マグネターからの重力波

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sources of GWs

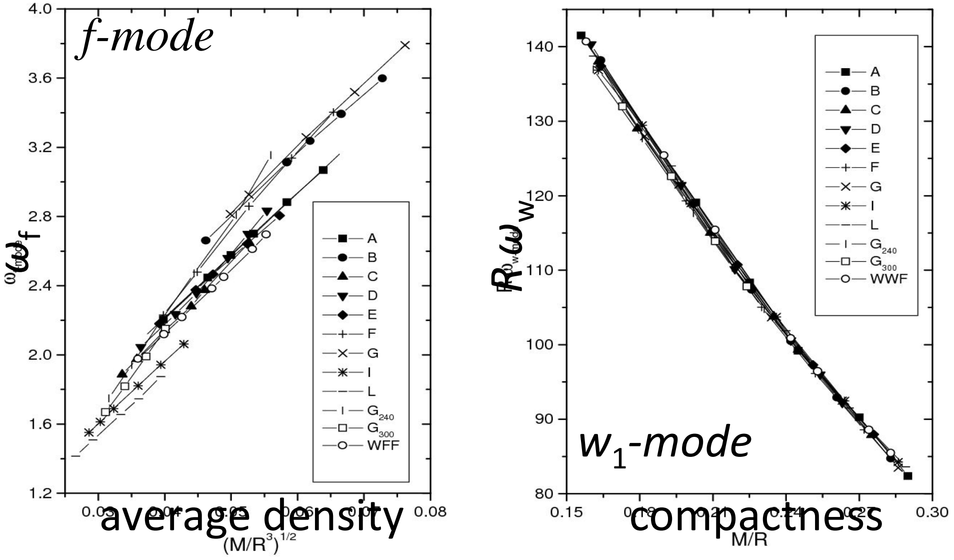
- binary mergers
  - BH, NS, WD
- supernovae
- *r*-mode instability
- NS mountain
- NS oscillations

QNMs

- Quasi Normal Modes (QNMs)
  - GWs bring out the oscillation energy
  - damped oscillation → QNMs (complex frequencies)
  - Re(ω): oscillation frequency, Im(ω): damping rate
- QNMs (polar parity) in NSs
  - fluid modes
    - * fundamental mode (f -mode) ··· ~ kHz
    - * pressure mode (p-mode) ··· > a few kHz
    - * gravity mode (g-mode) ··· < a few 100 Hz
    - * rotational mode (r-mode) ··· ~ rotation frequency
  - relativistic modes
    - * spacetime mode (w-mode) ··· > a few tens kHz
- QNMs (axial parity) in NSs
  - relativistic modes; w-mode ··· > a few tens kHz
  - fluid modes; torsional mode (t-mode) ··· > ten Hz

GWs asteroseismology

- seismic waves in Earth → interior structure of Earth (seismology)
- oscillations in Sun → interior structure of Sun (helioseismology)
- osicllations in NSs → interior structure of NSs (GWs asteroseismology)
  - – obtain the astronomical data about NSs via observations of GWs
  - – “rosetta stone” to know the interior structure of NSs



Andersson & Kokkotas (1998)

- empirical formula

$$\omega_f \approx 0.78 + 1.64\left[\left(\frac{M}{1.4M_{\oplus}}\right)\left(\frac{10\text{km}}{R}\right)^3\right]^{1/2}$$

$$\omega_w \approx \left(\frac{10\text{km}}{R}\right)\left[20.92 - 9.14\left(\frac{M}{1.4M_{\oplus}}\right)\left(\frac{10\text{km}}{R}\right)\right]$$

- with two relations, one can know *M* & *R* with less than10% accuracy, *independent of EOS*.

how about magnetars ?

- sudden localized energy releases could excite non-radial NS oscillations
- precise sky locations & trigger times from EM bursts allow us to reduce the false-alarm rate and increase sensitivity relative to all-sky all-time searches
- the closest of potential GW burst sources

