2015年11月30日

談話会＠名古屋大学

宇宙の構造形成：
ΛCDMモデルとその向こう

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内容

宇宙の標準モデルの成り立ち

ΛCDMモデルの向こうへ

精密観測時代の宇宙論：理論ツールのリノベーション

collaboration:

F. Bernardeau, S. Colombi, T. Nishimichi

今回の集中講義

「宇宙大規模構造と精密宇宙論」

11/9	宇宙大規模構造の線形理論
11/16	宇宙大規模構造の観測量
	宇宙大規模構造の非線形進化（摂動計算手法）
11/30	種々の非線形性

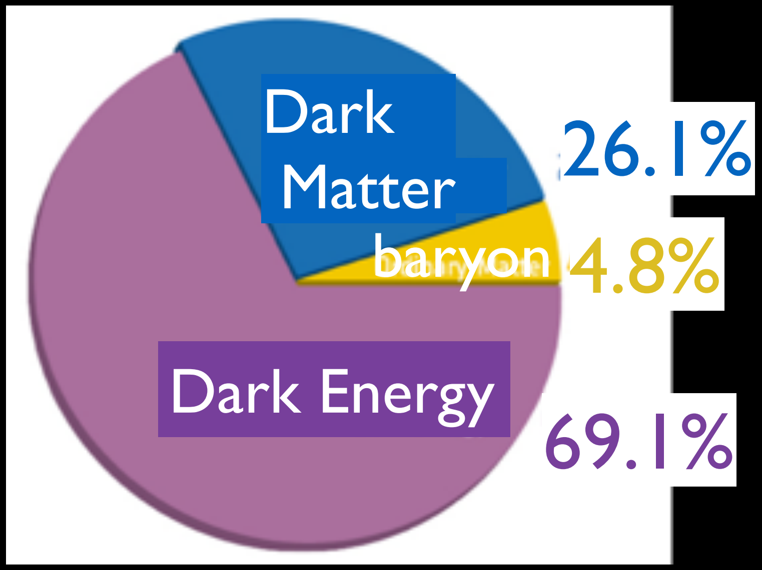
宇宙大規模構造の観測から、宇宙の「**標準モデル**」を超える手がかりを得るための理論的基礎と最近の進展

ΛCDMモデル

現在の宇宙論の標準モデル

- 宇宙項入りの曲率ゼロの平坦宇宙
- ６個のパラメーターで記述されるミニマムモデル

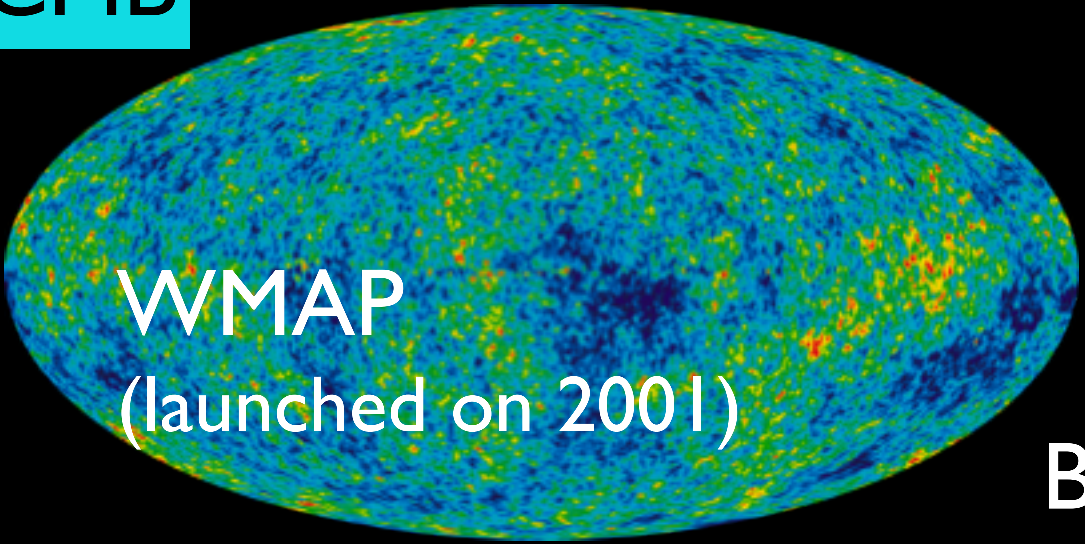
⇨ 現在の観測精度で宇宙膨張と構造形成を無矛盾に説明



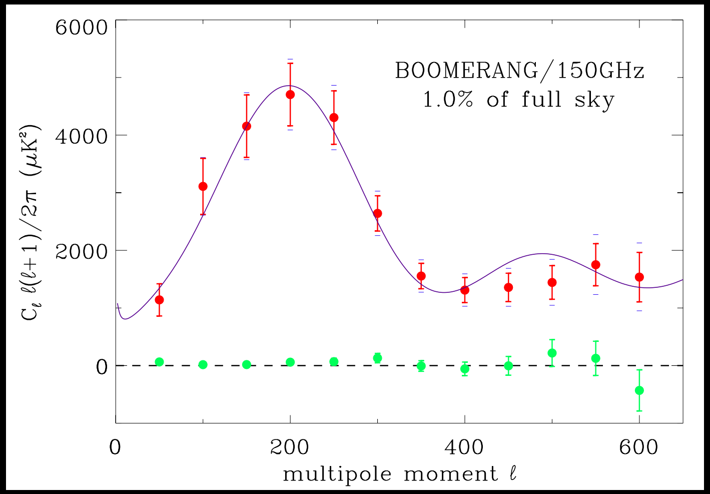
$\Omega_\mathrm{b}h^2$	: baryon density	宇宙膨張
$\Omega_\mathrm{c}h^2$	: CDM density	
θ_MC	: distance ratio to last scattering surface	
n_s	: scalar spectral index	原始密度ゆらぎ
A_s	: amplitude of curvature fluctuation	
τ	: reionization optical depth	初期天体形成

ΛCDMモデル確立の立役者

CMB



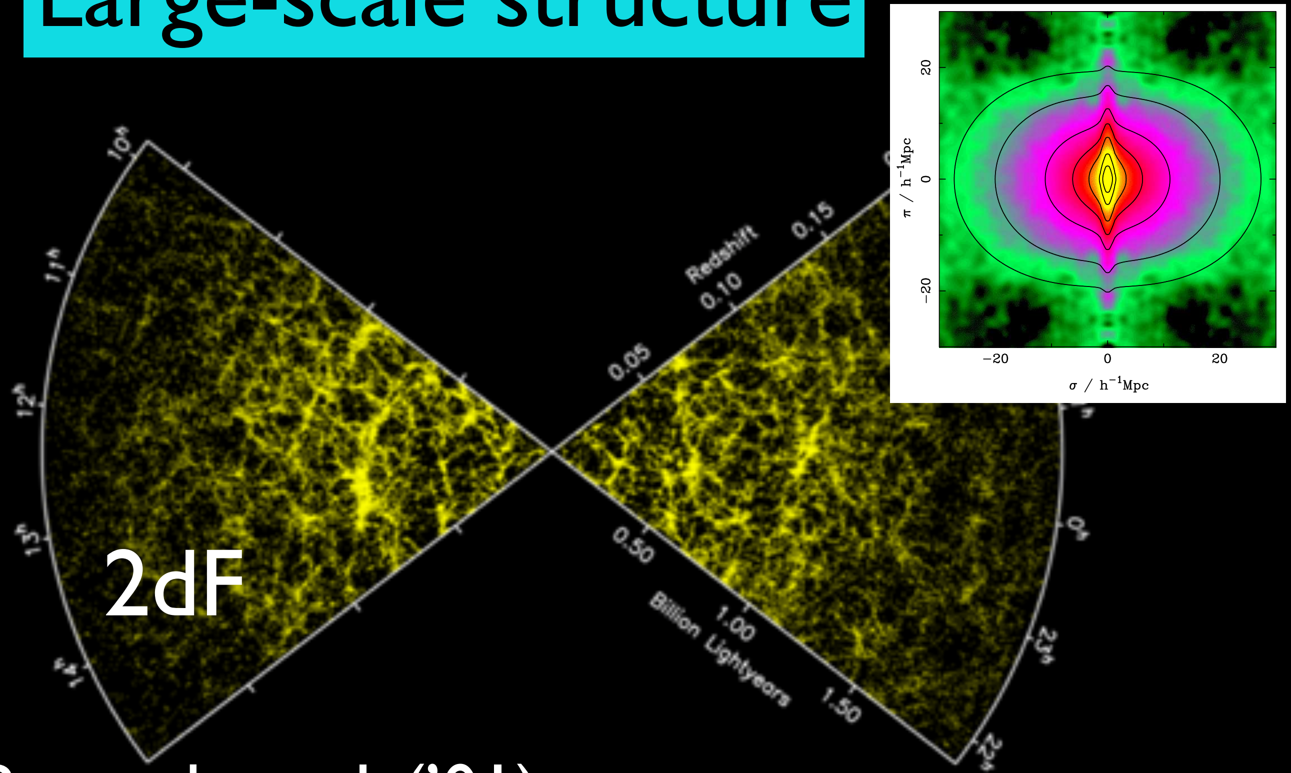
WMAP
(launched on 2001)



BOOMERanG

de Barnardis et al. (’00)

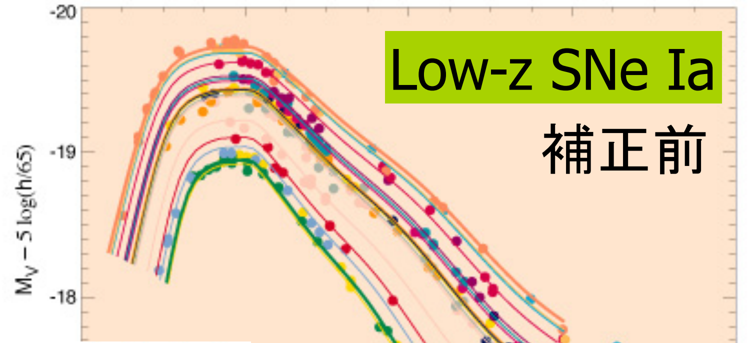
Large-scale structure

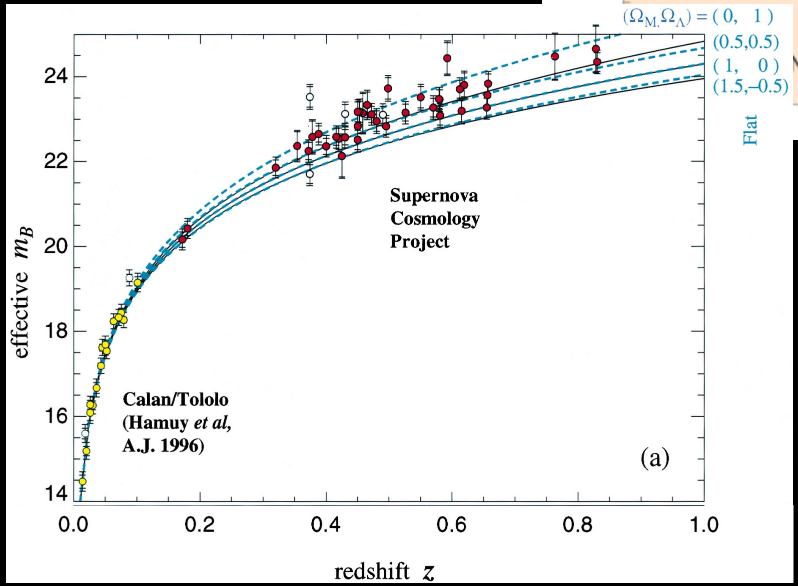


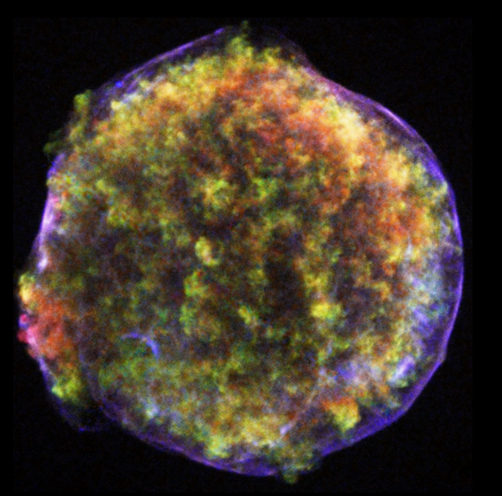
2dF

Peacock et al. (’01)

Type Ia SNe



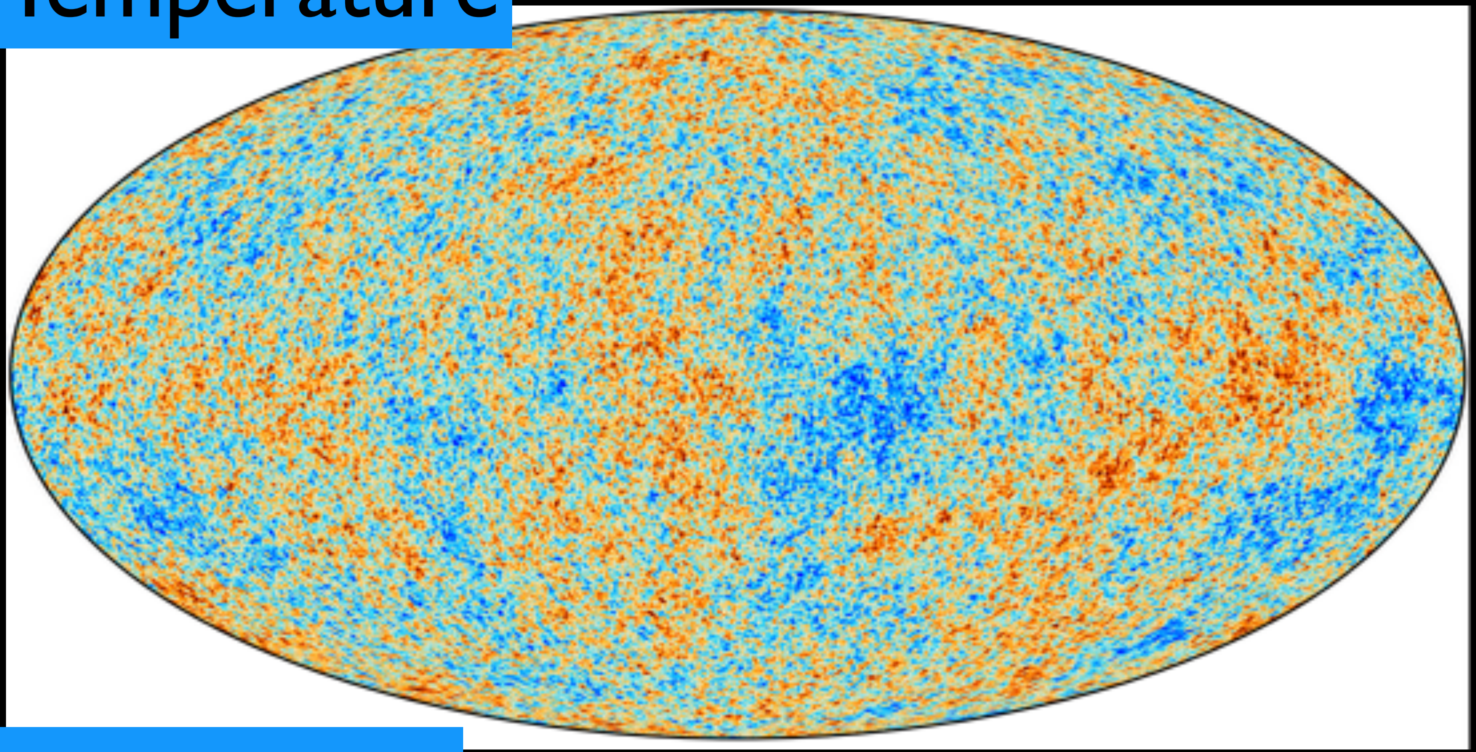


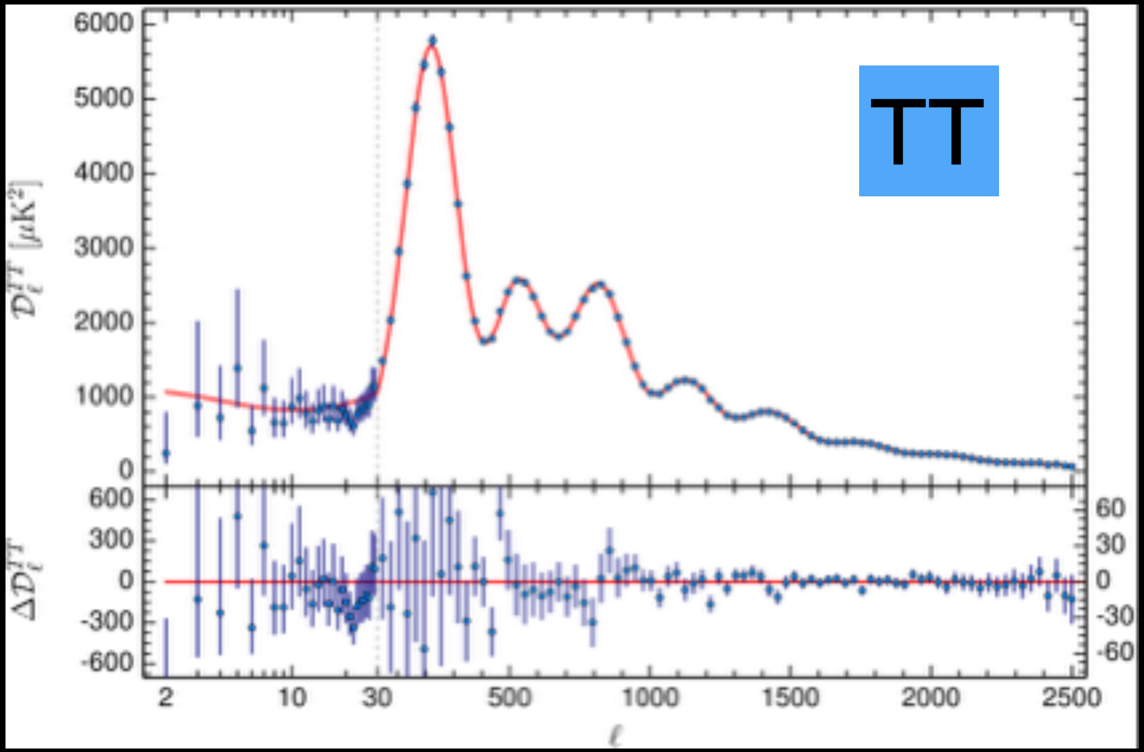


Perlmutter et al. (’99)

