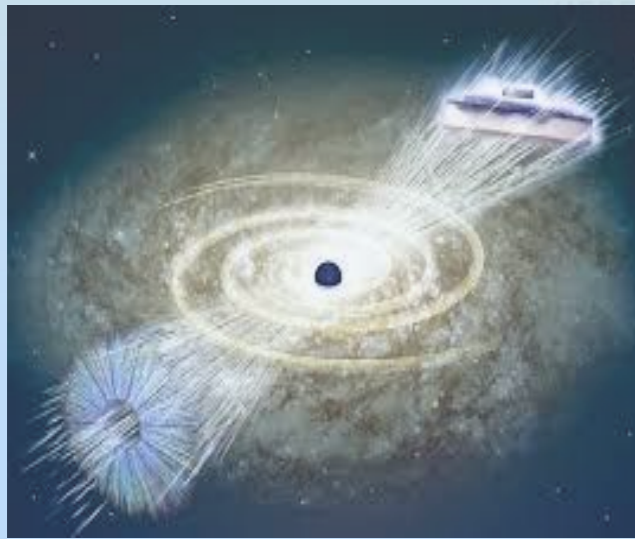


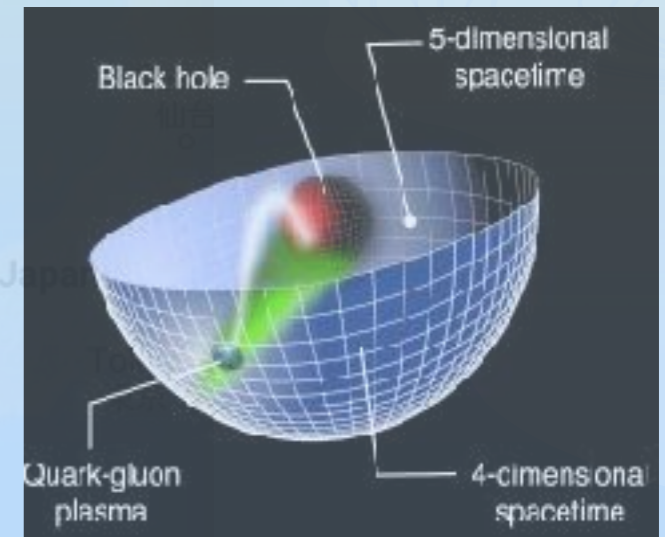
Holographic Model of the Dark Universe & the Late Time Evolution



by Yun-Long Zhang (張, 雲龍)

Yukawa Institute for Theoretical Physics
Nov. 2018 - @ YITP, Kyoto, Japan

Asia Pacific Center for Theoretical Physics
(2016-2018 @APCTP, Pohang, Korea)



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}) = \kappa_4 T_{\mu\nu}$$

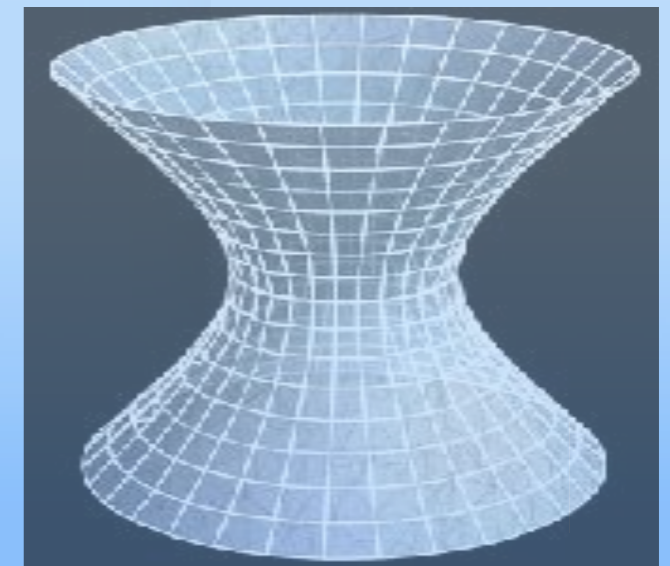
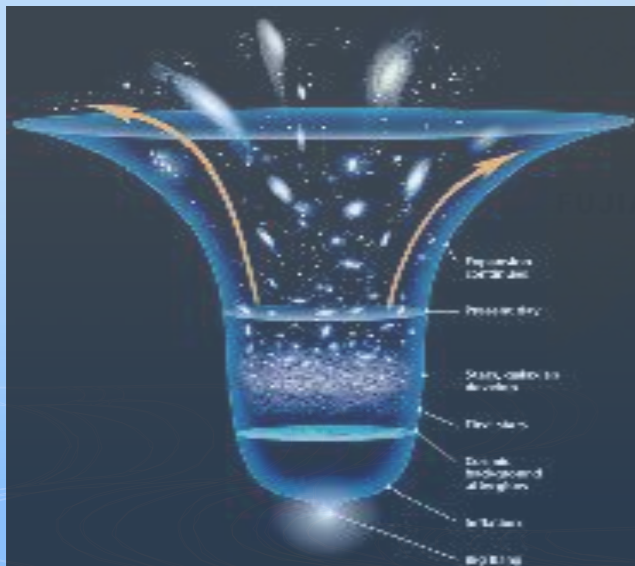
Ref: [JHEP 1810 \(2018\) 009 \[arXiv: 1712.09326\]](#)

by: Rong-Gen **Cai** (ITP/ Beijing)

Sichun **Sun** (NTU/ Taipei)

Yun-Long Zhang (APCTP/ Pohang)

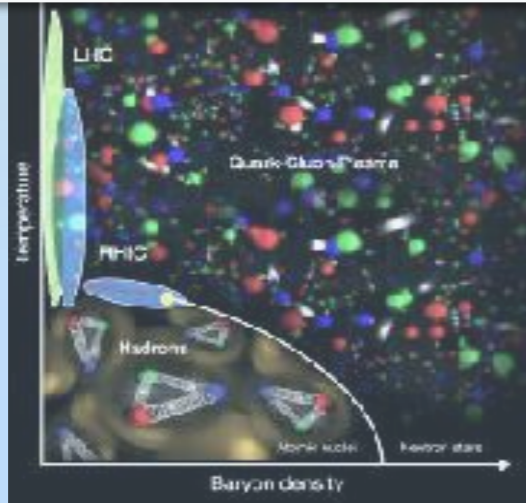
& Fitting with SNIa [arXiv: [1812.11105](#)]
with Bum-Hoon **Lee** (Sogang U./ Seoul)
Sunly **Khimphun** (Phnom Penh)
Gansukh Tumurtushaa (IBS/Daejeon)



Motivations — Holographic Hydrodynamics

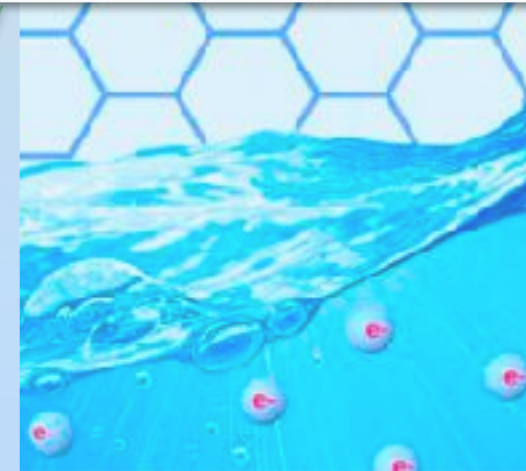
Quark Gluon Plasma

RHIC ['08] & LHC ['16]



Quantum Critical Liquid

Graphene ['09] & Semi-Metal ['16]



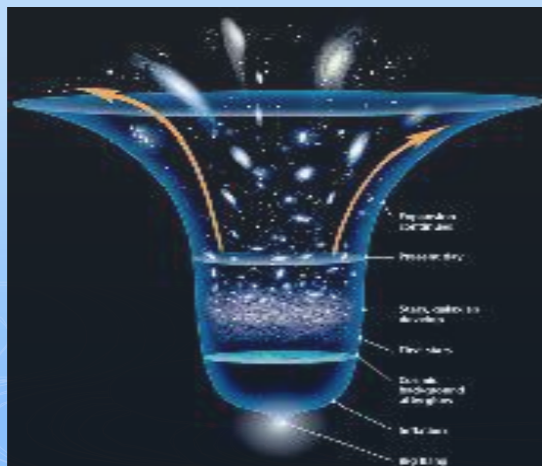
Black Holes
Membrane Fluid [KSS,05']
Rindler Fluid [BKLS,11']

$$\frac{\eta}{s} \simeq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$

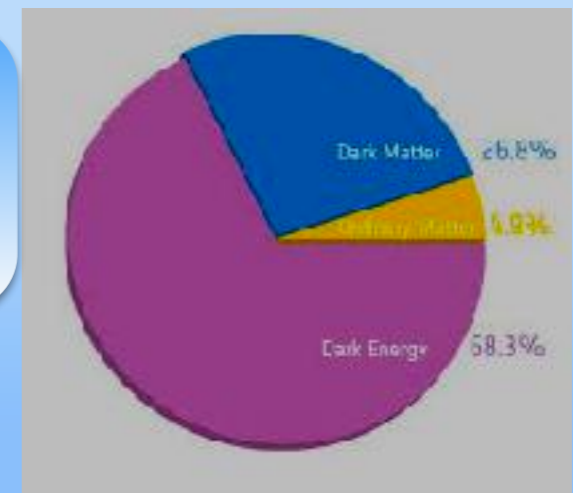
$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda \left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4} \right)}$$

$$\Omega_D^2 \simeq \frac{1}{2} \Omega_\Lambda (\Omega_D - \Omega_B)$$



Cosmological Fluid ['00,'17]
Dark Radiation & Energy & Matter

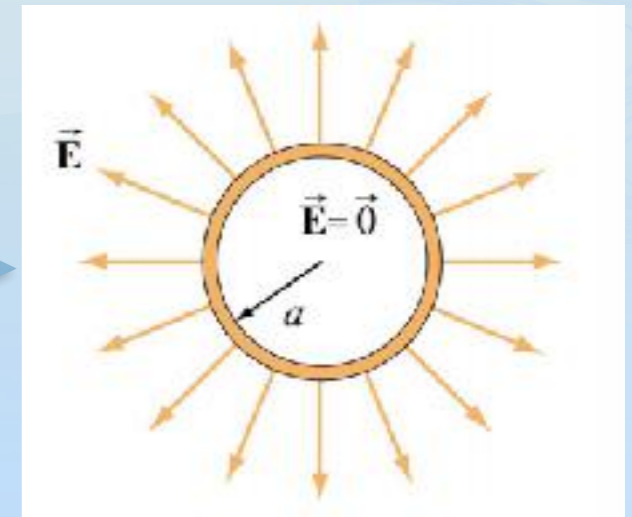
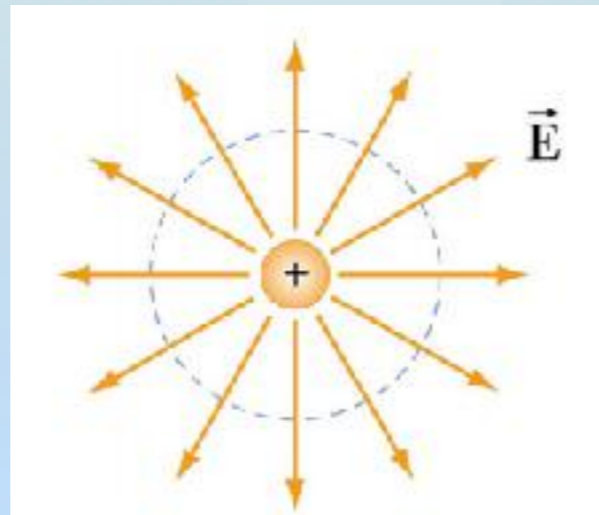
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{L} (\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu}) = \kappa_4 T_{\mu\nu}$$