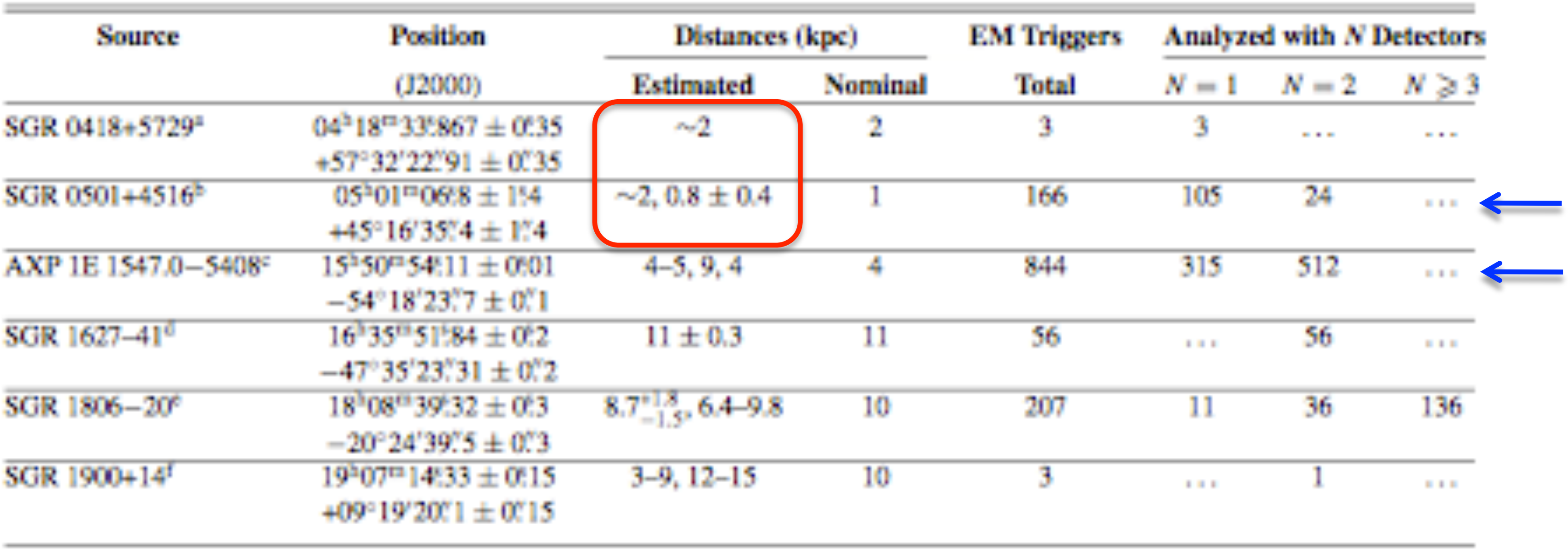
↓

- magnetars could be one of promising candidates

search for GWs from magnetars sensitive to f mode ringdowns

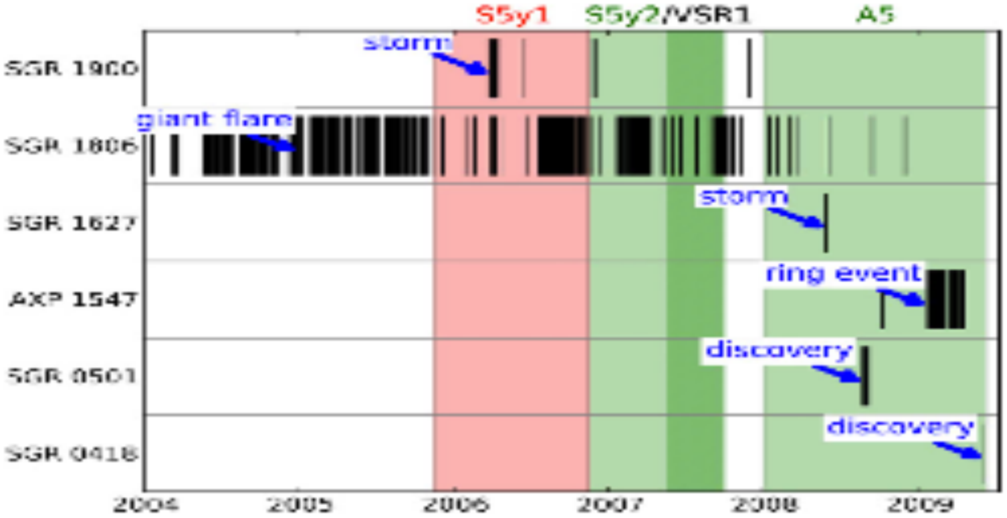
- 1st: until Nov. 2006 (SGR 1806-20 & 1900+14) (Abbott et al. 2008)
  - 1806-20 GF, 2006 storm burst from SGR 1900+14, 188 other evens from two SGRs
  - upper limits on f mode GW energy emission @1090Hz : **2.4×10^{48} erg – 2.6×10^{51} erg**
  - upper limits on band- & time-limited white noise bursts @100-200Hz : **3.1×10^{45} erg – 7.3×10^{47} erg**
- 2nd: focus on the 2006 SGR 1900+14 storm (Abbott et al. 2009)
  - upper limit f mode emission @1090 Hz : **1.2×10^{48} erg / burst**

- 3rd: 1271 soft GRBs from 6 magnetars (Abbott et al. 2011)

