QED in de Sitter Space

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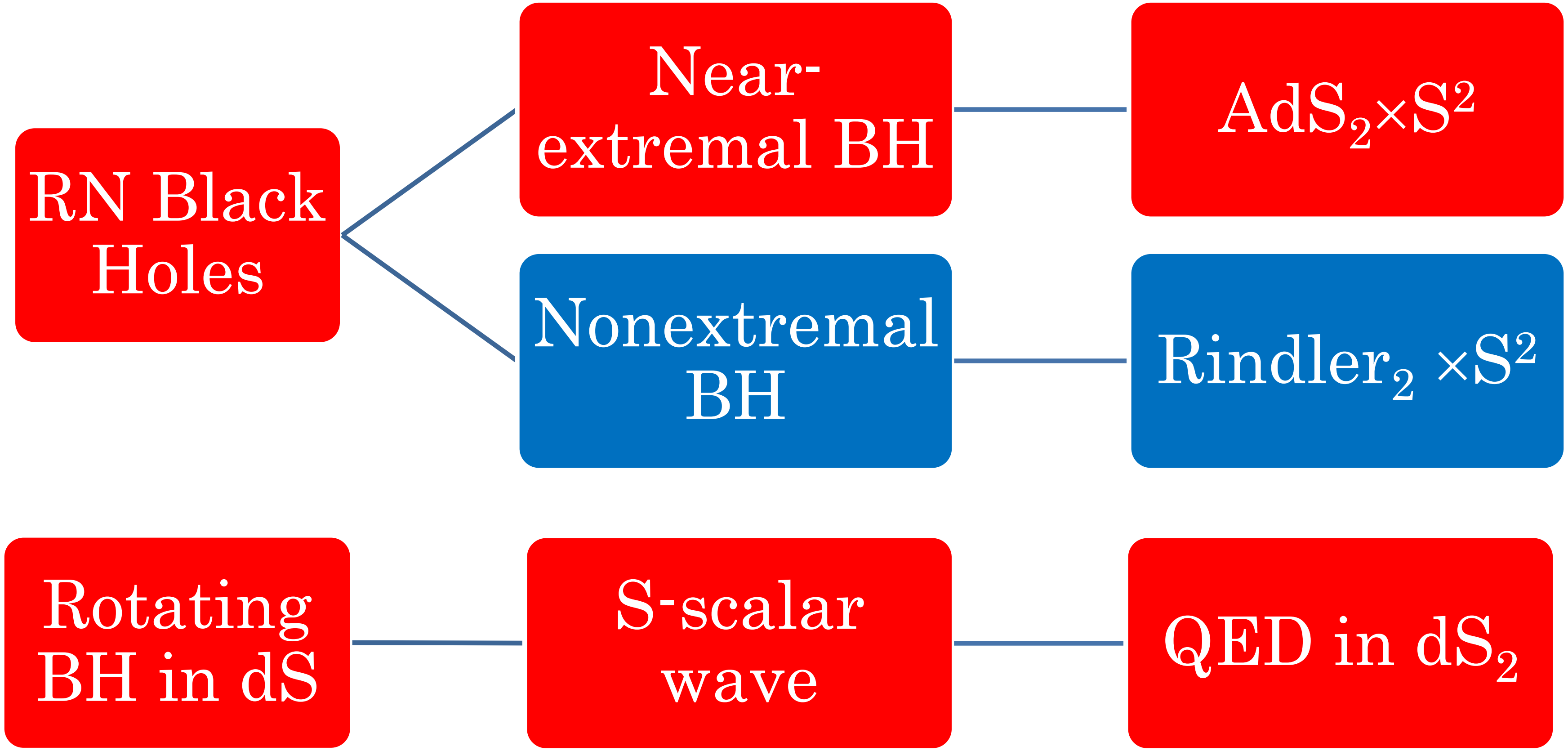
Contents

Motivation

- Near extremal charged black holes
  - Near horizon geometries are $AdS_2 \times M_{D-2}$
- Supercritical-field neutron stars (magnetars):
  - Magnetars with B 100 times or higher than the critical field B_C.
  - Unstable magnetars gravitationally collapse to form black holes.
- Mergers of neutron stars:
  - GW170817: gravitational waves from a merger of neutron stars
  - QED phenomena (QED vacuum polarization, Schwinger pair production and QED cascade) in strong gravity
- Magnetogenisis
  - Astrophysical magnetic fields
  - Cosmological magnetic fields