Toy Duality: Field & Surface Matter

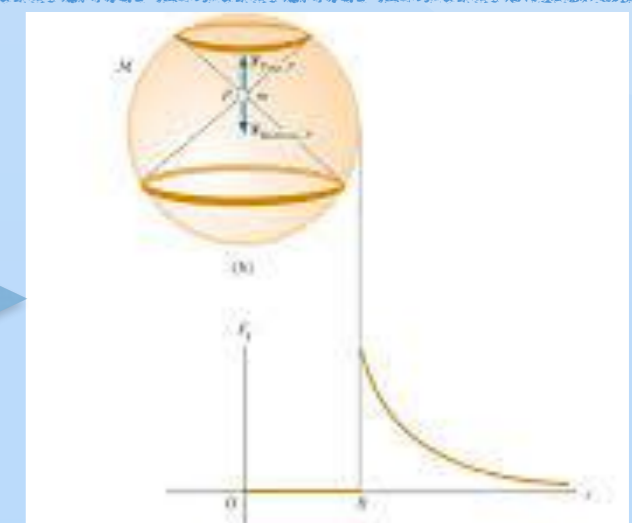
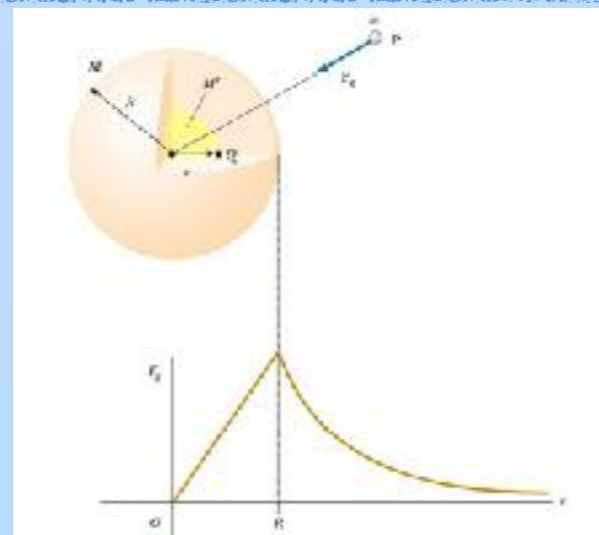
I. Electromagnetism

Electric Field &
Surface Charge



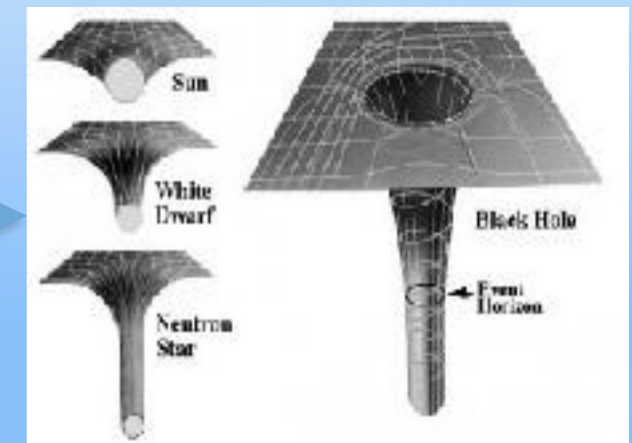
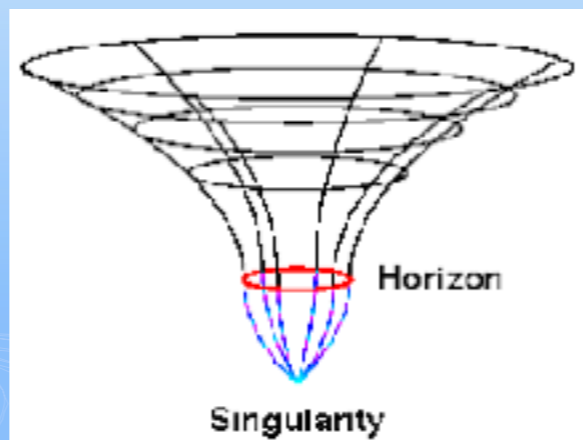
II. Newton Gravity

Massive Star &
Surface Shell



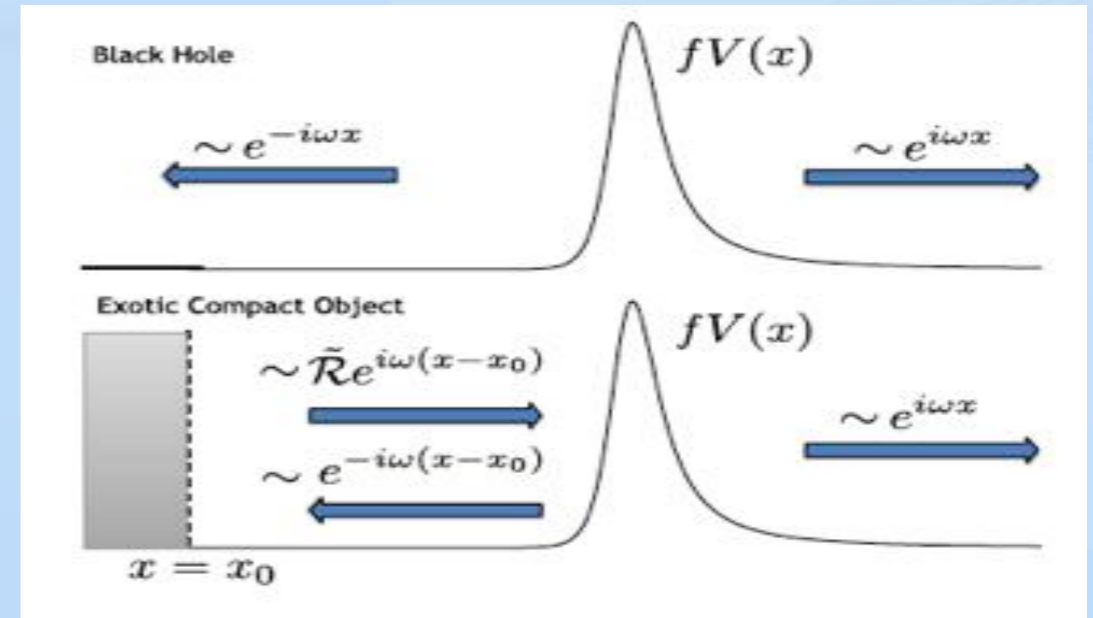
III. Einstein Gravity

Black Hole &
Surface Fluid



Membrane Duality: Black Hole & Surface Fluid

T. Doumer & K. Thorne, (1980s-)



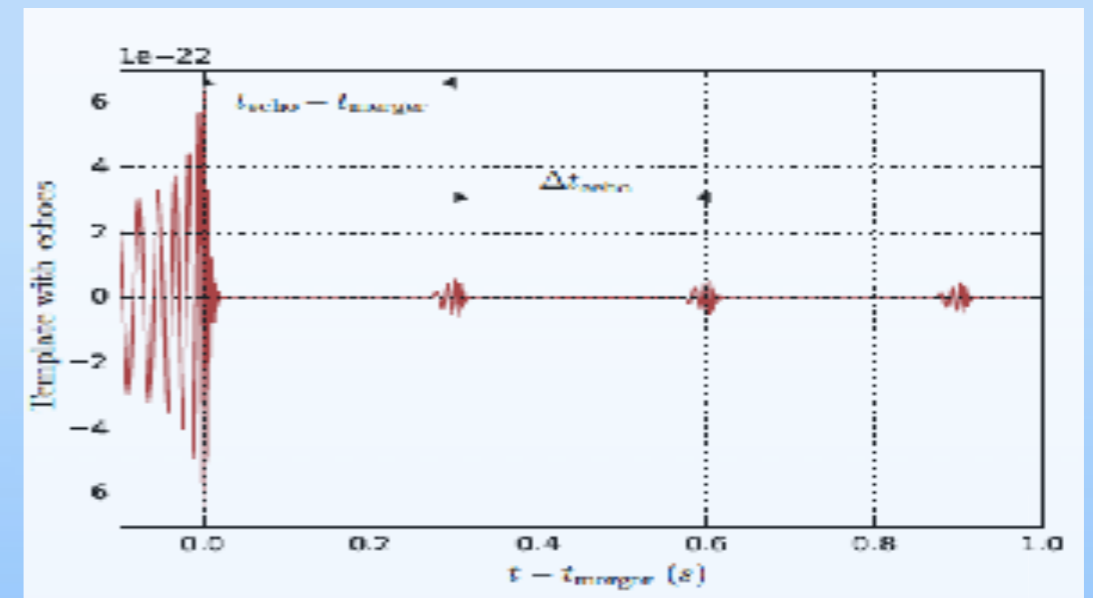
Effective Description

$$\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab})$$

Membrane on Stretched horizon

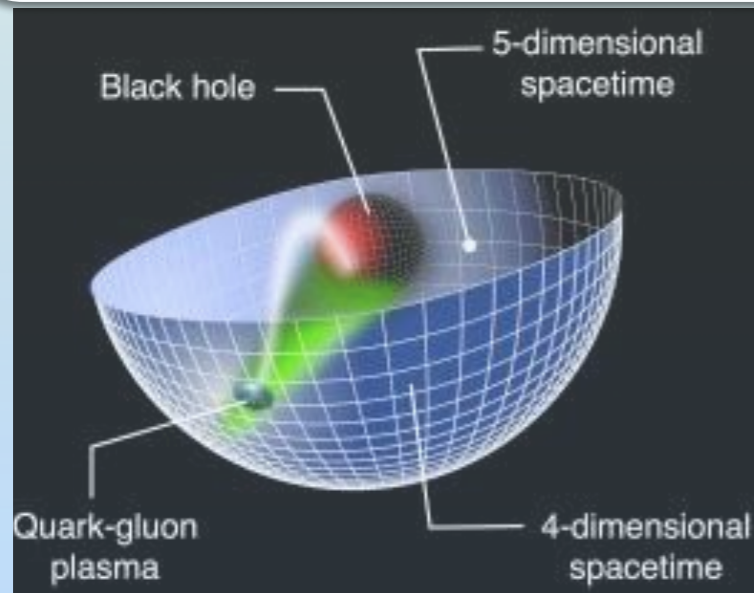
Viscosity & Conductivity

Echoes from Compact Objects [1706.06155 PRD'17]



Echoes from the Abyss [1612.00266 PRD'17]

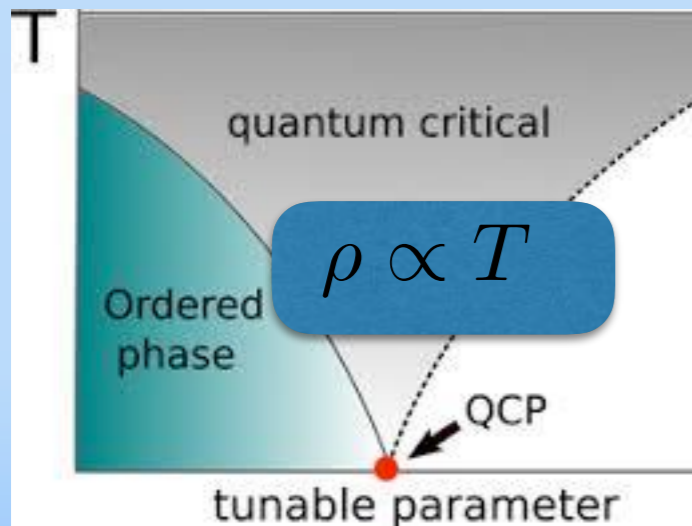
AdS/CMT Duality: Geometry & Quantum Matters