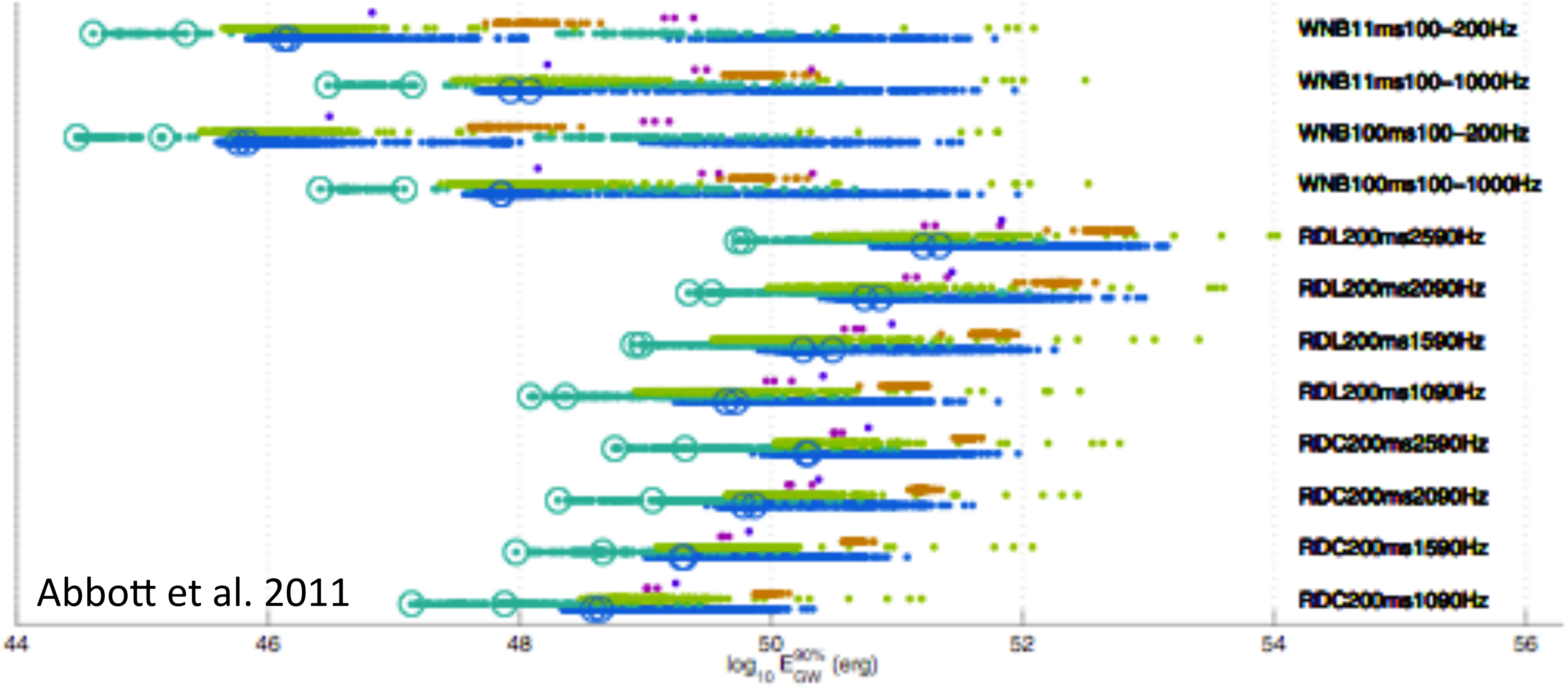


Source	Position	Distances (kpc)		EM Triggers	Analyzed with N Detectors		
	(J2000)	Estimated	Nominal	Total	$N = 1$	$N = 2$	$N \geqslant 3$
SGR 0418+5729[a]	$04^{h}18^{m}33^{s}.867 \pm 0^{s}.35$ $+57^{\circ}32'22''.91 \pm 0''.35$	~2	2	3	3	...	...
SGR 0501+4516[b]	$05^{h}01^{m}06^{s}.8 \pm 1^{s}.4$ $+45^{\circ}16'35''.4 \pm 1''.4$	~2, 0.8 ± 0.4	1	166	105	24	...
AXP 1E 1547.0−5408[c]	$15^{h}50^{m}54^{s}.11 \pm 0^{s}.01$ $-54^{\circ}18'23''.7 \pm 0''.1$	4–5, 9, 4	4	844	315	512	...
SGR 1627−41[d]	$16^{h}35^{m}51^{s}.84 \pm 0^{s}.2$ $-47^{\circ}35'23''.31 \pm 0''.2$	11 ± 0.3	11	56	...	56	...
SGR 1806−20[e]	$18^{h}08^{m}39^{s}.32 \pm 0^{s}.3$ $-20^{\circ}24'39''.5 \pm 0''.3$	$8.7^{+1.8}_{-1.5}$, 6.4–9.8	10	207	11	36	136
SGR 1900+14[f]	$19^{h}07^{m}14^{s}.33 \pm 0^{s}.15$ $+09^{\circ}19'20''.1 \pm 0''.15$	3–9, 12–15	10	3	...	1	...

SGR 0501+4516 might be associated with SN remnant HB9 (Gaensler & Chatterjee 08)



S5y2: 3LIGO (Louisiana+2Washington)
VSR1: 3LIGO + Virgo
A5: LIGO(2km) + GEO600



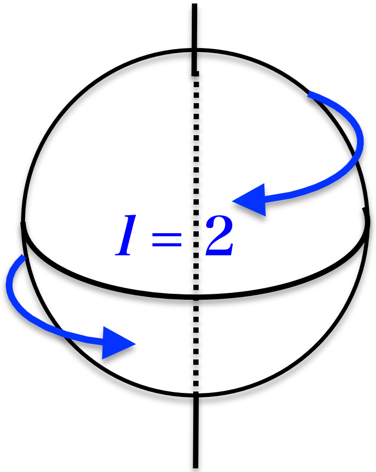
Abbott et al. 2011

- no evidence of GW signal in any of the signal regions analyzed
- whether E_{EM} & E_{GW} are correlated is still unknown
- the best SGR 0501+4516 f mode limit @1090Hz : **1.4×10^{47} erg**
- the best 100-200Hz white noise burst limit : **3.5×10^{44} erg** ~ E_{EM}^{GF}

