On the extension of TNS model to cross power spectra

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Extending TNS model to describe power spectrum in redshift space, we derive the analytic expressions for cross power spectrum in redshift space, assuming linear galaxy and velocity biases.

I. PRELIMINARIES

Throughout the report, we work with the distant-observer limit, and assume that the line-of-sight direction is parallel to the z-axis. Then the *observed* redshift space may be written as

$$\boldsymbol{s}=\boldsymbol{r}-f\,u_z(\boldsymbol{r})\hat{z}, \tag{1}$$

where the quantity u_z is the normalized velocity field along the line-of-sight, defined by $u_z\equiv -v_z/(aH\,f)$. The density field in observed redshift space, $\delta^{(\mathrm{S})}$, is expressed in Fourier space as

$$\delta^{(\mathrm{S})}(\boldsymbol{k})=\int d^3\boldsymbol{r}\Big\{\delta(\boldsymbol{r})+f\,\nabla_z u_z(\boldsymbol{r})\Big\}\,e^{i\,\{\boldsymbol{k}\cdot\boldsymbol{r}-k\mu\,f\,u_z\}} \tag{2}$$

with $\mu\equiv k_z/k$.

We are particularly interested in the cross correlation between the different samples (with different bias parameter). We denote the number density fluctuation of the objects A and B by δ_{A} and δ_{B}. Also, we consider that the velocity for each object do not simply trace the underlying mass density field, and is labeled as $u_{\mathrm{A,B}}$. Then, the cross power spectrum is expressed as

$$P^{(\mathrm{S})}(\boldsymbol{k})=\int d^3\boldsymbol{x}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,\Big\langle e^{-i\,k\mu\,(f\Delta u_z+\Delta\epsilon)}\Big[\delta_{\mathrm{A}}(\boldsymbol{r})+f\nabla_z u_{\mathrm{A},z}(\boldsymbol{r})\Big]\Big[\delta_{\mathrm{B}}(\boldsymbol{r}')+f\nabla_z u_{\mathrm{B},z}(\boldsymbol{r}')\Big]\Big\rangle \tag{3}$$

with $\boldsymbol{x}=\boldsymbol{r}-\boldsymbol{r}'$. We here define

$$\Delta u_z\equiv u_{\mathrm{A},z}(\boldsymbol{r})-u_{\mathrm{B},z}(\boldsymbol{r}'). \tag{4}$$

II. MODELING REDSHIFT-SPACE CROSS POWER SPECTRUM AT WEAKLY NONLINEAR REGIME

To derive the expression relevant at weakly nonlinear regime, we follow Ref. [1], and rewrite Eq. (3) with

$$P^{(\mathrm{S})}(\boldsymbol{k})=\int d^3\boldsymbol{x}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\Big\langle e^{j_1\,A_1}\,A_2\,A_3\Big\rangle \tag{5}$$

with the quantities j_1, A_i given by

$$\begin{aligned}
j_1&=-i\,k\mu,\\
A_1&=f\,\Delta u_z\\
A_2&=\delta_{\mathrm{A}}(\boldsymbol{r})+f\nabla_z\,u_{\mathrm{A},z}(\boldsymbol{r}),\\
A_3&=\delta_{\mathrm{B}}(\boldsymbol{r}')+f\nabla_z\,u_{\mathrm{B},z}(\boldsymbol{r}').
\end{aligned}$$

Then, with a help of cumulant expansion theorem, we obtain

$$P^{(\mathrm{S})}(\boldsymbol{k})=\int d^3\boldsymbol{x}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\exp\Big\{\langle e^{j_1A_1}\rangle_c\Big\}\,\Big[\Big\langle e^{j_1A_1}A_2A_3\Big\rangle_c+\Big\langle e^{j_1A_1}A_2\Big\rangle_c\Big\langle e^{j_1A_1}A_3\Big\rangle_c\Big]. \tag{6}$$

Here, $\langle\cdots\rangle_c$ indicates the cumulant.