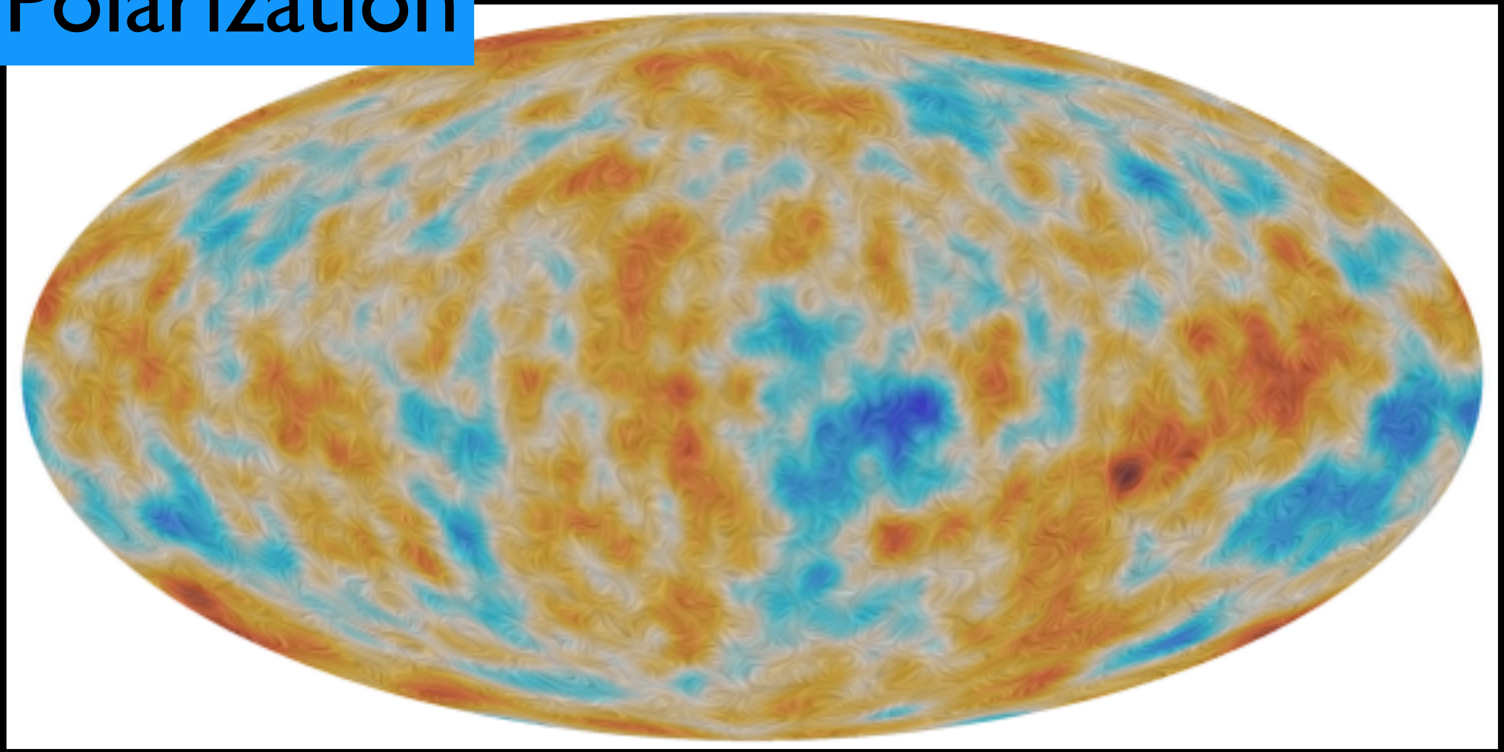
Planck 2015

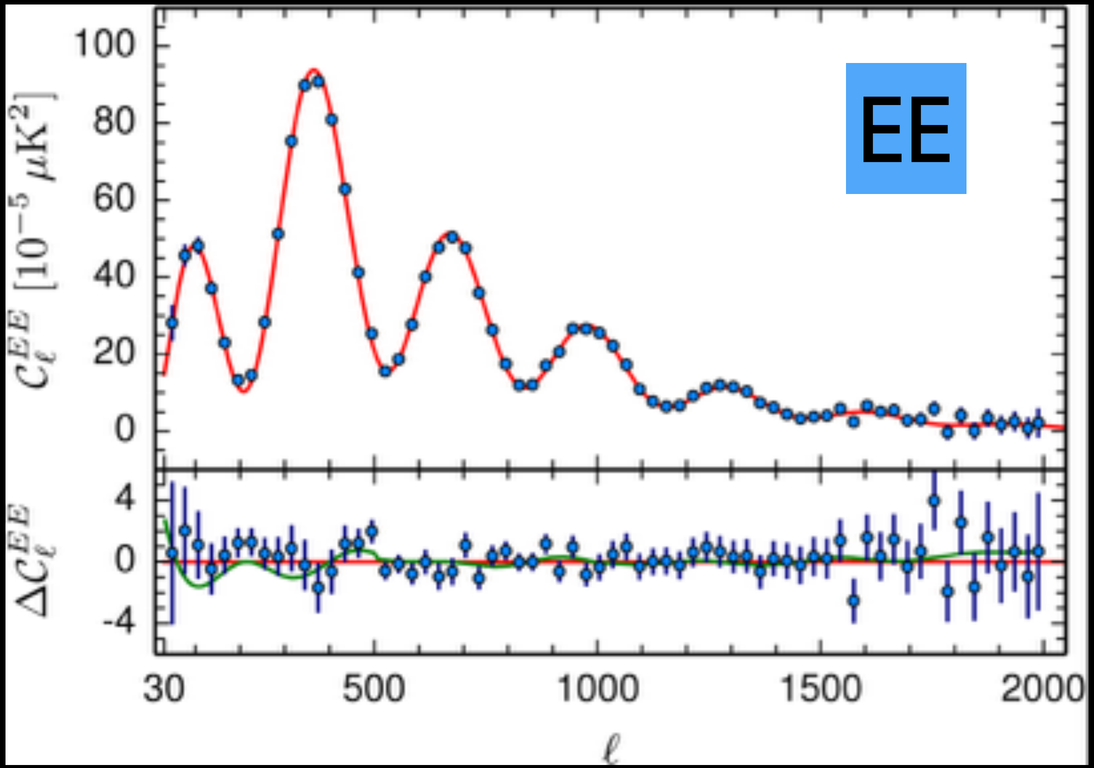
Temperature





Polarization

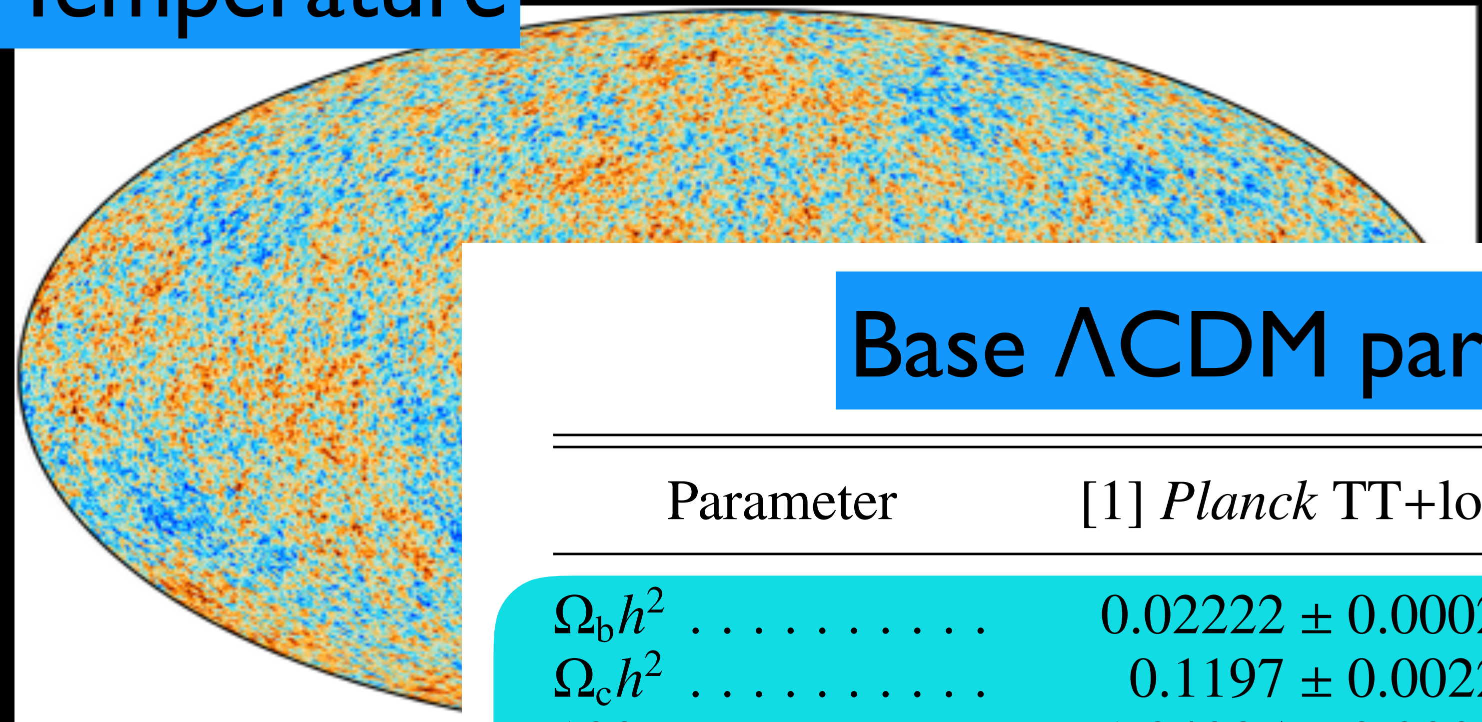


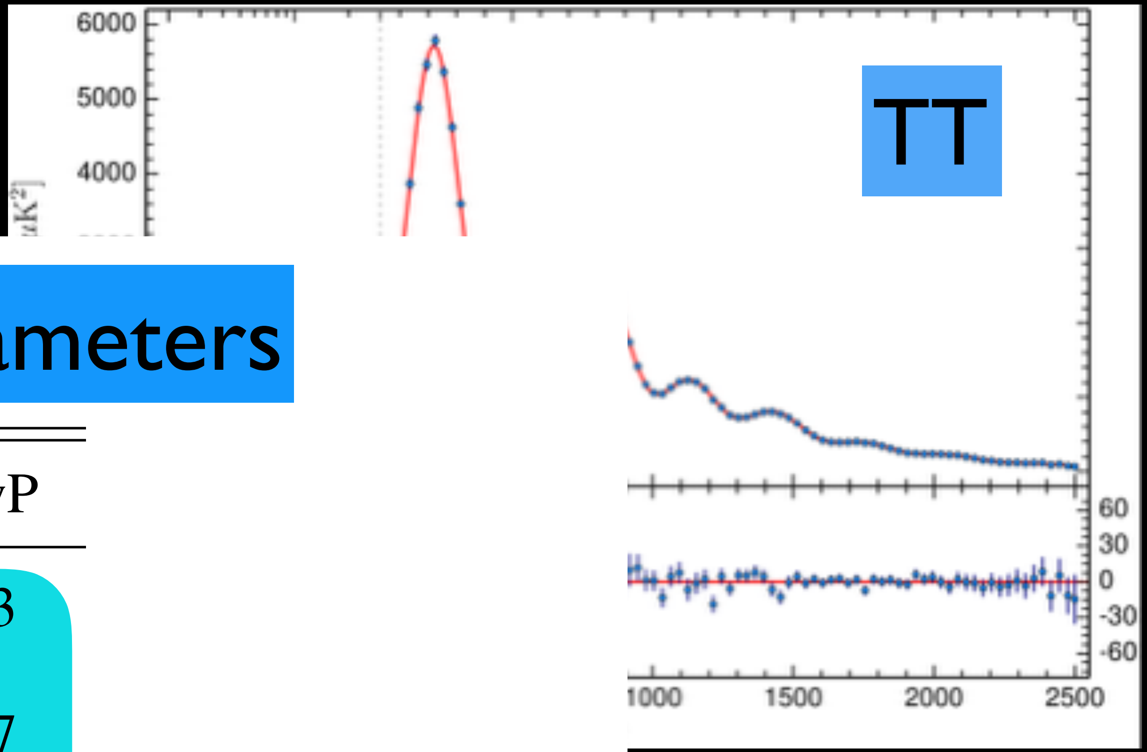


http://www.cosmos.esa.int/web/planck/

Planck 2015

Temperature



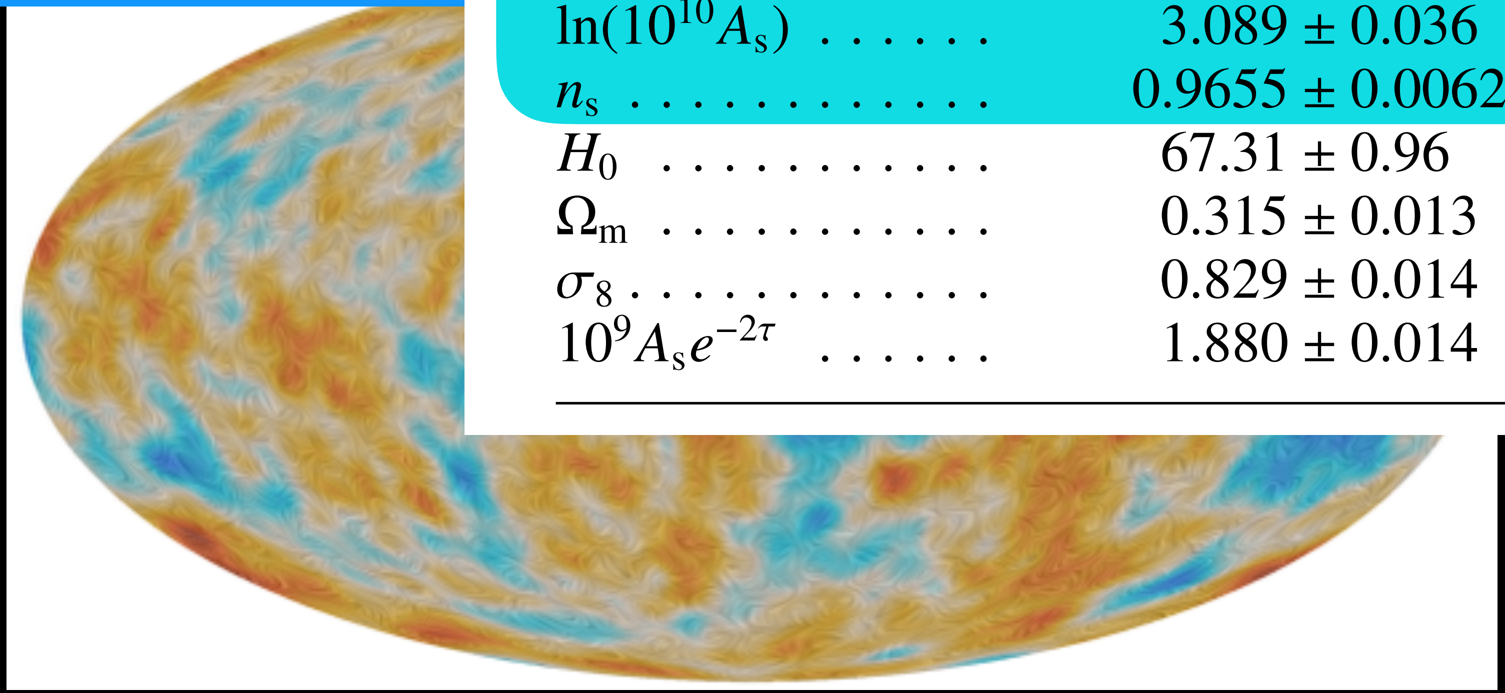


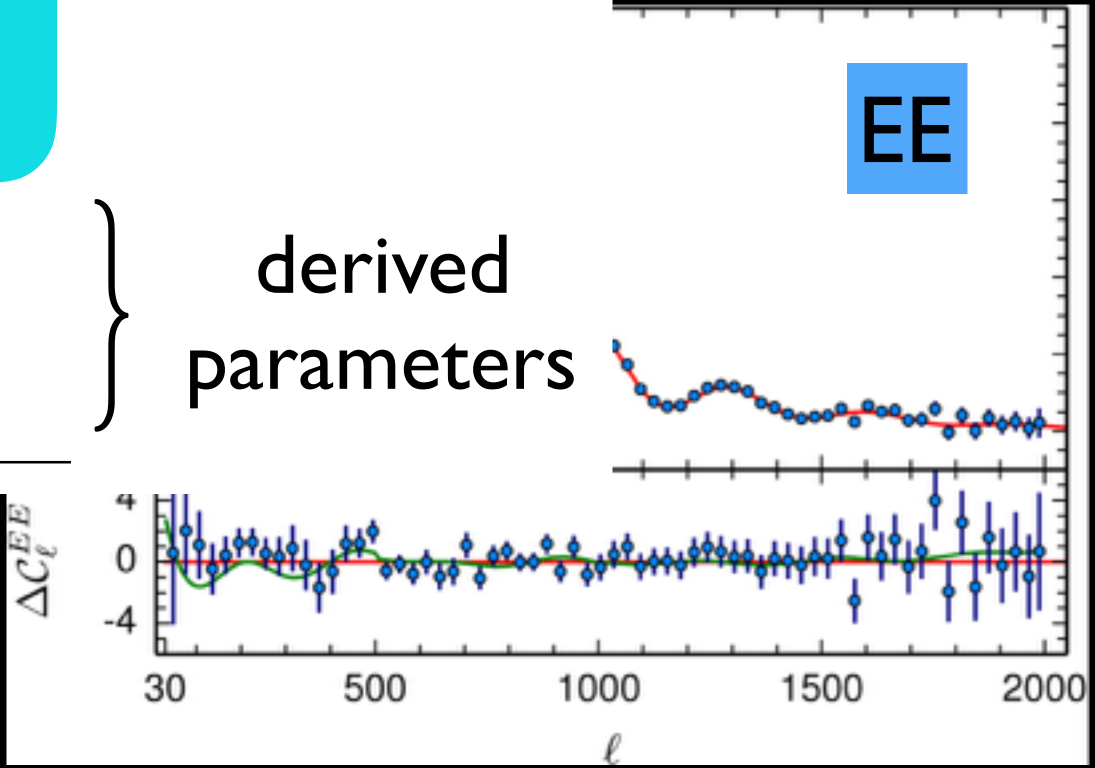
Base ΛCDM parameters

Parameter	[1] *Planck* TT+lowP
$\Omega_b h^2$	0.02222 ± 0.00023
$\Omega_c h^2$	0.1197 ± 0.0022
$100\theta_{MC}$	1.04085 ± 0.00047
τ	0.078 ± 0.019
$\ln(10^{10} A_s)$	3.089 ± 0.036
n_s	0.9655 ± 0.0062
H_0	67.31 ± 0.96
Ω_m	0.315 ± 0.013
σ_8	0.829 ± 0.014
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014

derived parameters

Polarization

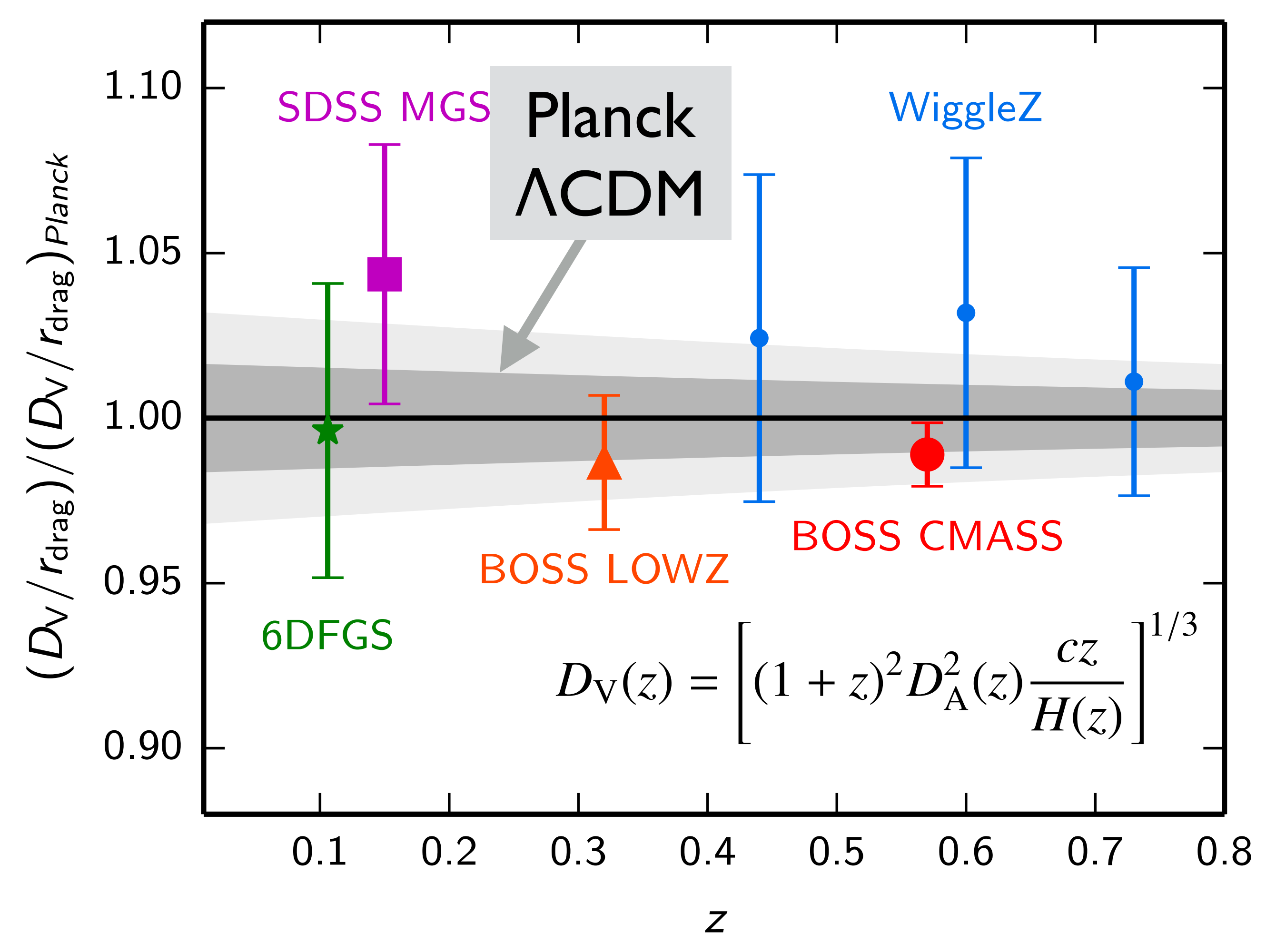




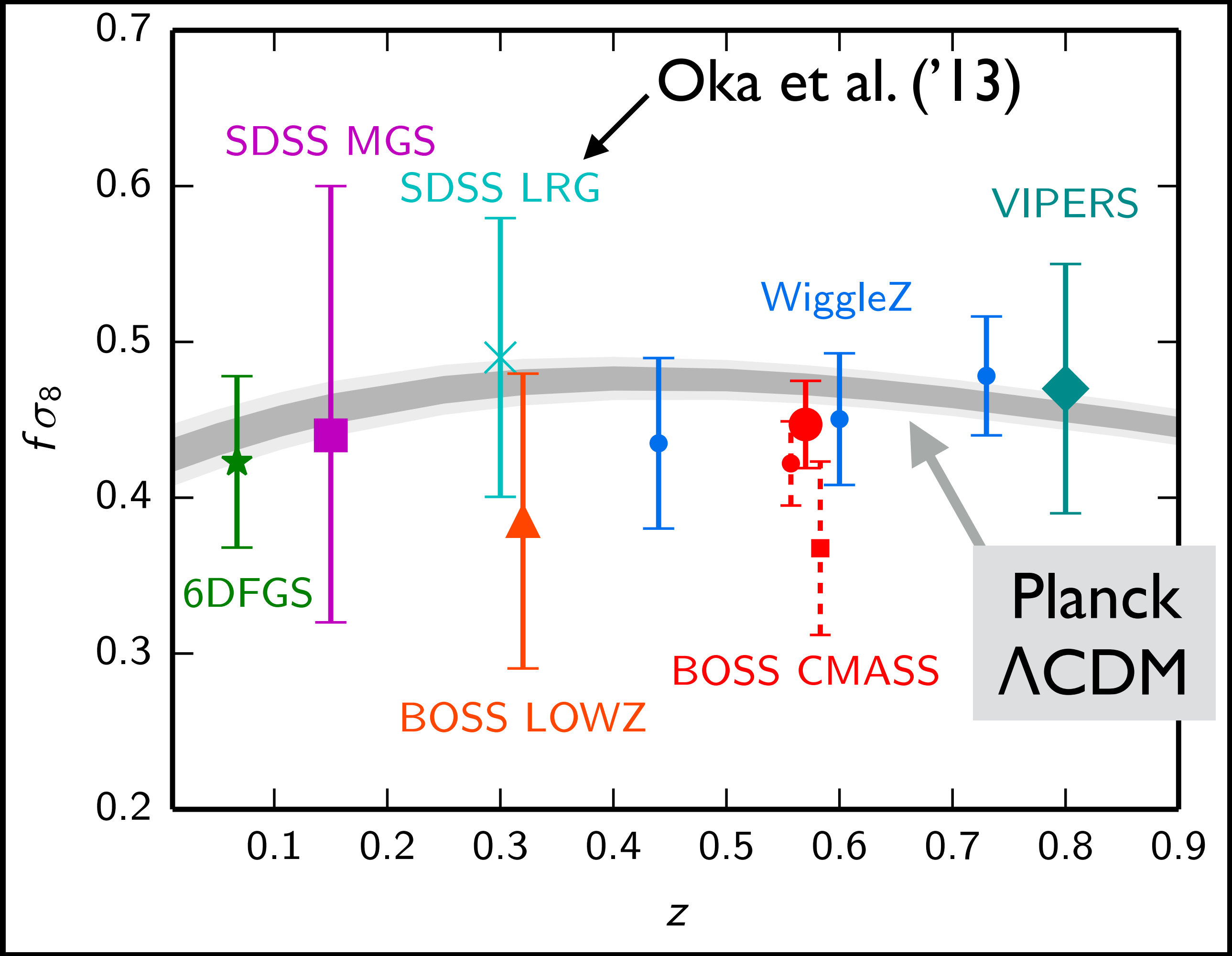
http://www.cosmos.esa.int/web/planck/

大規模構造観測との整合性

バリオン音響振動による
宇宙論的距離の測定



赤方偏移空間ゆがみによる
構造成長率の測定



Planck 2015 XIII.

現時点で深刻な矛盾は見いだせていない
今後は大規模構造の観測が主導してさらなる検証が進む

ΛCDMモデル

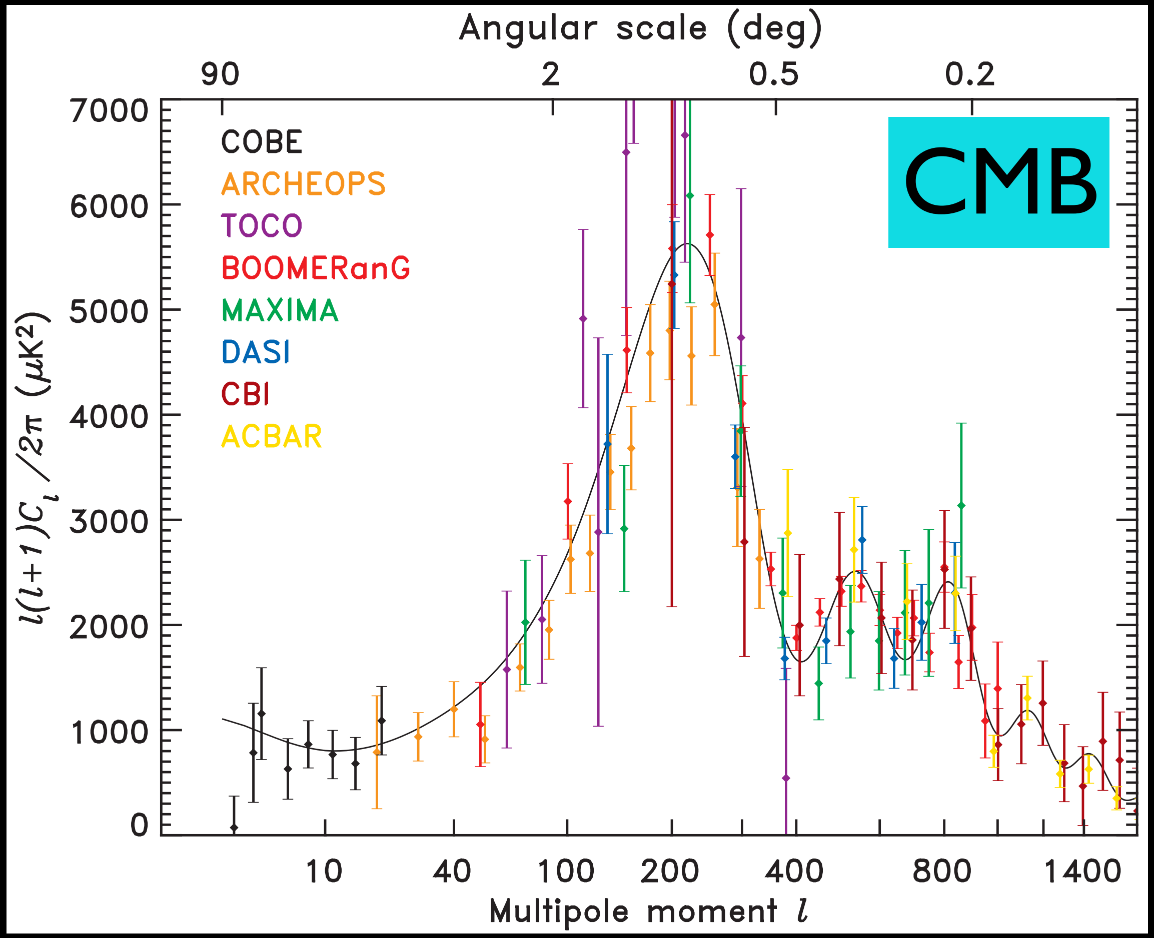
ミニマムモデル
＝仮定の積み重ね

- 曲率ゼロの平坦宇宙 ＋ 宇宙項（ダークエネルギー）
（物質成分としてはダークマター、バリオンのみ）
- インフレーションと無矛盾な断熱ゆらぎ
（ベキ型パワースペクトル）
- ゆらぎの初期条件はガウス統計に従う
- 一般相対論にもとづく宇宙の大域的進化
（宇宙膨張＋密度ゆらぎ）
- 宇宙原理が成り立つ（宇宙は大域的に一様・等方）