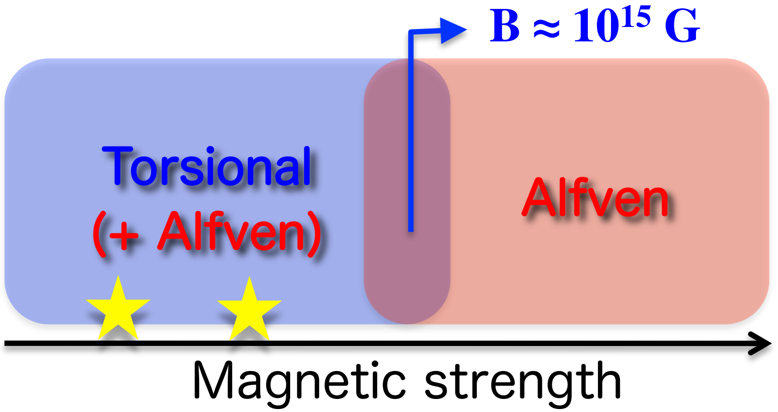
oscillations in NSs (magnetars)

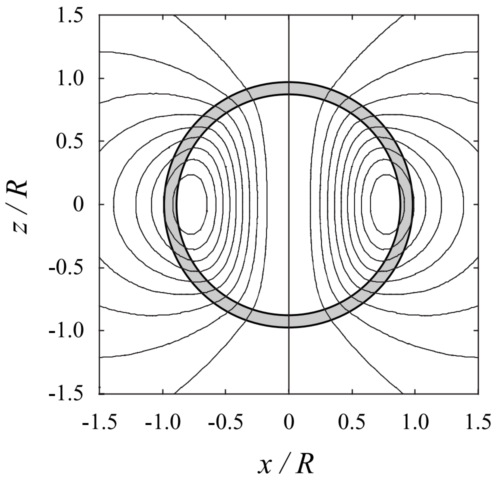
- $\delta A(t,r,\theta,\phi) = \delta A(t,r) Y_{lm}(\theta,\phi)$
- axisymmetric oscillations ($m = 0$)
  - axial oscillations ($\delta u^r = 0$)
    - incompressible
  - polar oscillations ($\delta u^r \neq 0$)
    - stellar deformation
    - density perturbations
- non-axisymmetric oscillations ($m \neq 0$)
  - axial oscillations can be coupled with polar oscillations

axial Alfven oscillations



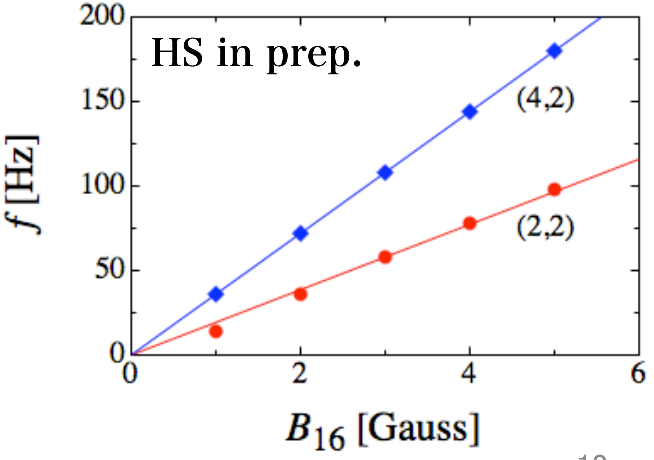
- continuum spectrum
  - upper & lower QPOs
- stronger magnetic field than 10^{15} G
  - only Alfven oscillations can be excited
- weaker magnetic field than 10^{15} G
  - crust torsional oscillations can be excited near surface
  - Alfven oscillations are confined in the core region





non-axisymmetric oscillations

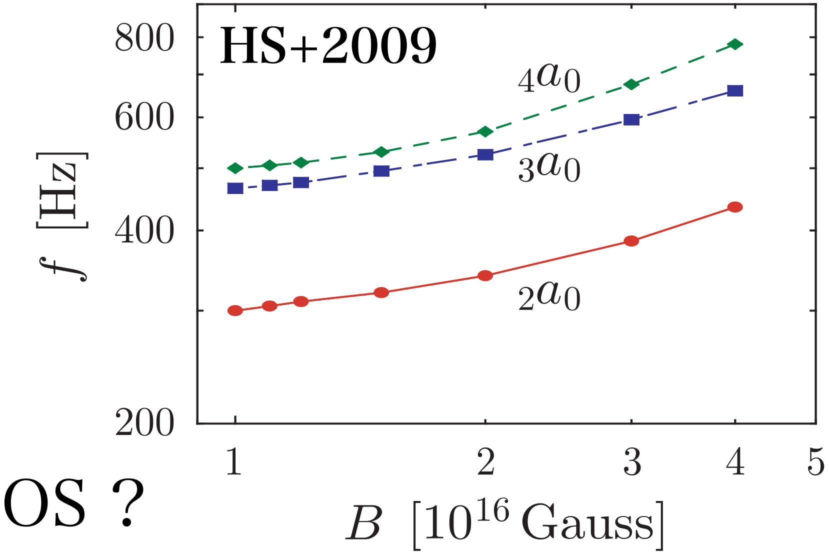
- axial oscillations can be coupled with polar ones
  - As a first step, we consider the only axial type oscillations.
- we find that…
  - non-axisymmetric axial Alfven oscillations are discrete oscillations.
  - It could be excited that the both crust and Alfven oscillations ??
  - Those frequencies are smaller than that of axisymmetric axial type Alfven oscillations.
  - axisymmetric case;
    minimum frequency is around 15 Hz for $B=4\times10^{15}$G
  - non-axisymmetric case;
    f_{22}=7.7 and f_{42}=14.4Hz for $B=4\times10^{15}$G
  - to fit the possible stellar model with the observations in SGRs, it is necessary to produce more oscillation frequencies with different value of (l,m) for the stellar models constructed with different EOSs.