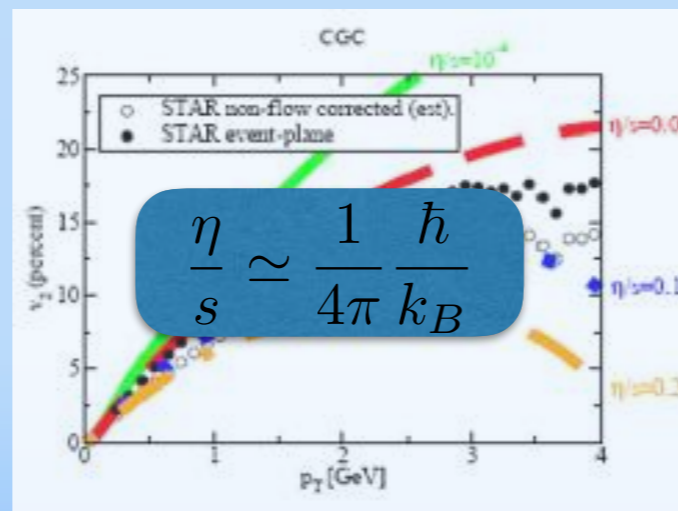
$$Z_{CFT} = \langle e^{S_{CFT}} \rangle \stackrel{AdS/CFT}{\simeq} e^{S_{AdS}}$$

Holographic Condensed Matters

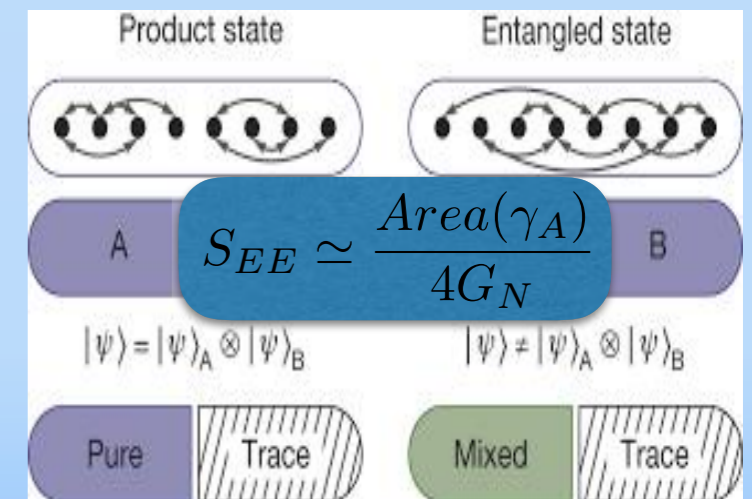
Holographic Strange Metal (2008-)



Holographic Hydrodynamics (2003-)



Holographic Entanglement (2006-)



DIRAC MEDAL

ICTP Dirac Medal 2018

Subir Sachdev
(Quantum & Phase)

Dam Thanh Son
(Nuclear & Fluid)

Xiao-Gang Wen
(Entanglement & Order)

Fluid/Gravity Duality (2008-)

AdS Black Brane

$$ds^2 = -2 u_\mu dx^\mu dr + r^2 (\eta_{\mu\nu} + [1 - f(r/\pi T)] u_\mu u_\nu) dx^\mu dx^\nu + 2r \left[\frac{r}{\pi T} F(r/\pi T) \sigma_{\mu\nu} + \frac{1}{3} u_\mu u_\nu \partial_\lambda u^\lambda - \frac{1}{2} u^\lambda \partial_\lambda (u_\nu u_\mu) \right] dx^\mu dx^\nu,$$

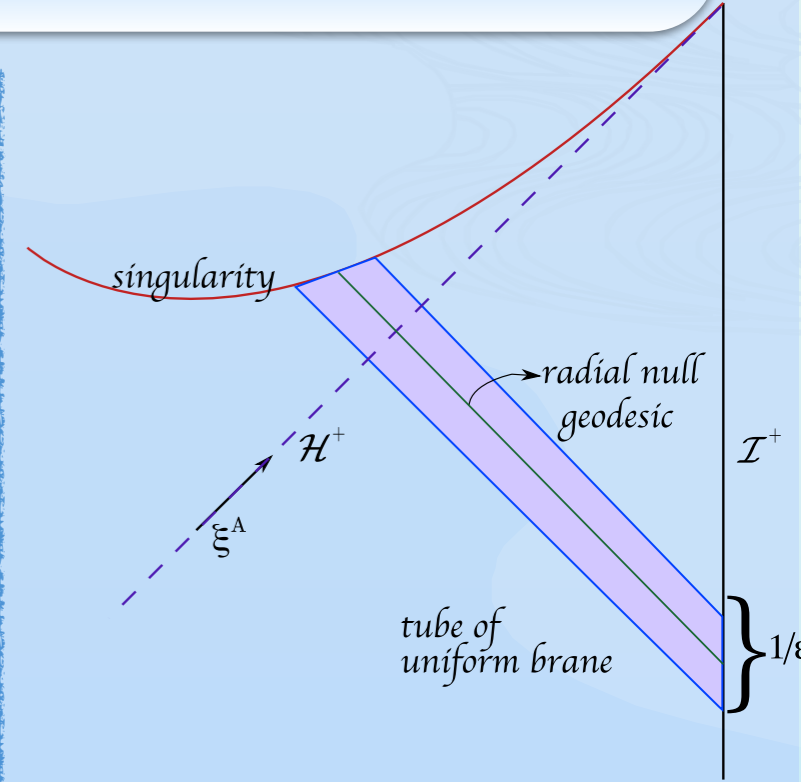
$$g_{ab} = \sum_{k=0}^{\infty} \epsilon^k g_{ab}^{(k)}, \quad T = \sum_{k=0}^{\infty} \epsilon^k T^{(k)}, \quad u_\mu = \sum_{k=0}^{\infty} \epsilon^k u_\mu^{(k)}.$$

$$T^{\mu\nu} = \lim_{r_c \rightarrow \infty} \frac{r_c^{d-2}}{16\pi G_N} \left[K^{\mu\nu} - K \gamma^{\mu\nu} - (d-1) \gamma^{\mu\nu} - \frac{1}{d-2} \left(R^{\mu\nu} - \frac{1}{2} R \gamma^{\mu\nu} \right) \right]$$

Conformal Fluid

$$T^{\mu\nu} = P (\gamma^{\mu\nu} + d u^\mu u^\nu) - 2 \eta \sigma^{\mu\nu} + 2 \eta [\tau_1 u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu} - \tau_\epsilon (\omega^\mu{}_\lambda \sigma^{\lambda\nu} + \omega^\nu{}_\lambda \sigma^{\lambda\mu})] + \xi_C C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \xi_\sigma [\sigma^\mu{}_\lambda \sigma^{\lambda\nu} - \frac{P^{\mu\nu}}{d-1} \sigma_{\alpha\beta} \sigma^{\alpha\beta}] + \xi_\omega [\omega^\mu{}_\lambda \omega^{\lambda\nu} + \frac{P^{\mu\nu}}{d-1} \omega_{\alpha\beta} \omega^{\alpha\beta}],$$

[Bhattacharyya, Hubeny, Minwalla, Rangamani et al]



$$P = \frac{1}{16\pi G_N} \left(\frac{4\pi T}{d} \right)^d$$

$$\eta = \frac{s}{4\pi} = \frac{1}{16\pi G_N} \left(\frac{4\pi T}{d} \right)^{d-1}$$

$$\tau_1 = \frac{d}{4\pi T} \left(1 - \int_1^\infty dy \frac{y^{d-2} - 1}{y(y^d - 1)} \right)$$

$$\tau_\epsilon = \frac{d}{4\pi T} \int_1^\infty dy \frac{y^{d-2} - 1}{y(y^d - 1)}$$

$$\xi_\sigma = \xi_C = \frac{d}{4\pi T} 2\eta$$

$$\xi_\omega = 0.$$

Moving the Holographic Screen to the Finite Cutoff

Extremal Charged BH

$AdS_2/CFT_1 \times R_p$
& Non-Fermi Liquid

Near Horizon

Cutoff AdS /
Effective CMT?

Near Boundary

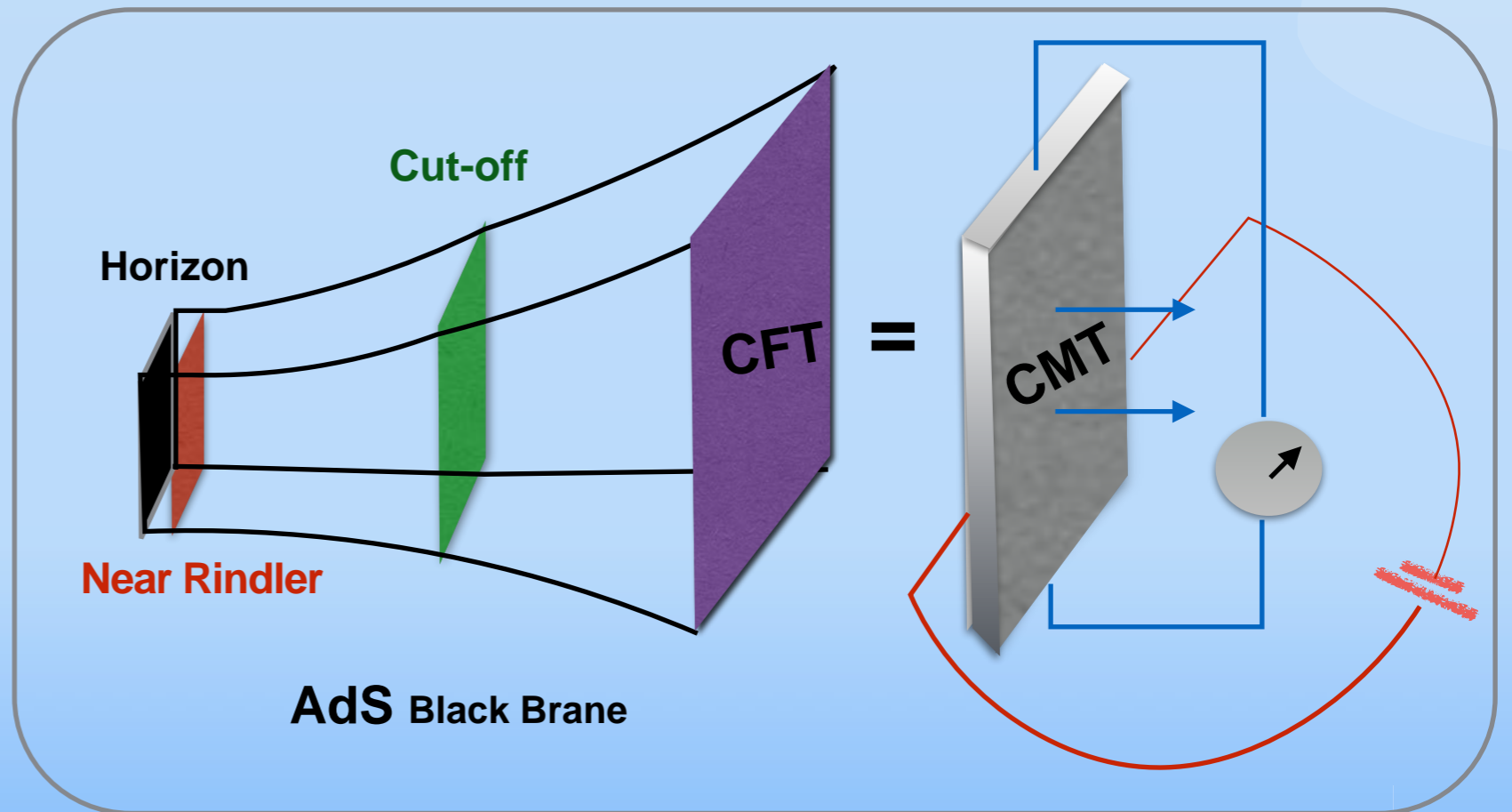
AdS /CFT&CMT

Finite Temperature

Rindler Space/
Special CMT



Membrane
Black Holes



Wilsonian Approach to Fluid/Gravity Duality [Bredberg, Keeler, Lysov, Strominger, JHEP 1103(2011)141]

Non-Relativistic Fluid Dual to Asymptotically AdS Gravity at Finite Cutoff Surface [Cai, Li, Zhang, JHEP 1107(2011)027]

Hydrodynamics of the Cutoff AdS Fluid

AdS Metric $ds_{p+2}^2 = -r^2 f(r) d\tau^2 + \frac{1}{r^2 f(r)} dr^2 + r^2 dx_i dx^i$