単純化とはいえ、これだけ仮定を積み重ねても観測と
無矛盾な結果が得られたことは逆に驚き？

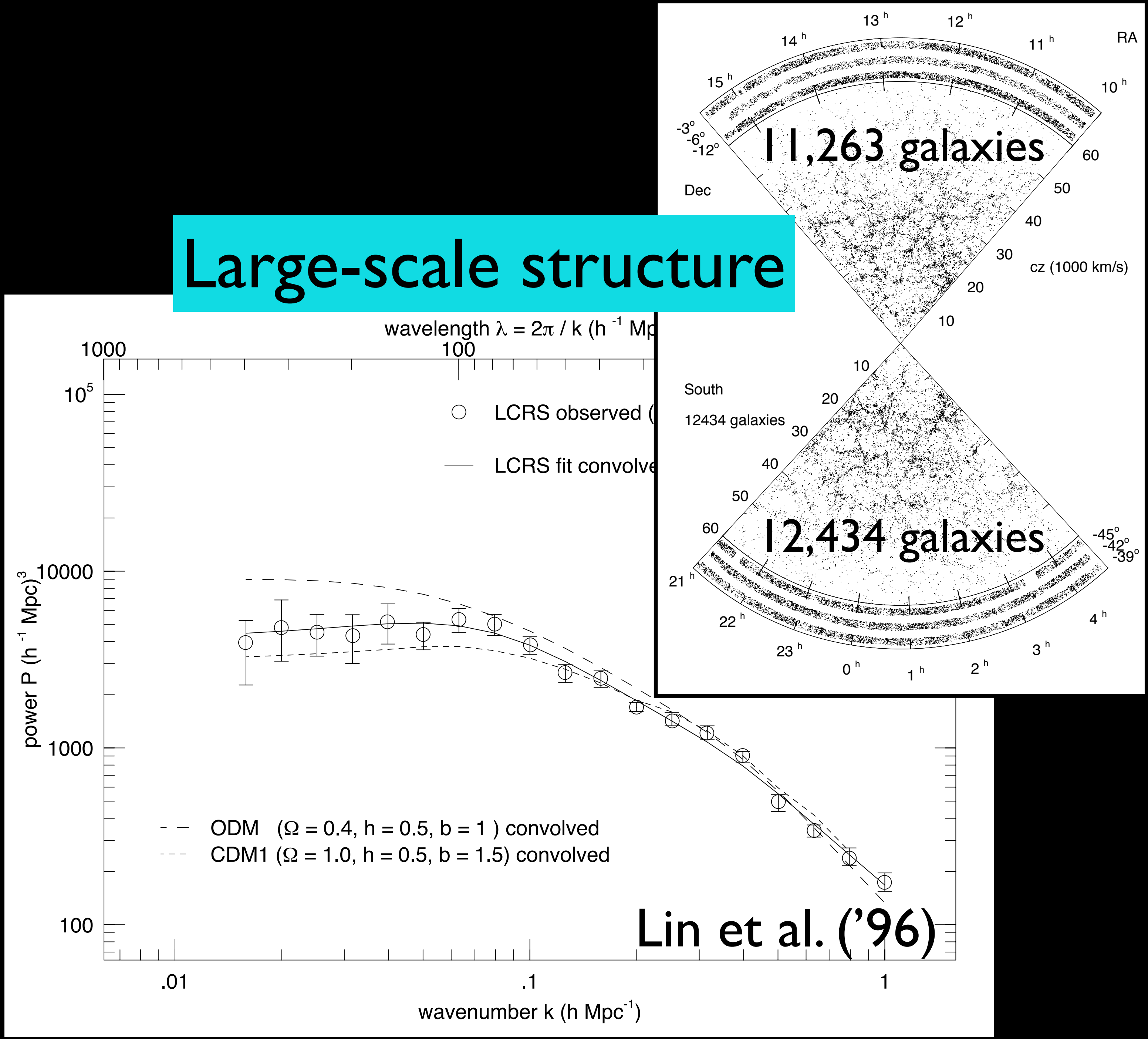
精密宇宙論前夜：~2000年以前

WMAP以前は高角度分解能
の全天観測がなかった



CMB

Hinshaw et al. ('01)



Large-scale structure

11,263 galaxies

12,434 galaxies

Lin et al. ('96)

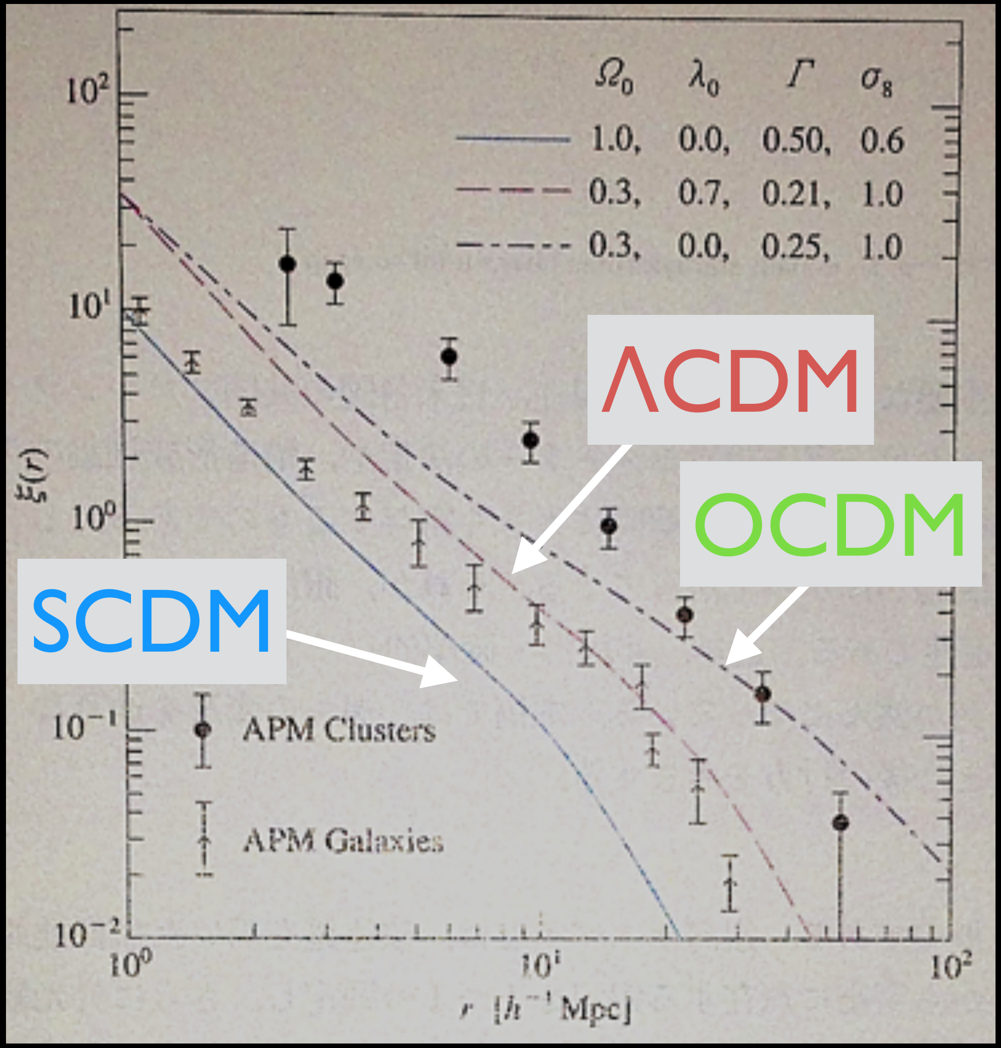
SDSS, 2dF 以前に最大規模だった
Las Campanas 赤方偏移サーベイ

標準モデルの候補たち

SCDM (Standard CDM) $(\Omega_{\mathrm{m}} = 1,\ \Omega_{\mathrm{DE}} = 0,\ h \simeq 0.5,\ \sigma_8 \simeq 0.6)$

OCDM (Open CDM) $(\Omega_{\mathrm{m}} \simeq 0.3,\ \Omega_{\mathrm{DE}} = 0,\ h \simeq 0.8,\ \sigma_8 \simeq 1.0)$

ΛCDM (Lambda CDM) $(\Omega_{\mathrm{m}} \simeq 0.3,\ \Omega_{\mathrm{DE}} \simeq 0.7,\ h \simeq 0.7,\ \sigma_8 \simeq 1.0)$



これら３つのモデルは、少なくとも当時は、同等にもっともらしかった

なぜΛCDMが標準モデルになりえたのか？

物理学会誌56巻 No.3, p.169, 2001年

線形パワースペクトル

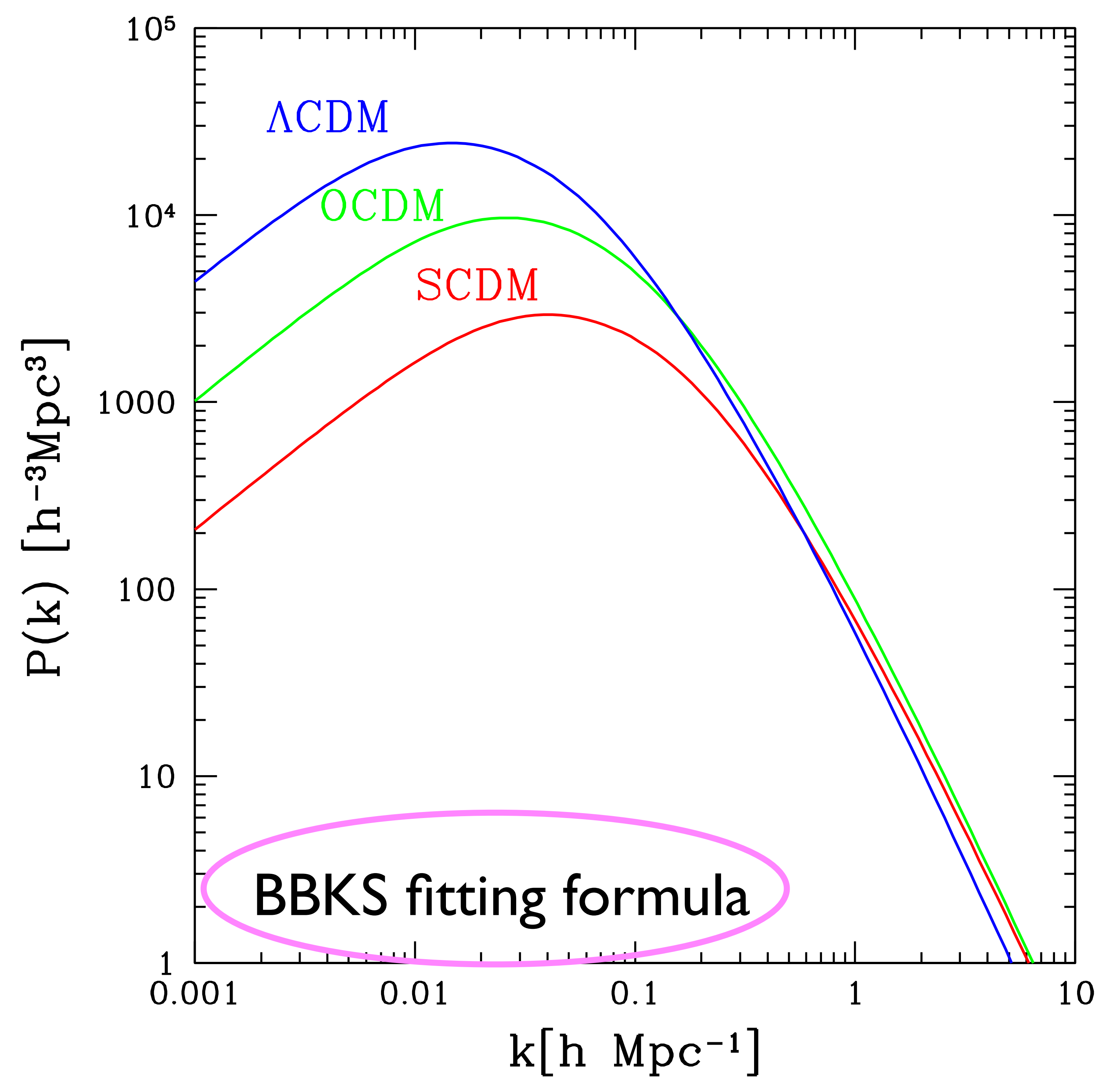
SCDM $(\Omega_{\rm m} = 1,\ \Omega_\Lambda = 0, \sigma_8 = 0.59)$

ΛCDM $(\Omega_{\rm m} = 0.272,\ \Omega_\Lambda = 0.728, \sigma_8 = 0.81)$

OCDM $(\Omega_{\rm m} = 0.45,\ \Omega_\Lambda = 0, \sigma_8 = 0.80)$

当時、よく描かれていた理論線

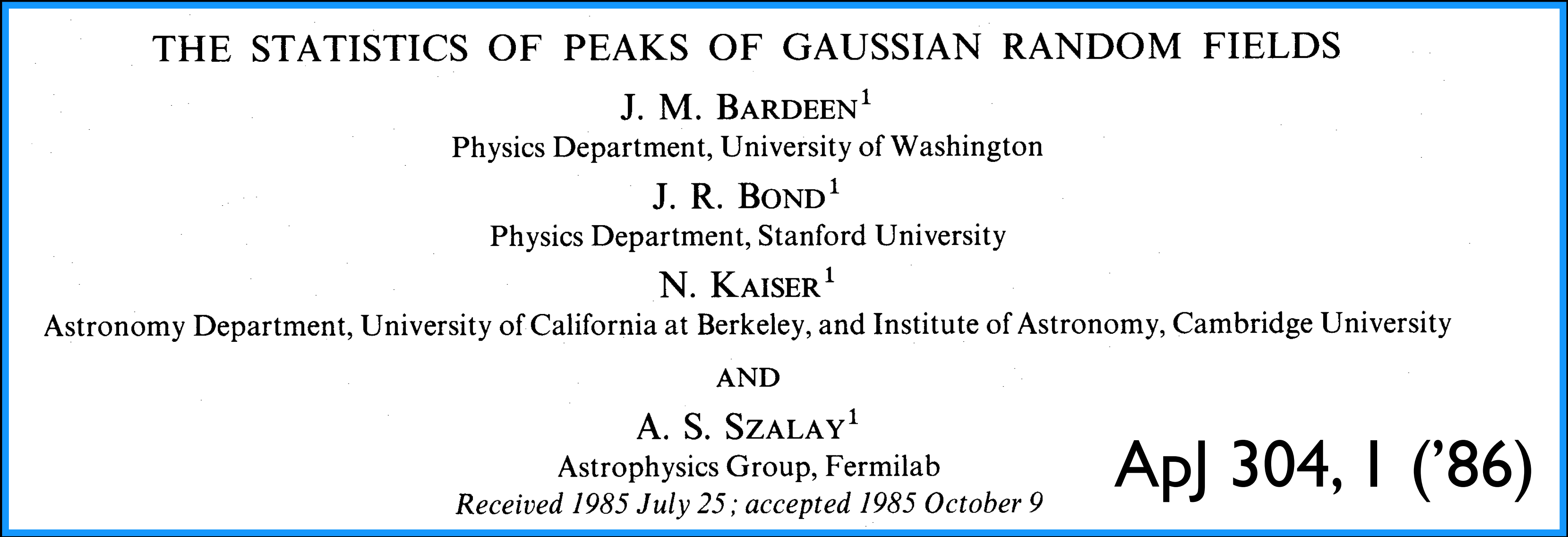
小さなスケールを見る限り、大きな違いはなさそう



BBKS fitting formula

BBKS fitting formula

遷移関数のフィッティング公式



THE STATISTICS OF PEAKS OF GAUSSIAN RANDOM FIELDS

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J. R. BOND[1]
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Received 1985 July 25; accepted 1985 October 9

ApJ 304, 1 ('86)

ランダムガウス場のピーク統計に関する有名な論文

Appendix G のフィッティング公式は特に重宝されて数多く引用されてきた

(全引用数 2613)

フィッティング公式

$$P_{\rm lin}(k) \propto k^{n_s}\{T_{\rm m}(k)\}^2 \ ; \ \ T_{\rm m}(k) = \frac{\ln(1+2.34q)}{2.34q}\left[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4\right]^{-1/4}$$

$q \equiv k/(\Gamma\, h\, {\rm Mpc}^{-1})$ ：シェイプパラメーター

$\Gamma = \Omega_0\, h \exp\left[-\Omega_b - (2h)^{1/2}\Omega_b/\Omega_0\right]$ ← Sugiyama ('95)

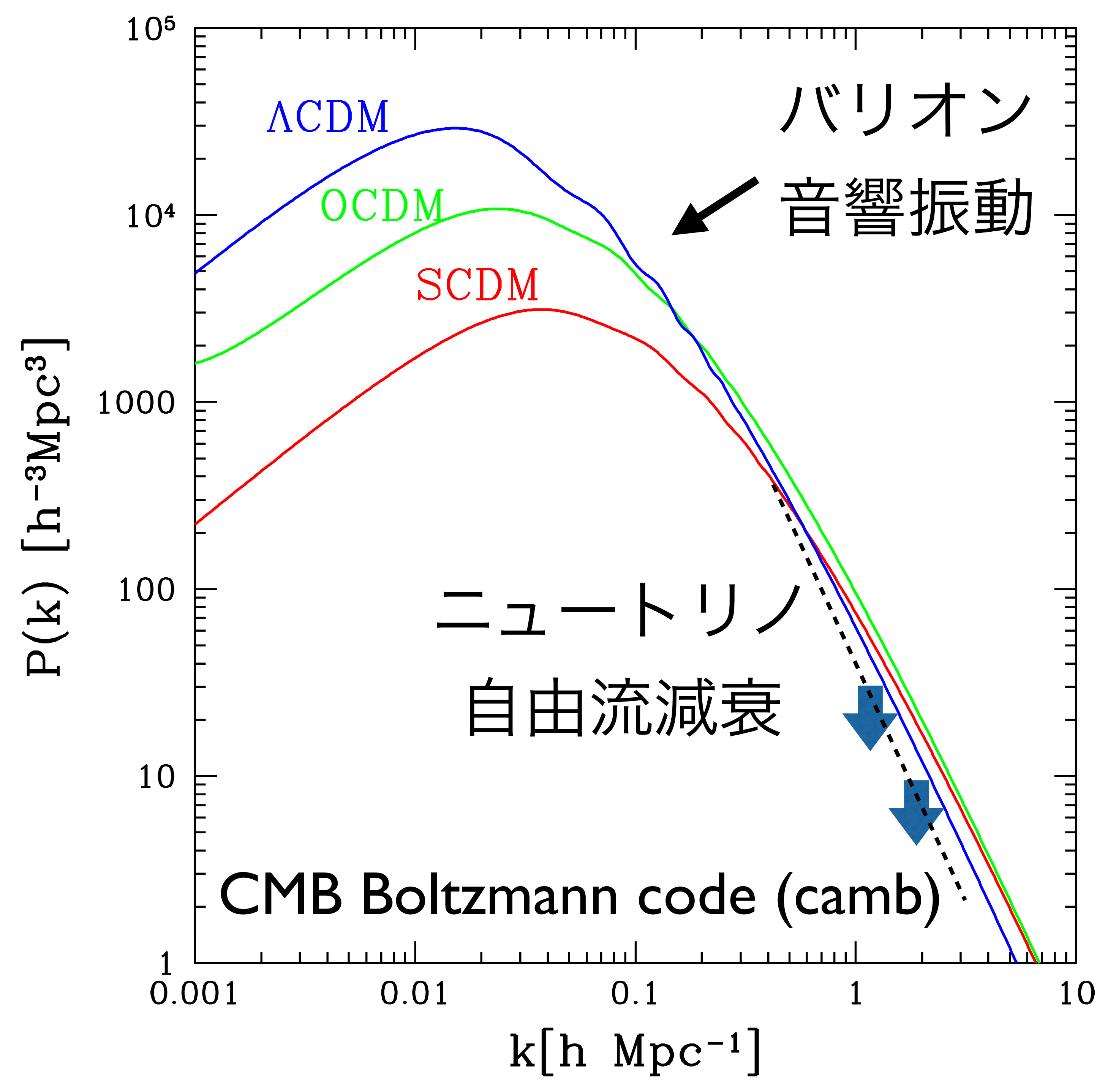
本来CDMの遷移関数だが、バリオンの影響を取り入れて質量密度ゆらぎの遷移関数としても多用されてきた

線形パワースペクトル

SCDM $(\Omega_\mathrm{m} = 1,\ \Omega_\Lambda = 0, \sigma_8 = 0.59)$

ΛCDM $(\Omega_\mathrm{m} = 0.272,\ \Omega_\Lambda = 0.728, \sigma_8 = 0.81)$

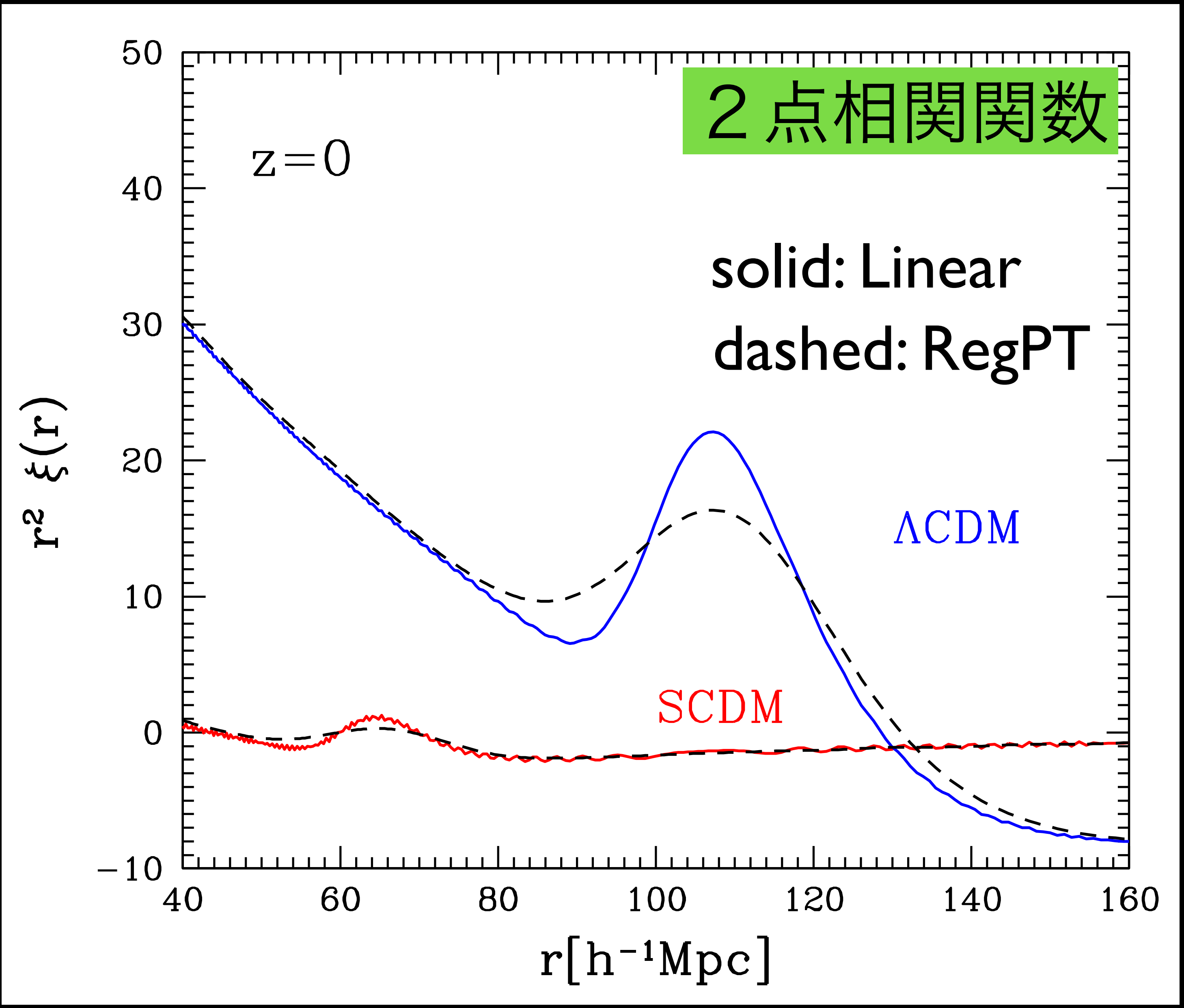
OCDM $(\Omega_\mathrm{m} = 0.45,\ \Omega_\Lambda = 0, \sigma_8 = 0.80)$