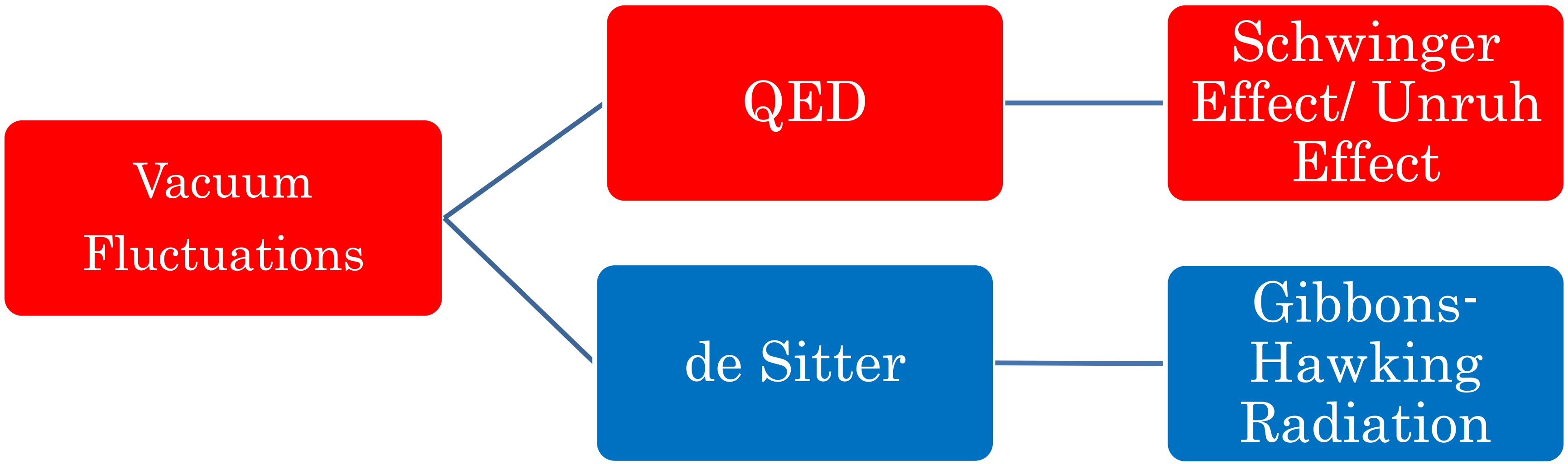
Why Schwinger Effect in (A)dS$_2$?

Near-Horizon Geometry of RN BHs



Schwinger Effect in (A)dS$_2$

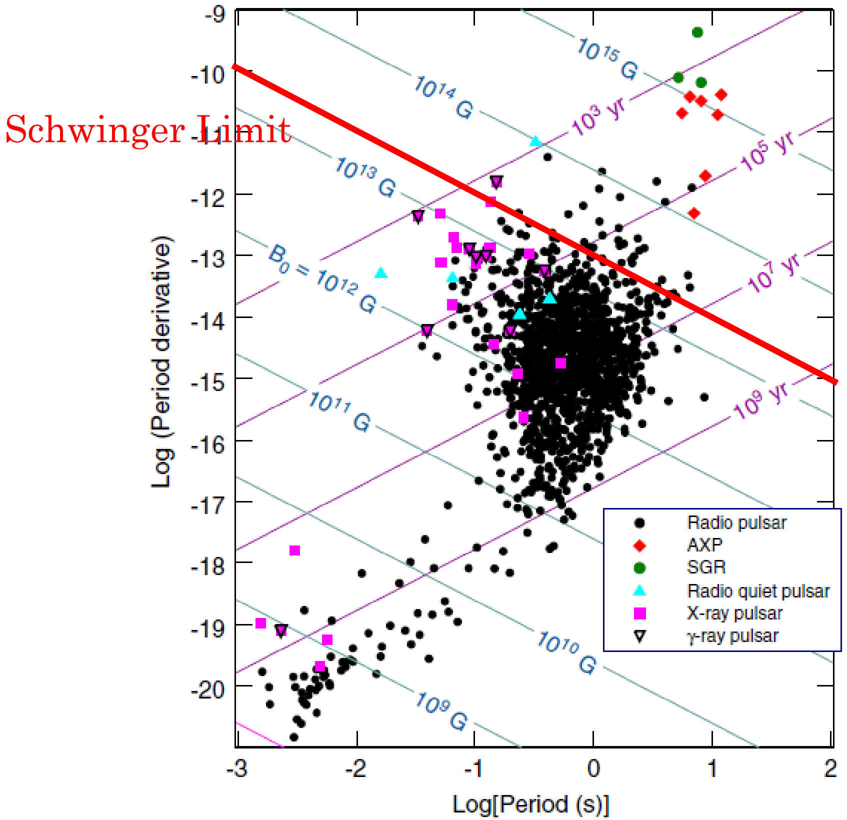
[Cai, SPK, JHEP 09 ('14)]



$$T_{\mathrm{eff}} = T_U + \sqrt{T_U^2 + \left(T_{GH}^2 \ or\ \frac{R_{AdS}}{8\pi^2}\right)}\,, \qquad T_U = \frac{qE/m}{2\pi}\,, \qquad T_{GH} = \frac{H}{2\pi}$$