As it is clear from the expression, the exponential prefactor $\exp\big\{\langle e^{j_1A_1}\rangle_c\big\}$ can be non-perturbative, and it lead to a strong damping even at large scales. We thus keep it untouched. But, at weakly nonlinear scales, we may expand the rest of the terms regarding j_1 as a small expansion parameter. Up to the order of $\mathcal{O}(j_1^2)$, we obtain

$$P^{(\mathrm{S})}(\boldsymbol{k})\simeq\int d^3\boldsymbol{x}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\exp\Big\{\langle e^{j_1A_1}\rangle_c\Big\}\,\Big[\Big\langle A_2A_3\Big\rangle_c+j_1\Big\langle A_1A_2A_3\Big\rangle_c+j_1^2\Big\langle A_1A_2\Big\rangle_c\Big\langle A_1A_3\Big\rangle_c+\cdots\Big]. \tag{7}$$

Here, the term $\frac{1}{2}j_1^2\langle A_1^2 A_2 A_3\rangle_c$ is ignored according to Ref. [1]. For more simplification, we shall assume that $\exp\{\langle e^{j_1 A_1}\rangle_c\}$ is independent of separation x, and is expressed as (even) function of $k\mu$. With this assumption/ansatz, the model of redshift-space cross power spectrum, $P^{(\mathrm{S})}_{\mathrm{AB}}$, is given by

$$P^{(\mathrm{S})}_{\mathrm{AB}}(\boldsymbol{k}) = D_{\mathrm{FoG}}(k\mu\tilde{\sigma}_{\mathrm{v}})\left[\widetilde{P}_{\mathrm{Kaiser}}(\boldsymbol{k}) + \widetilde{A}(\boldsymbol{k}) + \widetilde{B}(\boldsymbol{k})\right] \tag{8}$$

with

$$\begin{aligned}
\widetilde{P}_{\mathrm{Kaiser}}(\boldsymbol{k}) &= \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\big\langle A_2 A_3\big\rangle_c,\\
\widetilde{A}(\boldsymbol{k}) &= j_1\int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\big\langle A_1 A_2 A_3\big\rangle_c,\\
\widetilde{B}(\boldsymbol{k}) &= j_1^2\int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\big\langle A_1 A_2\big\rangle\big\langle A_1 A_3\big\rangle_c.
\end{aligned} \tag{9}$$

Below, we explicitly write down the expression of each term in the bracket. In what follows, we assume the linear bias for δ_{A} and δ_{B}, and rewrite them with $b_{\mathrm{A}}\,\delta$ and $b_{\mathrm{B}}\,\delta$, respectively. Similarly, assuming the linear relation, we may write biased velocity field as $\boldsymbol{u}_{\mathrm{A,B}} = c_{\mathrm{A,B}}\,\boldsymbol{u}$. With the velocity-divergence field θ defined by $\theta = \nabla\cdot\boldsymbol{u} = -\nabla\cdot\boldsymbol{v}/(afH)$, we then have:

$$\widetilde{P}_{\mathrm{Kaiser}}(k,\mu) = b_{\mathrm{A}}b_{\mathrm{B}}\,P_{\delta\delta}(k) + f\,\mu^2(b_{\mathrm{A}}c_{\mathrm{B}} + b_{\mathrm{B}}c_{\mathrm{A}})P_{\delta\theta}(k) + f^2\,\mu^4\,c_{\mathrm{A}}c_{\mathrm{B}}\,P_{\theta\theta}(k), \tag{10}$$

$$\widetilde{A}(k,\mu) = k\mu\,f\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{p_z}{p^2}\Big\{c_{\mathrm{A}}\,\widetilde{B}_\sigma(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k}) - c_{\mathrm{B}}\,\widetilde{B}_\sigma(\boldsymbol{p},\boldsymbol{k},-\boldsymbol{k}-\boldsymbol{p})\Big\}, \tag{11}$$

$$\widetilde{B}(k,\mu) = (k\mu\,f)^2 c_{\mathrm{A}}c_{\mathrm{B}}\int\frac{d^3\boldsymbol{p}\,d^3\boldsymbol{q}}{(2\pi)^3}\delta_{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\,\tilde{F}_{\mathrm{A}}(\boldsymbol{p})\tilde{F}_{\mathrm{B}}(\boldsymbol{q}), \tag{12}$$

where the quantities $\widetilde{B}_\sigma$, $\tilde{F}_{\mathrm{X}}$ ($X=$A or B) are the cross bispectrum and power spectrum, respectively, defined by

$$\begin{aligned}
&(2\pi)^3\,\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\widetilde{B}_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\quad = \Big\langle\theta(\boldsymbol{k}_1)\big\{b_{\mathrm{A}}\,\delta(\boldsymbol{k}_2) + c_{\mathrm{A}}\,f\Big(\frac{k_{2,z}}{k_2}\Big)^2\theta(\boldsymbol{k}_2)\big\}\big\{b_{\mathrm{B}}\,\delta(\boldsymbol{k}_3) + c_{\mathrm{B}}\,f\Big(\frac{k_{3,z}}{k_3}\Big)^2\theta(\boldsymbol{k}_3)\big\}\Big\rangle.
\end{aligned} \tag{13}$$

$$\tilde{F}_{\mathrm{X}}(\boldsymbol{p}) = \frac{p_z}{p^2}\left\{b_{\mathrm{X}}\,P_{\delta\theta}(p) + c_{\mathrm{X}}\,f\left(\frac{p_z^2}{p^2}\right)^2 P_{\theta\theta}(p)\right\}. \tag{14}$$

We will derive below the explicit expressions for $\widetilde{A}$ and $\widetilde{B}$, which are given in powers of μ and f.