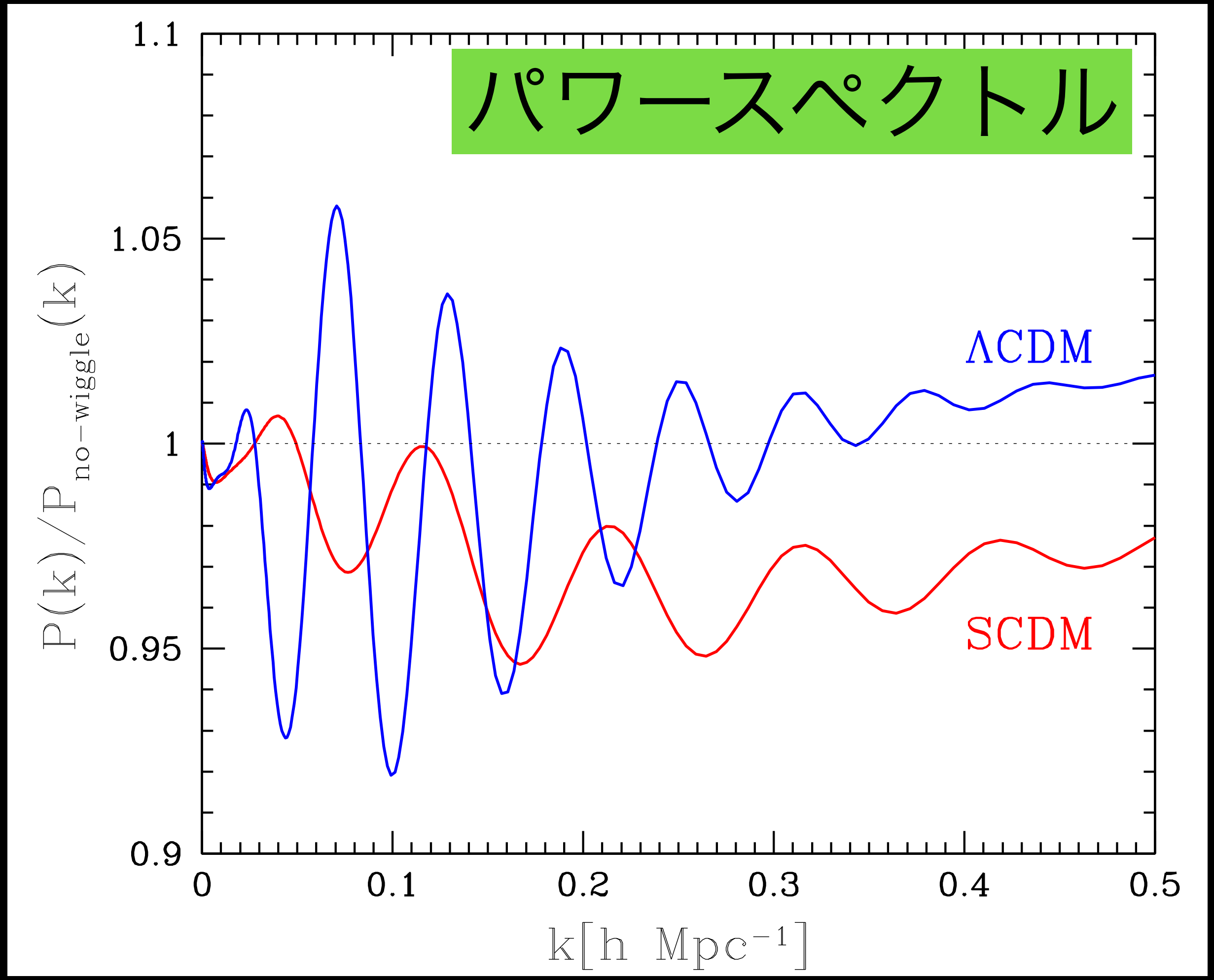


もし宇宙の標準モデルがΛCDMではなく、SCDMだったら？

バリオン音響振動

バリオン音響振動のシグナル
が小さすぎる！！



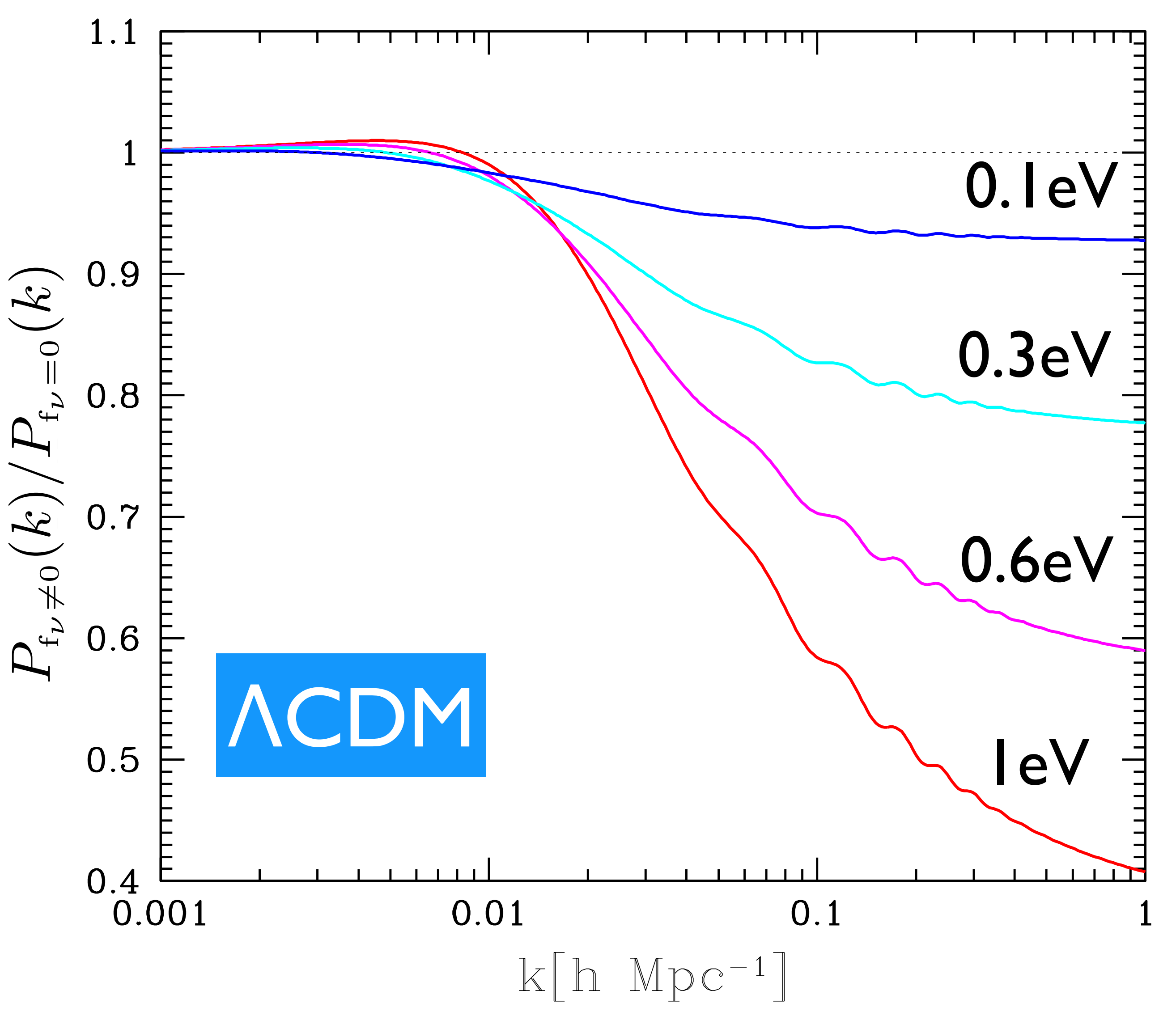


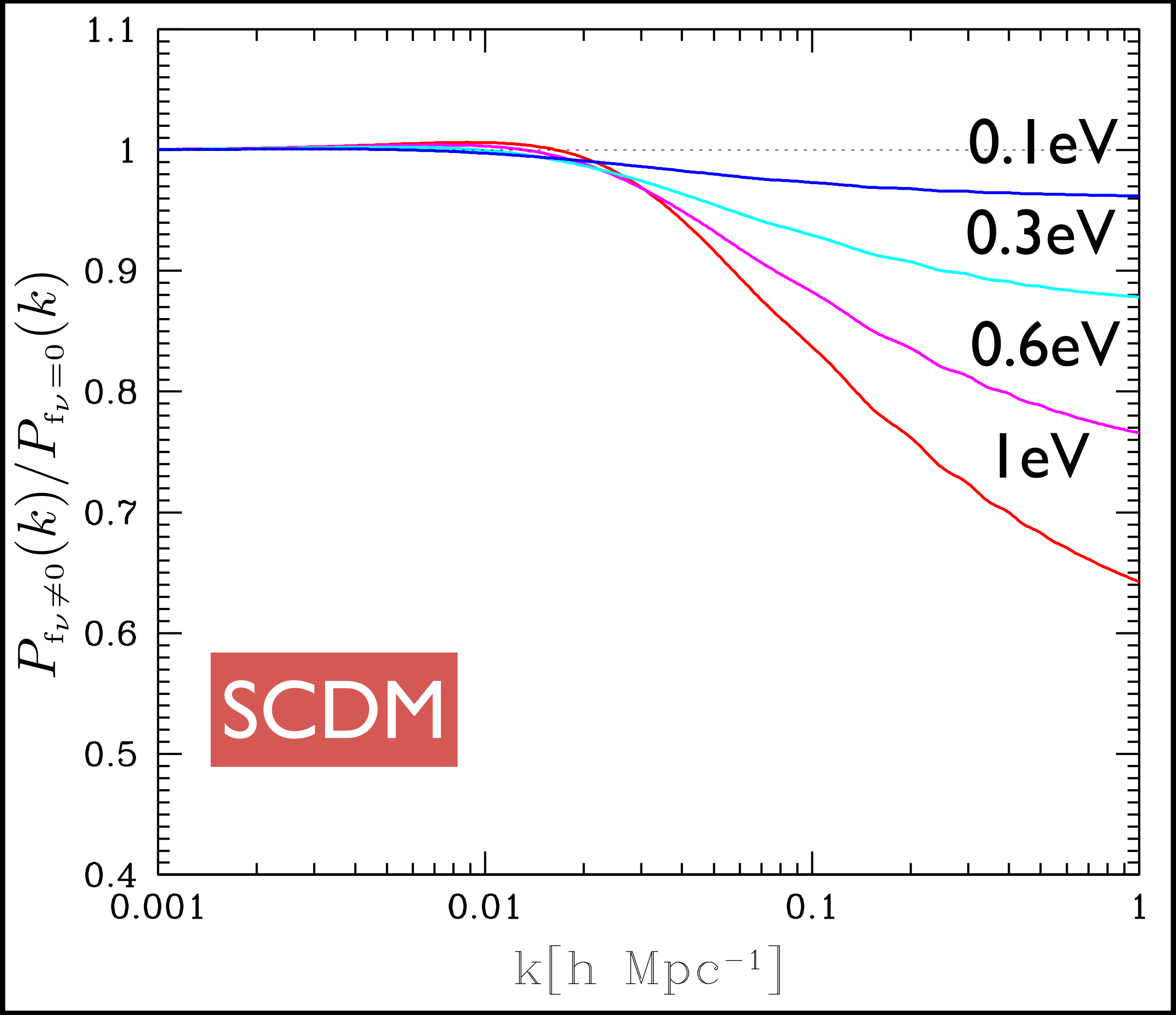
バリオン音響振動で宇宙膨張
を診断をするのは難しい？！

ニュートリノの自由流減衰

$$\frac{P_{\mathrm{f}_\nu \neq 0}(k)}{P_{\mathrm{f}_\nu = 0}(k)} \simeq 1 - 8\, f_\nu \quad ; \qquad f_\nu = \frac{\Omega_\nu}{\Omega_\mathrm{m}}$$

(e.g., Hu et al. '98)





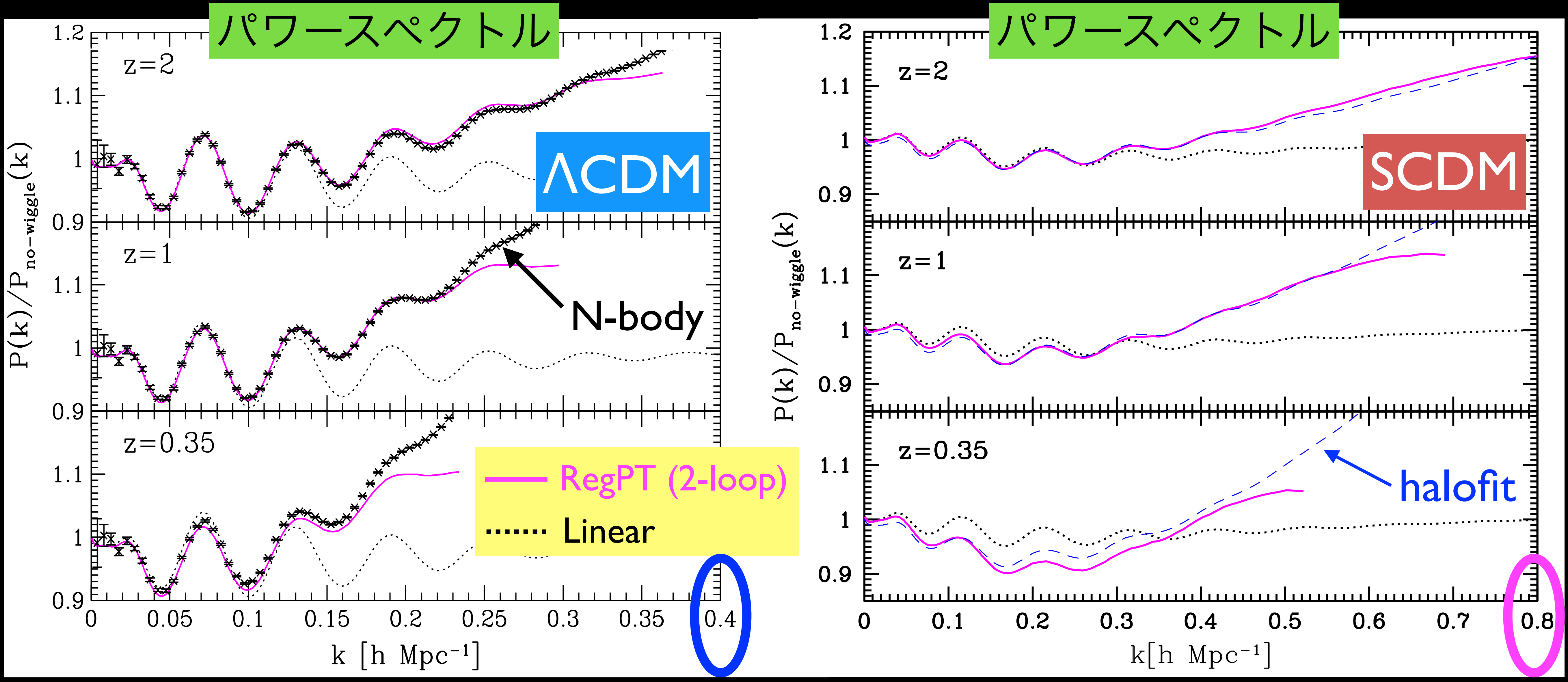
SCDMでは自由流減衰の効果が小さい

非線形重力進化

High-z だとかなり広い範囲を摂動計算でカバーできる

（ただし low-z では急激に悪化）

摂動論がもっと役に立っていた？　（標準摂動論でも精度は十分）



もし標準モデルがSCDMだったら？

〜まとめ〜

- バリオン音響振動：振幅が小さく検出も難しい
- ニュートリノ自由流減衰： 効果が小さく、質量検出は困難
- 非線形重力進化： 標準摂動論が威力を発揮

（くりこみ・再和法などの方法が発展しなかった？）

- 赤方偏移空間ゆがみ：

ゆらぎの成長率を使った重力テストは難しい？

$$\boxed{\text{赤方偏移空間} \atop \text{ゆがみの強さ}} \propto f(z) \equiv \frac{d\ln D_+(z)}{d\ln a} \simeq \{\Omega_{\rm m}(z)\}^{\gamma} \xrightarrow{\text{SCDM なら}} 1$$

なぜΛCDMかはわからないけど

標準宇宙モデルがΛCDMでよかったかもしれない！

ΛCDMモデルの向こうへ

次世代観測で探る宇宙論

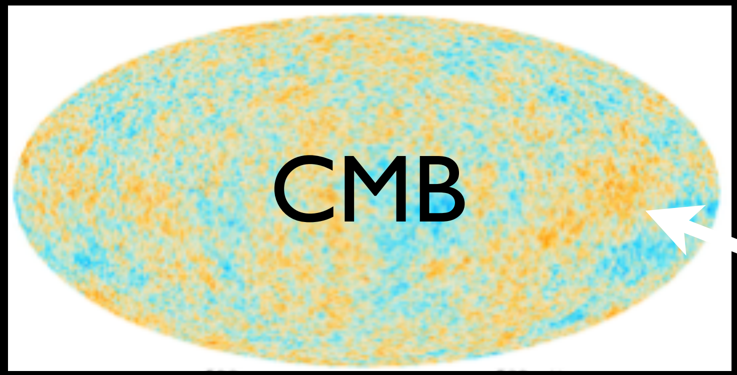
（→ なぜΛCDMが現在の標準モデルたりえたかを知る手がかり）

- ✓ 加速膨張の起源・正体
  - ダークエネルギーの状態方程式
  - 一般相対論のテスト・重力理論の検証
- ✓ 宇宙論的ニュートリノの質量検出
- ✓ ダークマターの正体・検出
- ✓ 原始（非）ガウス性の制限・検証
- ✓ 宇宙原理の観測的検証

ΛCDMが標準モデルだから
できるサイエンス

これからは宇宙大規模構造
の観測が主役

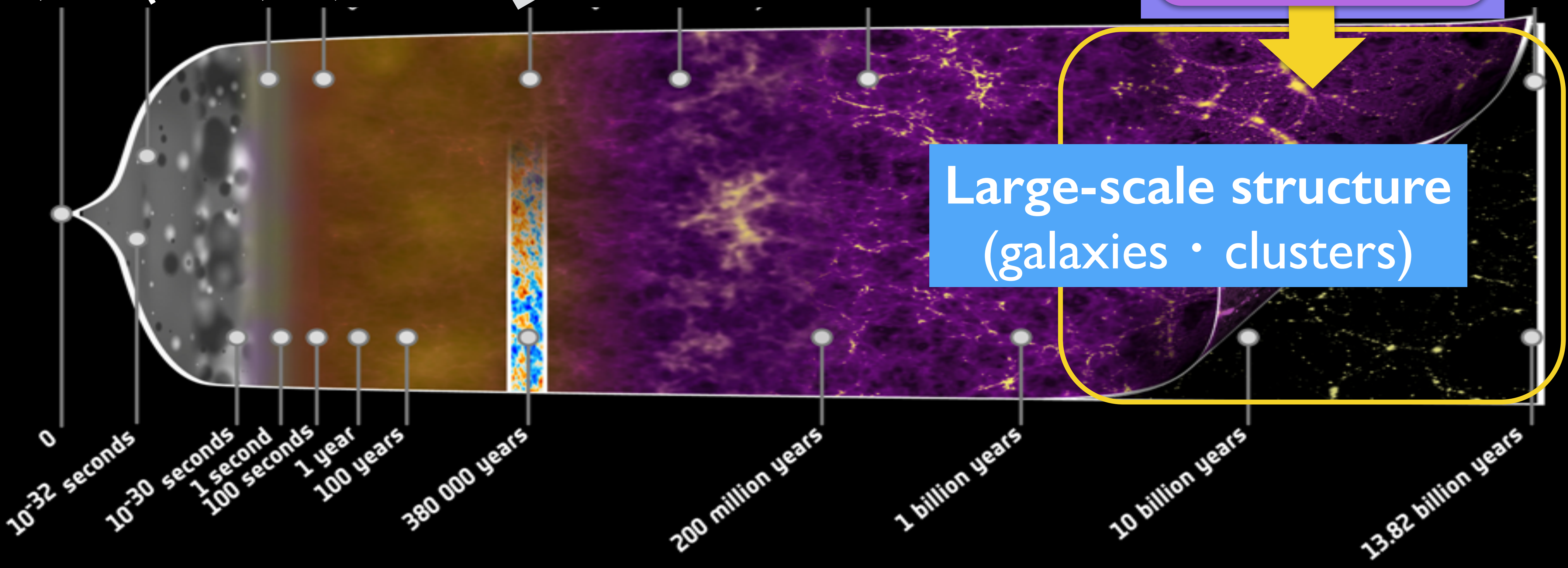
Timeline of the Universe



SuMIRe

Weak lensing survey with HSC

Galaxy redshift survey with PFS



Big-Bang

Inflation

Baryogenesis

Nucleaosynthesis

Last scattering surface

Dark ages

First-stars/galaxies

Now

Large-scale structure (galaxies・clusters)

380,000 yrs

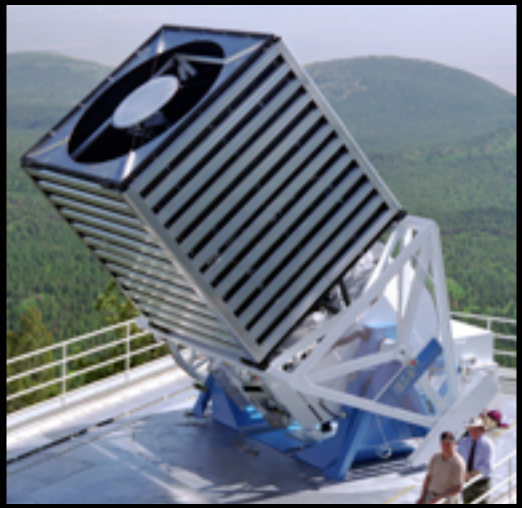
13.8G yrs

Word-wide competition

Multi-purpose ground- & space-based experiments

DES (欧米) (2013~)

HETDEX (米) (2015+)

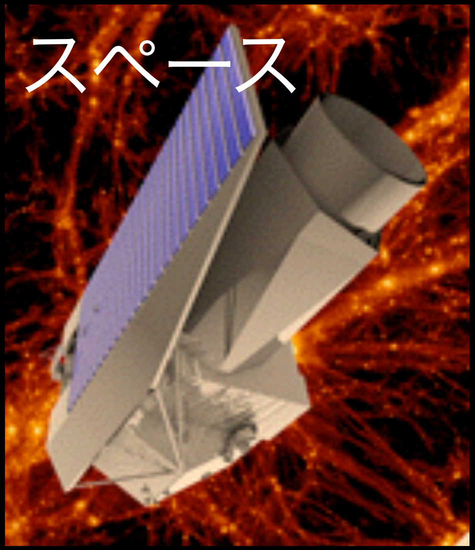


DESI (米)

(2018+)

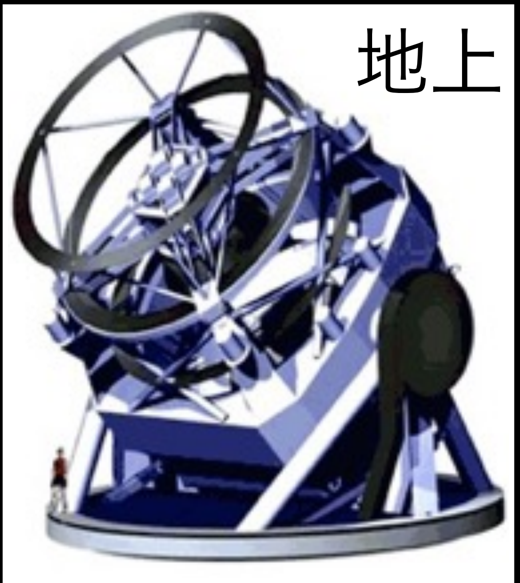
WFIRST (米)

(2024++)



eBOSS (米欧日) (2014~)

Euclid (欧) (2020)



LSST (米)

(2022++)