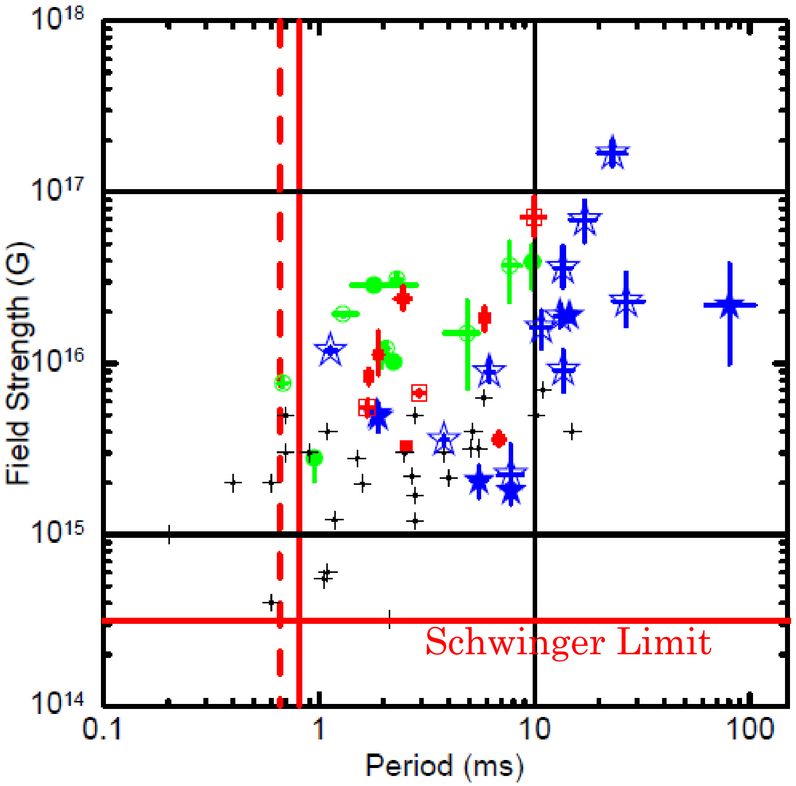
Neutron Stars and Magnetars

Neutron Stars



[Harding, Lai, Rep. Prog. Phys. 69 ('06)]

Strongest Magnetic Fields in the Universe



[Gompertz, PhD thesis ('15)]

blue: stable magnetars

green: unstable to collapse to black holes

Astrophysical/ Cosmological Magnetic Fields

- Astrophysical Magnetic Fields
  - Magnetic fields of μG amplitude observed in galaxies, clusters and redshift objects ($z < 4$).
  - To generate the galactic fields of μG amplitude by the flux conservation during the formation of galaxies, seed fields of about nG are required. What is the origin and related physics?
- Cosmological Magnetic Fields
  - Lower bound on magnetic field amplitude in the intergalactic medium from observation of blazars with gamma ray telescopes

$$B_{Mpc} > 6 \times 10^{-18} G \quad \text{[Vovk 1112.2534]}$$

  - The flux conservation would imply a huge magnetic field in the early universe.
  - What is the origin? after recombination (related to structure formation) or primordial?

Heisenberg-Euler/ Schwinger QED Action

Heisenberg-Euler/Schwinger QED Action

- QED in intense lasers (coherent, multi-photons): PW, EW, ZW

$$D_\mu = \partial_\mu - ieA_\mu \ , \qquad |p, N\rangle = e^{-N/2} \sum_{n=0}^{\infty} \frac{N^{n/2}}{\sqrt{n!}} |n\rangle$$

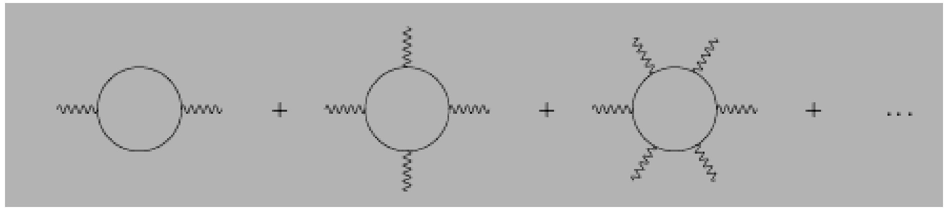
- Gauge invariant Maxwell scalar and pseudo-scalar

$$F = \frac{F^{\mu\nu}F_{\mu\nu}}{4} = \frac{(\vec{B}^2 - \vec{E}^2)}{2}, \qquad G = \frac{F^{\mu\nu}F^*_{\mu\nu}}{4} = \vec{B}\cdot\vec{E}\ , \qquad X = \sqrt{2(F+iG)} = |\vec{B} + i\vec{E}|$$

- QED one-loop action (nonlinear QED action) [Heisenberg-Euler, Z. Phys. ('36); Schwinger, Phys. Rev. ('51)]

$$\mathcal{L}_{eff} = -F - \frac{1}{8\pi^2}\int_0^\infty ds \frac{e^{-m^2 s}}{s^3}\left[(es)^2 G \frac{\Re e(\cosh eXs)}{\Im m(\cosh eXs)} - 1 - \frac{2}{3}(es)^2 F\right]$$

- Feynman diagrams: internal loop for fermion or bosons and external legs for photons



