Metric-affine Geometry and Scalar-tensor theories

Asian-Pacific Winter School and Workshop on Gravitation and Cosmology

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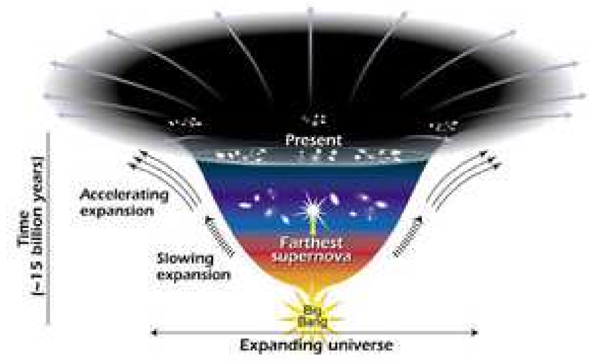
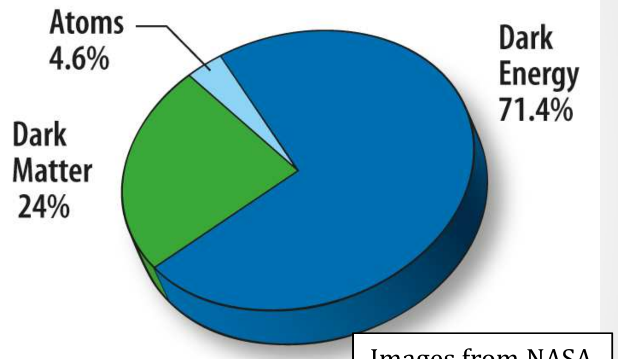
Based on (KS, Katsuki Aoki, Kei-ichi Maeda) arXiv:1812.0342[gr-qc]

PRD Submitted

Introduction





Images from NASA

✓ There are two tools in Cosmology
- ➢ GR + SM of Particle Physics

➢ Couple of Mysteries in the Universe:

Inflation? Dark Energy? Dark Matter?...etc…

- "Modified Gravity" as one approach

Finding alternatives to GR

✓ Metric Formalism (Riemann Geometry)
1. Introduce new d.o.f.
2. Higher Derivative/Curvature

✓ Metric-affine Formalism (Metric-affine Geometry)
1. Introduce new d.o.f.?
2. Higher Derivative/Curvature?

Introduction: Metric-affine Geometry

In (Pseudo)-Riemann geometry…

1. Riemann metric $g_{\mu\nu}$:

   Define inner product / length : $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$

   Symmetric 2-rank Tensor

2. Connection $\left\{ {\lambda \atop \mu\nu} \right\}$: Levi-Civita Connection

   Define parallel transport

   Symmetric ($\left\{ {\lambda \atop \mu\nu} \right\} = \left\{ {\lambda \atop \nu\mu} \right\}$)

   Metric-compatibility ($\nabla_\lambda g_{\mu\nu} = 0$)

Introduction: Metric-affine Geometry

In (Pseudo)-Riem ⇓ **Decided from the gravitational action**

1. Riemann metric $g_{\mu\nu}$:

   Define inner product / length : $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$

Geometrically different variables

2. Connection $\left\{ {\lambda \atop \mu\nu} \right\}$: Levi-Civita Connection

   Define parallel transport ⇑ **Decided by hand**

   Symmetric ($\left\{ {\lambda \atop \mu\nu} \right\} = \left\{ {\lambda \atop \nu\mu} \right\}$), Metric-compatibility ($\nabla_\lambda g_{\mu\nu} = 0$)

Introduction: Metric-affine Geometry

Thus metric-affine geometry extends the geometry as…

1. Riemann metric $g_{\mu\nu}$:

   Define inner product / length : $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$

   Symmetric 2-rank Tensor

2. Connection $\Gamma^\lambda{}_{\mu\nu}$: Arbitrary Affine Connection

   Define parallel transport

   ~~Symmetric ($\left\{ {\lambda \atop \mu\nu} \right\} = \left\{ {\lambda \atop \nu\mu} \right\}$)~~

   ~~Metric compatibility ($\nabla_\lambda g_{\mu\nu} = 0$)~~

Torsion: $T^\lambda{}_{\mu\nu} := \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}$

Non-metricity: $Q_\alpha{}^{\beta\gamma} := \nabla^\Gamma_\alpha g^{\beta\gamma}$