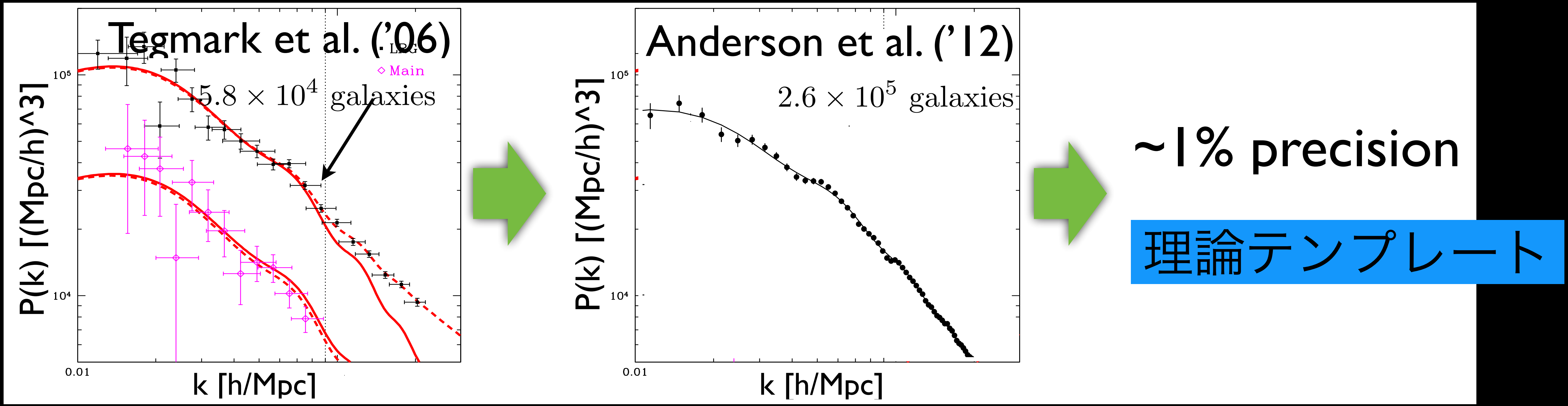
SuMIRe (日)

(2014~)

すばる望遠鏡

精密観測時代の宇宙論

観測データ主導だから　理論研究が重要



~1% precision

理論テンプレート

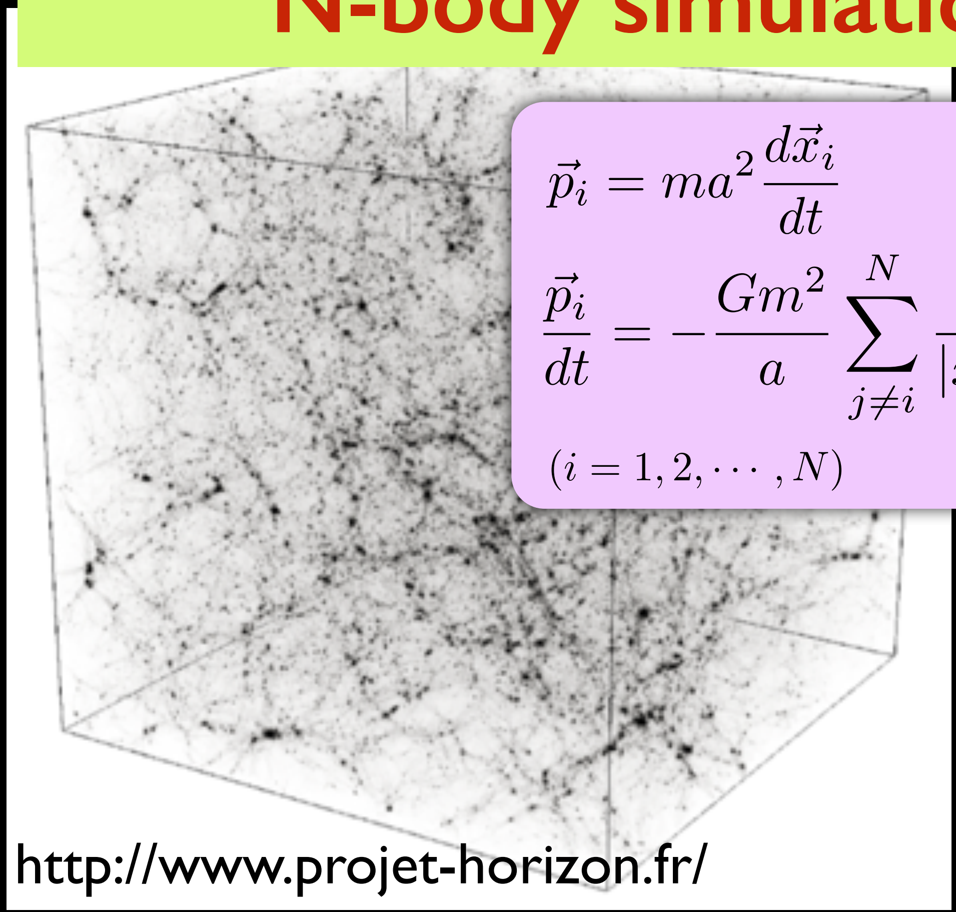
- ✓ 理論予言の高精度化
- ✓ 系統誤差のコントロール・低減 (e.g., 銀河バイアス)
- ✓ ΛCDMモデルを超える新しい物理の影響
  (how/warm dark matter, relativistic effect, modification to gravity, …)

さらに 宇宙論の標準解析ツールのリノベーションも必要

宇宙大規模構造の理論ツール

線形理論を越えて、ダークマター優勢宇宙の
構造形成を取り扱う代表的手法
(その派生・発展版にフィッティング公式、ハローモデルなど)

Cosmological N-body simulation



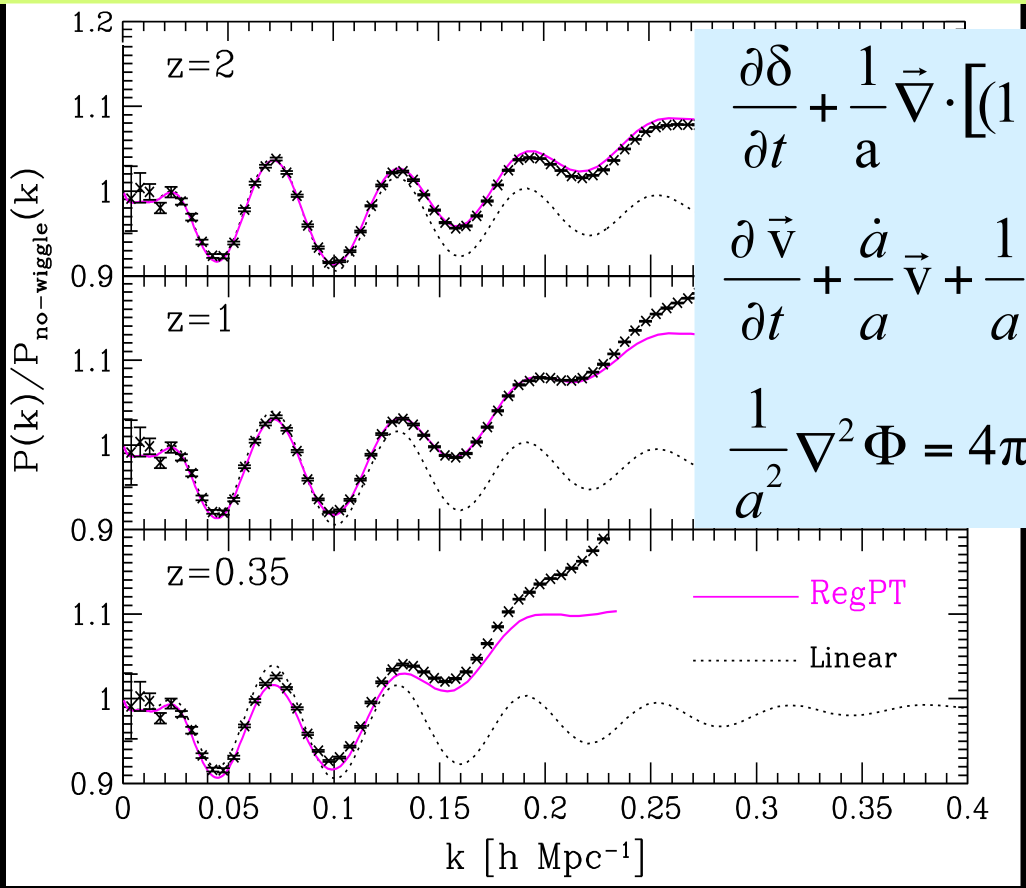
$$\vec{p}_i = m a^2 \frac{d\vec{x}_i}{dt}$$

$$\frac{\vec{p}_i}{dt} = -\frac{Gm^2}{a}\sum_{j\neq i}^{N}\frac{\vec{x}_i-\vec{x}_j}{|\vec{x}_i-\vec{x}_j|^3}$$

$$(i=1,2,\cdots,N)$$

http://www.projet-horizon.fr/

Perturbation theory (PT)
(based on fluid approx.)



$$\frac{\partial\delta}{\partial t}+\frac{1}{\mathrm{a}}\vec{\nabla}\cdot\left[(1+\delta)\vec{\mathrm{v}}\right]=0$$

$$\frac{\partial\,\vec{\mathrm{v}}}{\partial t}+\frac{\dot{a}}{a}\vec{\mathrm{v}}+\frac{1}{a}(\vec{\mathrm{v}}\cdot\vec{\nabla})\,\vec{\mathrm{v}}=-\frac{1}{a}\vec{\nabla}\Phi$$

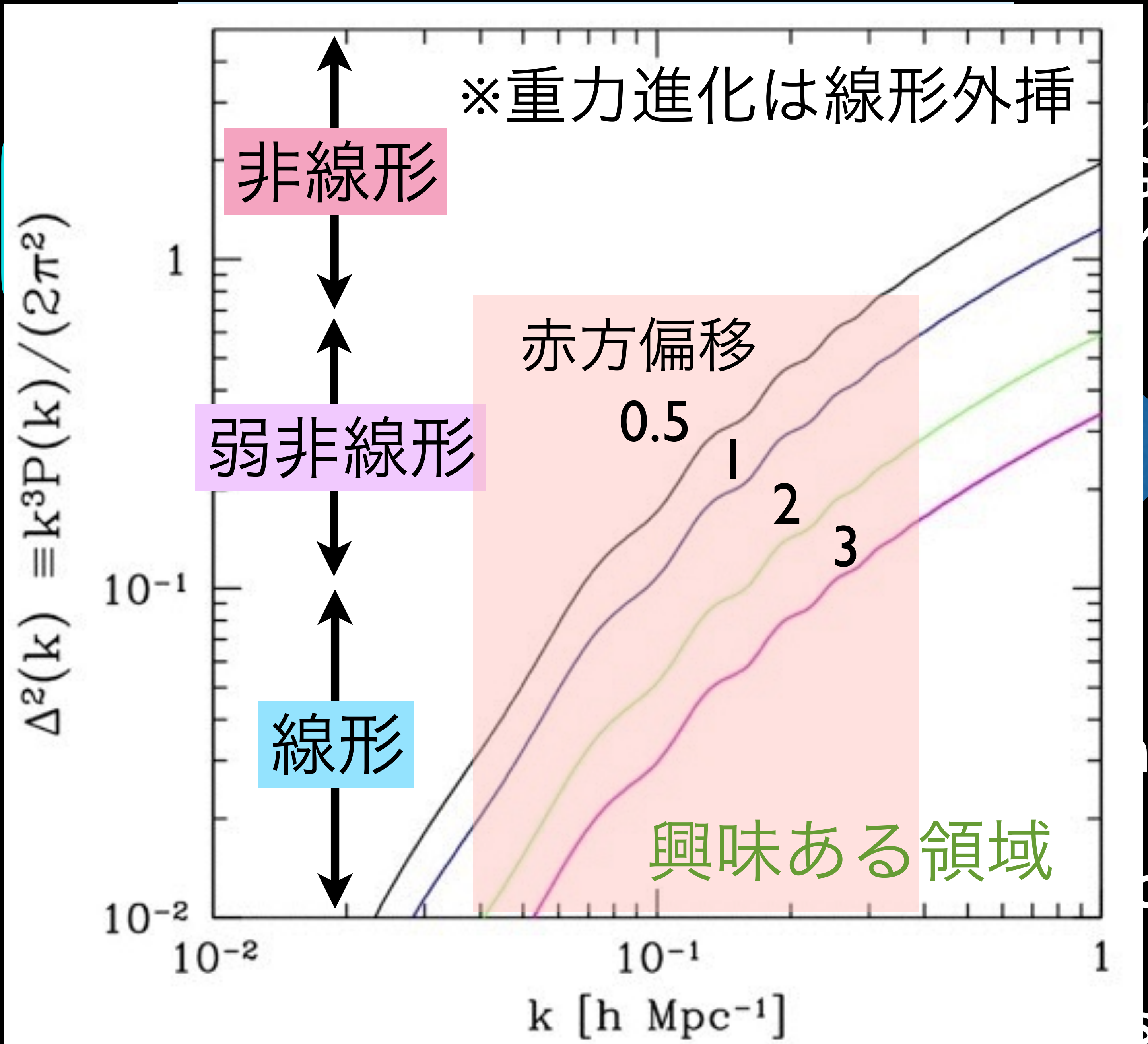
$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\,\bar{\rho}_{\mathrm{m}}\,\delta$$

AT et al. ('12)

Perturbation theory (PT)

CDM + baryon = pressureless & irrotational fluid

Single-stream approx. of collisionless Boltzmann eq.



uszkiewicz (’81), Vishniac (’83),
Goroff et al. (’86), Suto & Sasaki (’91),
Makino, Sasaki & Suto (’92), ...

Standard PT $(\delta_1 \ll 1)$

$$\delta = \delta_1 + \delta_2 + \delta_3 + \cdots$$

or renomalized PT treatment

ode (*RegPT*)

2-loop (next-to-next-to leading order)

ssive ν, modified gravity, halo bias,…)

Perturbation theory (PT)

CDM + baryon = pressureless & irrotational fluid

Single-stream approx. of collisionless Boltzmann eq.

Basic eqs.

$$\frac{\partial\delta}{\partial t}+\frac{1}{\mathrm{a}}\vec{\nabla}\cdot\left[(1+\delta)\vec{\mathrm{v}}\right]=0$$

$$\frac{\partial\,\vec{\mathrm{v}}}{\partial t}+\frac{\dot{a}}{a}\vec{\mathrm{v}}+\frac{1}{a}(\vec{\mathrm{v}}\cdot\vec{\nabla})\,\vec{\mathrm{v}}=-\frac{1}{a}\vec{\nabla}\,\Phi$$

$$\frac{1}{a^{2}}\nabla^{2}\Phi=4\pi\,G\,\overline{\rho}_{\mathrm{m}}\,\delta$$

Juszkiewicz ('81), Vishniac ('83),
Goroff et al. ('86), Suto & Sasaki ('91),
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Standard PT $(\delta_1 \ll 1)$

$$\delta=\delta_1+\delta_2+\delta_3+\cdots$$

Recent developments

- Improving accuracy by Resummation or renomalized PT treatment
- Higher-order calculation & fast PT code (*RegPT*)

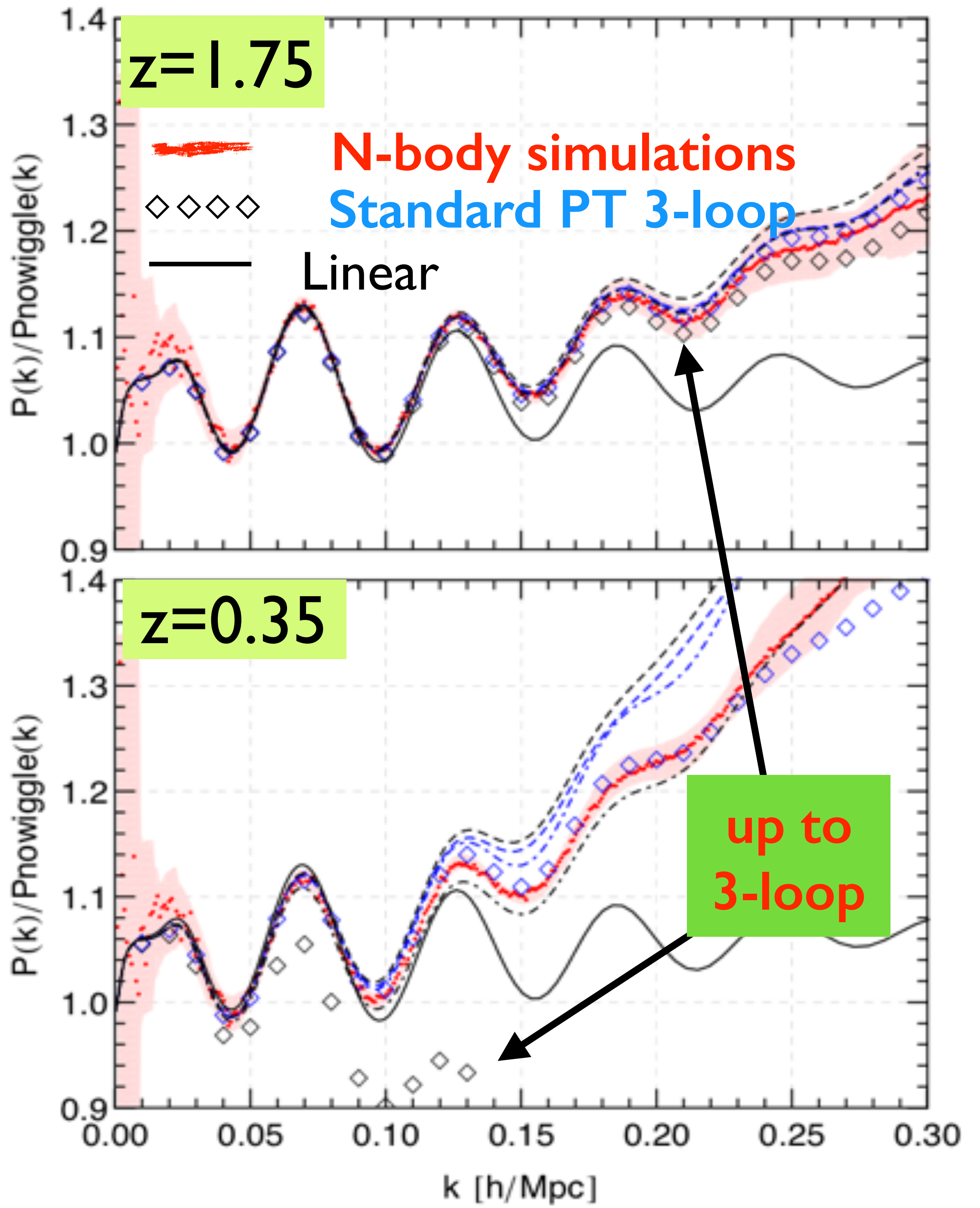
2-loop (next-to-next-to leading order)

- Incorporating other systematics (massive ν, modified gravity, halo bias,…)

摂動論におけるUV問題

摂動論の高次補正の次数を上げると (3-loop) 大きなUV補正

→ 摂動論の破綻？！



既存の再和法・くりこみ摂動でも

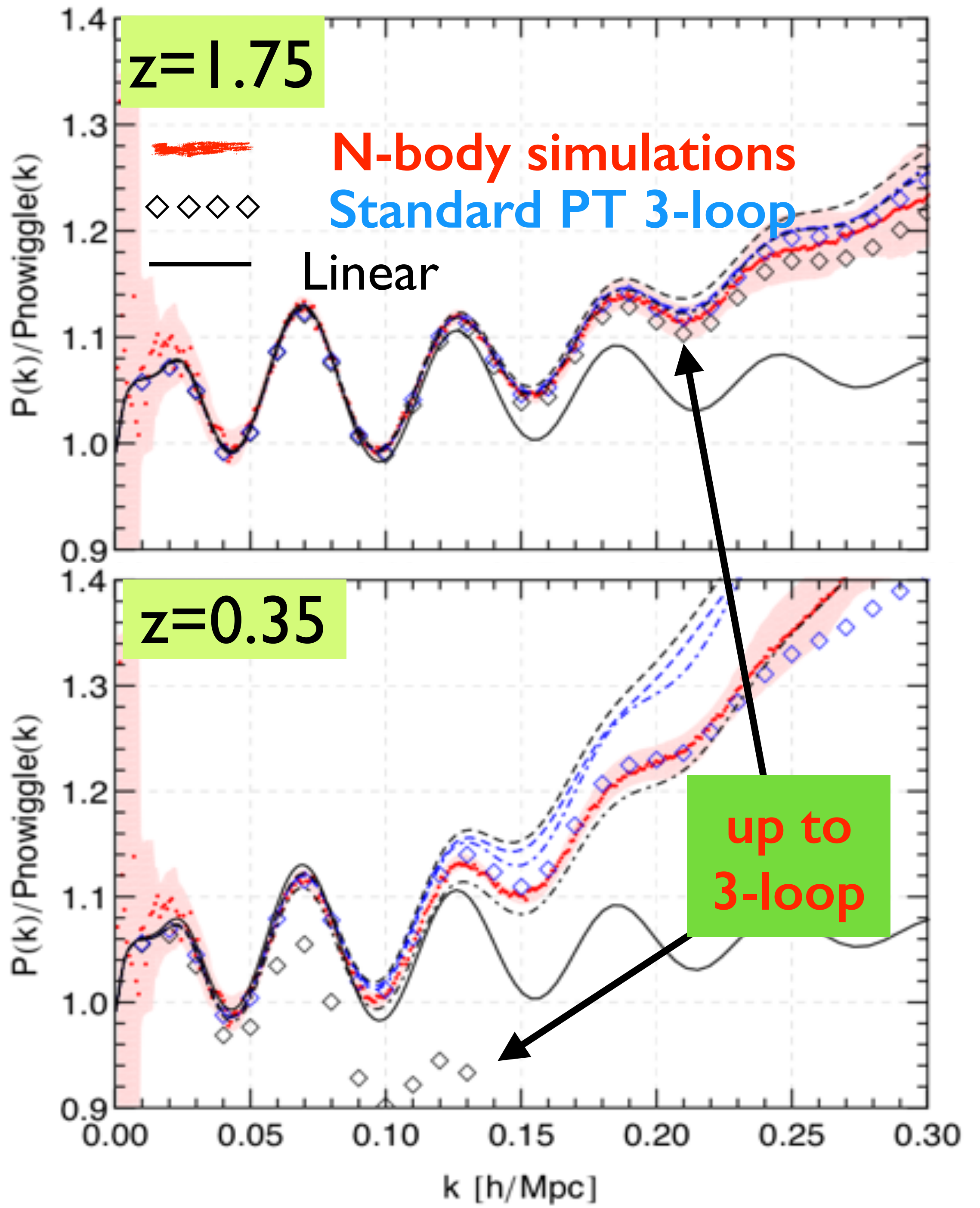
摂動次数を上げると現れる

（3-loop)

Blas et al. ('14)

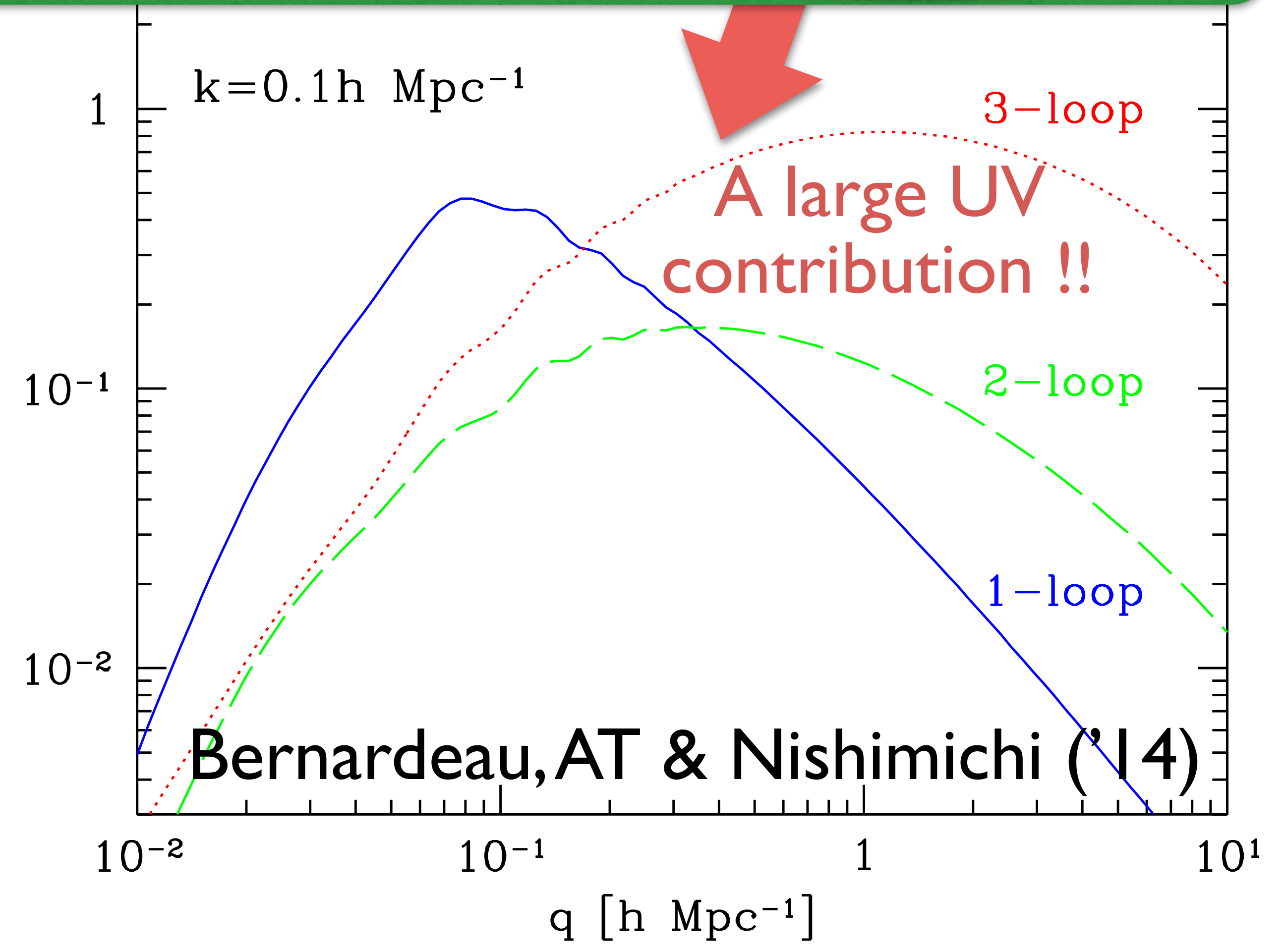
摂動論におけるUV問題

摂動論の高次補正の次数を上げると (3-loop) 大きなUV補正
→ 摂動論の破綻？！



Each higher-order term involves mode-coupling integral:

$$P_{\text{n-loop}}(k) \propto \int d\ln q\, K_{\text{n-loop}}(k,q)\, P_0(q)$$



A large UV contribution !!