Heisenberg-Euler/Schwinger QED Action

- When $G \neq 0$, in a Lorentz frame in which $\vec{E} \parallel \vec{B}$, the Heisenberg-Euler and Schwinger's one-loop action takes the form ($[\cdot] = 1$ for scalar QED)

$$\mathcal{L}_{eff}^{(1)} = -\frac{(eE)(eB)}{2(2\pi)^2}\int_0^\infty ds\,\frac{e^{-m^2 s}}{s^3}\left[s^2\frac{[\cosh(eBs)\cos(eEs)]}{\sinh(eBs)\sin(eEs)} - \frac{1}{(eE)(eB)} - \frac{s^2}{3\cdot[2]}\frac{B^2 - E^2}{EB}\right]$$

- The vacuum persistence amplitude for spinor QED (twice the imaginary part of effective action)

$$2\Im m\left(\mathcal{L}_{eff}^{(1)}\right) = \frac{(eE)(eB)}{(2\pi)^2}\sum_{n=1}^{\infty}\frac{1}{n}\coth\left(\frac{\pi B}{E}n\right)e^{-\frac{\pi m^2}{eE}n}$$

- The vacuum persistence amplitude for scalar QED

$$2\Im m\left(\mathcal{L}_{eff}^{(1)}\right) = \frac{(eE)(eB)}{2(2\pi)^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\operatorname{csch}\left(\frac{\pi B}{E}n\right)e^{-\frac{\pi m^2}{eE}n}$$

Gamma-Function Regularization & QED Action beyond Schwinger

Γ-Function Regularization

- Most of soluble models in QED and gravity have the Bogoliubov coefficients of the form

$$\alpha_k = A_k \prod \frac{\Gamma(a \pm ib)}{\Gamma(c \pm id)}, \beta_k = B_k \prod \frac{\Gamma(f \pm ig)}{\Gamma(h \pm ik)}$$

  – a, …, h: integers or half-integers depending on spins
  – Constants A_k and B_k to be regulated away

- Integral representation for gamma-function

$$\ln \Gamma(a \pm ib) = \int_0^\infty \frac{dz}{z} \left[\frac{e^{-(a \pm ib)z}}{1 - e^{-z}} - \frac{e^{-z}}{1 - e^{-z}} + (a \pm ib - 1) e^{-z} \right]$$

Complex One-Loop Effective Actions

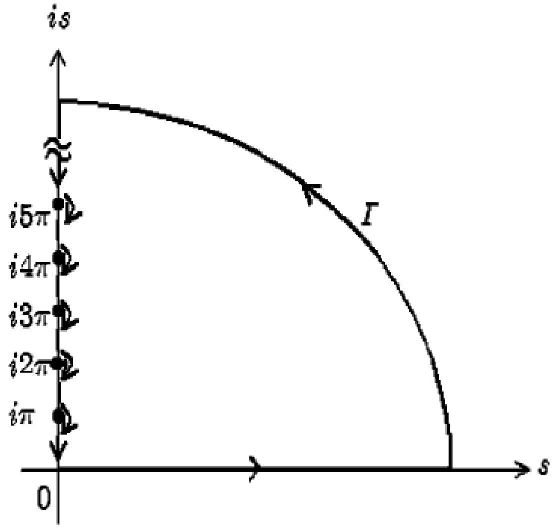
- One-loop effective action for scalars/fermions in the in-out formalism

$$\mathcal{L}^{(1)} = \pm i \sum_k D(k) \ln(\alpha_k^*)$$

- Γ- function regularization [SPK, Lee, Yoon, PRD 78 (‘08), 82 (‘10); SPK, PRD 84 (‘11)]

$$\int_0^\infty \frac{dz}{z} \frac{e^{-(a\pm ib)z}}{1-e^{-z}} = P\int_0^\infty \frac{ds}{s} \frac{e^{-(a\pm ib)(\mp is)}}{1-e^{\pm is}} \mp \pi i \sum_{n=1}^\infty \frac{e^{-(a\pm ib)(\mp 2n\pi i)}}{\mp 2n\pi i}$$

- Cauchy residue theorem



Magnetic Fields in GR

Melvin's Magnetic Universe

- One parameter family of solutions of Einstein-Maxwell theory [Melvin, Phys. Rev. 139 ('65)]
  - $ds^2 = \left(1 + \frac{1}{4}B^2\rho^2\right)^2 (-dt^2 + d\rho^2 + dz^2) + \left(1 + \frac{1}{4}B^2\rho^2\right)^{-2} \rho^2 d\phi^2$
  - $t, z \in (-\infty, +\infty), \rho \in [0, \infty), \phi \in [0,2\pi)$
- Non-null electromagnetic field with the complex self-dual Maxwell tensor
  - $F + i\tilde{F} = e^{-i\psi} B \left(dz \wedge dt + i\left(1 + \frac{1}{4}B^2\rho^2\right)^{-2} \rho d\rho \wedge d\phi\right)$
  - ψ parameterizes the freedom in duality rotation
  - $\psi = 0$: an electric field along the z-direction $[F = B dz \wedge dt]$
  - $\psi = \frac{\pi}{2}$: a pure magnetic field along the z-direction $\left[iF = B\left(1 + \frac{1}{4}B^2\rho^2\right)^{-2} \rho d\rho \wedge d\phi\right]$

