2015年11月9, 16, 30日

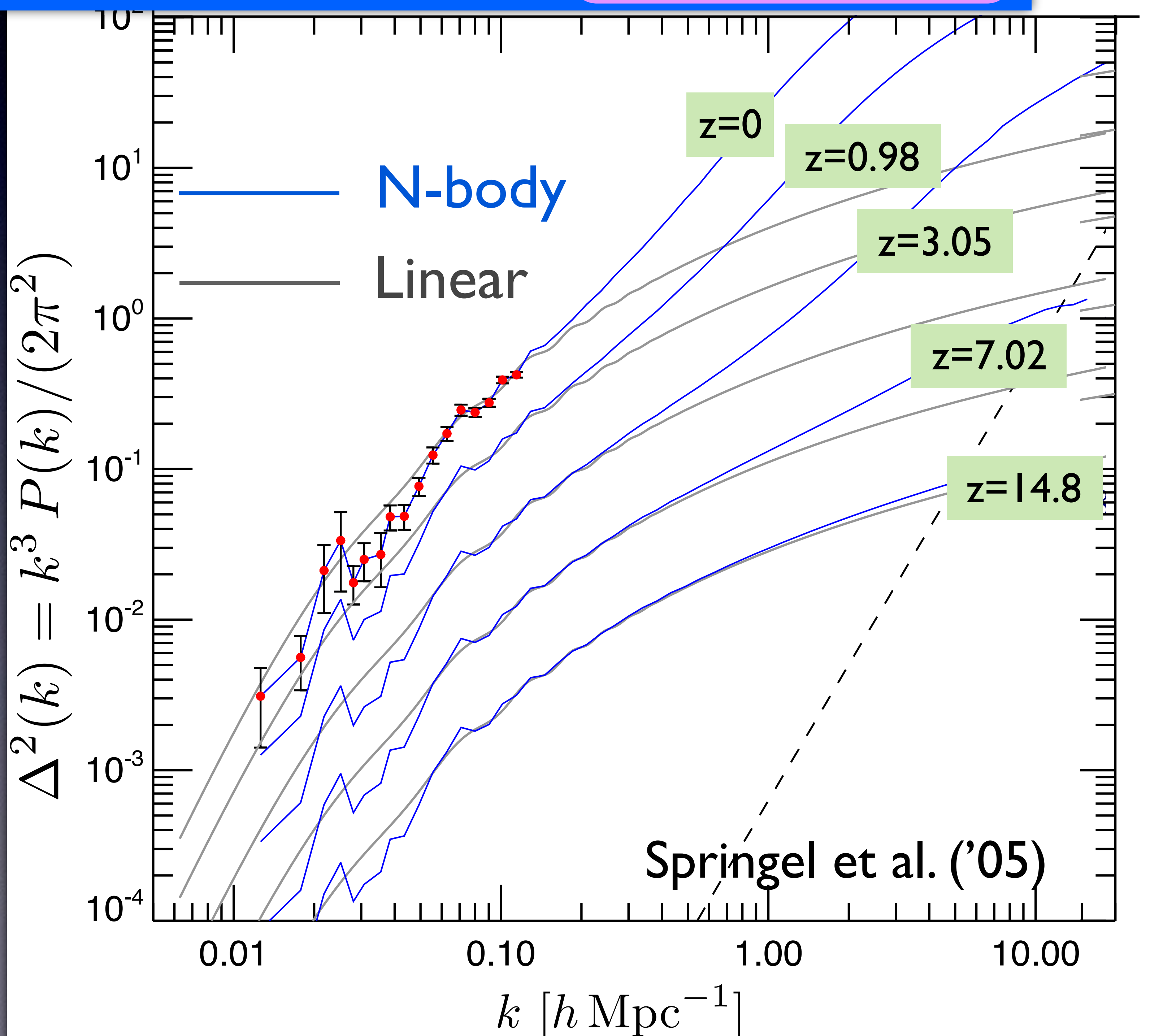
集中講義@名古屋大学

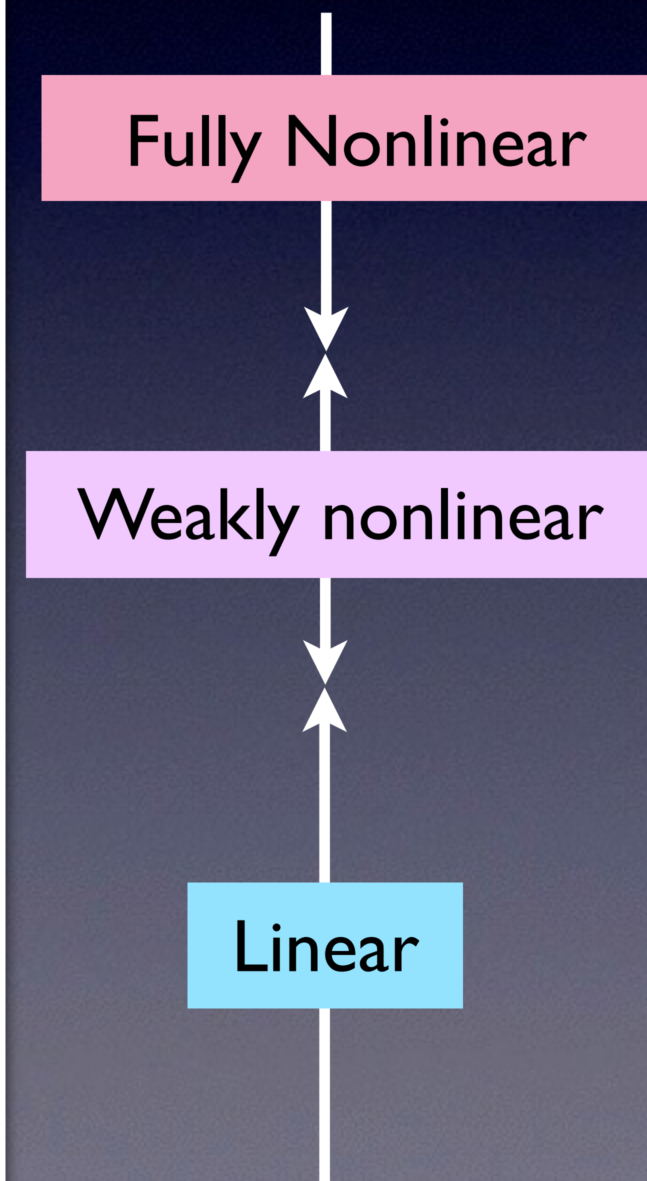
「宇宙大規模構造と精密宇宙論」

宇宙大規模構造の非線形進化

Nonlinear gravitational evolution

$$\delta(\vec{x}) \equiv \frac{\delta\rho_{\rm m}(\vec{x})}{\overline{\rho}_{\rm m}} = \frac{1}{\sqrt{V}}\sum_{\vec{k}}\delta(\vec{k})\,e^{i\vec{k}\cdot\vec{x}} \Rightarrow P(k) = \frac{1}{N_k}\sum_{|\vec{k}|=k}|\delta(\vec{k})|^2$$





Range of applicability

	Methods (Gravitational evolution)	Other systematics
Fully nonlinear $(\Delta^2 > 1)$	N-body simulation most powerful, but extensive & time-consuming **(c.f. fitting formula)**	*very difficult* **Baryon physics** (weak lensing)
weakly nonlinear $(\Delta^2 \lesssim 1)$	Perturbation theory limited range of application, but analytical & very fast	• **Galaxy bias** • **Redshift-space distortion** (galaxy surveys)
linear $(\Delta^2 \ll 1)$	Linear theory (CMB Boltzmann code)	*relatively easy*

Perturbation theory (PT)

Theory of large-scale structure based on gravitational instability

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Jain & Bertschinger ('94), ...

Cold dark matter + baryons = pressureless & irrotational fluid

Basic eqs.

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot \left[(1+\delta) \vec{v} \right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} + \frac{1}{a} (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{1}{a} \vec{\nabla} \Phi$$

$$\frac{1}{a^2} \nabla^2 \Phi = 4\pi G \bar{\rho}_{\rm m} \delta$$

Single-stream approx. of collisionless Boltzmann eq.

standard PT $|\delta| \ll 1$

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots \qquad \langle \delta(\boldsymbol{k};t) \delta(\boldsymbol{k}';t) \rangle = (2\pi)^3 \delta_{\rm D}(\boldsymbol{k}+\boldsymbol{k}') P(|\boldsymbol{k}|;t)$$

Equations of motion