Bernardeau, AT & Nishimichi ('14)

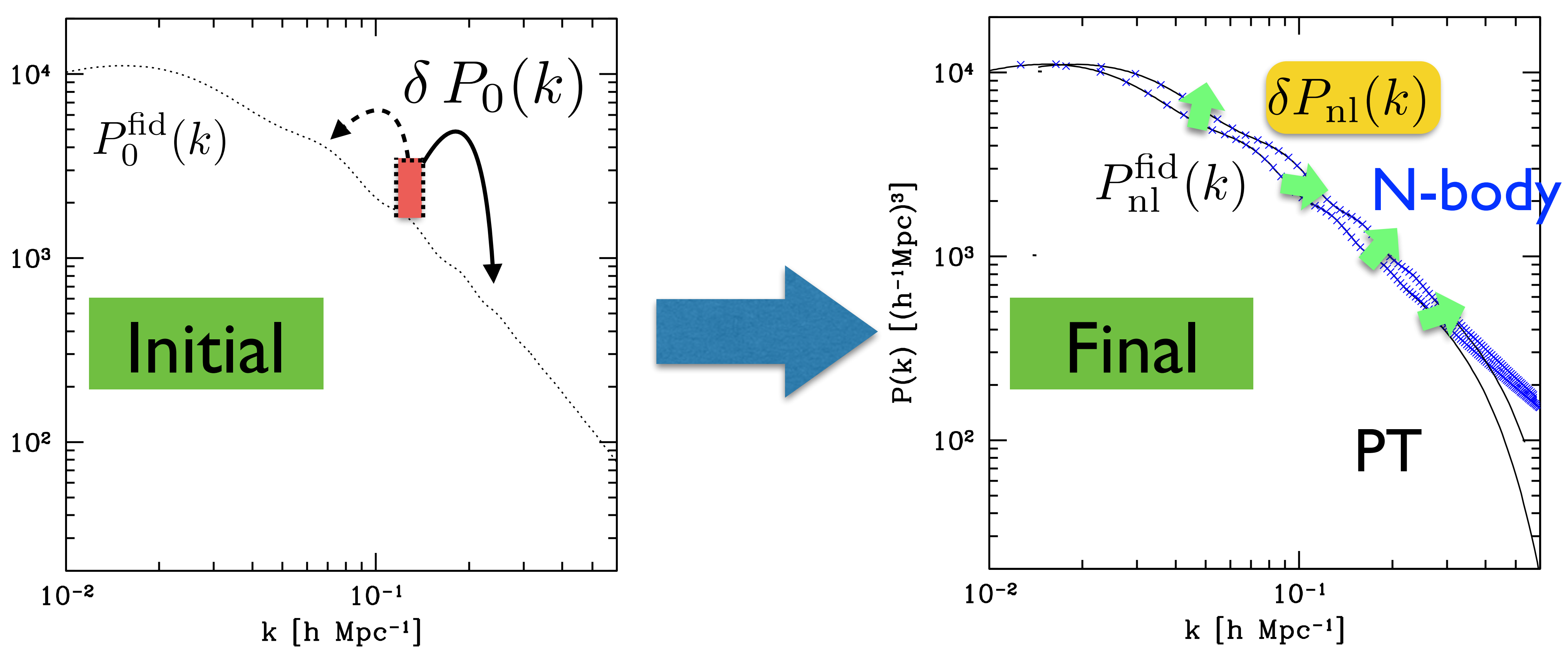
Nature of nonlinear response

Nishimichi, Bernardeau & AT (arXiv:1411.2970)

Q How does the mode-coupling structure look like in reality ?

Nonlinear response we will measure
$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

How the small disturbance added in initial power spectrum can contribute to each Fourier mode in final power spectrum



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How the small disturbance added in <u>initial power spectrum</u> can contribute to each Fourier mode in <u>final power spectrum</u>

Alternative definition

$$K(k,q) = q\,\frac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$$

(discretized) estimator

$$\widehat{K}(k_i,q_j)\,P_0(q_j) \equiv \frac{P^+_{\rm nl}(k_i) - P^-_{\rm nl}(k_i)}{\Delta\ln P_0\,\Delta\ln q}$$

$$\Delta\ln q = \ln q_{j+1} - \ln q_j$$

name	box	particles	$z_{\rm start}$	soft	mass	bins	runs	total
L9-N10	512	1024^3	63	25	0.97	5	1	10
L9-N9	512	512^3	31	50	7.74	15	4	120
L9-N8	512	256^3	15	100	61.95	13	4	104
L10-N9	1024	512^3	31	100	61.95	15	1	30

Run many simulations…

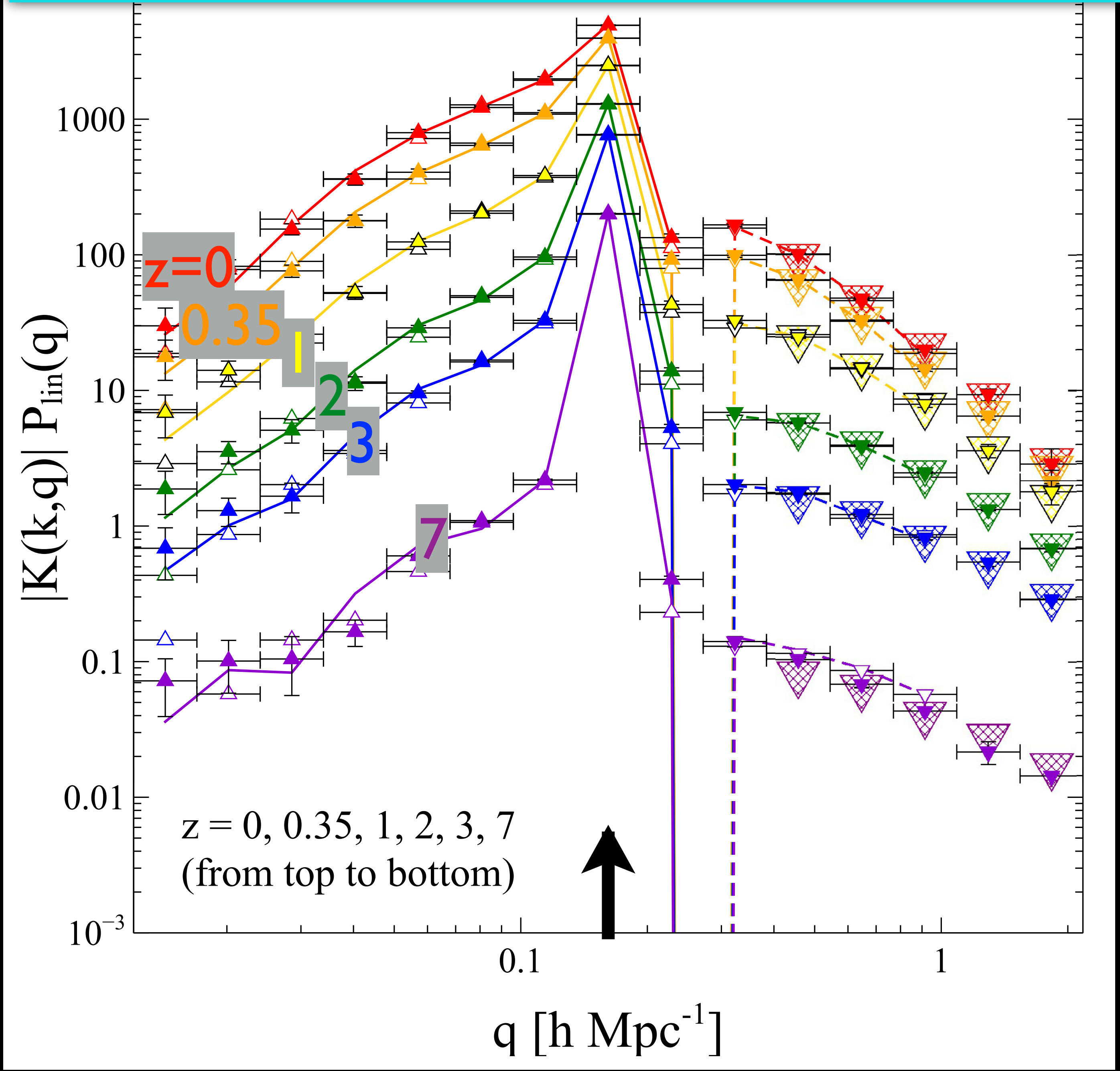
by T.Nishimishi

Measurement result

Nishimichi, Bernardeau & AT (arXiv:1411.2970)

Nonlinear response to a small initial variation in P(k):

$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$



Measured at
k=0.162 [h/Mpc]

▲ or —— : positive

▼ or ------ : negative

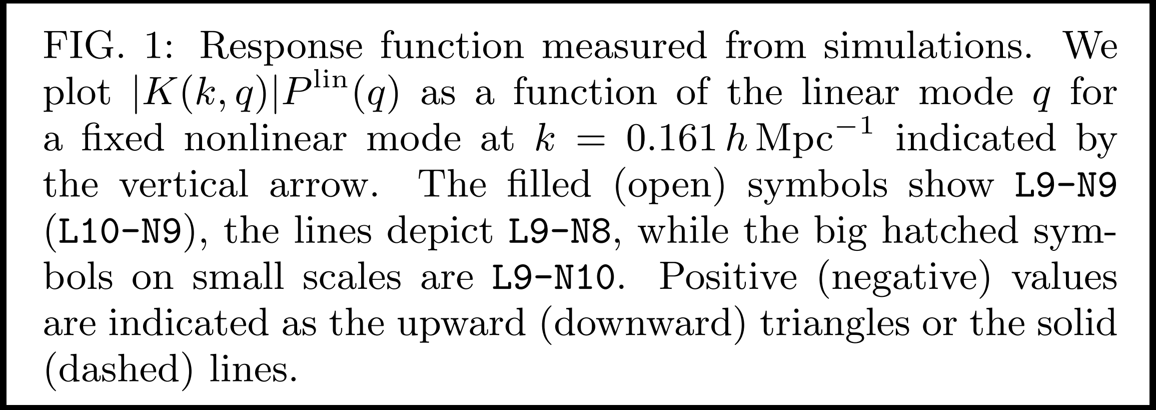
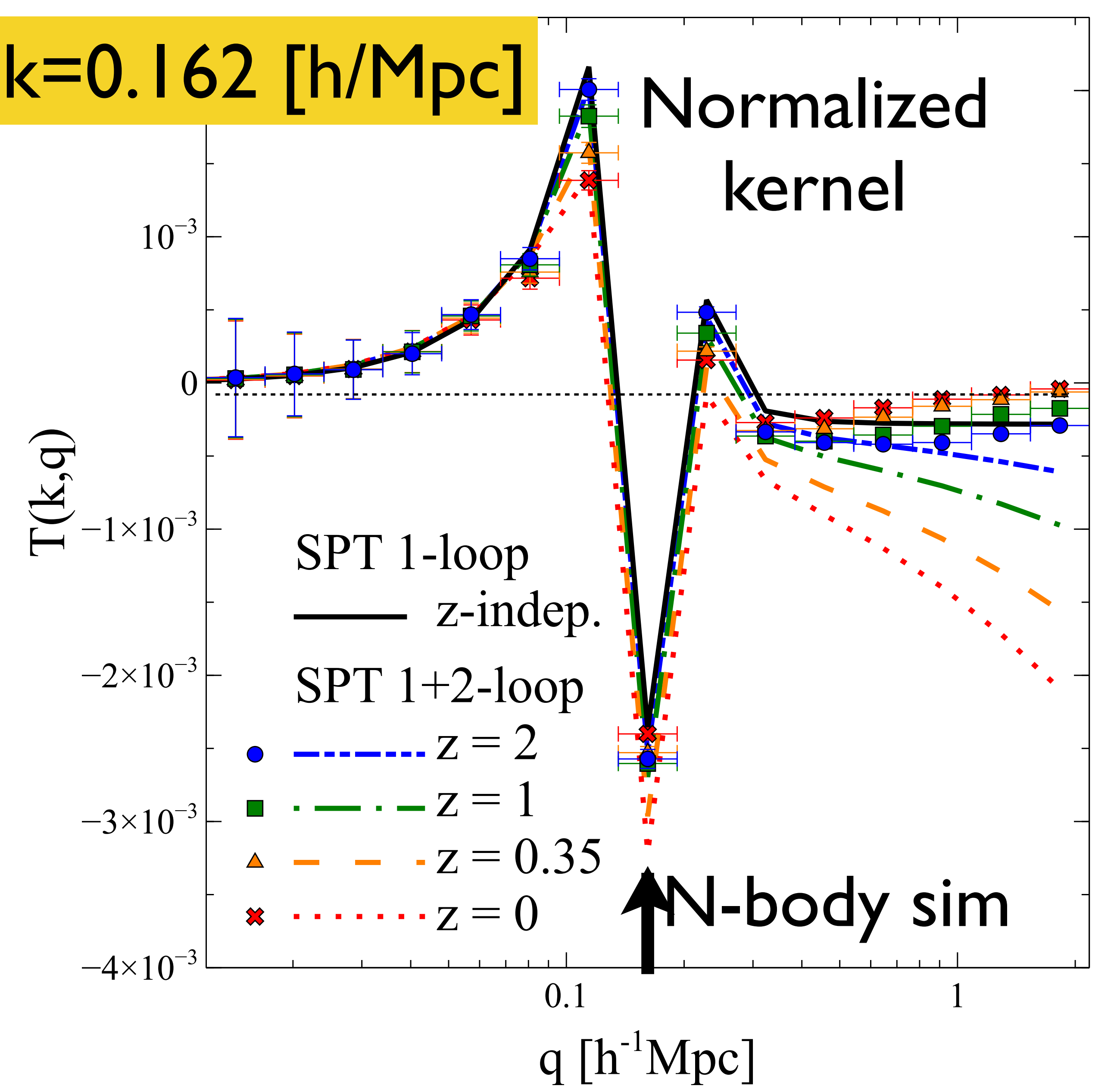


FIG. 1: Response function measured from simulations. We plot $|K(k,q)|P^{\rm lin}(q)$ as a function of the linear mode q for a fixed nonlinear mode at $k = 0.161\,h\,{\rm Mpc}^{-1}$ indicated by the vertical arrow. The filled (open) symbols show `L9-N9` (`L10-N9`), the lines depict `L9-N8`, while the big hatched symbols on small scales are `L9-N10`. Positive (negative) values are indicated as the upward (downward) triangles or the solid (dashed) lines.

Response function in simulations

Nishimichi, Bernardeau & AT (arXiv:1411.2970)

$$T(k,q) = [K(k,q) - K_{\rm lin}(k,q)]/[q\,P_{\rm lin}(k)]$$



Black solid : Standard PT 1-loop (z-indept.)

Blue, Green, Orange, Red : 2-loop

q<k : reproduce simulation well

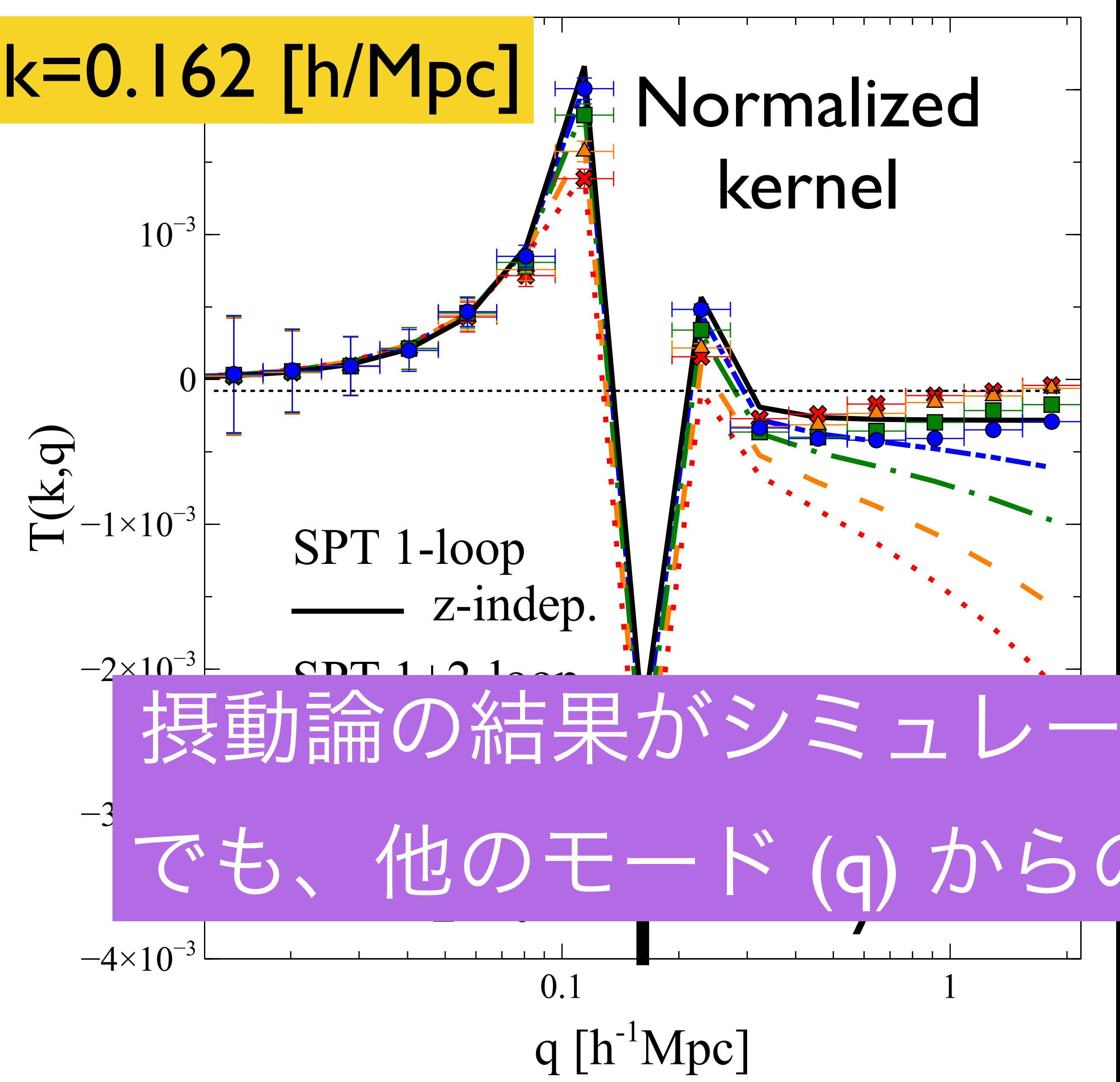
q>k : discrepancy is manifest (particularly large at low-z)

UV contribution is suppressed in N-body simulation!!

Response function in simulations

Nishimichi, Bernardeau & AT (arXiv:1411.2970)

$$T(k,q) \\ = [K(k,q) - K_{\rm lin}(k,q)]/[q\, P_{\rm lin}(k)]$$



Black solid：Standard PT 1-loop (z-indept.)

Blue, Green, Orange, Red：2-loop

q<k ： reproduce simulation well

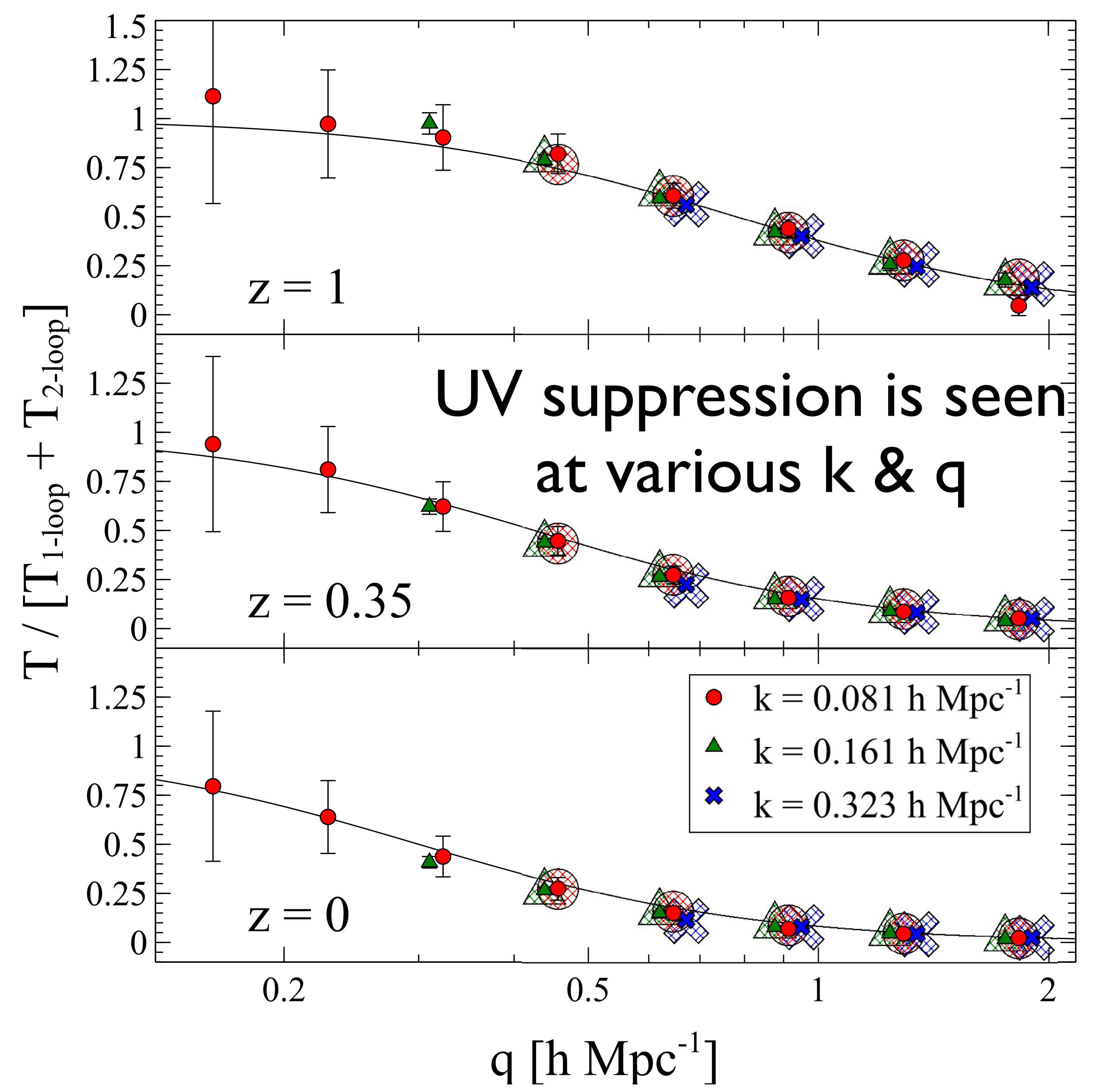
摂動論の結果がシミュレーションとよく合う波数領域 (k) でも、他のモード (q) からの応答をみると「ずれ」がある

UV contribution is suppressed in N-body simulation!!