A. $\widetilde{A}$ term

The bispectrum $\widetilde{B}_\sigma$ given at Eq. (13) is related to the real-space matter bispectra, B_{abc}, defined by $\langle\Phi_a(\boldsymbol{k}_1)\Phi_b(\boldsymbol{k}_2)\Phi_c(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B_{abc}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ with doublet $\Phi_a = (\delta,\theta)$. It is given by

$$\begin{aligned}
\widetilde{B}_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= b_{\mathrm{A}}b_{\mathrm{B}}\,B_{211}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + c_{\mathrm{A}}c_{\mathrm{B}}\,f^2\Big(\frac{k_{2,z}}{k_2}\Big)^2\Big(\frac{k_{3,z}}{k_3}\Big)^2 B_{222}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\quad + b_{\mathrm{A}}c_{\mathrm{B}}\,f\Big(\frac{k_{3,z}}{k_3}\Big)^2 B_{212}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + b_{\mathrm{B}}c_{\mathrm{A}}\,f\Big(\frac{k_{2,z}}{k_2}\Big)^2 B_{221}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\equiv \widetilde{B}^{(\mathrm{sym})}_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \widetilde{B}^{(\mathrm{non\text{-}sym})}_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)
\end{aligned} \tag{15}$$

Note that the first line at RHS or $\widetilde{B}^{(\mathrm{sym})}_\sigma$ is symmetric under $\boldsymbol{k}_2\leftrightarrow\boldsymbol{k}_3$, but the second line or $\widetilde{B}^{(\mathrm{non\text{-}sym})}_\sigma$ is not, and can become symmetric only in the auto-power spectrum (i.e., $b_{\mathrm{A}} = b_{\mathrm{B}}$ and $c_{\mathrm{A}} = c_{\mathrm{B}}$). This asymmetry gives rise to non-trial contribution, which makes the $\widetilde{A}$ term different from that in the auto-power spectrum case.

To derive the explicit expressions of the $\widetilde{A}$ term in powers of μ and f, we rewrite Eq. (11) as

$$\begin{aligned}
\widetilde{A}(k,\mu) &= k\mu\,f\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\Big\{c_{\mathrm{A}}\,\frac{p_z}{p^2}\,\widetilde{B}^{(\mathrm{sym})}_\sigma(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k}) + c_{\mathrm{B}}\,\frac{k_z-p_z}{|\boldsymbol{k}-\boldsymbol{p}|^2}\,\widetilde{B}^{(\mathrm{sym})}_\sigma(\boldsymbol{k}-\boldsymbol{p},\boldsymbol{p},-\boldsymbol{k})\Big\}\\
&\quad + k\mu\,f\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\Big\{c_{\mathrm{A}}\,\frac{p_z}{p^2}\,\widetilde{B}^{(\mathrm{non\text{-}sym})}_\sigma(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k}) + c_{\mathrm{B}}\,\frac{k_z-p_z}{|\boldsymbol{k}-\boldsymbol{p}|^2}\,\widetilde{B}^{(\mathrm{non\text{-}sym})}_\sigma(\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k},\boldsymbol{p})\Big\}
\end{aligned} \tag{16}$$

where $\widetilde{B}_\sigma^{(\mathrm{sym})}$ and $\widetilde{B}_\sigma^{(\mathrm{non\text{-}sym})}$ are defined below:

$$\begin{aligned}\widetilde{B}_\sigma^{(\mathrm{sym})}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= b_{\rm A}b_{\rm B}\,B_{211}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + c_{\rm A}c_{\rm B}\,f^2\Big(\frac{k_{2,z}}{k_2}\Big)^2\Big(\frac{k_{3,z}}{k_3}\Big)^2 B_{222}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3),\\ \widetilde{B}_\sigma^{(\mathrm{non\text{-}sym})}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= b_{\rm A}c_{\rm B}\,f\Big(\frac{k_{3,z}}{k_3}\Big)^2 B_{212}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + b_{\rm B}c_{\rm A}\,f\Big(\frac{k_{2,z}}{k_2}\Big)^2 B_{221}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).\end{aligned} \tag{17}$$