Magnetic Black Hole

- Charged RN and (dyonic) Kerr-Newman black holes from the Einstein-Maxwell theory are known.
- No solutions are known yet for the Einstein-Maxwell+QED theory
  - $G_{\mu\nu} = 8\pi\left(T_{\mu\nu}^{Maxwell} + T_{\mu\nu}^{QED}\right)$
- Magnetic black hole studied in the background of Maxwell+Heisenberg-Euler-Schwinger action [SPK, Page, Italian-Korean Sympoisum 2003, JKPS Suppl. 45 ('04)]; action in constant B field
  - $L_{MQ} = -\frac{B^2}{8\pi}\left(1 - \frac{e^2}{\pi} I(b)\right)$, $b = \frac{eB}{m^2} = \frac{B}{B_c}$
  - $I(b) = \int_0^\infty dx \frac{e^{-bx}}{x^3}\left(1 + \frac{x^2}{3} - x \coth x\right)$
  - $\varrho = T_{\hat{0}\hat{0}} = -L$, $P_r = T_{\hat{r}\hat{r}} = L$, $P_\perp = T_{\hat{\theta}\hat{\theta}} = \frac{B^2}{8\pi}\left[1 - \frac{e^2}{\pi}\left(I(b) + b\frac{dI}{db}\right)\right]$
- QED vacuum polarization causes all magnetic black holes to be unstable emitting smaller magnetic black holes.

Schwinger Mechanism in E-B & dS Space

Bavarsad, SPK, Stahl, Xue,

Phys. Rev. D 97 ('18) & EPJ Web Conf. 168 ('18)

Schwinger Effect in E ∥ B & dS_4

- Assumptions:
  - ✓ density of created pairs smaller than vacuum density

$$\frac{(qE)^2|\gamma|}{2(2\pi)^2}e^{-2\pi|\gamma|} \ll 3H^2$$

  - ✓ massive charged particle production

$$|\gamma|^2 \equiv \left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \left(\frac{3}{2}\right)^2 \geq 0$$

- Conformal metric for 4-dimensional dS and parallel E & B

$$ds^2 = \Omega^2(\tau)(-d\tau^2 + dx^2 + dy^2 + dz^2)\,,\ \Omega(\tau) = -\frac{1}{H\tau}\,,\ \tau \in (-\infty, 0)$$

$$A_\mu = -\frac{E}{H^2\tau}\delta_\mu^3 + By\delta_\mu^1$$

Schwinger Effect in E ∥ B & dS_4

- Positive and negative solutions at $\tau = -\infty$ and$\tau = 0$ for KG equation for charges
  - ✓ In-vacuum (Hadamard state)

$$\varphi_{in}^{(+)} = \frac{e^{i\pi\kappa/2}}{\sqrt{2k\Omega}} e^{i\vec{x}\cdot\vec{\tilde{k}}_y} h_n(y_+) W_{\kappa,\gamma}\left(2e^{-i\pi/2}p\right)$$

$$\varphi_{in}^{(-)} = \frac{e^{-i\pi\kappa/2}}{\sqrt{2k\Omega}} e^{-i\vec{x}\cdot\vec{\tilde{k}}_y} h_n(y_-) W_{\kappa,-\gamma}\left(2e^{i\pi/2}p\right)$$

  - ✓ Out-vacuum

$$\varphi_{out}^{(+)} = \frac{e^{-i\pi\gamma/2}}{\sqrt{4|\gamma|k\Omega}} e^{i\vec{x}\cdot\vec{\tilde{k}}_y} h_n(y_+) M_{\kappa,\gamma}\left(2e^{-i\pi/2}p\right)$$

$$\varphi_{out}^{(-)} = \frac{e^{i\pi\gamma/2}}{\sqrt{4|\gamma|k\Omega}} e^{-i\vec{x}\cdot\vec{\tilde{k}}_y} h_n(y_-) M_{\kappa,-\gamma}\left(2e^{i\pi/2}p\right)$$

$$y_\pm = \sqrt{qB}y \pm \frac{k_x}{\sqrt{qB}}, \qquad h_n(y_\pm) = \left(\frac{qB}{\pi(2^n n!)^2}\right)^{1/4} e^{-y_\pm^2/2} H_n(y_\pm), \qquad p = -\tau k$$

Schwinger Effect in E ∥ B & dS$_4$

- Bogoliubov coefficients

$$\alpha = \frac{(2|\gamma|)^{1/2}\Gamma(2\gamma)}{\Gamma\left(\frac{1}{2}+\kappa+\gamma\right)} e^{i\pi(\kappa-\gamma)/2}, \qquad \beta = -i\frac{(2|\gamma|)^{1/2}\Gamma(-2\gamma)}{\Gamma\left(\frac{1}{2}+\kappa-\gamma\right)} e^{i\pi(\kappa+\gamma)/2}$$

$$k = \sqrt{k_z^2 + qB(2n+1)}, \qquad \mu = \frac{m}{H}, \qquad \lambda = \frac{qE}{H^2}$$

$$\kappa = i\lambda\frac{k_z}{k}, \qquad \gamma = \sqrt{\left(\frac{3}{2}\right)^2 - (\lambda^2+\mu^2)}, \qquad l = qB\tau^2$$