polar Alfven oscillations

- GWs due to axial oscillations are quite weak.
- polar oscillations in NSs
  - fundamental mode ~ kHz
  - Alfven modes ~ a few 100Hz

  → targets for DECIGO & KAGRA
- Kashiyama & Ioka (2011)
  - if GWs are radiated for long term, one can detect the GWs with 2nd and 3rd generation GW detectors.
- how about dependence on EOS ?
- how strong GWs radiate ?
- how about dependence on magnetic configuration ?



coupling EMWs with GWs

- Einstein-Maxwell equations

$$G_{\mu\nu} = 8\pi(T_{\mu\nu} + E_{\mu\nu}) \qquad T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + p g_{\mu\nu}$$

$$(T^{\mu\nu} + E^{\mu\nu})_{;\nu} = 0 \qquad E_{\mu\nu} = \frac{1}{4\pi}\left(g^{\alpha\beta}F_{\alpha\mu}F_{\beta\nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$F^{\mu\nu}{}_{;\nu} = 4\pi J^\mu$$

$$F_{\mu\nu,\lambda} + F_{\lambda\mu,\nu} + F_{\nu\lambda,\mu} = 0$$

- the deformation due to the magnetic pressure can be negligible, because $\varepsilon_m/\varepsilon_g \sim 10^{-4}(B/10^{16}G)^2$
  - to derive the stellar configuration, $E_{\mu\nu} = 0$
  - stellar configuration is spherically symmetric
- magnetic field can be determined from Maxwell eqs. with the obtained metric
  - in particular, we focus on dipole magnetic configuration

- add perturbations

$$g_{\mu\nu} = g^{(B)}_{\mu\nu} + h_{\mu\nu}$$

$$F_{\mu\nu} = F^{(B)}_{\mu\nu} + f_{\mu\nu}$$

- perturbation eqs.

$$\delta G_{\mu\nu} = 8\pi(\delta T_{\mu\nu} + \delta E_{\mu\nu})$$

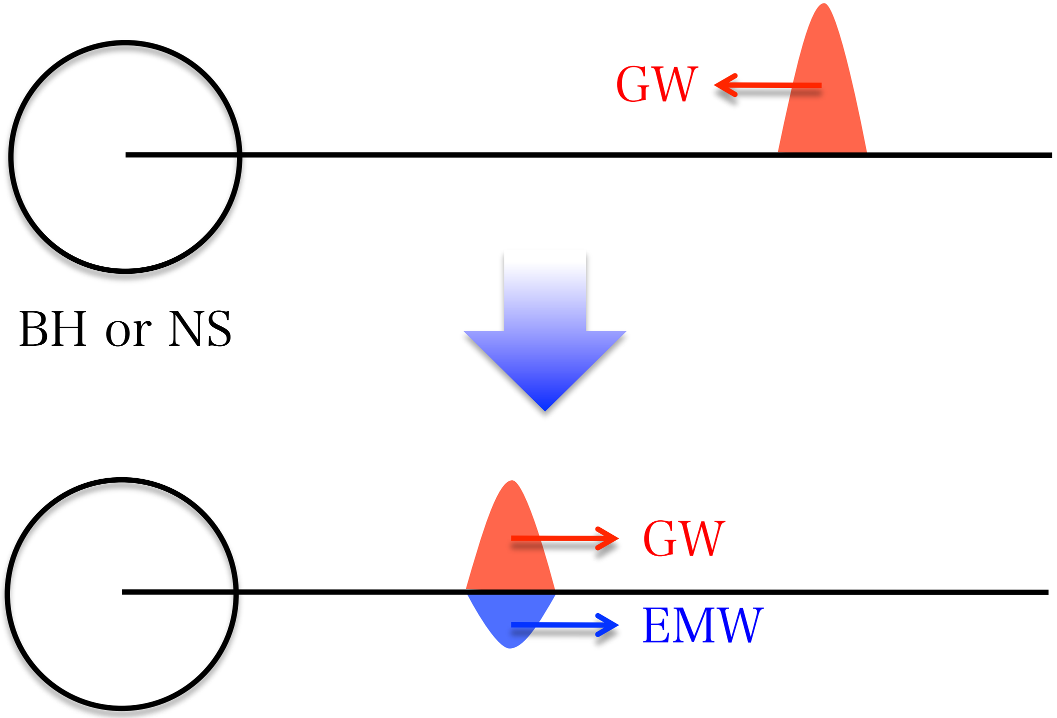
$$\delta(T^{\mu\nu} + E^{\mu\nu})_{;\nu} = 0$$