Induced Metric $ds_{p+1}^2 = -r_c^2 f(r_c) d\tau^2 + r_c^2 dx_i dx^i$

Dual Tensor $\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab} + C\gamma_{ab})$

Constraint equations $2G_{\mu b} n^\mu|_{r_c} = 2\partial^a (K_{ab} - \gamma_{ab} K) = 0 \Rightarrow \partial^a T_{ab} = 0$

Our contributions in the Holographic Cutoff AdS Fluid

Holographic Cutoff AdS Fluid in Non-relativistic limit

[Cai, Li, Zhang, JHEP 1107(2011)027]

$$\partial_r \sim \epsilon^0, \quad \partial_i \sim v_i \sim \partial_i \phi \sim \epsilon^1, \quad \partial_\tau \sim P \sim \epsilon^2$$

Holographic Forced Fluid Dynamics on finite Cutoff

[Cai, Li, Nie, Zhang, NPB 864 (2012) 260]

$$\partial_i v^i = 0, \quad \partial_\tau v_i + v^j \partial_j v_i + \partial_i P - \nu \partial^2 v_i = f_i^\phi + f_i^q$$

Incompressible Navier-Stokes from Chern-Simons Modified Gravity

Cai, Li, Qi, Zhang, PRD 86 (2012) 086008]

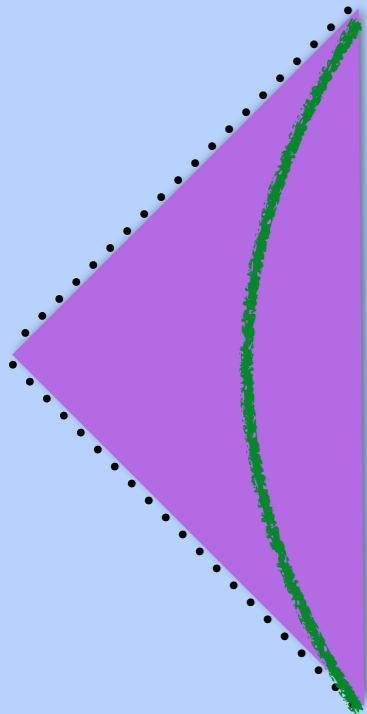
$$\partial_\tau v_i + v^j \partial_j v_i + \partial_i P - \nu \partial^2 v_i - (\tilde{\nu} \epsilon_{ij} \partial^2 v_j + \tilde{\zeta} \epsilon^{jk} \partial_i \partial_j v_k) = f_i$$

Holographic Charged Fluid with Anomalous Current at Finite Cutoff

[Bai, Hu, Lee, Zhang, JHEP 1211 (2012) 054]

$$\xi_B = c \left(\mu - \frac{1}{2} \frac{n\mu^2}{\rho+p} \right), \quad \xi_V = c \left(\mu^2 - \frac{2}{3} \frac{n\mu^3}{\rho+p} \right)$$

Cutoff AdS Fluid with Momentum Relaxation



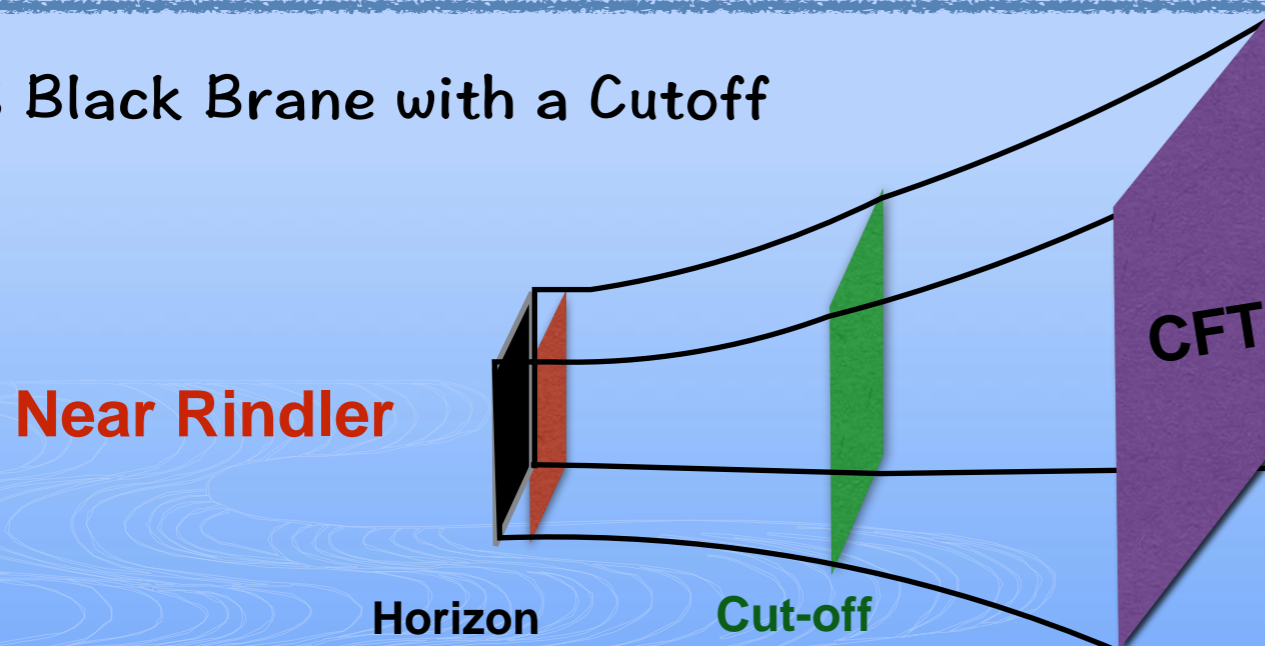
Ward Identity

$$\partial_t \langle \mathcal{T}^t_i \rangle + \partial_i \tilde{\mathcal{P}} = -\bar{\tau}_c^{-1} \langle \mathcal{T}^t_i \rangle - \ell_c k^2 \partial_t v_i + \dots, \quad \bar{\tau}_c^{-1} = \frac{k^2 \tilde{s}_c}{4\pi(\tilde{\mathcal{E}} + \tilde{\mathcal{P}})}.$$

Thermal Conductivity & Relaxation Rate

$$\tilde{\kappa}_\omega = \frac{1}{1 - i\omega\tau_c} \frac{4\pi \tilde{s}_c \tilde{T}_c}{k^2}, \quad \tau_c^{-1} = \frac{k^2}{4\pi \tilde{T}_c} \left[1 - \frac{\ell_c T_c}{s_c} \frac{k^2}{T_c^2} \right] + O(k^6).$$

AdS Black Brane with a Cutoff



Sub-Leading Correction

$$\xi_c \equiv \frac{\ell_c T_c}{s_c} = (p+1) \left[\tilde{\xi}_p(r_c) - \frac{r_c \tilde{\xi}'_p(r_c)}{(p-1)} \right],$$

Running From Conformal Fluid to Rindler Fluid

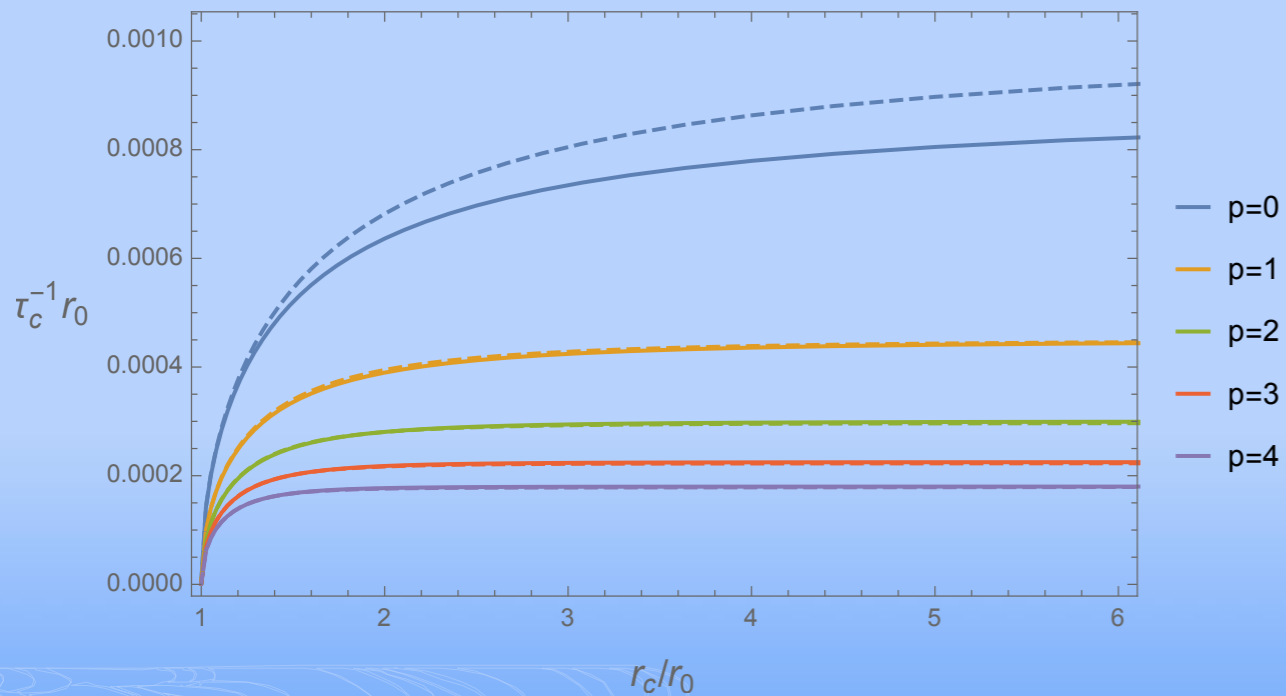


$$S_{\text{Rindler}} = S_{\text{CFT}} - S_{\text{AdS}}|_{r_0+\epsilon}^{\infty}$$

$$S_{\text{Cutoff}} = S_{\text{CFT}} - S_{\text{AdS}}|_{r_c}^{\infty}$$

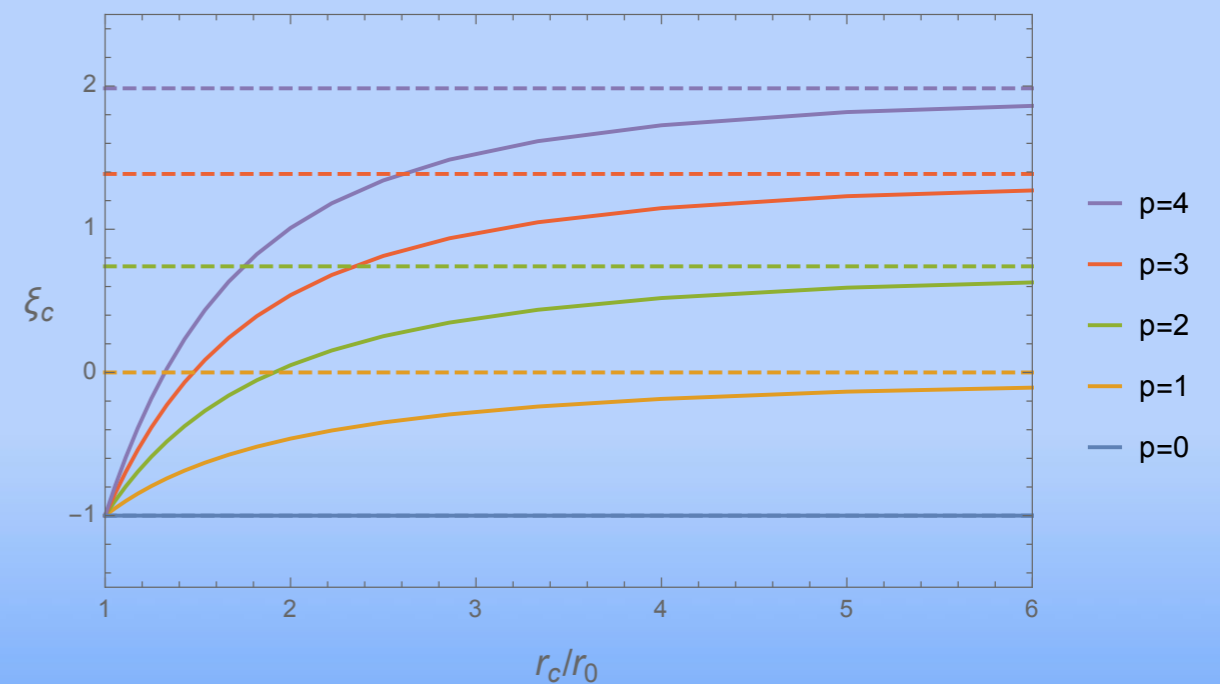
$$S_{\text{AdS}} = S_{\text{CFT}}$$

Momentum Relaxation Rate



$$\tau_c^{-1} = \frac{k^2}{4\pi\tilde{T}_c} \left(1 - \xi_c \frac{k^2}{\tilde{T}_c^2} \right), \quad \xi_c = \frac{\ell_c T_c}{s_c}$$