Characterizing UV suppression

Nishimichi, Bernardeau & AT (arXiv:1411.2970)

$$T(k,q) = [K(k,q) - K^{\mathrm{lin}}(k,q)]/[qP^{\mathrm{lin}}(k)]$$



UV suppression is seen at various k & q

ratio of measured response function to PT prediction

Fitting formula

$$K_{\mathrm{eff}}(k,q) \qquad q_0(z) = 0.3/D_+^2(z)\ [h\,\mathrm{Mpc}^{-1}]$$

$$= \left[K^{\text{1-loop}}(k,q) + K^{\text{1-loop}}(k,q)\right] \frac{1}{1+(q/q_0)^2}$$

$K^{\text{1-loop}},\ K^{\text{1-loop}}$: Standard PT kernel

Some physical mechanism works, and controls the mode transfer

EFT cures PT predictions ?

UV suppression is definitely attributed to small-scale physics, which cannot be described by current PT treatment

(formation & merging processes of dark matter halos, …)

Effective field theory (EFT) of large-scale structure

Phenomenologically introduce <u>viscousity & anisotropic stress</u> to characterize deviations from pressureless & irrotational fluid

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{v}] = 0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + H\,\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\cdot\boldsymbol{v} = -\frac{1}{a}\nabla\psi - \boxed{\frac{1}{\rho_{\rm m}}\frac{1}{a}\nabla\tau_{ij}}$$

$$\frac{1}{a^2}\nabla^2\psi = \frac{\kappa^2}{2}\,\rho_{\rm m}\,\delta$$

Baumann et al. ('12), Carrasco, Herzberg & Senatore ('12), Carrasco et al. ('13ab), Porto, Senatore & Zaldarriaga ('14), …

but need a calibration with N-body simulation

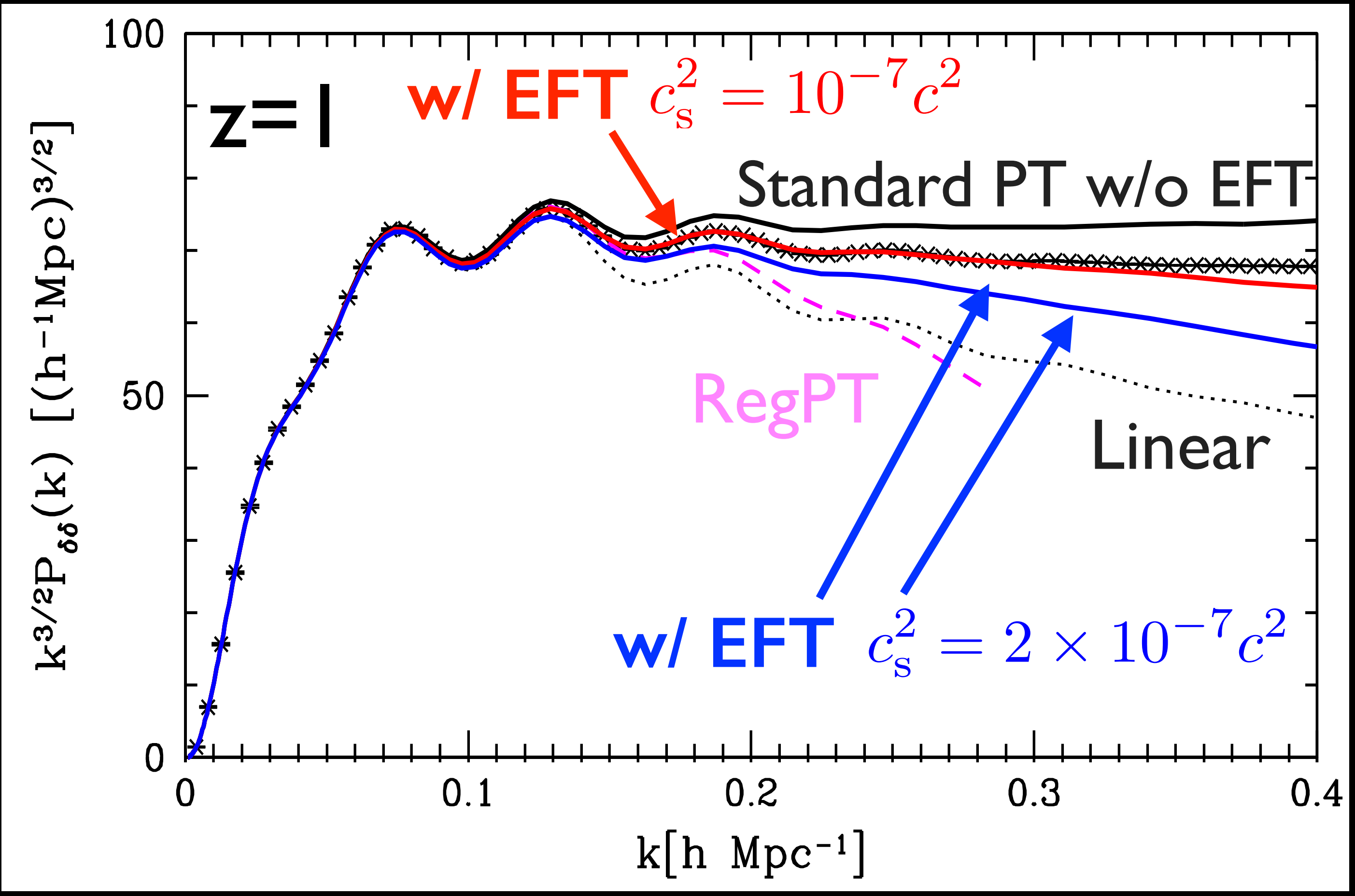
Testing EFT approach

Leading-order EFT corrections

e.g., Herzberg ('14)

$$\tau_{ij}=\rho_{\rm m}\left[\left(c_{\rm s}^2\,\delta-\frac{c_{\rm bv}^2}{aH}\,\nabla\cdot\boldsymbol{v}\right)\delta_{ij}-\frac{3}{4}\frac{c_{\rm sv}^2}{aH}\left\{\partial_j v_i+\partial_i v_j-\frac{2}{3}(\nabla\cdot\boldsymbol{v})\delta_{ij}\right\}\right]$$

Does this really help PT prediction ?



At 1-loop (next-to-leading) order,

corrections are approximately described by single-parameter:

$$c_{\rm s}^2+f\,(c_{\rm bv}^2+c_{\rm sv}^2)$$

Testing EFT approach

Leading-order EFT corrections

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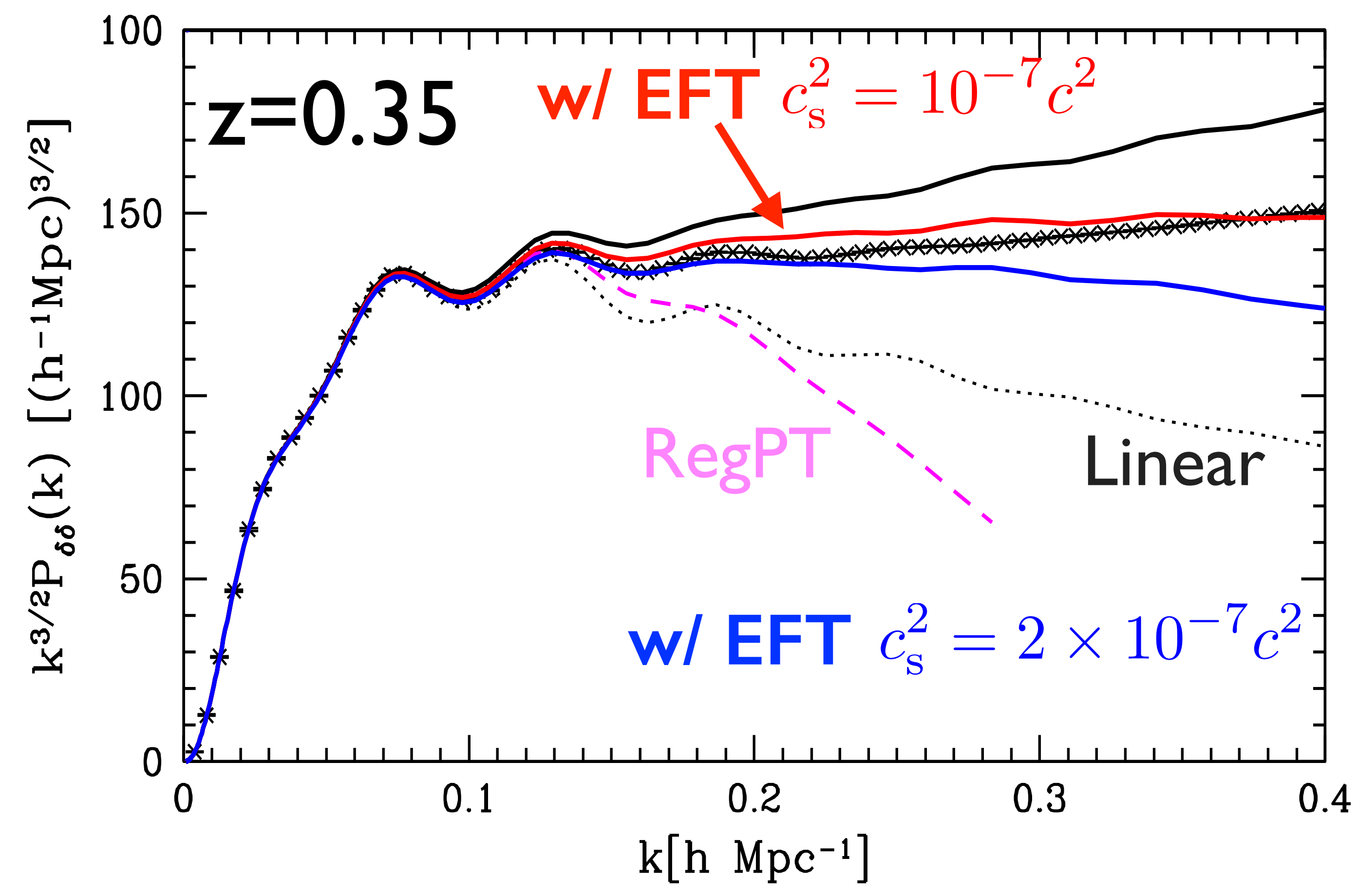


At 1-loop (next-to-leading) order,

corrections are approximately described by single-parameter:

$$c_{\rm s}^2 + f\,(c_{\rm bv}^2 + c_{\rm sv}^2)$$

Allowing c_s to be free, EFT 1-loop reproduce N-body results, but resultant c_s depends on redshift and cosmology… furthermore,

Testing EFT approach

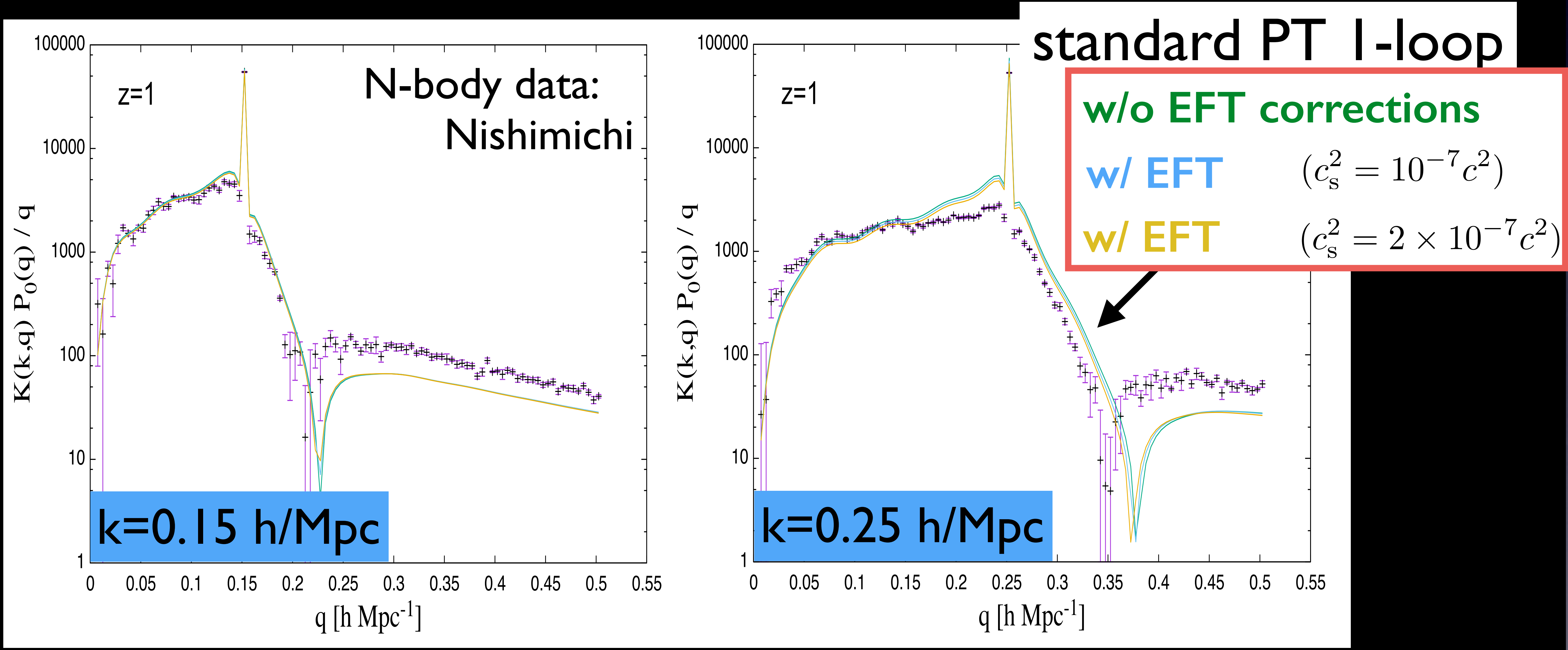
Response function of P(k)

nonlinear

linear (initial)

$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

Nishimichi, Bernardeau & AT
arXiv:1411.2970



standard PT 1-loop

w/o EFT corrections

w/ EFT $(c_{\rm s}^2 = 10^{-7}c^2)$

w/ EFT $(c_{\rm s}^2 = 2\times10^{-7}c^2)$

At 1-loop, PT predictions with EFT do not so much differ from the one w/o EFT, which does not perfectly match simulations

Testing EFT approach

Response function of P(k)

nonlinear *linear (initial)*

$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

Nishimichi, Bernardeau & AT
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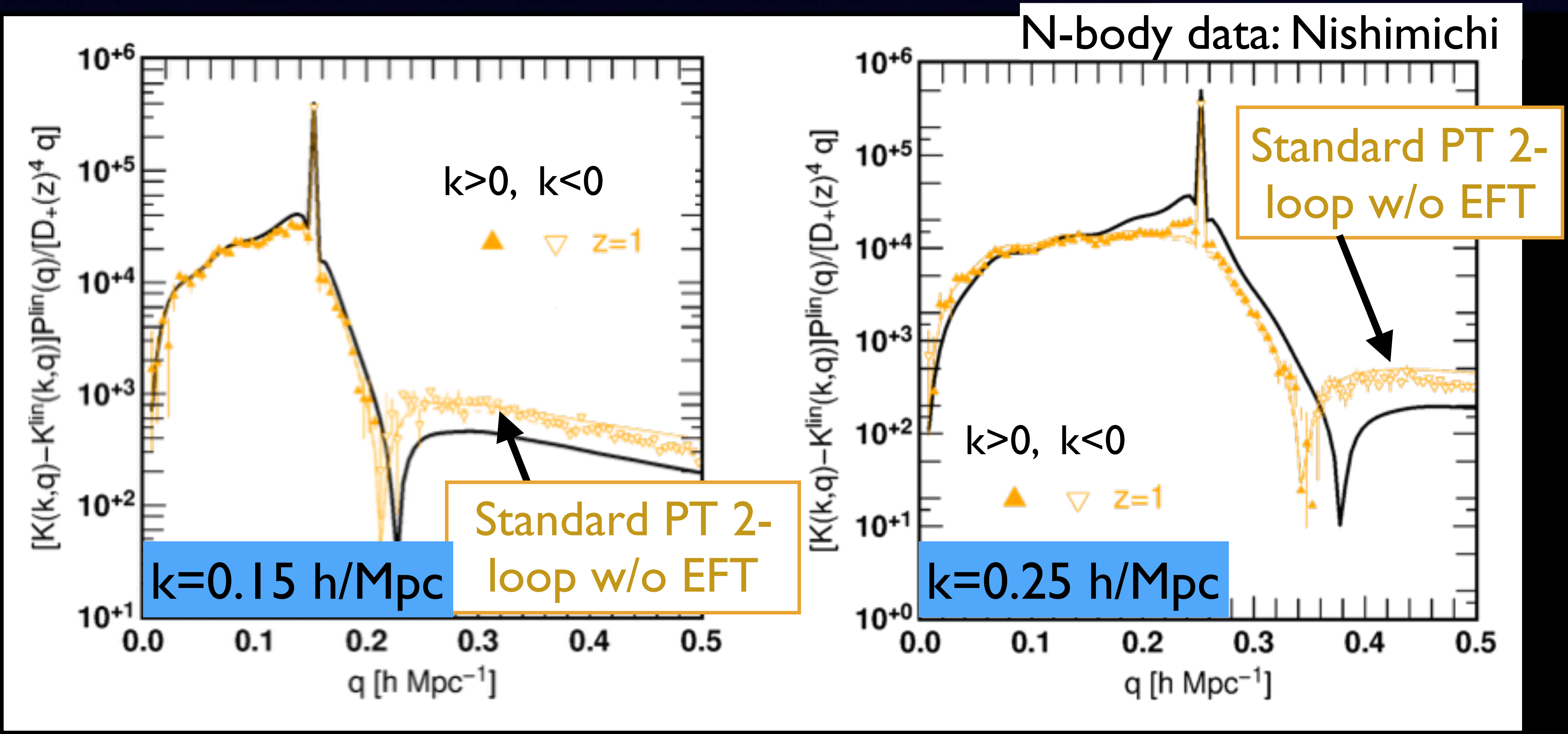


N-body data: Nishimichi

Standard PT 2-loop w/o EFT

k=0.15 h/Mpc

k=0.25 h/Mpc

Simply adding standard PT 2-loop w/o EFT apparently looks better (although it starts to fail at k>0.4 h/Mpc)

Vlasov-Poisson: back to the source

My personal viewpoint

- EFT is far more than complete treatment
- No more than the revival of the old ideas

(e.g., Adhesion model by Gurvatov et al. '89)

To understand what is going on,
we have to go back to a more fundamental description :

Vlasov-Poisson system

$$\left[a\frac{\partial}{\partial t}+\frac{\boldsymbol{v}}{a}\cdot\frac{\partial}{\partial \boldsymbol{x}}-a\frac{\partial\phi}{\partial \boldsymbol{x}}\cdot\frac{\partial}{\partial \boldsymbol{v}}\right]f(\boldsymbol{x},\,\boldsymbol{v};\,t)=0$$

$$\nabla^2\phi(x;\,t)=4\pi\,G\,a^2\int d^3\boldsymbol{v}\,f(\boldsymbol{x},\,\boldsymbol{v};\,t)$$

Vlasov-Poisson system

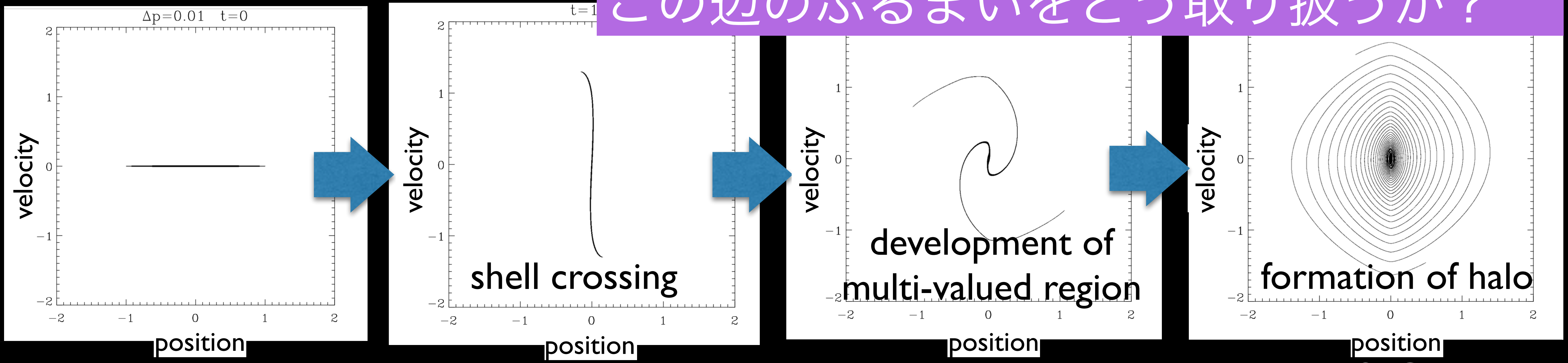
- N→∞ limit of self-gravitating N-body system (assuming that particles are not correlated with each other)
- Can be reduced to a <u>pressureless fluid</u> system if we assume single-stream flow:

$$f(\boldsymbol{x},\,\boldsymbol{v};\,t)\,\rightarrow\,\overline{\rho}(t)\,\left\{1+\delta(\boldsymbol{x};\,t)\right\}\,\delta_{\rm D}\left(\boldsymbol{v}-\boldsymbol{v}(\boldsymbol{x};\,t)\right)$$

But, single-stream flow is violated at small scales

<u>Example: 1D collapse</u>

この辺のふるまいをどう取り扱うか？



By S. Colombi

Post-collapse perturbation theory

Going beyond shell-crossing, a new analytical framework needs to be developed:

Post-collapse PT Colombi ('15), AT & Colombi (in prep.)