Schwinger Effect in E ∥ B & dS_4

- Pair production rate

$$\Gamma = \frac{1}{\Omega^4 T L_y} \sum_{n=0}^{\infty} \int \frac{dk_z}{(2\pi)} \frac{dk_y}{(2\pi)} |\beta(k_z, n)|^2$$

- Under the semiclassical condition

$$\Gamma = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{qB\Omega^{-2}}{2\pi}\right) \sum_{n=0}^{\infty} \left[\frac{e^{2\pi|\kappa_n|} - 1}{e^{2\pi|\gamma|} - e^{-2\pi|\gamma|}} + \frac{1}{e^{2\pi|\gamma|} - 1}\right]$$

$$\kappa_n = \frac{\lambda|\gamma|}{\sqrt{|\gamma|^2 + l(2n+1)}}$$

- Riemann zeta-function prescription

$$\Gamma = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{1}{e^{4\pi|\gamma|} - 1}\right)\left[\frac{1}{2} + \sum_{n=0}^{\infty} e^{2\pi(|\gamma| + |\kappa_n|)}\right]$$

Induced Current

- Induced current

$$J = \frac{qH^3 l}{(2\pi)^2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dk_z}{k} (\lambda - k_z \tau) e^{i\pi\kappa} \left| W_{\kappa,\lambda}(2ik\tau) \right|^2$$

- In the weak magnetic field regime $l \ll min(1, \lambda, \mu)$

$$J = \left(\frac{qH^3}{(2\pi)^2} \right) \frac{l\gamma \sinh(2\pi\lambda)}{\sin(2\pi\gamma)}$$

- In the weak magnetic field and strong electric field

$$J_{reg} = \frac{q}{H} \left(\frac{qB\Omega^{-2}}{2\pi} \right) \left(\frac{qE}{2\pi} \right) e^{-\frac{\pi m^2}{|qE|}}$$

- Hyperconductivity in the infrared regime $l \ll \mu \ll \lambda \ll 1$

$$J_{reg} = \frac{9}{2} \left(\frac{qB\Omega^{-2}}{2\pi} \right) \left(\frac{qE}{2\pi} \right) \frac{qH^3}{(qE)^2 + (mH)^2}$$

Induced Current

- Hyperconductivity in the infrared regime $l \ll \mu \ll \lambda \ll 1$

$$J_{reg} = \frac{9}{2}\left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{qE}{2\pi}\right)\frac{qH^3}{(qE)^2+(mH)^2}$$

- In the strong magnetic field regime $l \gg max(1, \lambda, \mu)$

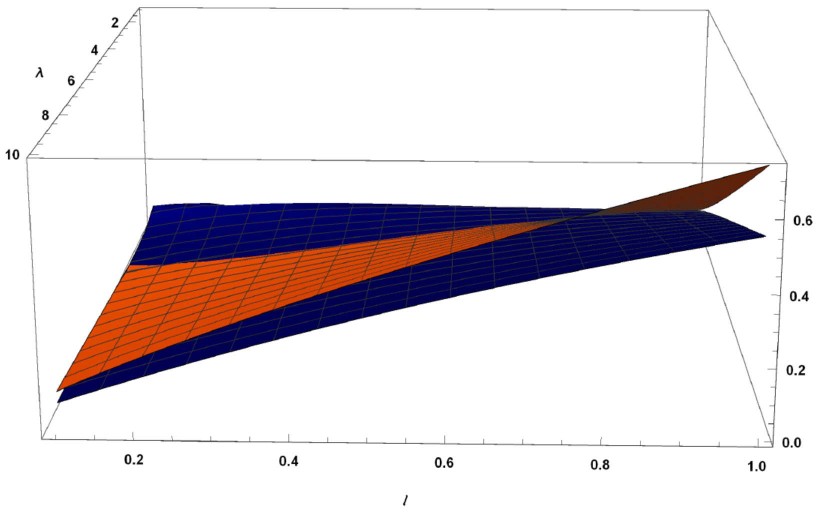
$$J_{reg} \cong (\gamma_{Euler} + \ln 4)\frac{qH^3 l\lambda}{2(2\pi)^2}$$

$$\approx \frac{q}{H}\left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{qE}{2\pi}\right)$$