$$\partial_\tau \delta + \partial_i \left[(1+\delta)v^i\right] \;=\; 0\,,$$

$$\partial_\tau v^i + \mathcal{H}\, v^i_l + \partial^i \phi + v^j_l\, \partial_j v^i \;= 0$$

$$\triangle\, \phi \;=\; \frac{3}{2}\mathcal{H}^2 \Omega_m\, \delta\,.$$

τ: conformal time
(adτ = dt)

$$\int_{\boldsymbol{q}} \;\equiv\; \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}$$

Fourier expansion

$$\partial_\tau \delta(\boldsymbol{k},\tau) + \theta(\boldsymbol{k},\tau) \;=\; -\int_{\boldsymbol{q}} \alpha(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\,\theta(\boldsymbol{q},\tau)\delta(\boldsymbol{k}-\boldsymbol{q},\tau)\,,$$

$$\partial_\tau \theta(\boldsymbol{k},\tau) + \mathcal{H}\,\theta(\boldsymbol{k},\tau) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(\boldsymbol{k},\tau)$$

$$\theta \equiv \nabla\cdot\boldsymbol{v}$$

$$= \; -\int_{\boldsymbol{q}} \beta(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\,\theta(\boldsymbol{q},\tau)\theta(\boldsymbol{k}-\boldsymbol{q},\tau)$$

$$\alpha(\boldsymbol{q}_1,\boldsymbol{q}_2) \;\equiv\; \frac{\boldsymbol{q}_1\cdot(\boldsymbol{q}_1+\boldsymbol{q}_2)}{q_1^2}\,, \qquad \beta(\boldsymbol{q}_1,\boldsymbol{q}_2) \;\equiv\; \frac{1}{2}\,(\boldsymbol{q}_1+\boldsymbol{q}_2)^2\,\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{q_1^2\, q_2^2}\,.$$

Standard perturbation theory

$$\delta(\boldsymbol{k},a) = \sum_{i=1}^{\infty}\delta_{(i)}(\boldsymbol{k},a)\,, \qquad \theta(\boldsymbol{k},a) = -\mathcal{H}\,f(a)\sum_{i=1}^{\infty}\theta_{(i)}(\boldsymbol{k},a)$$

$$f(a) \equiv d\ln D_1/d\ln a$$

$D_1(a)$: Linear growth factor

Adopting the E-dS approximation,

$$\delta_{(n)}(\boldsymbol{k},a) = \underline{D_1^n(a)}\,\delta_n(\boldsymbol{k})\,, \qquad \theta_{(n)}(\boldsymbol{k},a) = \underline{D_1^n(a)}\,\theta_n(\boldsymbol{k})\,.$$

