Kinetic equation for Lifshitz scalar

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Based on

1812.10983
with S. Mukohyama

Outline

- Motivation
- Kinetic eq. from action
- Kinetic eq. for Lifshitz scalar

Intro: Hořava-Lifshitz (HL) gravity

Hořava 0901.3775

- GR: non-renormalizable

  HL: a candidate of quantum gavity

  achieved by Lorentz breaking $\omega^2 \simeq \dfrac{p^{2z}}{M^{2z-2}}$ @ $p \gtrsim M$

  Propagator $\sim \dfrac{1}{\omega^2 - p^{2z}/M^{2z-2}}$: more convergent

  $z \geq d$ in $(d+1)$ dim

  Renormalizability has been proven in minimal setup $N = N(t)$

  Barvinsky, Blas, Herrero-Valea, Sibiryakov, Steinwachs 1512.02250

- Foliation-preserving diffeo.

$$t \to t'(t), \qquad \vec{x} \to \vec{x}'(t, \vec{x})$$

- No local Hamiltonian constraint
- 2+1 DoF: scalar graviton behaves as dark matter

Mukohyama 0905.3563

HL gravity as an alternative to inflation

- HL has some properties of inflation
  - Scale-invariant perturbation — Mukohyama 0904.2190
  - Solves Horizon problem
  - Solves Flatness problem — Bramberger, Coates, Magueijo, Mukohyama, Namba, YW 1706.06809
- Isotropy problem

Vector perturbation around flat FLRW in GR

$$(\text{Vorticity}) \propto a^{-\frac{1}{2}(1-9w)}$$

w: EOS param. of matter

Albrecht, Magueijo 9811018

Kodama, Sasaki (1984) PTP Suppl.

If $w > 1/9$, vorticity grows: isotropy problem

cf. Inflation ($w \simeq -1$) is a solution

- This can be used as a test for an alternative to inflation

Does HL solve the isotropy problem?: Goal

This talk is 1st step

Motivation

- Consider the action of HL gravity & vector field A_i
  - Photons are described by distribution func. $f(x^\mu, p_i)$
    Need evolution eq. for f
  - In ordinary cases f obeys the Boltzmann eq.
    derivable from 1st principle

What is the Boltzmann eq. for Lifshitz vector?

Outline

- **Previous work**

  Derived Boltzmann eq for Lorentz scalar

  on curved space

- **Purpose**

  Derive Boltzmann eq for Lifshitz vector

  on curved space

- **Result**

  Derived Boltzmann eq for Lifshitz scalar

  on curved space

Derivation of kinetic eq.

- Relativistic kinetic eq.

$$\left[p^\mu \partial_\mu + p^\mu p_\nu \Gamma^\nu_{\mu i} \frac{\partial}{\partial p_i}\right] f \simeq \text{(interactions, corrections)}$$

- Method: use Wigner func.

de Groot, van Leeuwen, van Weert (1980)

$$\tilde{f}_{\text{flat}}(x^\mu, p_i) = \int d^3 r \, e^{-\frac{i}{\hbar} r^i p_i} \left\langle : \phi\left(x + \frac{r}{2}\right) \phi\left(x - \frac{r}{2}\right) : \right\rangle$$

  - Known to systematically derive interaction, quantum correction & field-theoretic correction terms
  - Formalism for curved spacetime is developed

Winter (1985)
Calzetta, Habib, Hu (1988)
Fonarev 9309005
Antonsen 9701182

Review: for relativistic real scalar

Friedrich, Prokopec 1805.02767
based on 3+1 decomposition

- Wigner func. on curved spacetime

$$F_{XY}(x^\mu, p_i) = \sqrt{\gamma}\int d^3r\, e^{-\frac{i}{\hbar}r^i p_i}\left\langle : \left[e^{\frac{r^i}{2}\nabla_i^H} X(x^\mu)\right]\left[e^{-\frac{r^i}{2}\nabla_i^H} Y(x^\mu)\right] : \right\rangle$$

$$= \left(1 + \frac{r^i}{2}\nabla_i + \frac{r^i r^j}{8}\nabla_i\nabla_j + \cdots\right) X$$