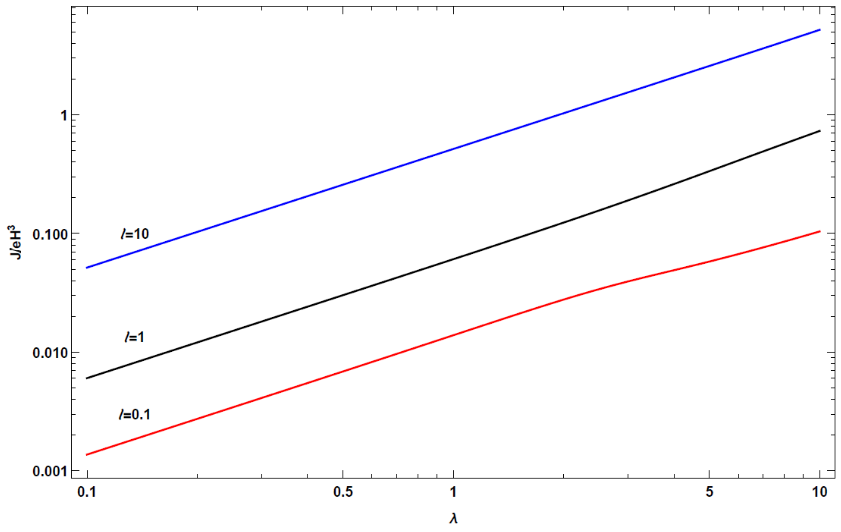
Induced Current

Induced current J/eH^3 vs

Semiclassical J_{sem}/eH^3 (blue)



Induced current J/eH^3

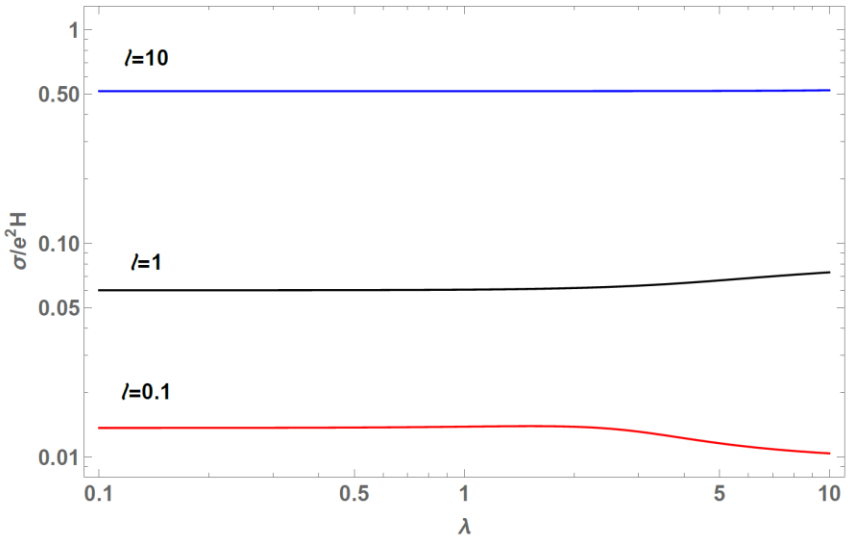


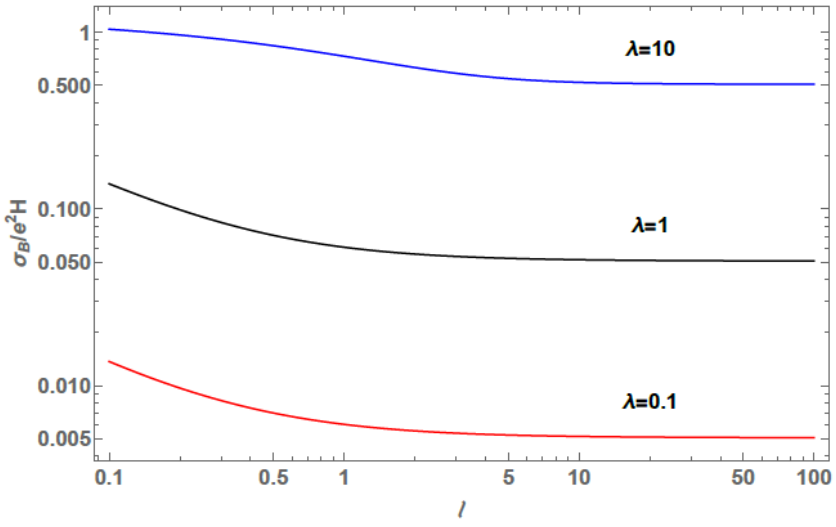
$l = eB\tau^2$, $\lambda = \frac{eE}{H^2}$ in the Lowest Landau Level

Induced Current

Electric conductivity $\sigma = J/E$
& Normalized one σ/e^2H

Magnetic conductivity $\sigma_B = J/B\Omega^{-2}$
& Normalized one σ_B/e^2H





$$l = eB\tau^2,\ \lambda = \frac{eE}{H^2}$$

QED Action in E-B & dS Space

Bavarsad, SPK, Stahl, Xue, in progress

QED Action in E in dS_4

- One-loop effective action for scalar QED

$$\mathcal{L}_{sc}^{(1)} = \frac{H^2|\gamma|}{2(2\pi)} \int \frac{d^2k_\perp}{(2\pi)^2} P \int_0^\infty \frac{ds}{s} \left[e^{-2|\gamma|s} \frac{\cos(s/2)}{\sin(s/2)} - e^{-(|\gamma|+\kappa_k(0))s} \frac{1}{\sin(s/2)} - \cdots \right]$$

$$\kappa_k(0) = -\frac{qE}{H^2} \frac{\left({}^{qE}/_{H}\right)}{\sqrt{\left({}^{qE}/_{H}\right)^2 + \vec{k}_\perp^2}}$$