Linear density field (Gaussian)

$$\delta_n(\boldsymbol{k}) = \int_{\boldsymbol{q}_1}\cdots\int_{\boldsymbol{q}_n}(2\pi)^3\delta_D^{(3)}(\boldsymbol{k}-\boldsymbol{q}_1\ldots-\boldsymbol{q}_n)\,\boxed{F_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)}\,\delta_0(\boldsymbol{q}_1)\ldots\delta_0(\boldsymbol{q}_n)\,,$$

$$\theta_n(\boldsymbol{k}) = \int_{\boldsymbol{q}_1}\cdots\int_{\boldsymbol{q}_n}(2\pi)^3\delta_D^{(3)}(\boldsymbol{k}-\boldsymbol{q}_1\ldots-\boldsymbol{q}_n)\,\boxed{G_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)}\,\delta_0(\boldsymbol{q}_1)\ldots\delta_0(\boldsymbol{q}_n)\,,$$

standard PT kernel $(F_1 = G_1 = 1)$

Recursion relation for PT kernels

$$\mathcal{F}_a^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\equiv\left[\begin{array}{c}F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\\ G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\end{array}\right]$$

$$\mathcal{F}_a^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)=\sum_{m=1}^{n-1}\sigma_{ab}^{(n)}\,\gamma_{bcd}(\boldsymbol{q}_1,\boldsymbol{q}_2)\,\mathcal{F}_c^{(m)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_m)\,\mathcal{F}_d^{(n-m)}(\boldsymbol{k}_{m+1},\cdots,\boldsymbol{k}_n)$$

$$\boldsymbol{q}_1=\boldsymbol{k}_1+\cdots+\boldsymbol{k}_m$$
$$\boldsymbol{q}_2=\boldsymbol{k}_{m+1}+\cdots+\boldsymbol{k}_n$$

$$\sigma_{ab}^{(n)}=\frac{1}{(2n+3)(n-1)}\left(\begin{array}{cc}2n+1 & 2\\ 3 & 2n\end{array}\right)$$

$$\gamma_{abc}(\boldsymbol{k}_1,\boldsymbol{k}_2)=\begin{cases}\frac{1}{2}\left\{1+\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_1}{|\boldsymbol{k}_2|^2}\right\}; & (a,\,b,\,c)=(1,\,1,\,2)\\ \frac{1}{2}\left\{1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{|\boldsymbol{k}_1|^2}\right\}; & (a,\,b,\,c)=(1,\,2,\,1)\\ \frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)|\boldsymbol{k}_1+\boldsymbol{k}_2|^2}{2|\boldsymbol{k}_1|^2|\boldsymbol{k}_2|^2}; & (a,\,b,\,c)=(2,\,2,\,2)\\ 0; & \text{otherwise.}\end{cases}$$

Note—. repetition of the same subscripts (a,b,c) indicates the sum over all multiplet components

PT kernels constructed from recursion relation should be *symmetrized*

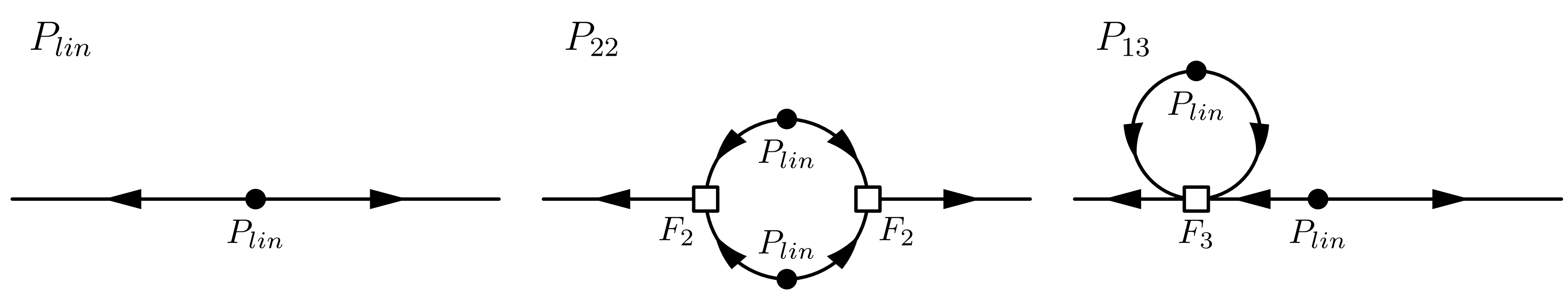
Power spectrum

$$\langle \delta(\boldsymbol{k}_1, a)\delta(\boldsymbol{k}_2, a)\rangle \equiv (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)\, P(k_1, a)$$

$$\Rightarrow \quad P_{SPT}(k) = \underbrace{P_{lin}(k)}_{\text{linear}} + \underbrace{P_{22}(k) + P_{13}(k)}_{\text{1-loop}} + \text{ higher order loops} .$$

$$P_{22}(k) = 2\int_{\boldsymbol{q}} P_{lin}(q) P_{lin}(|\boldsymbol{k}-\boldsymbol{q}|)\, F_2^2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) ,$$

$$P_{13}(k) = 6 P_{lin}(k) \int_{\boldsymbol{q}} P_{lin}(q)\, F_3(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}) ,$$