Lagrangian-based PT that can follow post-collapse dynamics

Outline

- Work in Lagrangian space (*q*) : $x(q;\,t) = q + S(q;\,t)$ *displacement field*
- Taylor-expand displacement around shell-crossing region (at *q*$_0$):

$$x_{\rm coll}(q;t_0) \simeq A(q_0,t) - B(q_0,t)\,(q-q_0) + C(q_0,t)\,(q-q_0)^3 + \cdots$$

time-dependent 3rd-order polynomial function of q

1. Force calculation at multi-valued region
2. Corrections to velocity & position :

$$\Delta v(q;t,\,t_q) = -\int_{t_q}^{t} dt'\,\nabla_x\Phi(x_{\rm coll}(q,t');\,t') \qquad \Delta x(q;t,\,t_q) = \int_{t_q}^{t} dt'\,\Delta v(q;t',\,t_q)$$

Post-collapse perturbation theory

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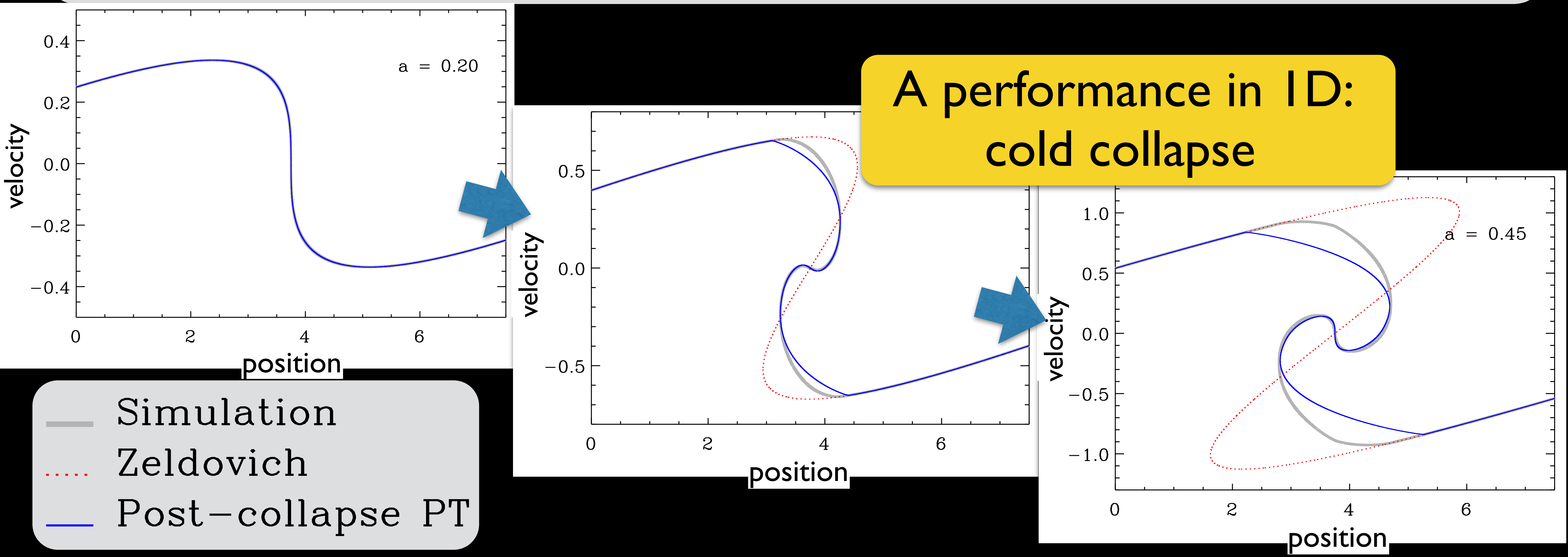
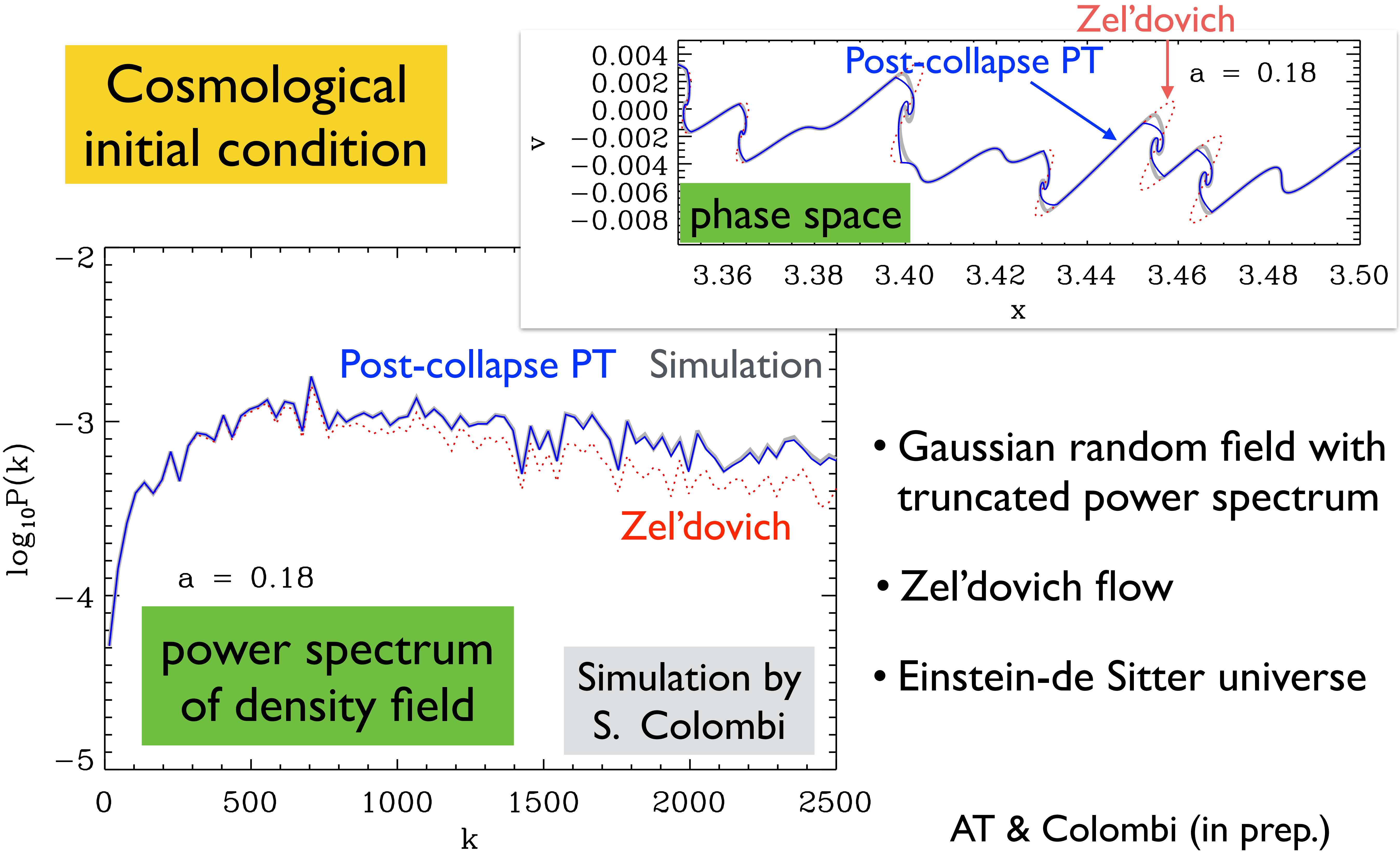


A performance in 1D: cold collapse

Note—. Zel'dovich solution is exact in 1D before shell crossing

Performance in 1D cosmology

Cosmological initial condition



phase space

power spectrum of density field

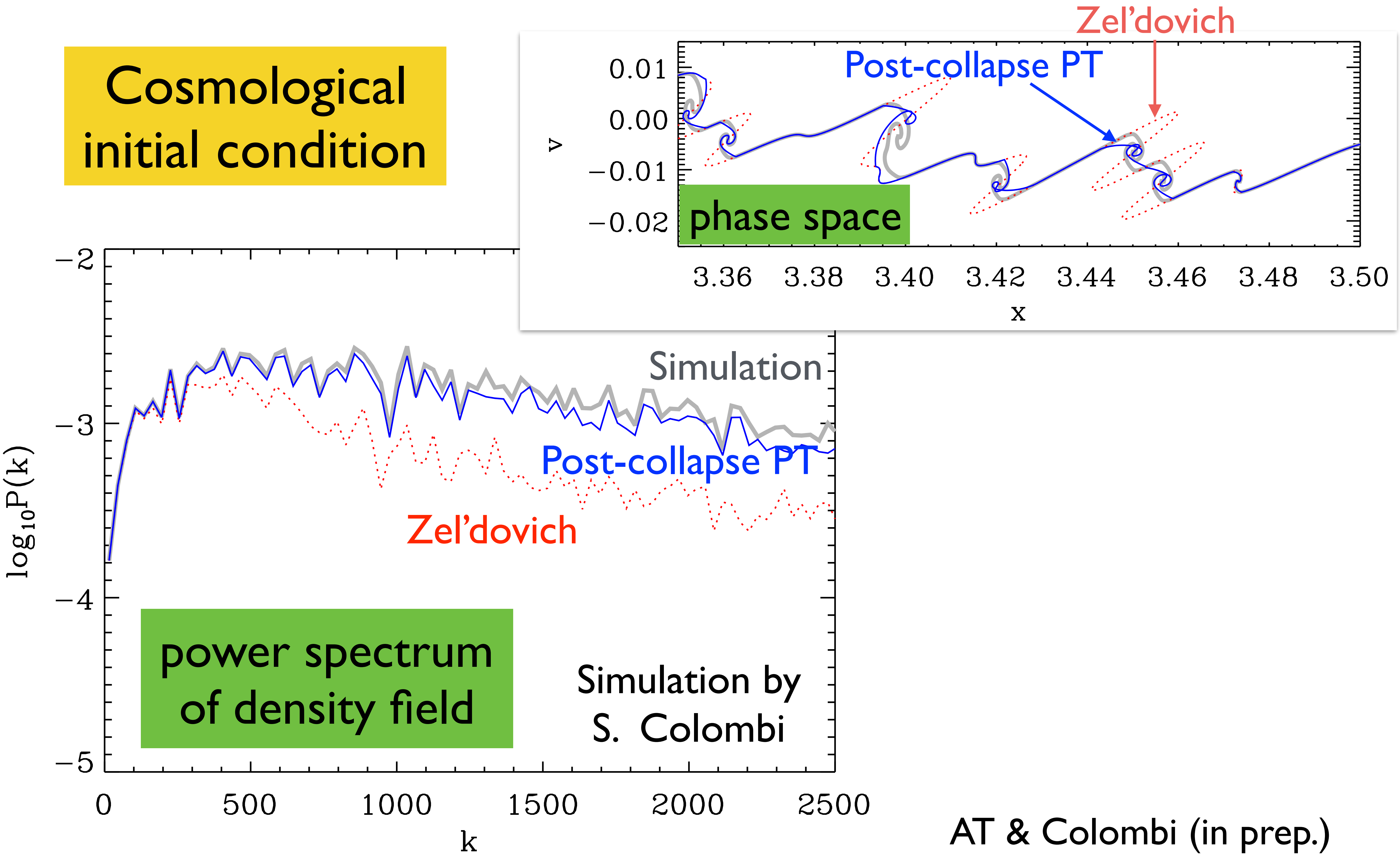
Simulation by S. Colombi

- Gaussian random field with truncated power spectrum
- Zel'dovich flow
- Einstein-de Sitter universe

AT & Colombi (in prep.)

Performance in 1D cosmology

Cosmological initial condition



Simulation by S. Colombi

AT & Colombi (in prep.)

Toward practical method

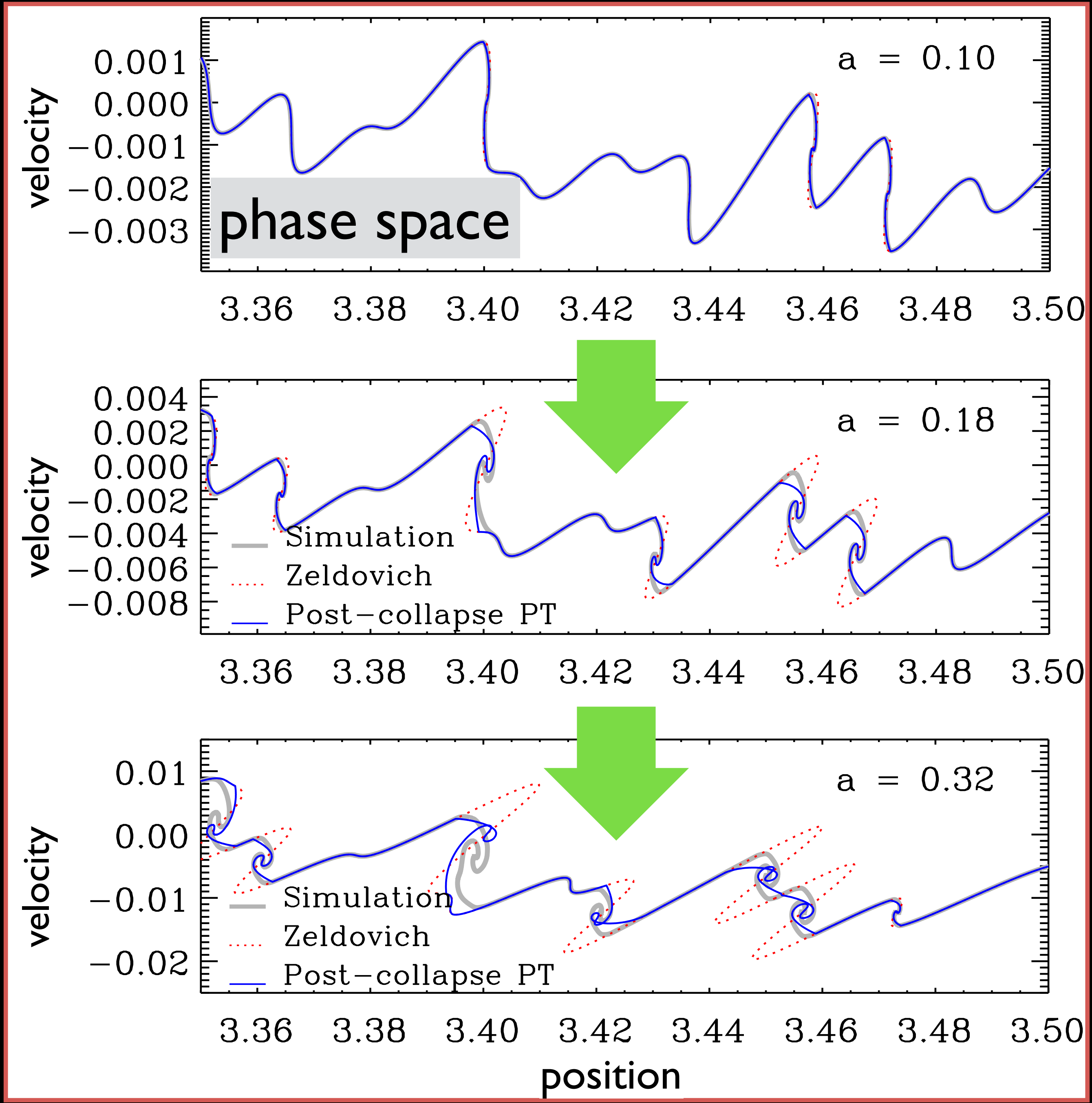
既存の取り扱いを超える摂動計算ができた！

摂動計算の適用範囲がさらに広がる可能性

課題

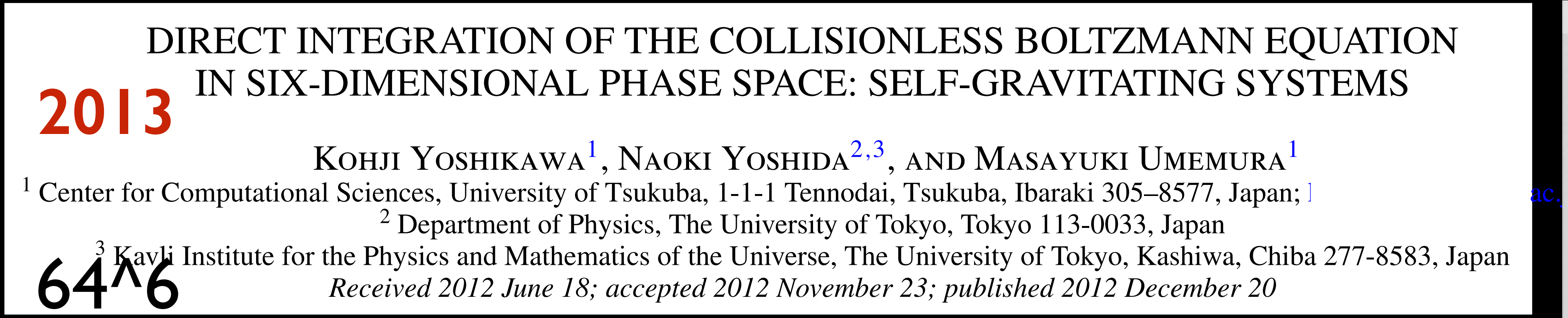
- 統計量（e.g., パワースペクトル）の解析計算法の確立
- １次元から３次元への拡張
- 計算の高速化

まだまだ未成熟だが今後発展が期待される 6D Vlasovコードを比較・検証する上でも重要



State-of-the-art 6D Vlasov code

relevant for cosmology



2013

DIRECT INTEGRATION OF THE COLLISIONLESS BOLTZMANN EQUATION
IN SIX-DIMENSIONAL PHASE SPACE: SELF-GRAVITATING SYSTEMS

Kohji Yoshikawa[1], Naoki Yoshida[2,3], and Masayuki Umemura[1]
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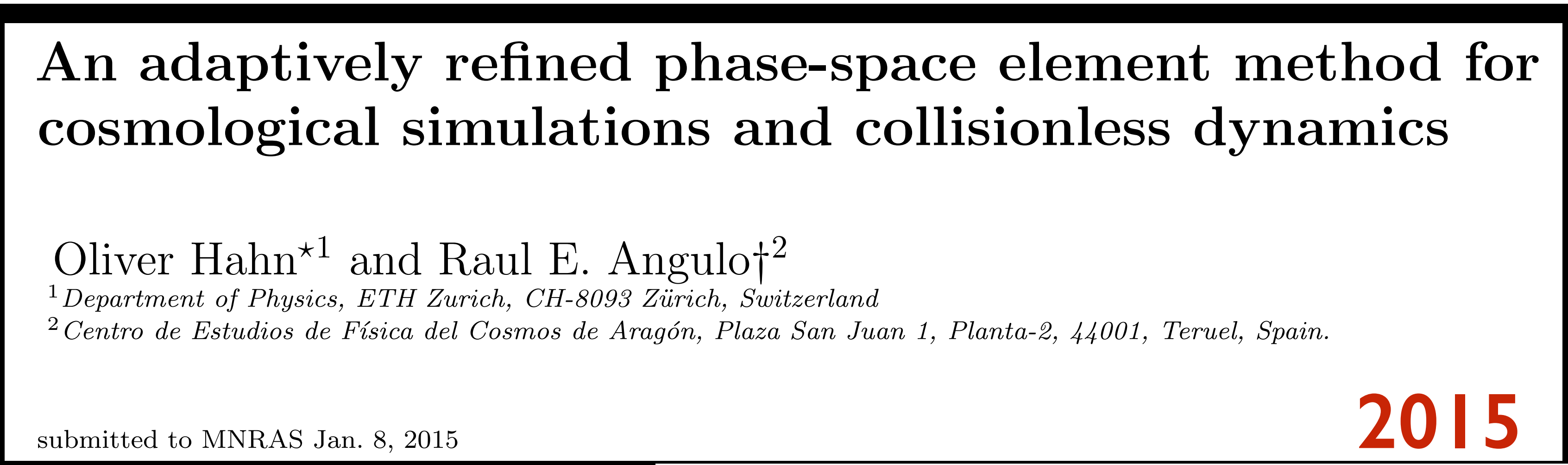
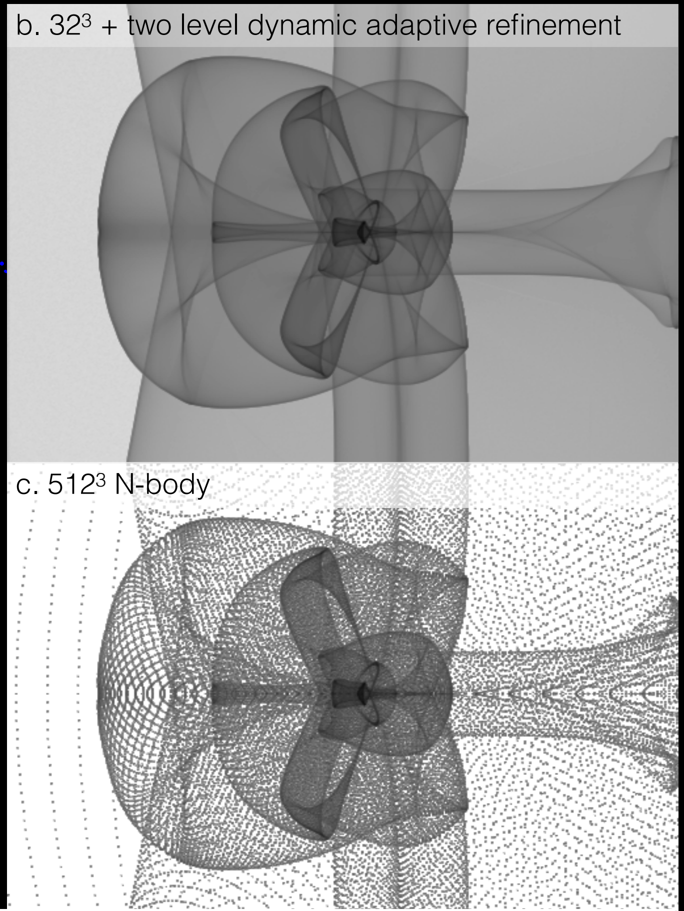


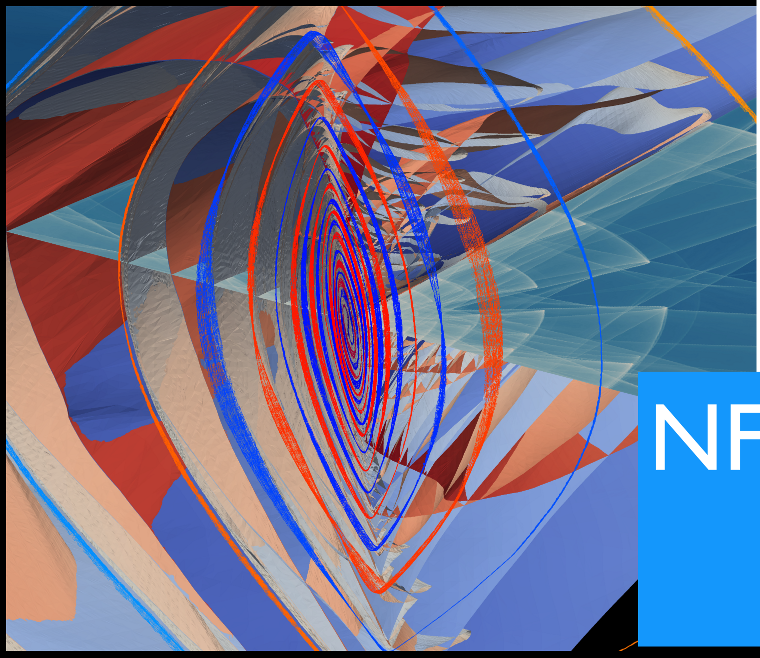
An adaptively refined phase-space element method for cosmological simulations and collisionless dynamics

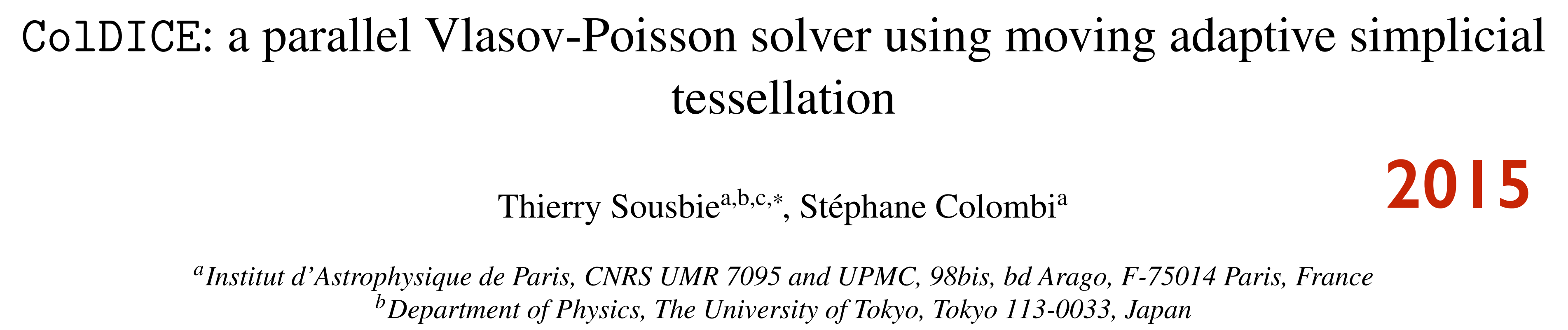
Oliver Hahn[⋆1] and Raul E. Angulo[†2]
[1] *Department of Physics, ETH Zurich, CH-8093 Zürich, Switzerland*
[2] *Centro de Estudios de Física del Cosmos de Aragón, Plaza San Juan 1, Planta-2, 44001, Teruel, Spain.*

submitted to MNRAS Jan. 8, 2015

2015







`ColDICE`: a parallel Vlasov-Poisson solver using moving adaptive simplicial tessellation

2015

Thierry Sousbie[a,b,c,*], Stéphane Colombi[a]
[a] *Institut d'Astrophysique de Paris, CNRS UMR 7095 and UPMC, 98bis, bd Arago, F-75014 Paris, France*
[b] *Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*

NFW プロファイルの起源の解決、ダークマター分布の速度構造の理解や観測への応用

まとめ

宇宙の構造形成：ΛCDMモデルとその向こう

標準モデルとしてのΛCDMモデル

ΛCDMモデルの向こうへ：宇宙大規模構造の精密観測

構造形成理論の精密化とリノベーション：

摂動論の発展と課題：
$\left\{\begin{array}{l}\text{大規模構造の応答関数}\\ \text{Post-collapse 摂動論}\end{array}\right.$

シミュレーション：粒子法から6次元Vlasovへ

構造形成の深い理解へ

精密観測がもたら宇宙論研究の新たな発展に期待