$$\delta(F^{\mu\nu}{}_{;\nu}) = 4\pi\delta J^{\mu}$$

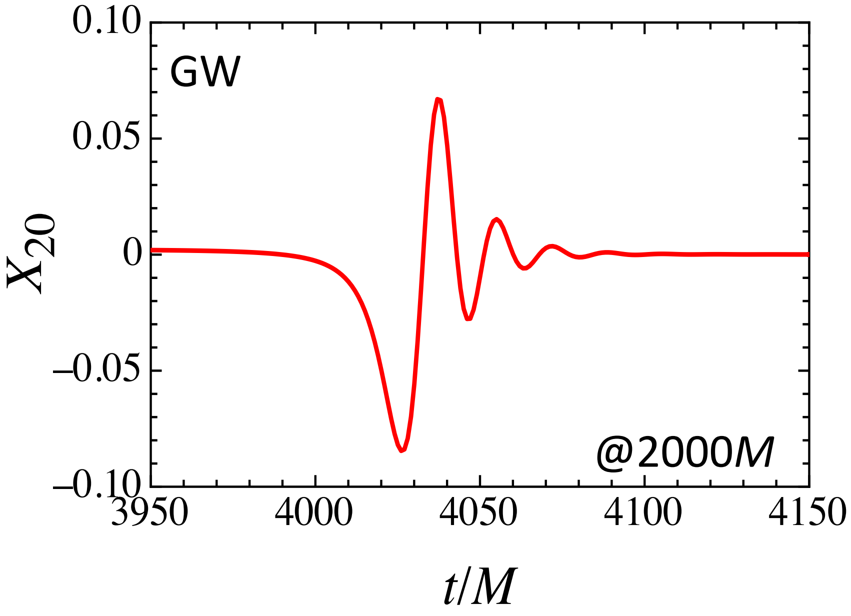
$$f_{\mu\nu,\lambda} + f_{\lambda\mu,\nu} + f_{\nu\lambda,\mu} = 0$$

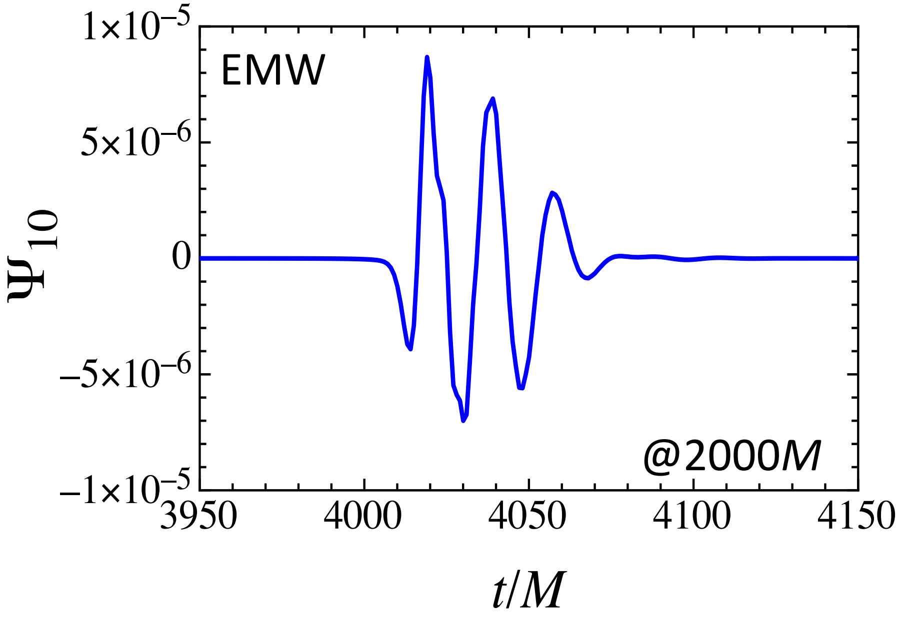
- for simplicity, we omit $\delta E_{\mu\nu}$
  - GWs : independent of EMWs
  - EMWs : coupled with GWs
- one can show
  - **dipole polar EMWs will be driven by quadrupole axial GWs**
  - **dipole axial EMWs will be driven by quadrupole polar GWs**