Metric-affine Gravity

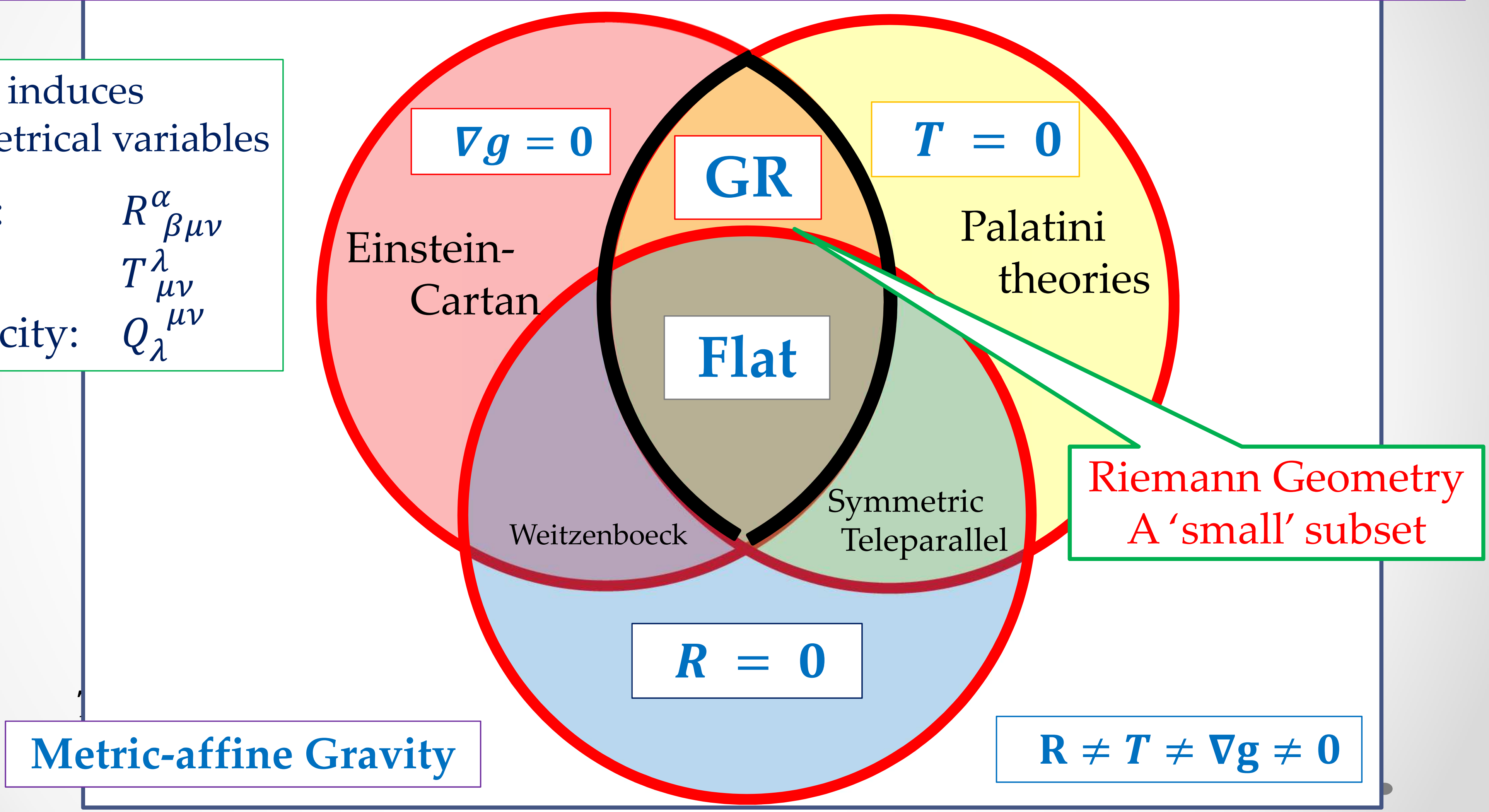
The Landscape of Metric-affine Gravity

Connection induces
three geometrical variables

Curvature: $R^{\alpha}{}_{\beta\mu\nu}$

Torsion: $T^{\lambda}{}_{\mu\nu}$

Non-metricity: $Q_{\lambda}{}^{\mu\nu}$



$$R^{\alpha}{}_{\beta\mu\nu}(\Gamma) \equiv \partial_\mu \Gamma^{\alpha}{}_{\nu\beta} - \partial_\nu \Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\alpha}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\beta} - \Gamma^{\alpha}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\beta}$$

EH action in Metric-affine Formalism

- Consider applying Metric-affine formalism to the EH action (Einstein, 1925):

Riemann metric $g_{\mu\nu}$ and the connection $\Gamma^{\lambda}_{\sigma\rho}$ are independent