$$f_1^+ = \frac{1}{2\hbar}\left[\frac{\omega}{\hbar}F_{\phi\phi} + \frac{\hbar}{\omega}F_{\phi_\perp\phi_\perp}\right]$$

$$f_1^- = \frac{i}{2\hbar}\left[F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right]$$

$$f_2 = \frac{1}{2\hbar}\left[\frac{\omega}{\hbar}F_{\phi\phi} - \frac{\hbar}{\omega}F_{\phi_\perp\phi_\perp}\right]$$

$$f_3 = \frac{1}{2\hbar}\left[F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right]$$

$$X, Y = \{\phi, \phi_\perp = n^\mu \partial_\mu \phi\}$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N}\right)$$

Review: for relativistic real scalar

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$$I = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} \left[-\frac{1}{2} g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi \right]$$

$$\langle n^\mu n^\nu T_{\mu\nu} \rangle = \frac{1}{\sqrt{\gamma}} \int \frac{d^3p}{(2\pi\hbar)^3} \omega f_1 + \mathcal{O}(\hbar^2)$$

$$f_1 = f_1^+ + f_1^-$$

$$\langle n^\mu T_{\mu i} \rangle = \frac{1}{\sqrt{\gamma}} \int \frac{d^3p}{(2\pi\hbar)^3} p_i f_1 + \mathcal{O}(\hbar)$$

$$\langle T_{ij} \rangle = \frac{1}{\sqrt{\gamma}} \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p_i p_j}{\omega} f_1 + \mathcal{O}(f_2) + \mathcal{O}(\hbar)$$

➡ f_1: classical distribution func.

f_2, f_3: quantum, field-theoretic corrections

remain small if so initially, as seen later

Time evolutions

$$D_i = \nabla_i + \Gamma^k_{ij} p_k \frac{\partial}{\partial p_j}$$

- Time derivatives of Wigner func. using EOM: $\Box\phi$=(int.)

$$\partial_t F_{\phi\phi} \simeq \left(N^i D_i - (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} + NK \right) F_{\phi\phi} + \mathcal{O}(\hbar)\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) + N\left(F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right)$$

$$\partial_t F_{\phi_\perp\phi_\perp} \simeq \left(N^i D_i - (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} - NK \right) F_{\phi_\perp\phi_\perp} - \left(\frac{1}{\hbar} N p_i D^i \right) i\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) + \mathcal{O}\left(F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right)$$

$$\partial_t \frac{i}{2}\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) \simeq \left(N^i D_i - (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} \right) \frac{i}{2}\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) - \left(\frac{1}{\hbar} N p_i D^i \right) F_{\phi\phi} + \mathcal{O}\left(F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right)$$

$$f_1^+ = \frac{1}{2\hbar}\left[\frac{\omega}{\hbar} F_{\phi\phi} + \frac{\hbar}{\omega} F_{\phi_\perp\phi_\perp}\right]$$

$$f_1^- = \frac{i}{2\hbar}\left[F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right] \qquad f_1 = f_1^+ + f_1^-$$

$$\left[p^{(4)\mu}\partial_\mu + p^{(4)\mu} p^{(4)}_\nu \Gamma^{(4)\nu}_{\mu i} \frac{\partial}{\partial p_i}\right] f_1 \simeq \text{(interactions, corrections)}$$

Smallness of other Wigner func. f_2, f_3

$$\frac{\partial^2}{\partial\tau^2} f_2 = -\left(\frac{2\omega}{\hbar}\right)^2 \big(1 + \mathcal{O}(\hbar)\big) f_2$$

$$\frac{\partial^2}{\partial\tau^2} f_3 = -\left(\frac{2\omega}{\hbar}\right)^2 \big(1 + \mathcal{O}(\hbar)\big) f_3$$