EMWs driven by GWs



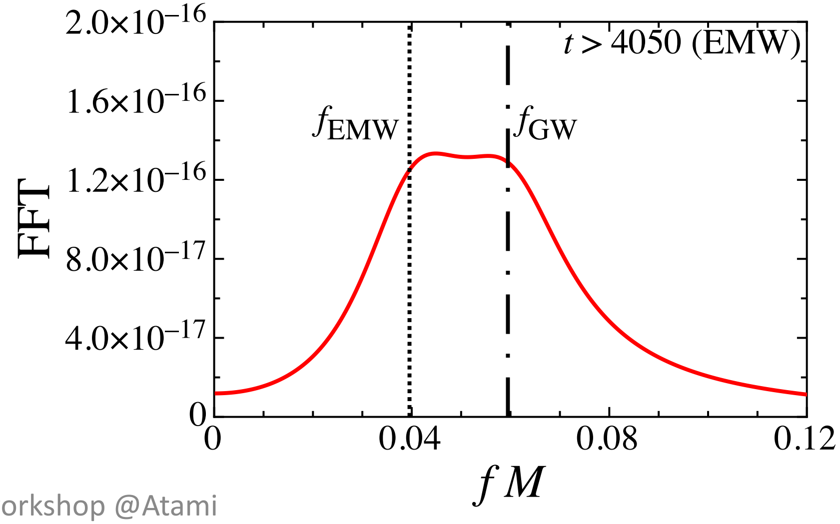
GWs & EMWs from BH



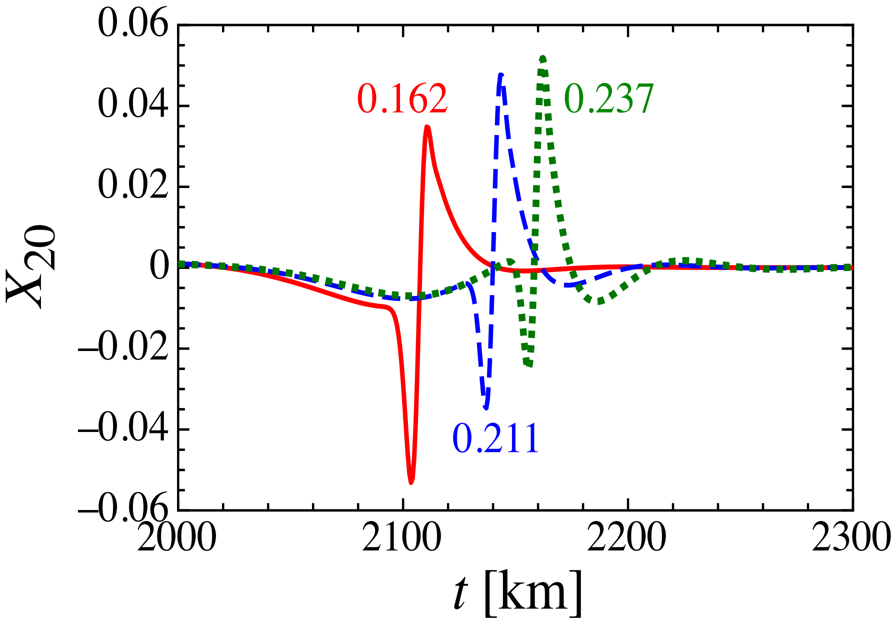


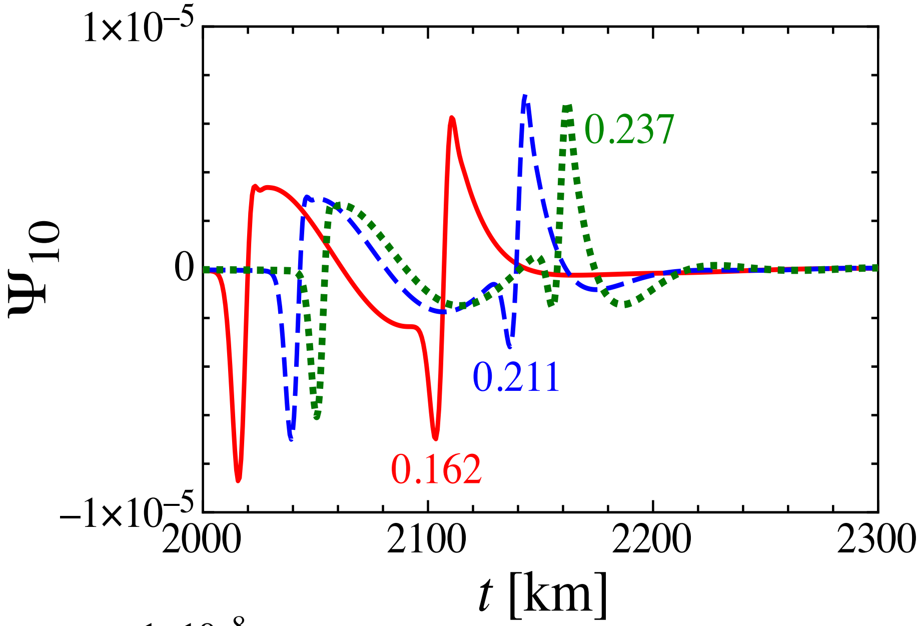
- $E_{EMW} = \alpha B_{15}^{2} E_{GW}$
  - $\alpha = 5.47 \times 10^{-6}$