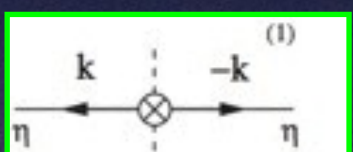


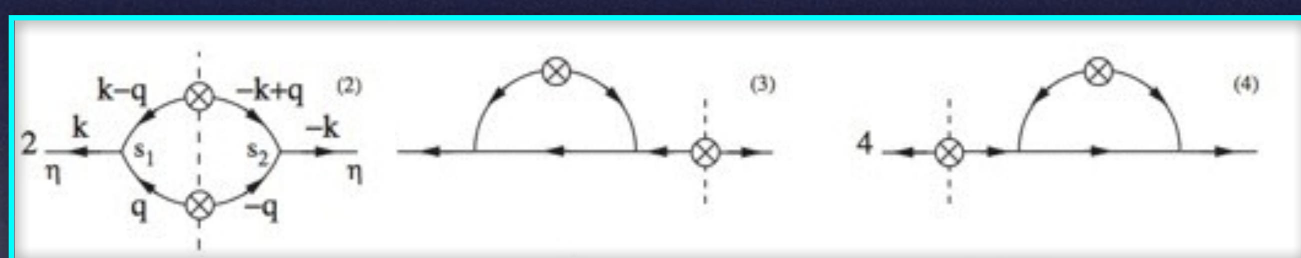
Next-to-next-to leading order

up to 2-loop order

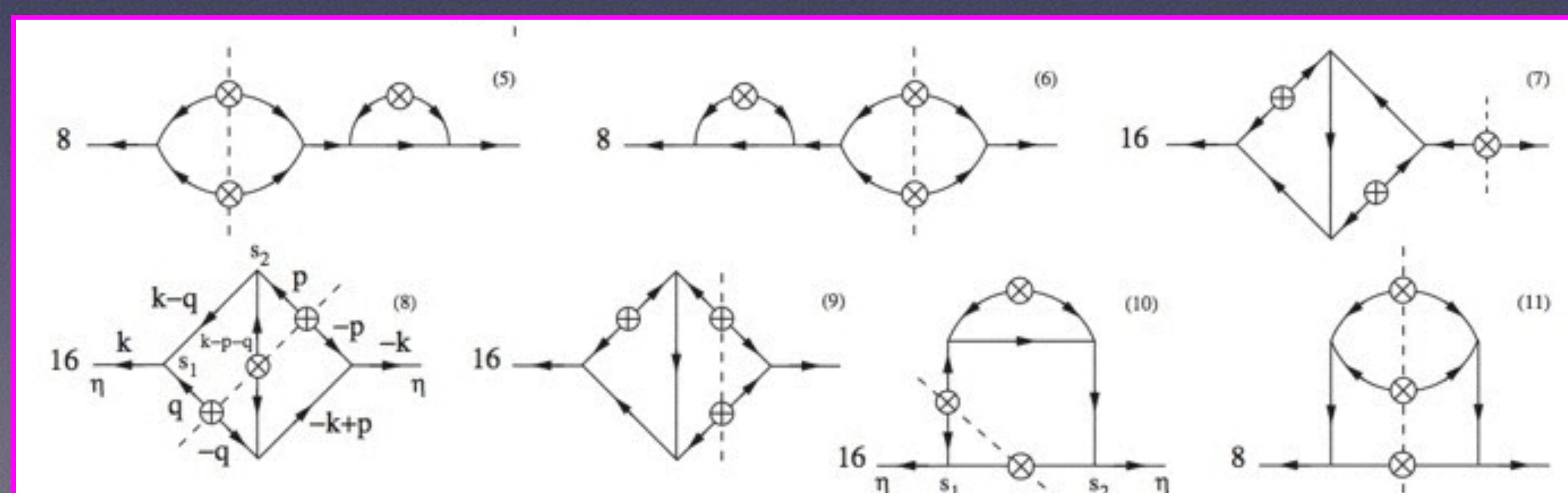
$$P^{(mn)} \simeq \langle \delta^{(m)} \delta^{(n)} \rangle$$

$$P(k) = \underbrace{P^{(11)}(k)}_{\text{Linear (tree)}} + \underbrace{\left(P^{(22)}(k) + P^{(13)}(k)\right)}_{\text{1-loop}} + \underbrace{\left(P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k)\right)}_{\text{2-loop}} + \cdots$$





Crocce & Scoccimarro ('06)



Calculation involves multi-dimensional numerical integration

(time-consuming)