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\, R(g,\Gamma) + S_M(g,\{\ \ \})$$

$$\Downarrow$$

$$\Gamma^{\lambda}_{\sigma\rho} = \left\{ \begin{matrix} \lambda \\ \sigma\rho \end{matrix} \right\} + U_\sigma \delta^{\lambda}_{\rho}$$

U_σ: Some arbitrary vector (Projective Mode)
The action is invariant under projective symmetry

$$R^\alpha{}_{\beta\mu\nu}(\Gamma) \equiv \partial_\mu \Gamma^\alpha{}_{\nu\beta} - \partial_\nu \Gamma^\alpha{}_{\mu\beta} + \Gamma^\alpha{}_{\mu\lambda}\Gamma^\lambda{}_{\nu\beta} - \Gamma^\alpha{}_{\nu\lambda}\Gamma^\lambda{}_{\mu\beta}$$

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$$\Downarrow$$

$$\Gamma^\lambda_{\sigma\rho} = \left\{ \begin{matrix} \lambda \\ \sigma\rho \end{matrix} \right\}$$

1. Choose $U_\sigma = 0$:
   Ex) $T = 0$, $\nabla g = 0$
2. Levi-Civita as a solution

The Equivalence between metric formalism and metric-affine formalism breaks in general for alternative theories of gravity

Application to Inflation

- Consider a "minimally coupled" scalar as an inflaton

$\bar{\Box}$: d'Alembertian operator in flat space

$$L_{\phi,flat} = \frac{1}{2}\phi\bar{\Box}\phi - V(\phi)$$

$$\parallel$$

Up to surface terms

$$-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

"Covariantization" to curved metric-affine geometry?

Application to Inflation

- In Flat space…

$$L_{\phi,flat} = \frac{1}{2}\phi\bar{\Box}\phi - V(\phi)$$

- In Metric-affine spacetime

Due to non-metricity..

$$\Box^{\Gamma} := \alpha\nabla^{\Gamma\mu}\nabla^{\Gamma}_{\mu} + (1-\alpha)\nabla^{\Gamma}_{\mu}\nabla^{\Gamma\mu}$$

KS, Aoki, Maeda(2018)

$$L(g,\Gamma,\phi) = \frac{M_{Pl}^2}{2}R(g,\Gamma) + \frac{1}{2}\phi\Box^{\Gamma}\phi - V(\phi)$$

⇐ We will consider this

$$\neq$$

$$L_{MA} = \frac{M_{Pl}^2}{2}R(g,\Gamma) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

Covariantization is not trivial!

$$\Box^{\Gamma} := \alpha \nabla^{\Gamma\mu} \nabla^{\Gamma}_{\mu} + (1-\alpha) \nabla^{\Gamma}_{\mu} \nabla^{\Gamma\mu}$$

Solving the Connection

Interesting results for inflationary scenario is obtained
when the connection does not have torsion $\left(T^{\lambda}{}_{\mu\nu} = 0\right)$

The inflaton action based on metric-affine formalism

$$L(g,\Gamma,\phi) = \frac{M_{Pl}^2}{2} R(g,\Gamma) + \frac{1}{2} \phi \Box^{\Gamma} \phi - V(\phi)$$

The Euler-Lagrangian equations are algebraic

$$\Gamma^{\lambda}{}_{\mu\nu} = \left\{ \begin{matrix} \lambda \\ \mu\nu \end{matrix} \right\} + \frac{\phi}{6M_{Pl}^2} \left[3(\alpha - 1) g_{\mu\nu} \partial^{\lambda} \phi + 2(\alpha + 1) \delta^{\lambda}_{(\mu} \partial_{\nu)} \phi \right]$$

The connection does not introduce new d.o.f.(does not propagate)

$\because$ In $L(g,\Gamma,\phi)$ there is no kinetic term for Γ

We may substitute the solution of Γ into the action

No Torsion is considered

Towards a Riemannian action…

Substituting the connection

$$\Box^{\Gamma} := \alpha \nabla^{\Gamma\mu} \nabla^{\Gamma}_{\mu} + (1-\alpha) \nabla^{\Gamma}_{\mu} \nabla^{\Gamma\mu}$$

$$L(g,\Gamma,\phi) = \frac{M_{Pl}^2}{2} R(g,\Gamma) + \frac{1}{2}\phi\Box^{\Gamma}\phi - V(\phi)$$

Since Γ does not propagate $\Downarrow$ $\Gamma^{\lambda}_{\mu\nu} = \{\ \ \} + \cdots$