Sub-leading Corrections

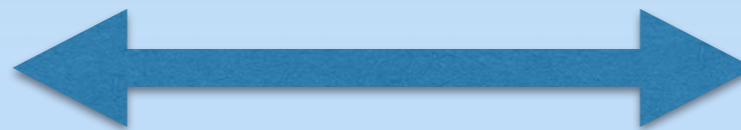


$$\tilde{\xi}_p(r) \equiv \int_{r_0}^r \frac{d\tilde{r} r_0^2}{\tilde{r}^3 f(\tilde{r})} \left(1 - \frac{r_0^{p-1}}{\tilde{r}^{p-1}} \right)$$

From AdS/CFT to Holographic Rindler Fluid

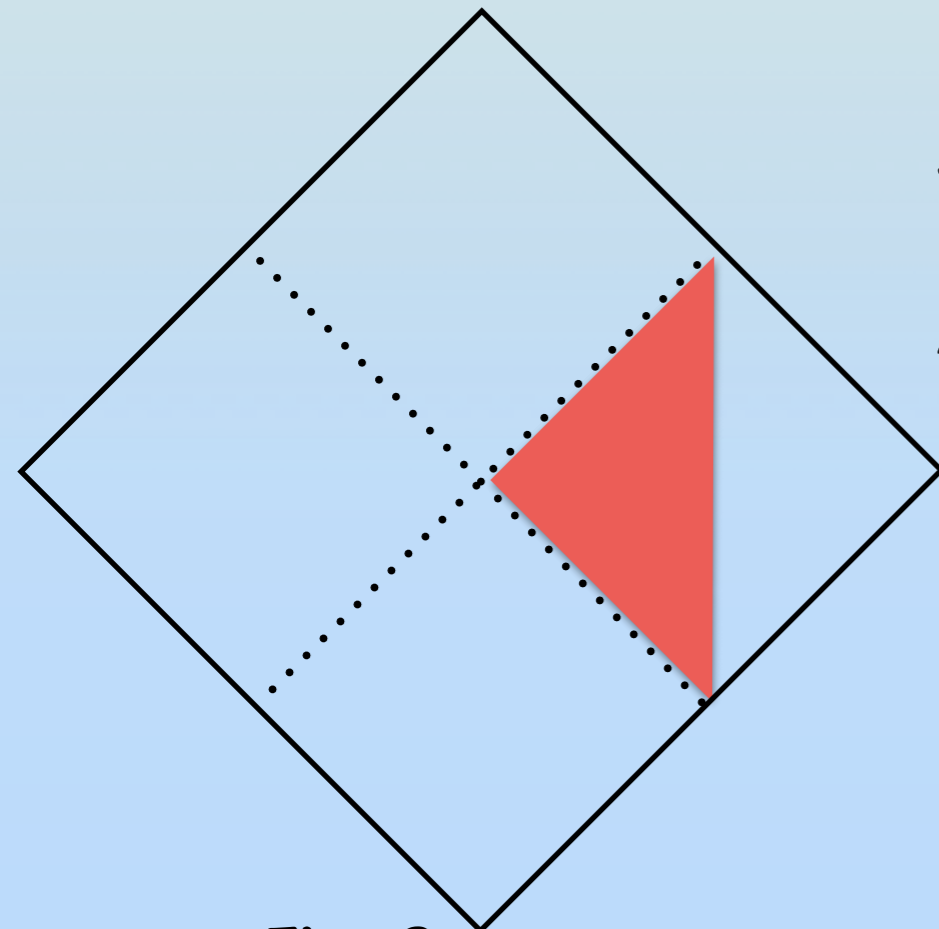
Holographic Screen
the Time-like boundary

$$\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab})$$

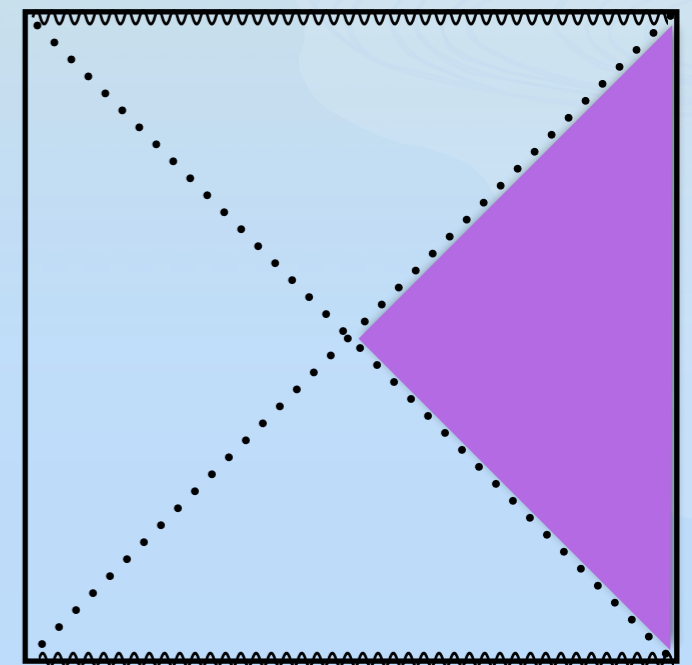


What is Rindler Fluid?

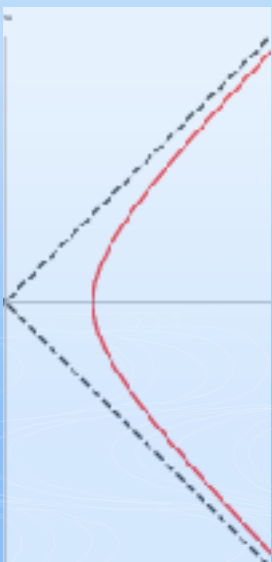
Fluid dual to Rindler spacetime



Flat Spacetime



AdS Spacetime



Navier–Stokes Equations:

Bredberg, Keeler, Lysov, Strominger ['10,'11]

Fluid/Gravity Expansion:

Compere, McFadden, Skenderis, Taylor ['11,'12]

Entropy Current and Constraint:

Chirco, Eling, Liberati, Meyer, Oz ['12,'13]

Comparison with AdS/Fluid:

Matsuo, Natsuume, Ohta, Okamura ['12,13]

Rindler Fluid and Recurrence Relation

Cai, Li, Yang, Zhang ['13,'14]

Rindler Fluid with Momentum Relaxation

Khimphun, Lee, Park, Zhang ['17]

Rindler Fluid in Relativistic Derivative Expansion

Rindler Metric with Perturbations

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -2\mathbb{P}u_a dx^a dr + g_{ab} dx^a dx^b,$$

$$\text{where } g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)} + g_{ab}^{(2)},$$

$$g_{ab}^{(0)} = -\mathbb{P}^2(r - r_c)u_a u_b + \gamma_{ab},$$

$$g_{ab}^{(1)} = 2\mathbb{P}(r - r_c)(u^c \partial_c \ln \mathbb{P} u_a u_b + 2a_{(a} u_{b)}),$$

$$g_{ab}^{(2)} = 2(r - r_c) \left[(\mathcal{K}_{cd} \mathcal{K}^{cd}) u_a u_b - 2u_{(a} h_{b)}^c \partial_d \mathcal{K}_c^d \right. \\ \left. - \mathcal{K}_a^c \mathcal{K}_{cb} + 2\mathcal{K}_{c(a} \Omega_{b)}^c - 2h_a^c h_b^d u^e \partial_e \mathcal{K}_{cd} \right] \\ + \mathbb{P}^2 (r - r_c)^2 \left\{ \left(\frac{1}{2} \mathcal{K}_{cd} \mathcal{K}^{cd} + a_c a^c \right) u_a u_b \right. \\ \left. + 2u_{(a} h_{b)}^c \left[\partial_d \mathcal{K}_c^d - (\mathcal{K}_{cd} + \Omega_{cd}) a^d \right] - \Omega_{ac} \Omega^c_b \right\} \\ + \mathbb{P}^4 (r - r_c)^3 \left(\frac{1}{2} \Omega_{cd} \Omega^{cd} \right) u_a u_b.$$

Constraint Equations

$$\partial_a u^a = 2\mathbb{P}^{-1} \mathcal{K}_{ab} \mathcal{K}^{ab} + O(\partial^3),$$

$$a_a + h_a^b \partial_b \ln \mathbb{P} = 2\mathbb{P}^{-1} h_a^c \partial_b \mathcal{K}_c^b + O(\partial^3).$$

Ref: Eling, Meyer, Oz, JHEP 1205 (2012) 116

Compère, McFadden, Skenderis, Taylor, JHEP 1203 (2012) 076

Holographic Fluid

$$T_{ab} = T_{ab}^{(0)} + T_{ab}^{(1)} + T_{ab}^{(2)} + O(\partial^3),$$

$$T_{ab}^{(0)} = \mathbb{P} h_{ab},$$

$$T_{ab}^{(1)} = \zeta' (u^c \partial_c \ln \mathbb{P}) u_a u_b - 2\eta \mathcal{K}_{ab},$$

$$T_{ab}^{(2)} = \mathbb{P}^{-1} \left\{ \left[d_1 \mathcal{K}_{ab} \mathcal{K}^{ab} + d_2 \Omega_{ab} \Omega^{ab} + d_3 (u^c \partial_c \ln \mathbb{P})^2 \right. \right. \\ \left. \left. + d_4 u^c \partial_c (u^d \partial_d \ln \mathbb{P}) + d_5 h^{cd} (\partial_c \ln \mathbb{P}) (\partial_d \ln \mathbb{P}) \right] u_a u_b \right. \\ \left. + \left[c_1 \mathcal{K}_{ac} \mathcal{K}_b^c + c_2 \mathcal{K}_{c(a} \Omega_{b)}^c + c_3 \Omega_{ac} \Omega^c_b + c_4 h_a^c h_b^d \partial_c \partial_d \ln \mathbb{P} \right. \right. \\ \left. \left. + c_5 \mathcal{K}_{ab} (u^c \partial_c \ln \mathbb{P}) + c_6 (h_a^c \partial_c \ln \mathbb{P}) (h_b^d \partial_d \ln \mathbb{P}) \right] \right\}.$$

Transport coefficients

$$\zeta' = 0, \quad \eta = 1,$$

$$d_1 = -2, \quad d_2 = d_3 = d_4 = d_5 = 0,$$

$$c_1 = -2, \quad c_2 = c_3 = c_4 = c_5 = -c_6 = -4.$$

Rindler Fluid and Recurrence Relation

Petrov Type I Condition

$$\mathbb{P}_{ab} \equiv \mathbf{n}^r h_a^c \mathbf{n}^r h_b^d C_{rcrd}$$

$$T_{ab}^{(0)} = \mathcal{E}u_a u_b + \mathcal{P}h_{ab} \xrightarrow{\mathbb{P}_{ab}=0} T_{ab}^{(1)} = -2\eta\sigma_{ab} + \dots \xrightarrow{\mathbb{P}_{ab}=0} T_{ab}^{(2)} = \dots$$

$$4\mathbb{P}_{ab} = h_a^m h_b^n \left[(T_{mc}T_{nd} - T_{mn}T_{cd}) u^c u^d - T_{mc}T_n^c \right. \\ \left. - 4u^c \partial_c T_{mn} + 4u^c \partial_{(m} T_{n)c} \right] \\ + p^{-2} \left[T(T + pT_{cd}u^c u^d) + 4p u^c \partial_c T \right] h_{ab}.$$

