \gg 1)$ with $1 + \frac{h^2}{9} \approx \frac{\epsilon_V}{\epsilon} \approx \frac{\eta_V}{2\epsilon - \eta}$

BISPECTRUM COMPUTATION

Tree-level 3-point function



$\sim e^{4x}$

Need to contract one decaying mode

Growing modes are purely real

Effective single-field cubic action

$O(1)$

$$S^{\text{EFT}}_{(3)}[\zeta] = \int d\tau d^3x \, \frac{a\,\epsilon^2}{H}\left(\frac{1}{c_s^2}-1\right)\left[\zeta'(\vec{\nabla}\zeta)^2 + \frac{A}{c_s^2}\zeta'^3\right]$$

Schinger-Keldysh formalism:

$$< \zeta_{\vec{k}_1}(t)\zeta_{\vec{k}_2}(t)\zeta_{\vec{k}_3}(t) >= 2\underline{\text{Im}}\left(\int_{t_0}^{t} dt' < 0|\zeta_{\vec{k}_1}(t)\zeta_{\vec{k}_2}(t)\zeta_{\vec{k}_3}(t)H^{(3)}(t')|0>\right)$$

$$f_{NL} \sim \frac{<\zeta^3>}{<\zeta^2>^2}$$ → No exponential enhancement of the bispectrum:

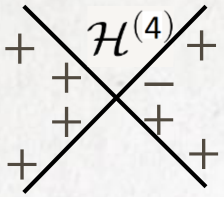
f_{NL}^{equil}	f_{NL}^{flat}
$\simeq \frac{10}{9}\left(\frac{1}{\lvert c_s\rvert^2}+1\right)\left(\frac{13A}{6}-\frac{5}{24}\right) \sim O(1)$	$\simeq \frac{5}{576}\left(\frac{1}{\lvert c_s\rvert^2}+1\right)\left(39(A-1)+12x^2+4(A+1)x^3\right) \sim O(10-100)$

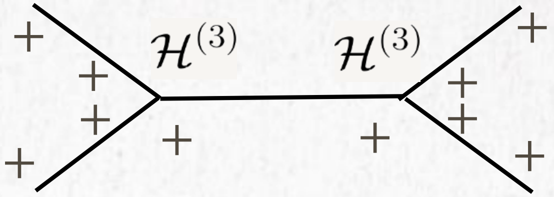
[S. Renaux-Petel, S. Garcia-Saenz 2018]

$x \sim 10$ in hyperinflation

COMPUTATION OF HIGHER-ORDER CORRELATIONS

- Trispectrum: contact and scalar-exchange diagrams



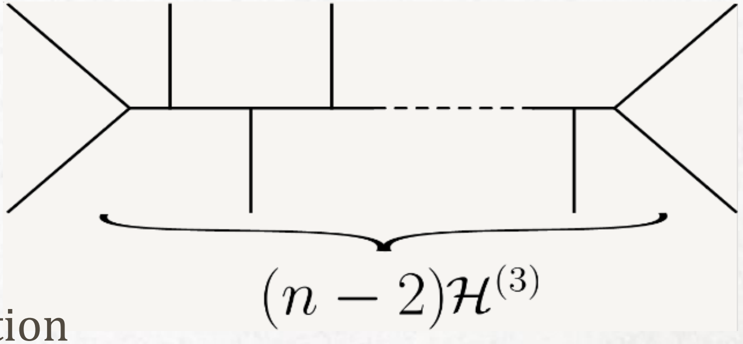


$$g_{NL} \sim \frac{<\zeta^4>}{<\zeta^2>^3} \sim e^{4x}$$

$$<\zeta^4>^c \sim 2\mathrm{Im}\left(\int_{t_0}^{t} dt' <0|\zeta^4(t)H^{(4)}(t')|0>\right) \sim e^{6x} \quad \ll \quad <\zeta^4>^{se} \sim \; -2\mathrm{Re}\left(\int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' <0|\zeta^4(t)H^{(3)}(t')H^{(3)}(t'')|0>\right) + \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' <0|H^{(3)}(t')\zeta^4(t)H^{(3)}(t'')|0> \sim e^{10x}$$

- Higher-order non-Gaussianities

$$f_{NL}^{(n-2)} \sim \frac{\langle\zeta^n\rangle}{\langle\zeta^2\rangle^{n-1}} \sim \left(\frac{\mathcal{P}_\zeta}{\mathcal{P}_0}\left(\frac{1}{|c_s|^2}+1\right)\right)^{n-2} \quad , n \geqslant 4$$



$\sim 10^{15}$ in hyperinflation

Perturbativity violation if $\frac{\mathcal{P}_\zeta}{\mathcal{P}_0}\left(\frac{1}{|c_s|^2}+1\right) \gtrsim 2\times 10^4$ ← Planck amplitude of $1/\zeta$