Comparison with simulations

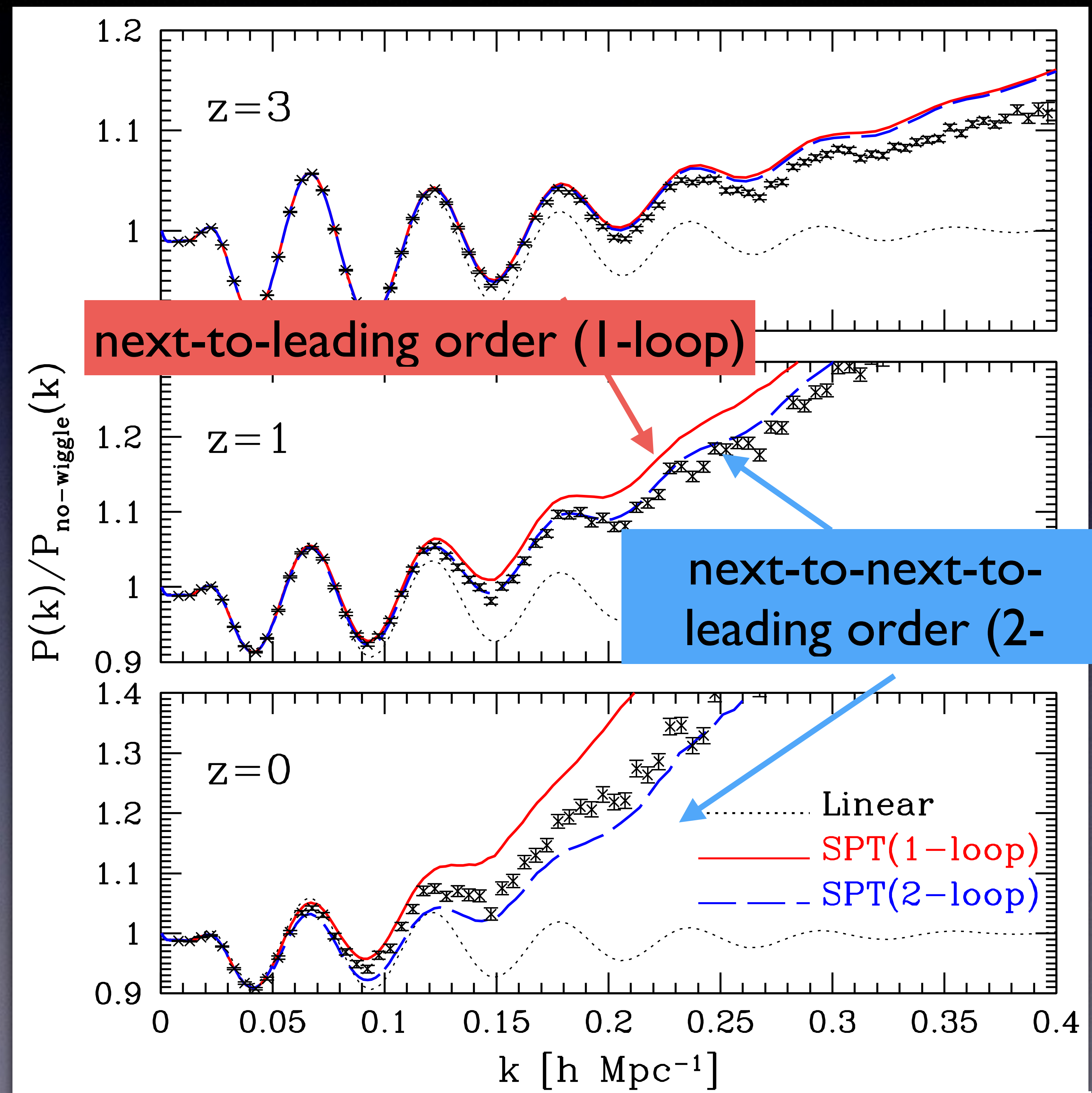
Standard PT qualitatively explains scale-dependent nonlinear growth, however,

1-loop :
overestimates simulations

2-loop :
overestimates at high-z, while it turn to underestimate at low-z

Standard PT produces ill-behaved PT expansion !!

… need to be improved



AT et al. ('09)

Improving PT predictions

Basic idea Reorganizing standard PT expansion by introducing non-perturbative statistical quantities

$\delta_0(\boldsymbol{k})$

initial density field (Gaussian)

$\delta(\boldsymbol{k};z)$

Evolved density field (non-Gaussian)

Initial power spectrum

$P_0(k)$

from linear theory

(CMB Boltzmann code)

Nonlinear mapping

Observables

$P(k;z)$

$B(k_1,k_2,k_3;z)$

$T(k_1,k_2,k_3,k_4;z)$

$\vdots$

of dark matter/galaxies/halos

Concept of ‘*propagator*’ in physics/mathematics may be useful

Propagator in physics

- ✦ Green's function in linear differential equations
- ✦ Probability amplitude in quantum mechanics

Schrödinger Eq.

$$\left(-i\hbar\frac{\partial}{\partial t} + H_x\right)\psi(x,t) = 0$$

$$G(x,t;x',t') \equiv \frac{\delta\psi(x,t)}{\delta\psi(x',t')}$$

$$\left(-i\hbar\frac{\partial}{\partial t} + H_x\right)G(x,t;x',t') = -i\hbar\,\delta_D(x-x')\delta_D(t-t')$$

$$\Rightarrow \quad \psi(x,t) = \int_{-\infty}^{+\infty} dx' G(x,t;x',t')\,\psi(x',t') \ ; \quad t > t'$$

Cosmic propagators

Propagator should carry information on
non-linear evolution & statistical properties

Crocce & Scoccimarro ('06)

Evolved (non-linear) density field

$$\left\langle \frac{\delta\,\delta_{\rm m}(\boldsymbol{k};t)}{\delta\,\delta_0(\boldsymbol{k}')} \right\rangle \equiv \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}')\boxed{\Gamma^{(1)}(k;t)}$$

Propagator

Initial density field

Ensemble w.r.t randomness of initial condition

Contain statistical information on *full-nonlinear* evolution

(Non-linear extension of Green's function)

Multi-point propagators

Bernardeau, Crocce & Scoccimarro ('08)

Matsubara ('11) ⟶ *integrated PT*

As a natural generalization,

Multi-point propagator

$$\left\langle \frac{\delta^n\,\delta_{\rm m}(\boldsymbol{k};t)}{\delta\,\delta_0(\boldsymbol{k}_1)\cdots\delta\,\delta_0(\boldsymbol{k}_n)}\right\rangle = (2\pi)^{3(1-n)}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k'})\,\Gamma^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)$$

With this multi-point prop.