  $(E_{GW} = 4.94 \times 10^{48}\, erg)$

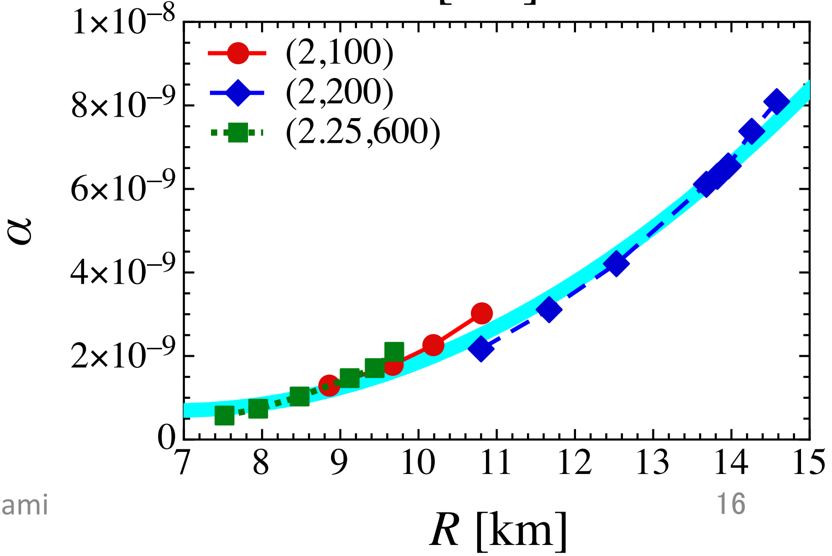


GWs & EMWs from NS

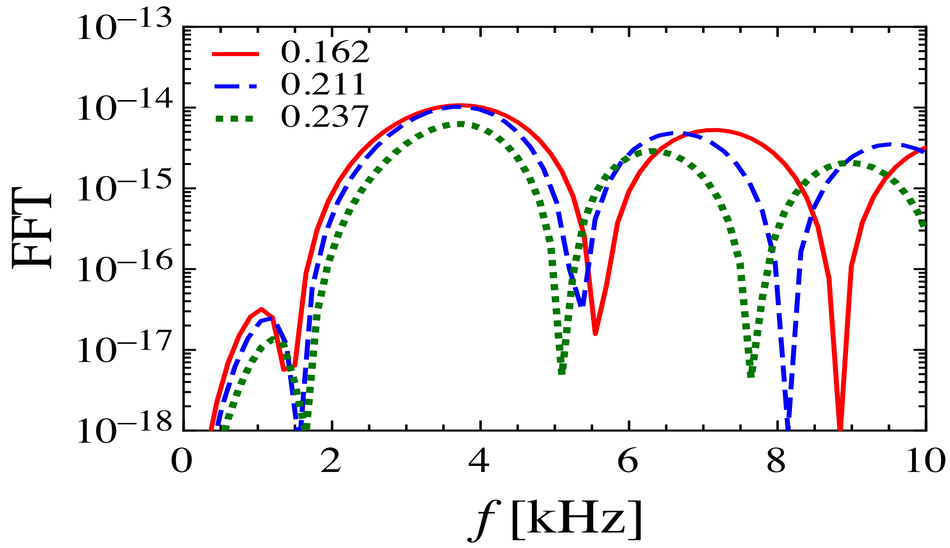


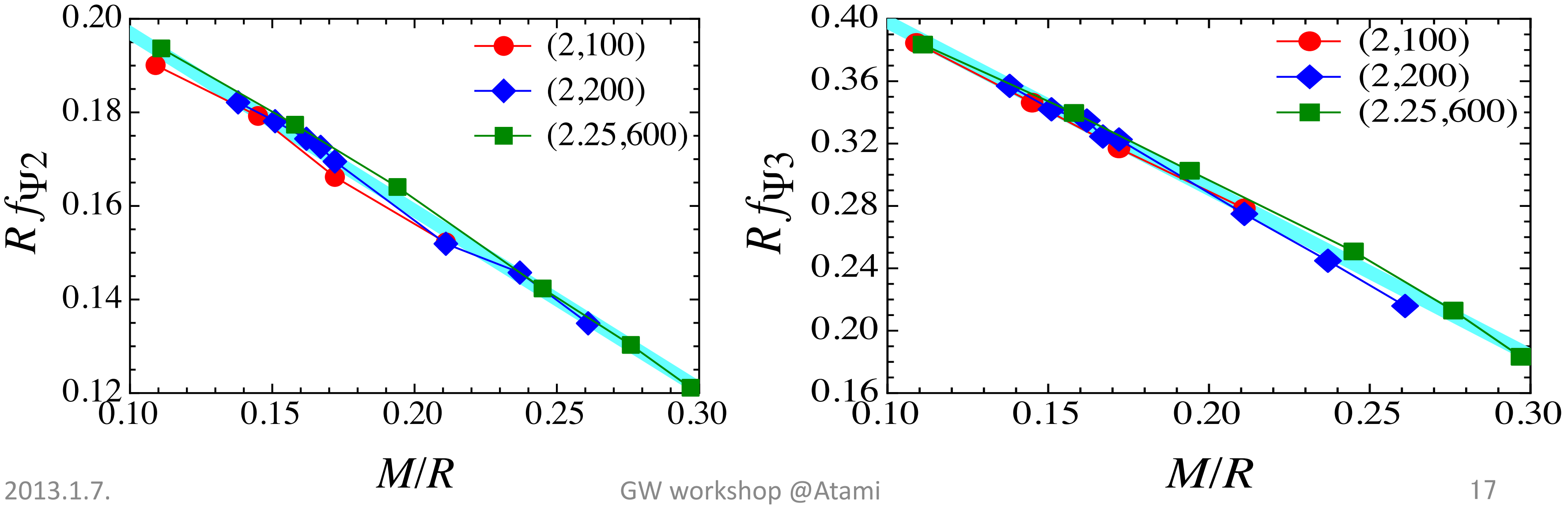


- α is almost independent of stellar EOS
  - $\alpha \sim 10^{-9}$-10^{-8}



FFT (preliminaly)





conclusion

- magnetars may be possible candidate for GWs emitter
  - observational upper limit on E_{GW} ~ 10^{47} erg for f mode
  - typical frequency of Alfven oscillations ~ 100 Hz
  - detectable via KAGRA and/or DECIGO ?
- we consider the coupling between GWs and EMWs
  - in particular, focus on EMWs driven by GWs
  - E_{EM} is proportional to B^2
  - for BH, $\alpha \sim 5\times10^{-6}$, which may depend on magnetic configuration
  - for NS, depending on stellar radius, $\alpha \sim 10^{-9}$-10^{-8}
  - f_{EM} (~ a few kHz) can be written as a function of M/R