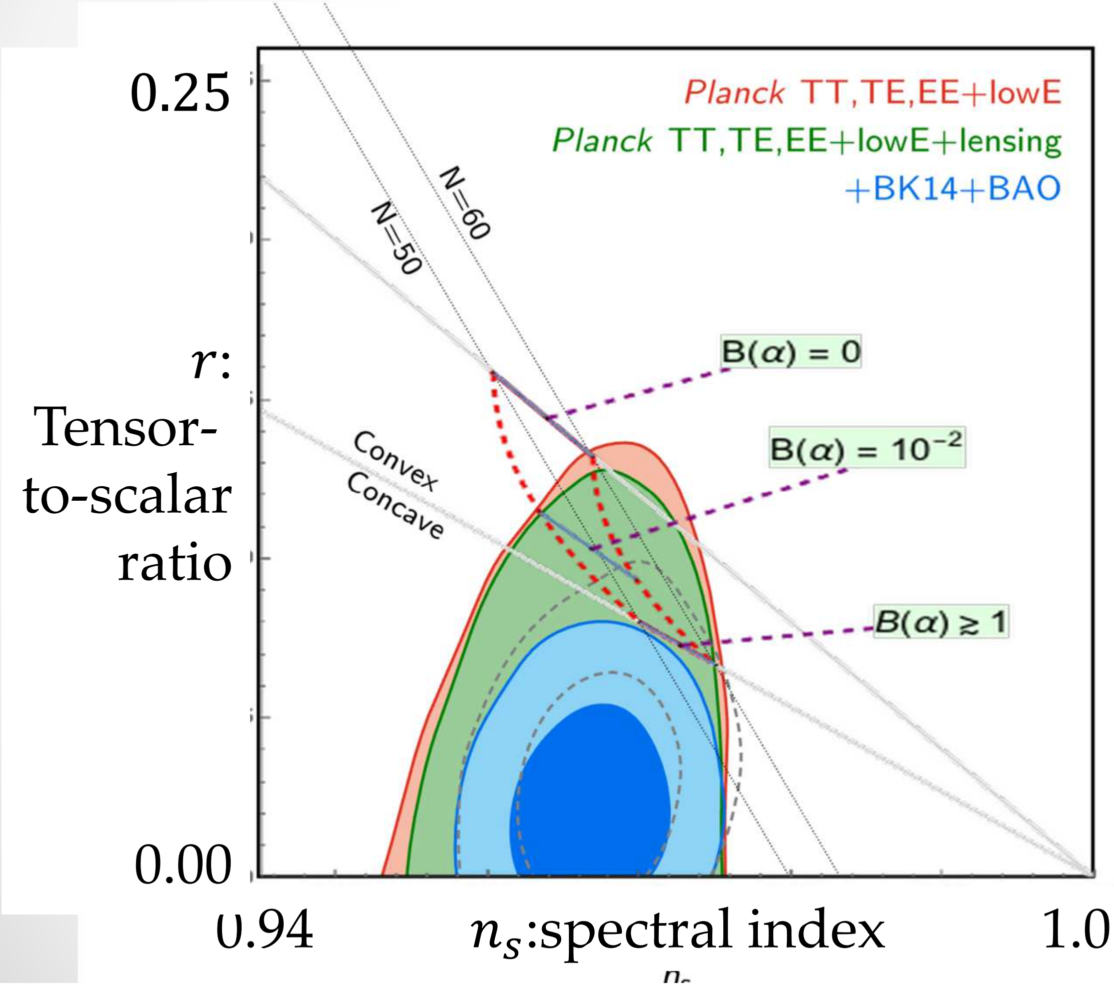
$$L(g,\phi) = \frac{M_{Pl}^2}{2} R(g,\{\}) - \frac{1}{2}\left(1 + B(\alpha)\frac{\phi^2}{M_{Pl}^2}\right)\partial^{\mu}\phi\partial_{\mu}\phi - V(\phi)$$

$$B(\alpha) = \frac{1}{6}(11\alpha^2 - 8\alpha - 1)$$

- Re-written into a equivalent "Riemann geometry based" action

$$\Box^{\Gamma} := \alpha \nabla^{\Gamma\mu}\nabla^{\Gamma}_{\mu} + (1-\alpha)\nabla^{\Gamma}_{\mu}\nabla^{\Gamma\mu}$$

Constraints from the CMB ($n_s - r$)



Comparing

Riemannian and metric-affine

using Planck2018 results.