- Building blocks of a new perturbative theory (PT) expansion

  ---------- Γ-expansion or Wiener-Hermite expansion

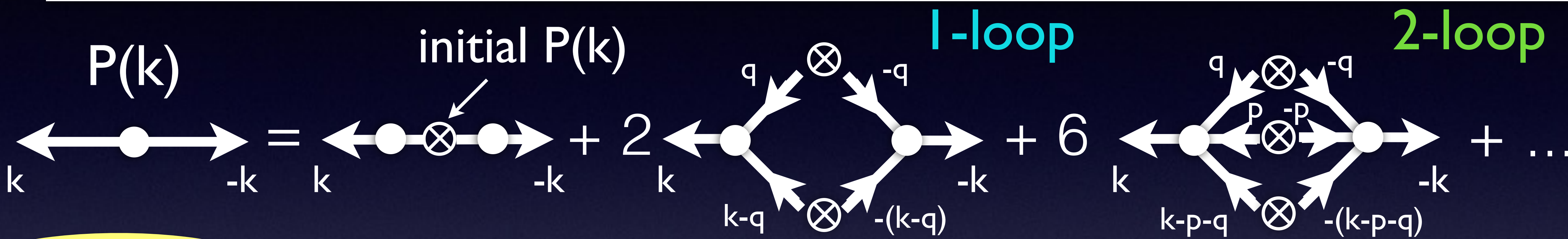
- A good convergence of PT expansion is expected

  (c.f. standard PT)

Power spectrum

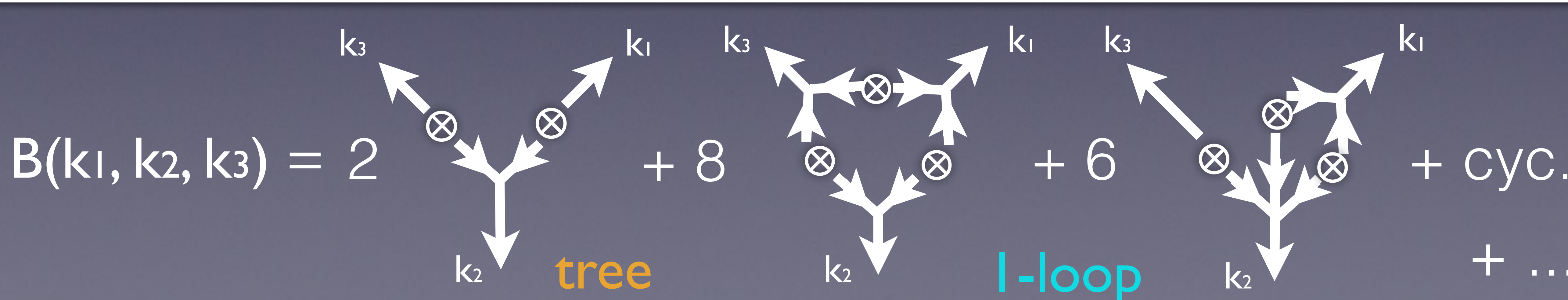
Initial power spectrum

$$P(k;t) = \left[\Gamma^{(1)}(k;t)\right]^2 P_0(k) + 2\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\Gamma^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q};t)\right]^2 P_0(q) P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$
$$+\,6\int \frac{d^6\boldsymbol{p}d^3\boldsymbol{q}}{(2\pi)^6}\left[\Gamma^{(3)}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q};t)\right]^2 P_0(p)P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|) + \cdots$$



Bispectrum

$$\begin{aligned} B(k_1,k_2,k_3) &= 2\,\Gamma^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\Gamma^{(1)}(k_1)\,\Gamma^{(1)}(k_2)\,P_0(k_1)P_0(k_2) + \text{cyc.} \\ &+ \Big[8\int d^3q\,\Gamma^{(2)}(\mathbf{k}_1-\mathbf{q},\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{q}-\mathbf{k}_1,-\mathbf{k}_2-\mathbf{q})P_0(|\mathbf{k}_1-\mathbf{q}|)P_0(|\mathbf{k}_2+\mathbf{q}|)P_0(q) \\ &+ \;6\int d^3q\,\Gamma^{(3)}(-\mathbf{k}_3,-\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2-\mathbf{q},\mathbf{q})\Gamma^{(1)}(\mathbf{k}_3)P_0(|\mathbf{k}_2-\mathbf{q}|)P_0(q)P_0(k_3) + \text{cyc.}\Big]. \end{aligned}$$