With the form given above, the $\widetilde{A}$ is expanded as

$$\begin{aligned}A(k,\mu) &= \frac{k^3}{(2\pi)^2}\sum_{n=1}^{3}\sum_{a,b}^{2}\mu^{2n}\,f^{a+b-1}\int_0^\infty dr\int_{-1}^{1}dx\\ &\times\Big\{A^n_{ab}(r,x)\,B_{2ab}(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k}) + \tilde{A}^n_{ab}(r,x)\,B_{2ab}(\boldsymbol{k}-\boldsymbol{p},\boldsymbol{p},-\boldsymbol{k}) + \hat{A}^n_{ab}(r,x)\,B_{2ab}(\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k},\boldsymbol{p})\Big\},\end{aligned} \tag{18}$$

where we define $r=p/k$ and $x=(\boldsymbol{k}\cdot\boldsymbol{p})/(kp)$. Then, according to Appendix B of [1], the coefficients A^n_{ab}, $\tilde{A}^a_{ab}$, and $\hat{A}^a_{ab}$ are derived, and the non-vanishing coefficients are expressed as follows:

$$\begin{aligned}
&A^1_{11} = r\,x\,b_{\rm A}b_{\rm B}c_{\rm A}, \quad A^1_{21} = -\frac{r^2(-2+3rx)(x^2-1)}{2(1+r^2-2rx)}\,b_{\rm B}c_{\rm A}^2, \quad A^2_{12} = r\,x\,b_{\rm A}c_{\rm A}c_{\rm B},\\
&A^2_{21} = \frac{r(2x+r(2-6x^2)+r^2x(-3+5x^2))}{2(1+r^2-2rx)}\,b_{\rm B}c_{\rm A}^2, \quad A^2_{22} = -\frac{r^2(-2+3rx)(x^2-1)}{2(1+r^2-2rx)}\,c_{\rm A}^2c_{\rm B},\\
&A^3_{22} = \frac{r(2x+r(2-6x^2+rx(-3+5x^2)))}{2(1+r^2-2rx)}\,c_{\rm A}^2c_{\rm B}\\
&\tilde{A}^1_{11} = -\frac{r^2(-1+rx)}{(1+r^2-2rx)}\,b_{\rm A}b_{\rm B}c_{\rm B}, \quad \tilde{A}^2_{22} = \frac{r^2(-1+3rx)(x^2-1)}{2(1+r^2-2rx)}\,c_{\rm A}c_{\rm B}^2, \quad \tilde{A}^3_{22} = \frac{r^2(-1+3rx+3x^2-5rx^3)}{2(1+r^2-2rx)}\,c_{\rm A}c_{\rm B}^2,\\
&\hat{A}^1_{12} = \frac{r^2(-1+3rx)(x^2-1)}{2(1+r^2-2rx)}\,b_{\rm A}c_{\rm B}^2, \quad \hat{A}^2_{12} = -\frac{r^2(1-3x^2+rx(-3+5x^2))}{2(1+r^2-2rx)}\,b_{\rm A}c_{\rm B}^2, \quad \hat{A}^2_{21} = -\frac{r^2(-1+rx)}{1+r^2-2rx}\,b_{\rm B}c_{\rm A}c_{\rm B}.
\end{aligned} \tag{19}$$

The contributions coming from the symmetric bispectrum $\widetilde{B}_\sigma^{(\mathrm{sym})}$, i.e., A^n_{11}, A^n_{22}, $\tilde{A}^n_{11}$, and $\tilde{A}^n_{22}$, coincide with those obtained in the auto-power spectrum case [2], but others do not necessarily reproduce the previous results. Nevertheless, summing up all contributions, the implemented code of the A term with Eqs. (18) and (19) reproduces the previous results if we set $b_{\rm A}=b_{\rm B}$ and $c_{\rm A}=c_{\rm B}$.

B. $\widetilde{B}$ term

We first rewrite Eq. (12) with

$$\widetilde{B}(k,\mu) = \frac{(k\mu\,f)^2}{2}c_{\rm A}c_{\rm B}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\Big[\tilde{F}_{\rm A}(\boldsymbol{p})\tilde{F}_{\rm B}(\boldsymbol{k}-\boldsymbol{p}) + \tilde{F}_{\rm A}(\boldsymbol{k}-\boldsymbol{p})\tilde{F}_{\rm B}(\boldsymbol{p})\Big]. \tag{20}$$