Petrov type I Spacetime and Dual Relativistic Fluids
[Cai, Yang, Zhang, PRD90 (2014) 041901(R)]

Rindler-Fluid		AdS Cutoff-Fluid		AdS-CFT Fluid
<p>Up to 2nd order How about Higher orders? [Cai, Yang, Zhang 1401.7792]</p>	→	<p>Up to to 0th order Modified Condition? [1306.5633]</p>	→	<p>Up to to 0th order AdS/Rindler correspondence?</p>

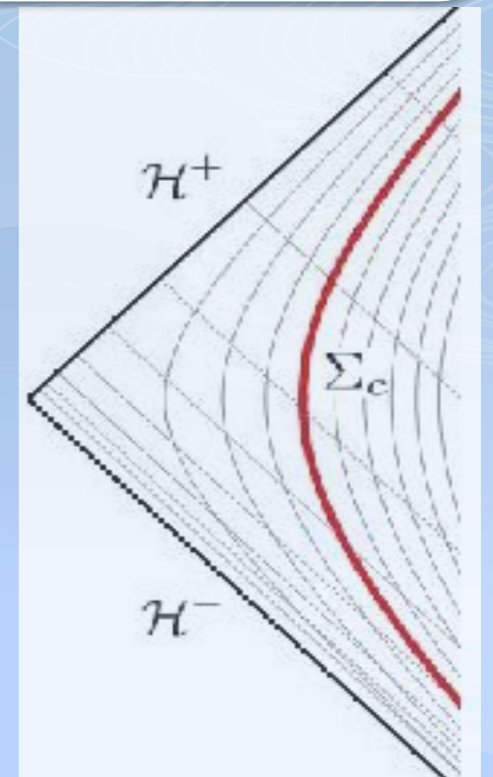
Petrov type I Condition and Rindler Fluid in Vacuum Einstein-Gauss-Bonnet Gravity
[Cai, Yang, Zhang, JHEP 1412 (2014) 147]

I. Flat Bulk/Rindler Fluid

Rindler Metric $ds^2 = -r d\tau^2 + \frac{1}{r} dr^2 + dx_i dx^i$

Induced Metric $ds^2 = -r_c d\tau^2 + dx_i dx^i$

Dual Tensor $\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab})$



Constraint equations

$$2G_{\mu b} n^\mu|_{r_c} = 2\partial^a (K_{ab} - \gamma_{ab} K) = 0 \Rightarrow \partial^a T_{ab} = 0$$

$$2G_{\mu\nu} n^\mu n^\nu|_{r_c} = (K^2 - K_{ab} K^{ab}) = 0 \Rightarrow T^2 - p T_{ab} T^{ab} = 0$$

Non-relativistic limit \rightarrow Incompressible Navier-Stokes Equations

$$\partial_i v^i = 0 \quad \partial_i P + \partial_\tau v_i + v^j \partial_j v_i - \eta \partial^2 v_i = 0$$

Bredberg, Keeler, Lysov, Strominger (JHEP 07 (2012) 146)

II. Flat Bulk/de-Sitter Fluid

1) Holographic Stress Tensor — Dark Sectors

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa_4 T_{\mu\nu} + \kappa_4 \langle \mathcal{T} \rangle_{\mu\nu}, \quad \langle \mathcal{T} \rangle_{\mu\nu} \equiv \frac{1}{\kappa_4 L} (\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu})$$

Modified Einstein equations

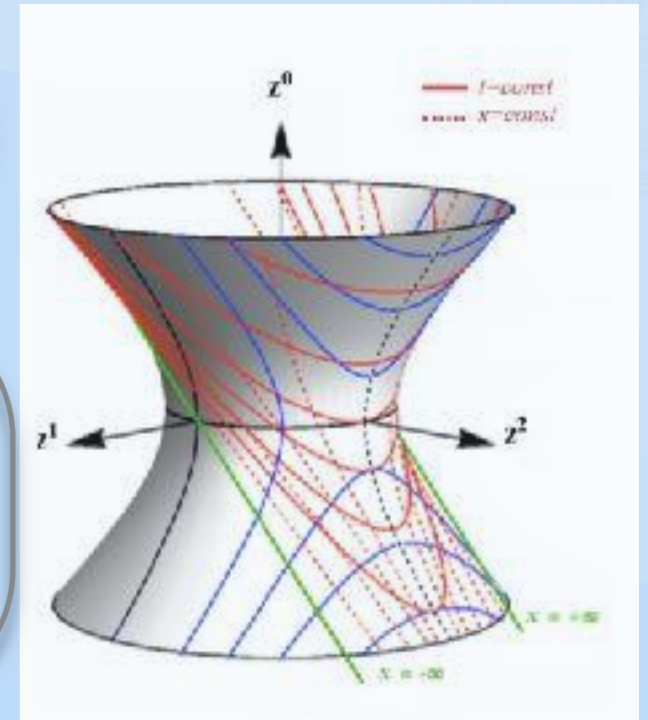
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L} (\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}) = \kappa_4 T_{\mu\nu}$$

Hamiltonian constraints

$$\mathcal{K}^2 - \mathcal{K}_{\mu\nu}\mathcal{K}^{\mu\nu} = R + 2G_{MN}^{(d+1)}\mathcal{N}^M\mathcal{N}^N,$$

$$\langle \mathcal{T}_{\mu\nu} \rangle = -\frac{\Lambda}{\kappa_4} g_{\mu\nu}$$

$$L = \frac{\kappa_5}{\kappa_4}$$



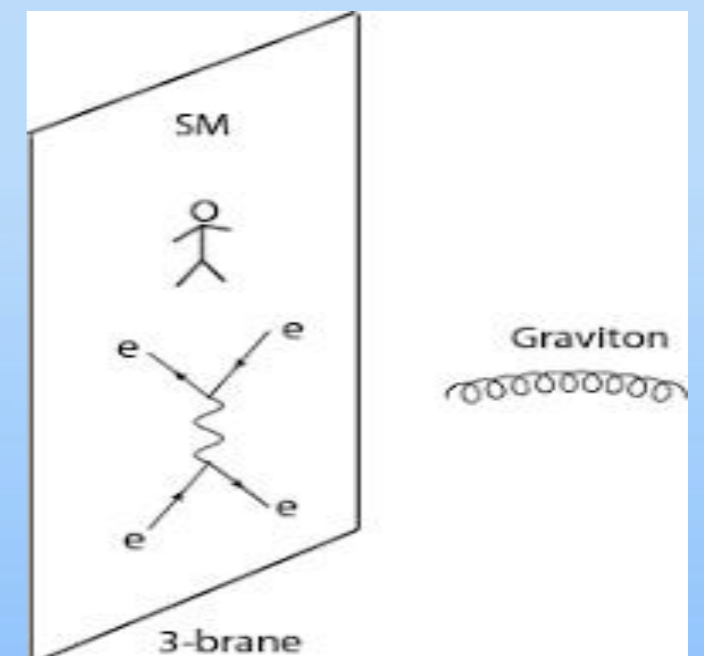
2) Embedding in higher dimensions — Brane World Models

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \mathcal{T}_{\mu\nu}^M + T_{\mu\nu}^B,$$

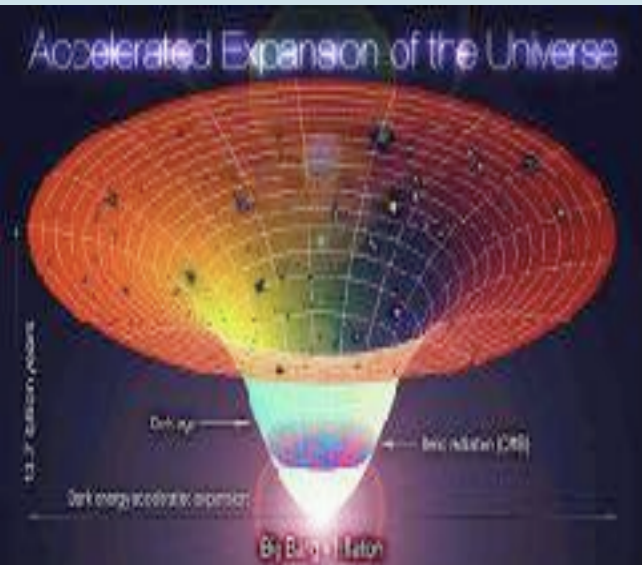
$$\mathcal{T}_{\mu\nu}^M \equiv (\mathcal{K}g_{\mu\sigma} - \mathcal{K}_{\mu\sigma})\mathcal{K}^\sigma{}_\nu + \mathcal{M}_{\mu\nu} - \frac{1}{2} (\mathcal{K}^2 - \mathcal{K}_{\rho\sigma}\mathcal{K}^{\rho\sigma}) g_{\mu\nu},$$

$$\mathcal{M}_{\mu\nu} \equiv g_\mu{}^M g_\nu{}^N R_{MN}^{(d+1)} - g_\mu{}^M \mathcal{N}^P g_\nu{}^N \mathcal{N}^Q R_{MPNQ}^{(d+1)}.$$

[Maeda, Mukohyama, Sasaki, Shiromizu, ..., ['99, '10]



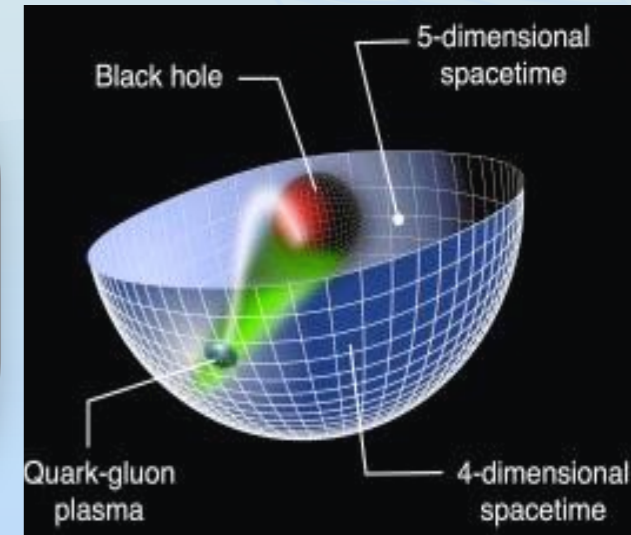
III. Flat Bulk/FRW Fluid



The Total Action

$$\mathcal{S}_{tot} = \int_{\mathcal{H}} d^4x \sqrt{-g} \left(\frac{1}{2\kappa_4} R + \mathcal{L}_m \right) + \mathcal{S}_5,$$

$$\mathcal{S}_5 \equiv \int_{\mathcal{M}} d^5x \sqrt{-\tilde{g}} \left(\frac{1}{2\kappa_5} \mathcal{R} \right) + \int_{\mathcal{H}} d^4x \sqrt{-g} \frac{1}{\kappa_5} \mathcal{K},$$



Einstein Field Equations

$$\frac{1}{\kappa_4} G_{\mu\nu} = T_{\mu\nu}^m + \langle \mathcal{T} \rangle_{\mu\nu}^d,$$

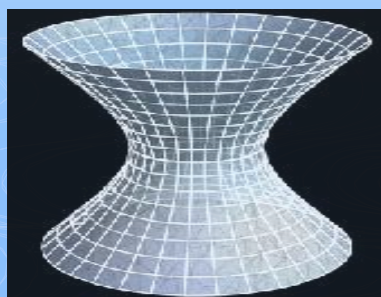
Holographic Dark Fluid?