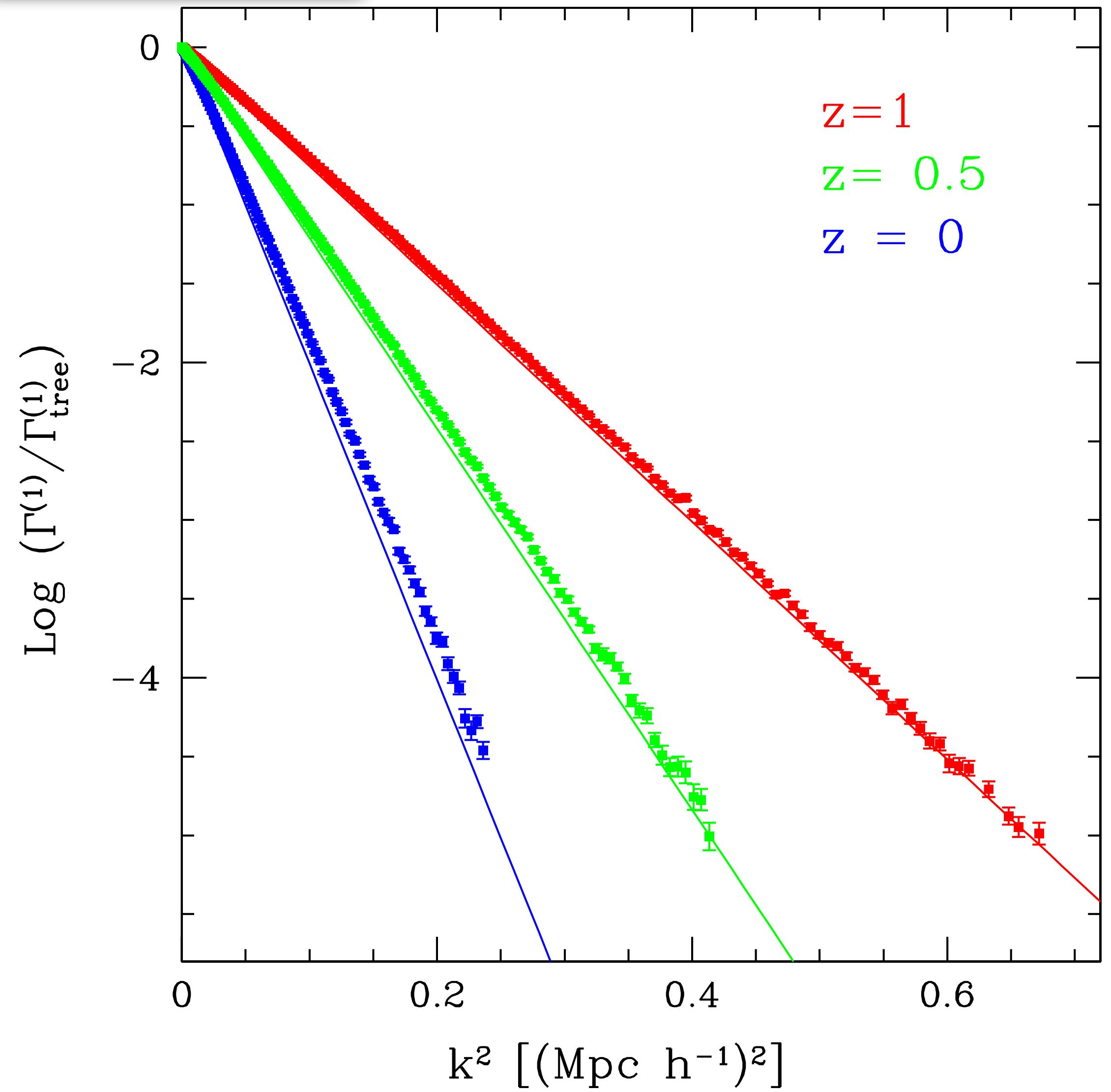


Generic property of propagators

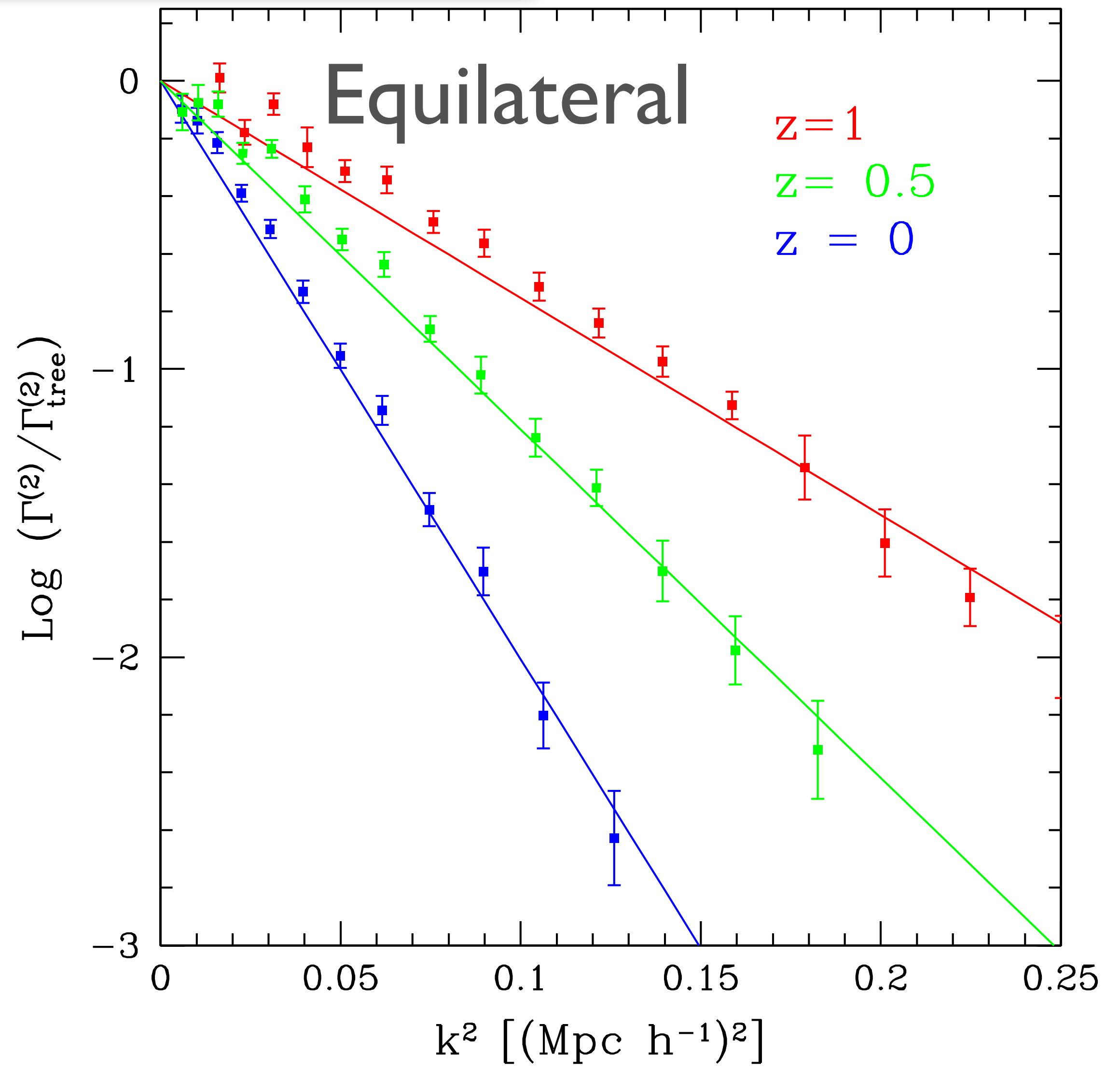
Crocce & Scoccimarro '06, Bernardeau et al. '08

$$\Gamma^{(n)} \xrightarrow{k \to +\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm v}^2/2} \quad ; \quad \sigma_{\rm v}^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)$$