For the potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

Constraints on "covariantization"

$$\alpha \gtrsim 0.84 \quad \text{or} \quad \alpha \lesssim -0.12$$

Chaotic inflation is '**revived**'

with metric-affine gravity

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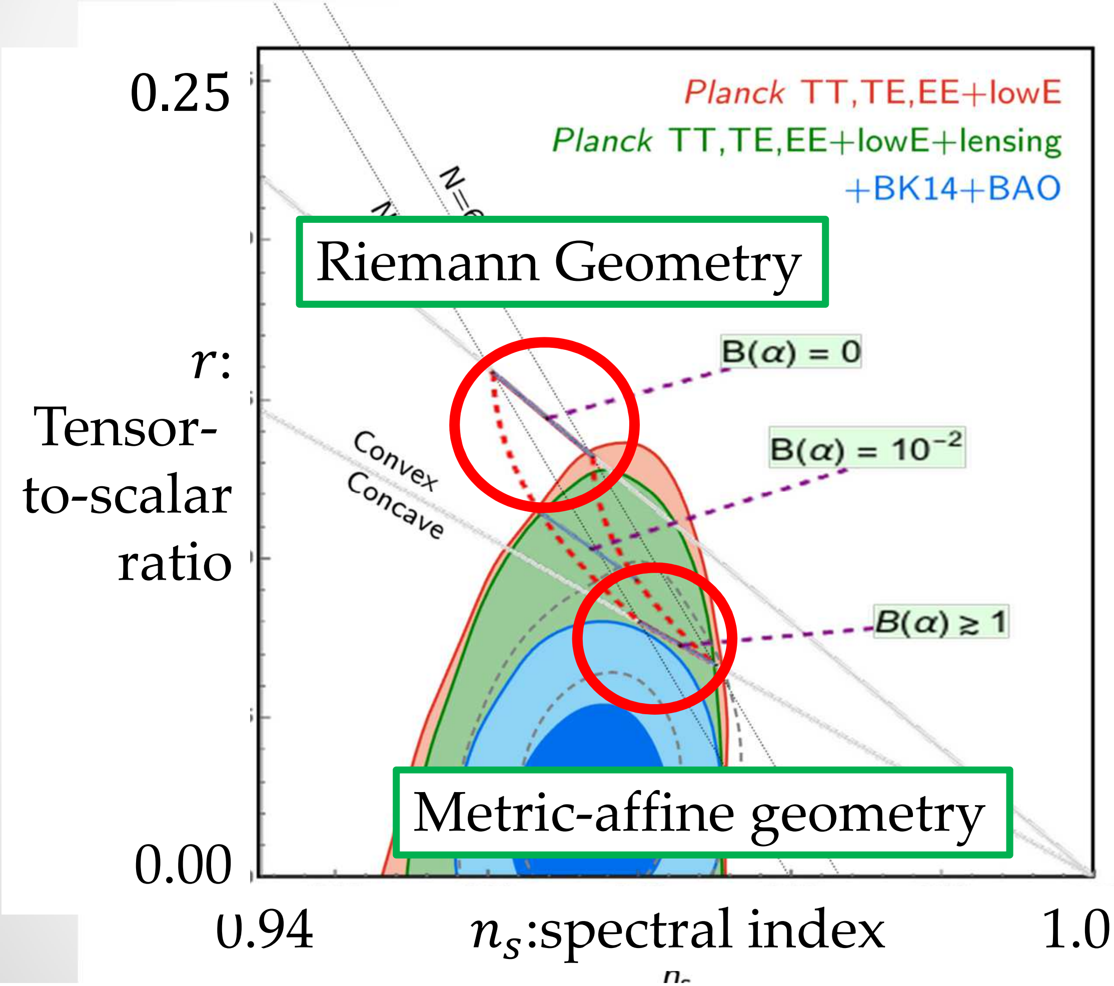

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Summary and Future Prospects

- ✓ Metric-affine formalism treats $g_{\mu\nu}$ and $\Gamma^{\lambda}_{\ \mu\nu}$ independently
- ✓ Covariantization is not trivial in metric-affine gravity
- ✓ When the connection does not propagate one may obtain a classically equivalent "*Riemannian geometry based*" action
- ✓ Inflationary models constructed on metric-affine geometry behaves differently compared to its Riemann counterpart

Summary and Future Prospects

? Further generalizations of scalar-tensor theories in metric-formalism?

? (Higher Derivatives/ Coupling with curvature)

? [Aoki, KS; PRD98(2018) no.4, 044038] / In preparation (Aoki, KS)

? What happens when the full connection propagates?

Extra d.o.f. from the connection. Is it Ghost-free?

? Standard model particles behaves differently

depending on spin in metric-affine geometry.

Different geodesics between Fermions and Bosons (Using WKB+Fermat)

Cosmological Applications?

Thank You for Attention

Questions and Discussions are very welcomed