- Vacuum persistence

$$2\Im\mathcal{L}_{sc}^{(1)} = \frac{H^2|\gamma|}{(2\pi)} \int \frac{d^2k_\perp}{(2\pi)^2} \ln\left(\frac{1+e^{-2\pi\left(|\gamma|+\kappa_k(0)\right)}}{1-e^{-4\pi|\gamma|}} \right) = \frac{H^2|\gamma|}{(2\pi)} \int \frac{d^2k_\perp}{(2\pi)^2} \ln\left(1 + N_{\vec{k}_\perp}\right)$$

QED Action in E ∥ B in dS$_4$

- One-loop effective action for scalar QED

$$\mathcal{L}_{sc}^{(1)} = \left(\frac{H^2|\gamma|}{2(2\pi)}\right)\left(\frac{qB\Omega^{-2}}{2\pi}\right)\sum_{n=0}^{\infty} P\int_0^{\infty}\frac{ds}{s}\left[e^{-2|\gamma|s}\frac{\cos(s/2)}{\sin(s/2)} - e^{-(|\gamma|+\kappa_n(0))s}\frac{1}{\sin(s/2)} - \cdots\right]$$

$$\kappa_n(0) = -\frac{qE}{H^2}\frac{\left({}^{qE}/_{H}\right)}{\sqrt{\left({}^{qE}/_{H}\right)^2 + qB(2n+1)}}$$

- Vacuum persistence

$$2\Im\mathcal{L}_{sc}^{(1)} = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{qB\Omega^{-2}}{2\pi}\right)\sum_{n=0}^{\infty}\ln\left(\frac{1+e^{-2\pi(|\gamma|+\kappa_n(0))}}{1-e^{-4\pi|\gamma|}}\right) = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{qB\Omega^{-2}}{2\pi}\right)\sum_{n=0}^{\infty}\ln(1+N_n)$$

Constraint on Primordial Magnetic Field Generation

Stahl, Nucl. Phys. B 939 (2019)

Bavarsad, SPK, Stahl, Xue, in progress

Astrophysical/Cosmological Magnetic Fields

Astrophysical Magnetic Fields

- Magnetic fields of μG amplitude observed in galaxies, clusters and redshift objects ($z < 4$).
- Observed correlation on scales of the order of the object size: hard to explain.
- To generate the galactic fields of μG amplitude by flux conservation during the formation of galaxies, seed fields of about nG are required.

Cosmological Magnetic Fields

- Lower bound on magnetic field amplitude in the intergalactic medium from observation of blazars with gamma ray telescopes

$$B_{Mpc} > 6 \times 10^{-18} G \quad \text{[Vovk 1112.2534]}$$

- The origin is not yet understood: after recombination (related to structure formation) or primordial?

Primordial Magnetic Field

- Assuming a primordial magnetic field in the universe could explain
  - Observations in all structure & at high redshift: astrophysical magnetic field
  - Lower bound in the intergalactic medium: cosmological magnetic field
- Many generation mechanisms proposed but none preferred:

Causal mechanism	Non causal mechanism
• Phase transition, MHD turbulence, charge & current density + vorticity, …	• Inflation: vacuum fluctuations generate EM field; after reheating, conductivity in the universe is very large, E-field dissipates away & B-field stays.
• Challenge: small correlation length & blue spectrum & too small seeds on cosmologically relevant scales	• Generation at all scales, spectrum can be red.

- Assume that a conformal symmetry breaking generates an EM-field during inflation.

Constraint on Primordial Magnetic Field

- Strong coupling problem
- Backreaction problem
- Schwinger effect

IFF model

- Need to break conformal invariance to generate EM-field
- A bound for the current magnetic field (Kobayashi 1408.4141)

$$B_0 \leq 10^{-28} G \left(\frac{k}{a_0} Mpc\right) \left(\frac{H_{inf}}{M_p}\right)^{1/2} \left(\frac{\sqrt{4\pi\alpha}}{|e|}\right)^3 I_{end}^2 Q$$

Tighter bound

- The result of Bavasard 1707.03975 updates that of Kobayashi

$$B_0 \leq 10^{-28} G \left(\frac{k}{a_0} Mpc\right)^2 \left(\frac{H_{inf}}{M_p}\right)^{1/2} \left(\frac{\sqrt{4\pi\alpha}}{|e|}\right)^3 I_{end}^2 \tilde{Q}$$

Conclusion

- Strong intertwinement of Maxwell theory (QED) and (quantum) gravity in (near-)extremal black holes
  - Spontaneous pair production
  - Vacuum polarization (one-loop action) via In-out formalism and Γ-regularization
- Collapse of magnetars or mergers of neutron stars
  - Astrophysical objects with supercritical-field
  - X-rays, gamma rays and optical spectrum following gravitational waves are affected by strong QED, gravity etc.
- QED phenomena in (anti-) de Sitter space (cosmology)
  - Effective temperature interpretation of Schwinger mechanism
  - Current in (A)dS
  - Constraint on primordial magnetogenesis