$\mathrm{d}\tau = N dt$: proper time

- Other Wigner func. f_2, f_3 are oscillators

$\rightarrow$ They remain small if they are set so initially

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- Previous work

  Derived Boltzmann eq for Lorentz scalar

  on curved space

- Purpose

  Derive Boltzmann eq for Lifshitz vector

  on curved space

- Result

  Derived Boltzmann eq for Lifshitz scalar

  on curved space

S. Mukohyama and YW, 1812.10983

For Lifshitz scalar

- As a first step, we consider a Lifshitz scalar

$$I = \frac{1}{2}\int dt d^3x\, N\sqrt{\gamma}[(\partial_\perp \phi)^2 + \phi\mathcal{F}(\Delta)\phi]$$

$$z = 2: \qquad \mathcal{F}(\Delta) = -\beta\frac{\hbar^2}{M^2}\Delta^2 + c_\phi^2\Delta - \frac{m^2}{\hbar^2}$$

$$z = 3: \qquad \mathcal{F}(\Delta) = \alpha\frac{\hbar^4}{M^4}\Delta^3 - \beta\frac{\hbar^2}{M^2}\Delta^2 + c_\phi^2\Delta - \frac{m^2}{\hbar^2}$$

with the "projectability condition" $N = N(t)$ for simplicity.

- EOM

$$\partial_\perp^2\phi + K\partial_\perp\phi - \mathcal{F}(\Delta)\phi = 0$$

S. Mukohyama and YW, 1812.10983

Time evolutions

$$D_i = \nabla_i + \Gamma^k_{ij} p_k \frac{\partial}{\partial p_j}$$

➢ Time derivatives of Wigner func. using EOM

$$\partial_t F_{\phi\phi} \simeq \left(N^i D_i - (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} + NK \right) F_{\phi\phi} + \mathcal{O}(\hbar)\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) + N\left(F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right)$$

$$\partial_t F_{\phi_\perp\phi_\perp} \simeq \left(N^i D_i - (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} - NK \right) F_{\phi_\perp\phi_\perp} - \left(\frac{N}{\hbar}\omega v_g^i D_i\right) i\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) + \mathcal{O}\left(F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right)$$

$$\partial_t \frac{i}{2}\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) \simeq \left(N^i D_i - (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} \right) \frac{i}{2}\left(F_{\phi_\perp\phi} - F_{\phi\phi_\perp}\right) - \left(\frac{N}{\hbar}\omega v_g^i D_i\right) F_{\phi\phi} + \mathcal{O}\left(F_{\phi_\perp\phi} + F_{\phi\phi_\perp}\right)$$

$$v_g^i = \frac{\partial\omega}{\partial p_i}$$: group velocity

for $z = 2$ scalar on curved geometry

& $z = 3$ scalar on spatially flat geometry $\gamma_{ij} = a^2(t)\delta_{ij}$

Result

We obtained the Boltzmann-like eq. w/ $N = N(t)$:

$$\left[\partial_t - N^i D_i + (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} + N v_g^i D_i\right] f_1(t, x^i, p_i) \simeq 0$$

$$D_i = \nabla_i + \Gamma^k_{ij} p_k \frac{\partial}{\partial p_j}$$

$$v_g^i = \frac{\partial \omega}{\partial p_i}$$: group velocity

for $z = 2$ Lifshitz scalar on curved geometry

& $z = 3$ Lifshitz scalar on spatially flat geometry $\gamma_{ij} = a^2(t)\delta_{ij}$

cf. for Lorentz invariant scalar

$$\left[\partial_t - N^i D_i + (\nabla_i N^j) p_j \frac{\partial}{\partial p_i} + N \frac{p^i}{\omega} D_i\right] f_1(t, x^i, p_i) \simeq 0$$

Summary

- HL: candidate of quantum gravity & alternative to inflation
  - Alternative to inflation must solve isotropy problem in vector perturbation

    Photons: described by distribution func. $f(x^{\mu}, p_i)$
  - Derived kinetic eq. for f using Wigner func. for Lifshitz scalar as 1st step
- Future work
  - Obtain kinetic eq. for f for $z = 3$ vector
  - Obtain eq. for vector pert. combining EOM for gravity
  - See whether vorticity grows or not

$\rightarrow$ whether HL gravity can be an alternative to inflation or not