Stress Energy Tensors

$$T_{\mu\nu}^m \equiv -\frac{2}{\sqrt{-g}} \frac{\delta(S_m)}{\delta g^{\mu\nu}}, \quad \langle \mathcal{T} \rangle_{\mu\nu}^d = -\frac{2}{\sqrt{-g}} \frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5} (\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu})$$

In the Bulk => Modified GR

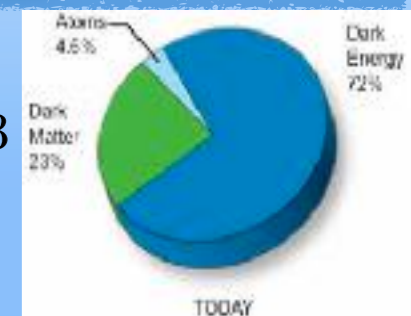
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{\kappa_4}{\kappa_5} (\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}) = \kappa_4 T_{\mu\nu}^m$$



Hamilton Constraint Equation
& Emergent de-Sitter Universe

$$\delta_{CSZ} \equiv \Omega_D^2 - \frac{1}{2} \Omega_\Lambda (\Omega_D - \Omega_B) \simeq -0.003$$

$$\delta_V \equiv \Omega_D^2 - \frac{4}{3} \Omega_B \simeq 0.004.$$



holographic Emergent Dark Universe (hEDU) & SNIa Data

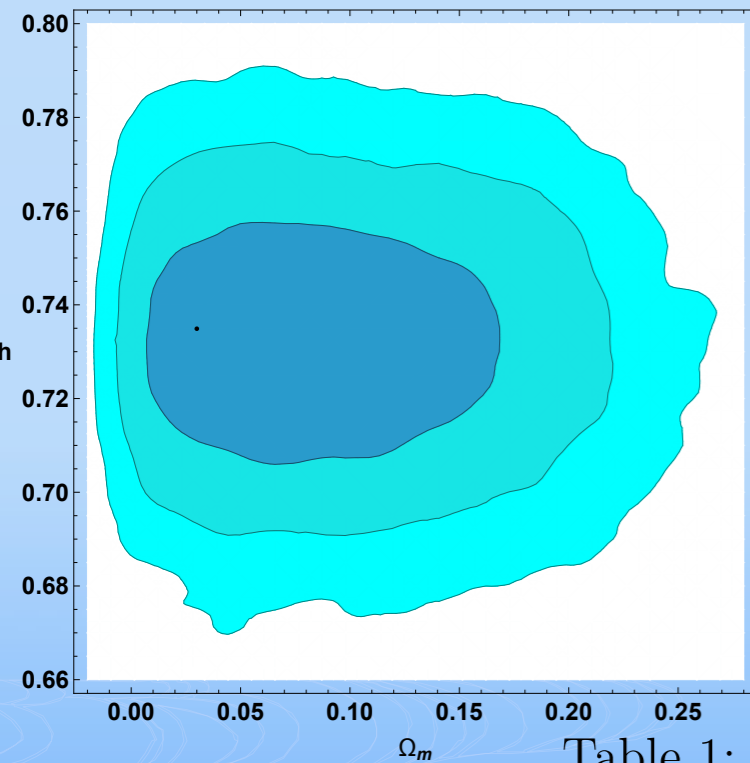
$$\text{LCDM} : \frac{H(z)^2}{H_0^2} = \Omega_\Lambda + \Omega_m(1+z)^3$$

Friedmann Equations

$$\text{sDGP} : \frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2} \sqrt{1 + \frac{4\Omega_m}{\Omega_\Lambda}(1+z)^3}$$

$$\text{hEDU} : \frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2} \sqrt{1 + \frac{4\Omega_m}{\Omega_\Lambda}(1+z)^3 + \frac{4\Omega_I}{\Omega_\Lambda}(1+z)^4}$$

$$\langle \mathcal{T} \rangle_{\mu\nu}^d = -\frac{2}{\sqrt{-g}} \frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5} (\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu})$$



Parameters	LCDM	hEDU
h	0.7330 ± 0.0180	0.7349 ± 0.0179
Ω_m	0.2969 ± 0.0352	0.0299 ± 0.0515
Ω_I	—	0.4382 ± 0.1317
α	0.1403 ± 0.0068	0.1409 ± 0.0068
β	3.1081 ± 0.0892	3.1144 ± 0.0896
χ_{\min}^2	695.063	694.321
ΔAIC	0	1.258
ΔBIC	0	5.866

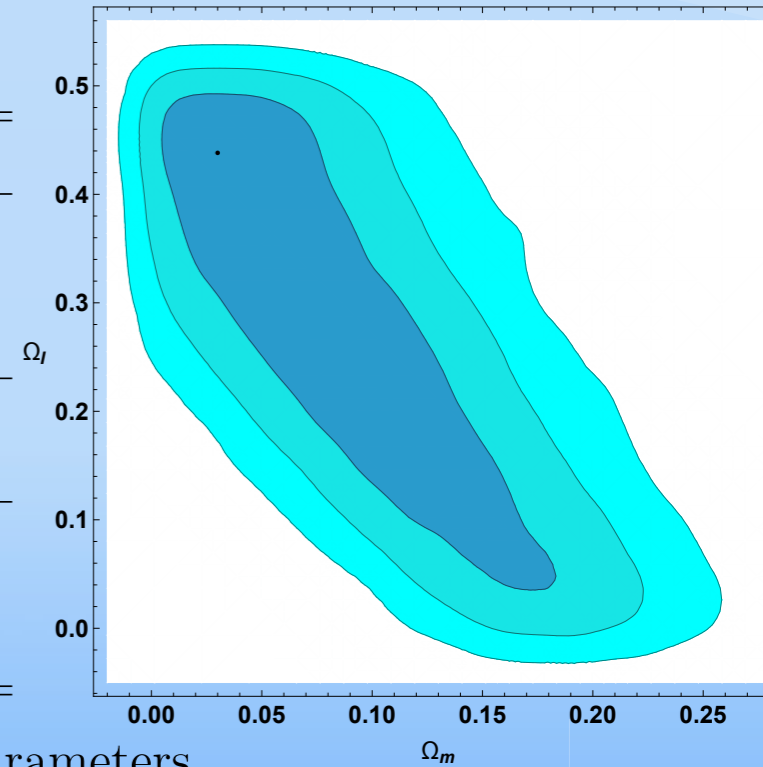
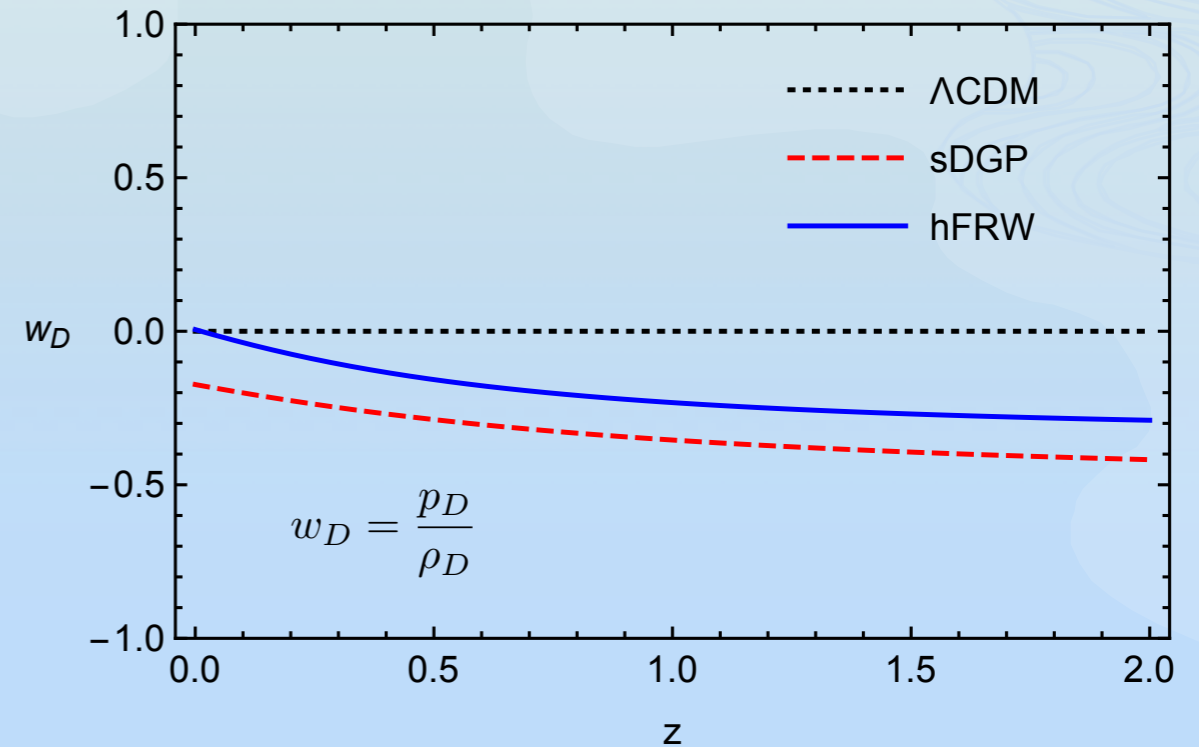
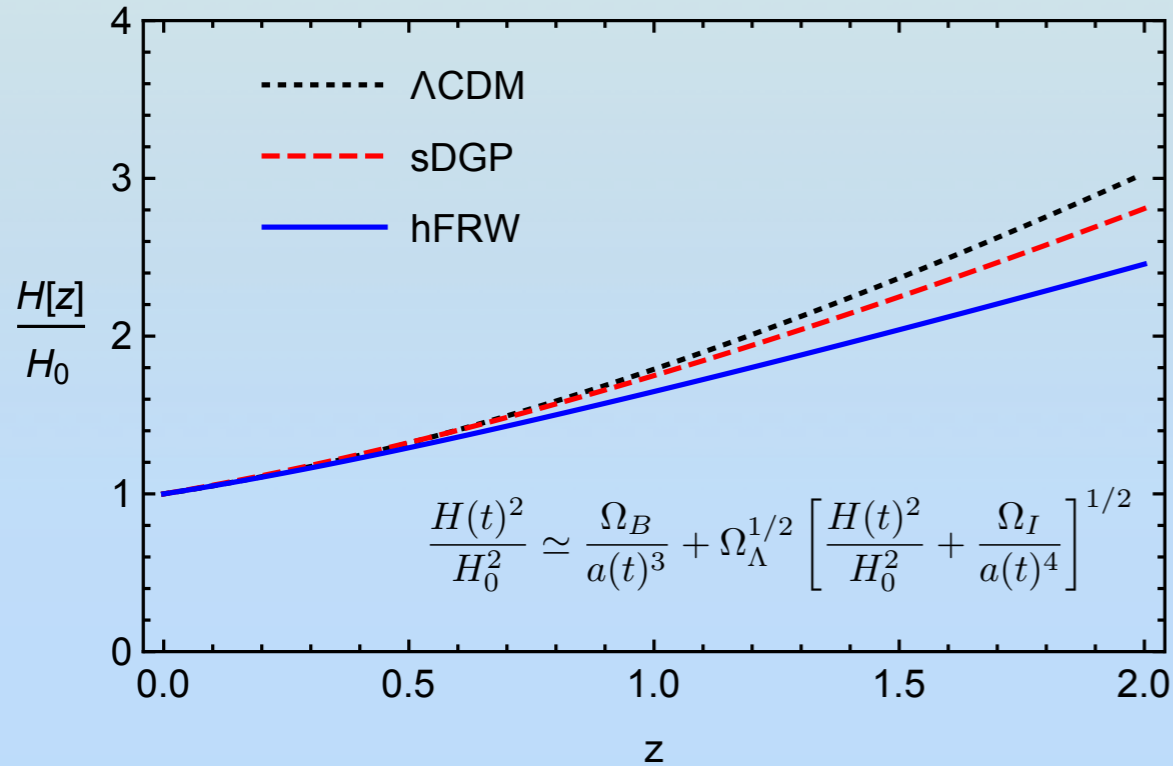


Table 1: Fitting values and uncertainties of the cosmological parameters.