The integrand of this expression is symmetric under $\boldsymbol{p}\leftrightarrow\boldsymbol{k}-\boldsymbol{p}$. Then, as similarly done in the auto-power spectrum case [1], we expand the $\widetilde{B}$ term in powers of f and μ, :

$$\widetilde{B}(k,\mu) = \frac{k^3}{(2\pi)^2}\sum_{n=1}^{4}\sum_{a,b=1}^{2}\mu^{2n}\,(-f)^{a+b}\int_0^\infty dr\int_{-1}^{1}dx\,\widetilde{B}^n_{ab}(r,x)\frac{P_{a2}(k\sqrt{1+r^2-2rx})\,P_{b2}(kr)}{(1+r^2-2rx)^a}. \tag{21}$$

Note again that $r\equiv p/k$ and $x=(\boldsymbol{p}\cdot\boldsymbol{k})/(pk)$. With the symmetric form of Eq. (20), the integral over r and x can be replaced with

$$\int_0^\infty dr\int_{-1}^{1}dx \;\longrightarrow\; 2\int_0^\infty dr\int_{-1}^{\mathrm{Min}[1,\,1/(2r)]}dx. \tag{22}$$

This would help to improve the convergence of numerical integration, avoiding poles. The coefficient $\tilde{B}^n_{ab}$ is derived based on Appendix B of [1], and the results are summarized below:

$$
\begin{aligned}
&\tilde{B}^1_{11}=\frac{r^2}{2}(x^2-1)\,b_{\rm A}b_{\rm B}c_{\rm A}c_{\rm B},\quad \tilde{B}^1_{12}=\frac{3r^2}{16}(x^2-1)^2\;c_{\rm A}c_{\rm B}(b_{\rm A}c_{\rm B}+b_{\rm B}c_{\rm A}),\quad \tilde{B}^1_{21}=\frac{3r^4}{16}(x^2-1)^2\;c_{\rm A}c_{\rm B}(b_{\rm A}c_{\rm B}+b_{\rm B}c_{\rm A}),\\
&\tilde{B}^1_{22}=\frac{5r^4}{16}(x^2-1)^3\;c_{\rm A}^2c_{\rm B}^2,\quad \tilde{B}^2_{11}=\frac{r}{2}(r+2x-3rx^2)c_{\rm A}c_{\rm B}b_{\rm A}b_{\rm B},\quad \tilde{B}^2_{12}=\frac{3r}{8}(x^2-1)(r+2x-5rx^2)c_{\rm A}c_{\rm B}(b_{\rm A}c_{\rm B}+b_{\rm B}c_{\rm A}),\\
&\tilde{B}^2_{21}=\frac{3r^2}{8}(x^2-1)(-2+r^2+6rx-5r^2x^2)c_{\rm A}c_{\rm B}(b_{\rm A}c_{\rm B}+b_{\rm B}c_{\rm A}),\quad \tilde{B}^2_{22}=\frac{3r^2}{16}(x^2-1)^2(-6+5r^2+30rx-35r^2x^2)c_{\rm A}^2c_{\rm B}^2,\\
&\tilde{B}^3_{11}=0,\quad \tilde{B}^3_{12}=\frac{r}{16}(4x(3-5x^2)+r(3-30x^2+35x^4))c_{\rm A}c_{\rm B}(b_{\rm A}c_{\rm B}+b_{\rm B}c_{\rm A})\quad,\\
&\tilde{B}^3_{21}=\frac{r}{16}(-8x+r(-12+36x^2+12rx(3-5x^2)+r^2(3-30x^2+35x^4)))c_{\rm A}c_{\rm B}(b_{\rm A}c_{\rm B}+b_{\rm B}c_{\rm A}),\\
&\tilde{B}^3_{22}=\frac{3r}{16}(x^2-1)(-8x+r(-12+60x^2+20rx(3-7x^2)+5r^2(1-14x^2+21x^4)))c_{\rm A}^2c_{\rm B}^2,\\
&\tilde{B}^4_{22}=\frac{r}{16}(8x(-3+5x^2)-6r(3-30x^2+35x^4)+6r^2x(15-70x^2+63x^4)+r^3(5-21x^2(5-15x^2+11x^4)))c_{\rm A}^2c_{\rm B}^2.
\end{aligned}
\tag{23}
$$

Setting $b_{\rm A}$, $b_{\rm B}$, $c_{\rm A}$ and $c_{\rm B}$ to unity, the above expressions exactly coincide with those presented in Ref. [1].

[1] A. Taruya, T. Nishimichi, and S. Saito, Phys.Rev. **D82**, 063522 (2010), 1006.0699.
[2] A. Taruya, T. Nishimichi, and F. Bernardeau, Phys. Rev. D **87**, 083509 (2013), 1301.3624.