Ref: [arXiv: 1812.11105] with Gansukh Tumurtushaa(IBS/Korea) et al

Late-Time Evolution of Holographic FRW Model



Hamiltonian Constraint from FRW Hypersurface

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda, \quad \tilde{\Omega}_D = \Omega_{\mathcal{H}}(t) - \Omega_\Lambda,$$

$$\tilde{w}_D = -1 - \frac{1}{3H(t)} \frac{\dot{\Omega}_{\mathcal{H}}(t)}{\Omega_{\mathcal{H}}(t) - \Omega_\Lambda},$$

$$\tilde{\Omega}_D^2 = \frac{\tilde{\Omega}_\Lambda}{2(1 + 3\tilde{w}_D)} [\tilde{\Omega}_D(1 - 3\tilde{w}_D) - \tilde{\Omega}_B].$$

$$\tilde{\Omega}_B \equiv \frac{\Omega_M}{a(t)^3} = \frac{H(t)^2}{H_0^2} - \Omega_{\mathcal{H}}(t),$$

$$\Omega_{\mathcal{H}}(t) \equiv \frac{\rho_{\mathcal{H}}}{\rho_c} = \Omega_\Lambda^{1/2} \left[\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4} \right]^{1/2}.$$

Late Time Universe

$$\delta_{CSZ} \equiv \Omega_D^2 - \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B) \simeq -0.003,$$

$$\delta_V \equiv \Omega_D^2 - \frac{4}{3}\Omega_B \simeq 0.004.$$

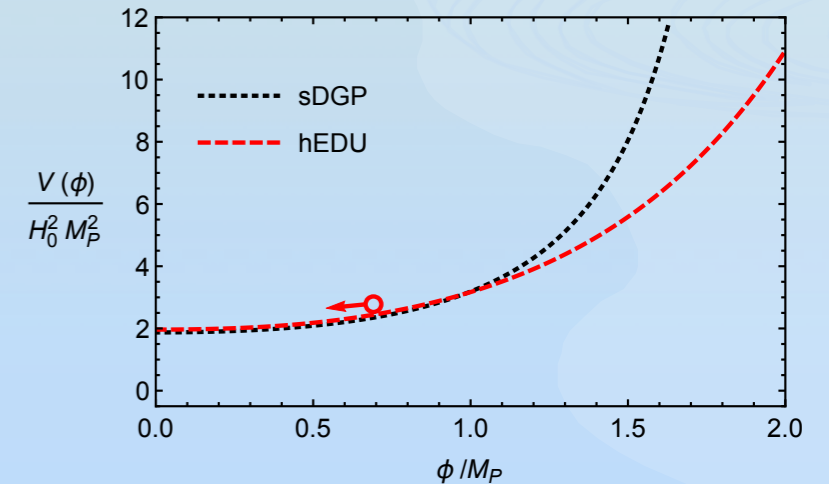
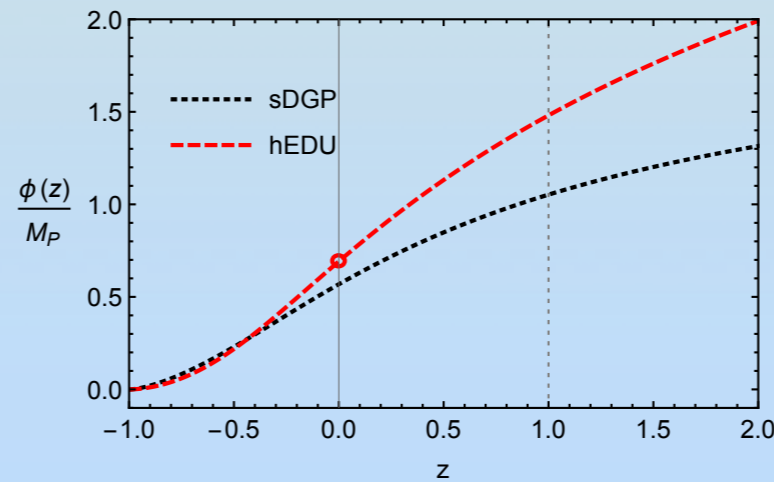
$$\Omega_\Lambda \simeq 0.685, \quad \Omega_D \simeq 0.265, \quad \Omega_B \simeq 0.050,$$

Effective Potential & Swampland Criteria

Effective Action $\mathcal{S}_{tot} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa_4} R + \mathcal{L}_m - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right].$

$$V[\phi(t)] = \frac{1}{2} [\rho_d(t) - p_d(t)],$$

$$\dot{\phi}(t) = -\sqrt{\rho_d(t) + p_d(t)}.$$



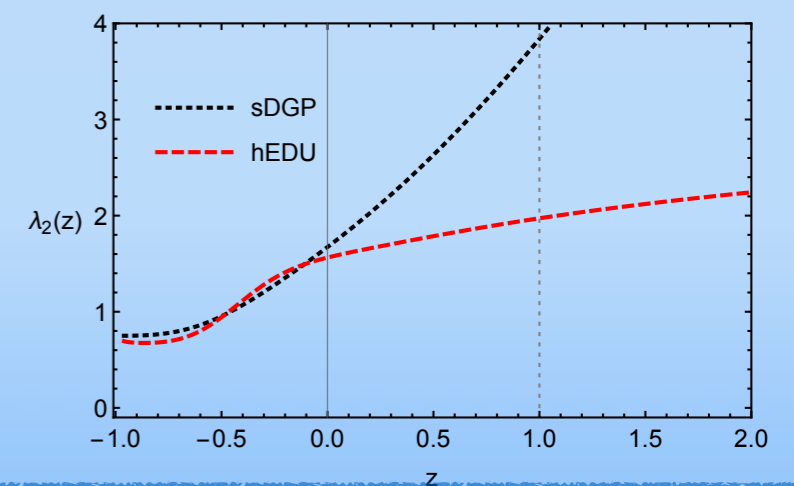
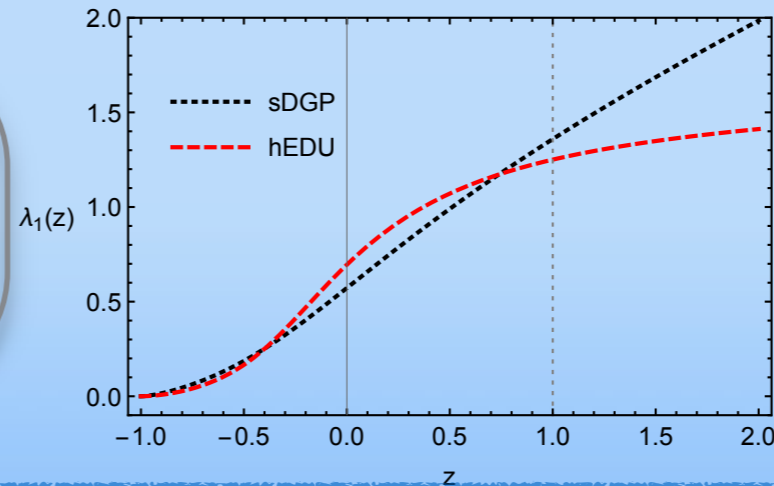
Swampland Criteria

Criterion 1 : $\frac{|\Delta\phi|}{M_P} \leq d_0,$ $d_0, c_1, c_2 \sim \mathcal{O}(1)$

Criterion 2 : $M_P |\nabla V| \geq c_1 V$ or $M_P^2 \min[\nabla_i \nabla_j V] \leq -c_2 V,$

$$\lambda_1 \equiv M_P \frac{V'}{V}, \quad V' \equiv \frac{dV(\phi)}{d\phi} = \frac{\dot{V}(t)}{\dot{\phi}(t)},$$

$$\lambda_2 \equiv M_P^2 \frac{V''}{V}, \quad V'' \equiv \frac{d}{d\phi} \frac{dV(\phi)}{d\phi} = \frac{1}{\dot{\phi}(t)} \frac{d}{dt} \left[\frac{\dot{V}(t)}{\dot{\phi}(t)} \right].$$



$$\frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} \sqrt{1 + \frac{4}{\Omega_\Lambda} \left[\Omega_m (1+z)^3 + \Omega_I (1+z)^4 \right]}$$

$$+ \frac{\Omega_\Lambda}{2} + \Omega_m (1+z)^3. \quad ($$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{L} (\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu}) = \kappa_4 T_{\mu\nu}$$

Summary & Outlook

$$\frac{\eta}{s} \simeq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$

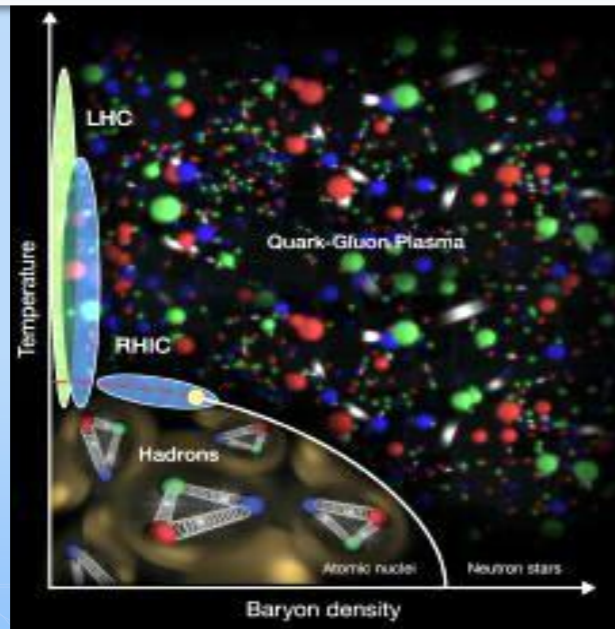


$$\Omega_D^2 \simeq \frac{1}{2} \Omega_\Lambda (\Omega_D - \Omega_B)$$

$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda \left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4} \right)}$$

Quark Critical Liquid
QGP [00s] & strange metal ['10s]

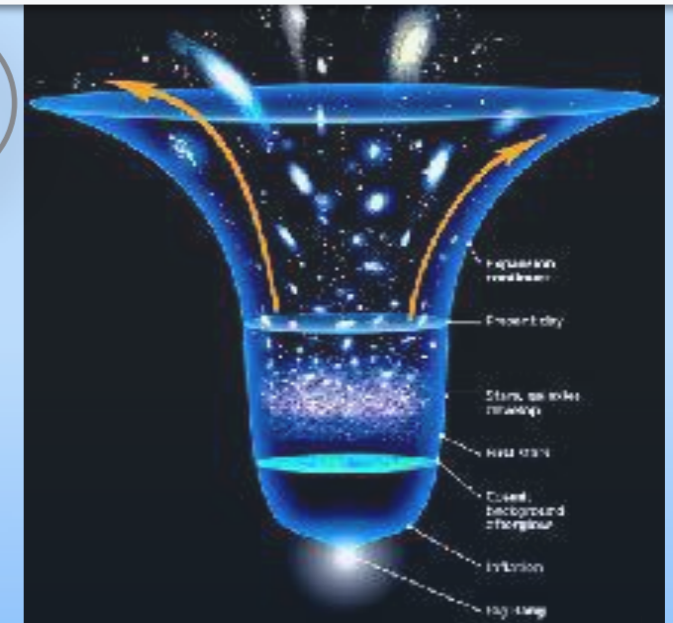
Cosmological Fluid
Dark Matter ['70s] & Energy ['90s]



$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{L} (\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu}) = \kappa_4 T_{\mu\nu}$$

$$\mathcal{S}_5 = \frac{1}{2\kappa_5} \int_{\mathcal{M}} d^5x \sqrt{-\tilde{g}} \mathcal{R} + \frac{1}{\kappa_5} \int_{\partial\mathcal{M}} d^4x \sqrt{-g} \mathcal{K},$$

$$\mathcal{S}_4 = \frac{1}{2\kappa_4} \int_{\partial\mathcal{M}} d^4x \sqrt{-g} R + \int_{\partial\mathcal{M}} d^4x \sqrt{-g} \mathcal{L}_M.$$



AdS/FRW (Gansukh's poster No.37)

Holographic Big Bang [Pourhasan, Afshordi, Mann, '14]

Thanks for Your Attention!