$\Gamma^{(1)}(k)$



$\Gamma^{(2)}(k_1, k_2, k_3)$

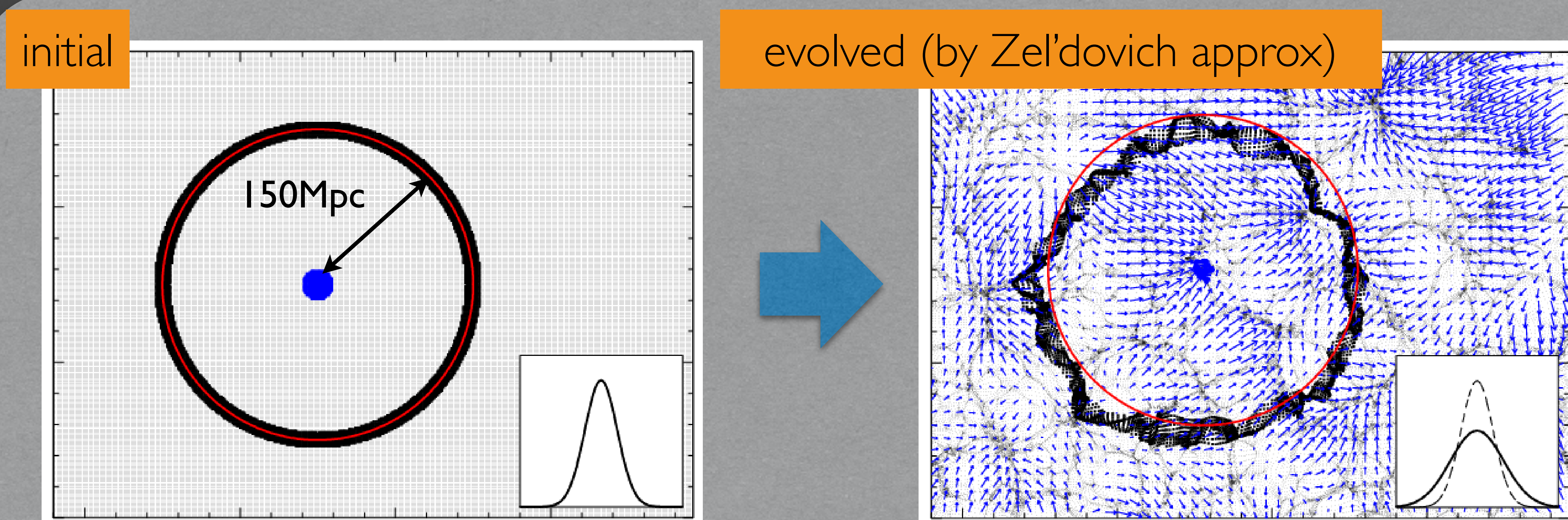


Origin of Exp. damping

For Gaussian initial condition,

$$\langle \delta_{\rm m}(\boldsymbol{k};t)\,\delta_0(\boldsymbol{k}')\rangle = \boxed{\Gamma^{(1)}(k;t)}\ \overbrace{\underbrace{\langle \delta_0(\boldsymbol{k})\delta_0(\boldsymbol{k}')\rangle}_{\text{initial power spectrum}}}^{=\,P_0(k)}$$

➡ Cross correlation between initial & evolved density fields



Padmanabhan et al. ('12)

Initial structure becomes blurred by the *local* cosmic flow

----- origin of Gaussian damping in propagator

Constructing regularized propagators

- UV property (k >>1) :

$$\Gamma^{(n)} \xrightarrow{k \to +\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm v}^2/2} \quad ; \quad \sigma_{\rm v}^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)$$

Bernardeau, Crocce & Scoccimarro ('08), Bernardeau, Van de Rijt, Vernizzi ('11)

- IR behavior (k<<1) can be described by standard PT calculations :

$$\Gamma^{(n)} = \Gamma^{(n)}_{\rm tree} + \Gamma^{(n)}_{\rm 1\text{-}loop} + \Gamma^{(n)}_{\rm 2\text{-}loop} + \cdots$$

Importantly, each term behaves like $\Gamma^{(n)}_{p\text{-loop}} \xrightarrow{k \to +\infty} \dfrac{1}{p!}\left(-\dfrac{k^2\,\sigma_{\rm v}^2}{2}\right)^p \Gamma^{(n)}_{\rm tree}$

A regularization scheme that reproduces both UV & IR behaviors

Bernardeau, Crocce & Scoccimarro ('12)

Regularized propagator

Bernardeau, Crocce & Scoccimarro ('12)

A global solution that satisfies both UV (k>>1) & IR (k<<1) properties:

$$\Gamma^{(n)}_{\rm reg} = \left[\Gamma^{(n)}_{\rm tree}\left\{1+\underbrace{\frac{k^2\sigma_{\rm v}^2}{2}}_{\text{counter term}}\right\}+\Gamma^{(n)}_{\text{1-loop}}\right]\exp\left\{-\frac{k^2\sigma_{\rm v}^2}{2}\right\}; \qquad \sigma_{\rm v}^2=\int\frac{dq}{6\pi^2}P_{\theta\theta}(q)$$

…………… *IR behavior is valid at 1-loop level*

Precision of IR behavior can be systematically improved by including higher-loop corrections and adding counter terms

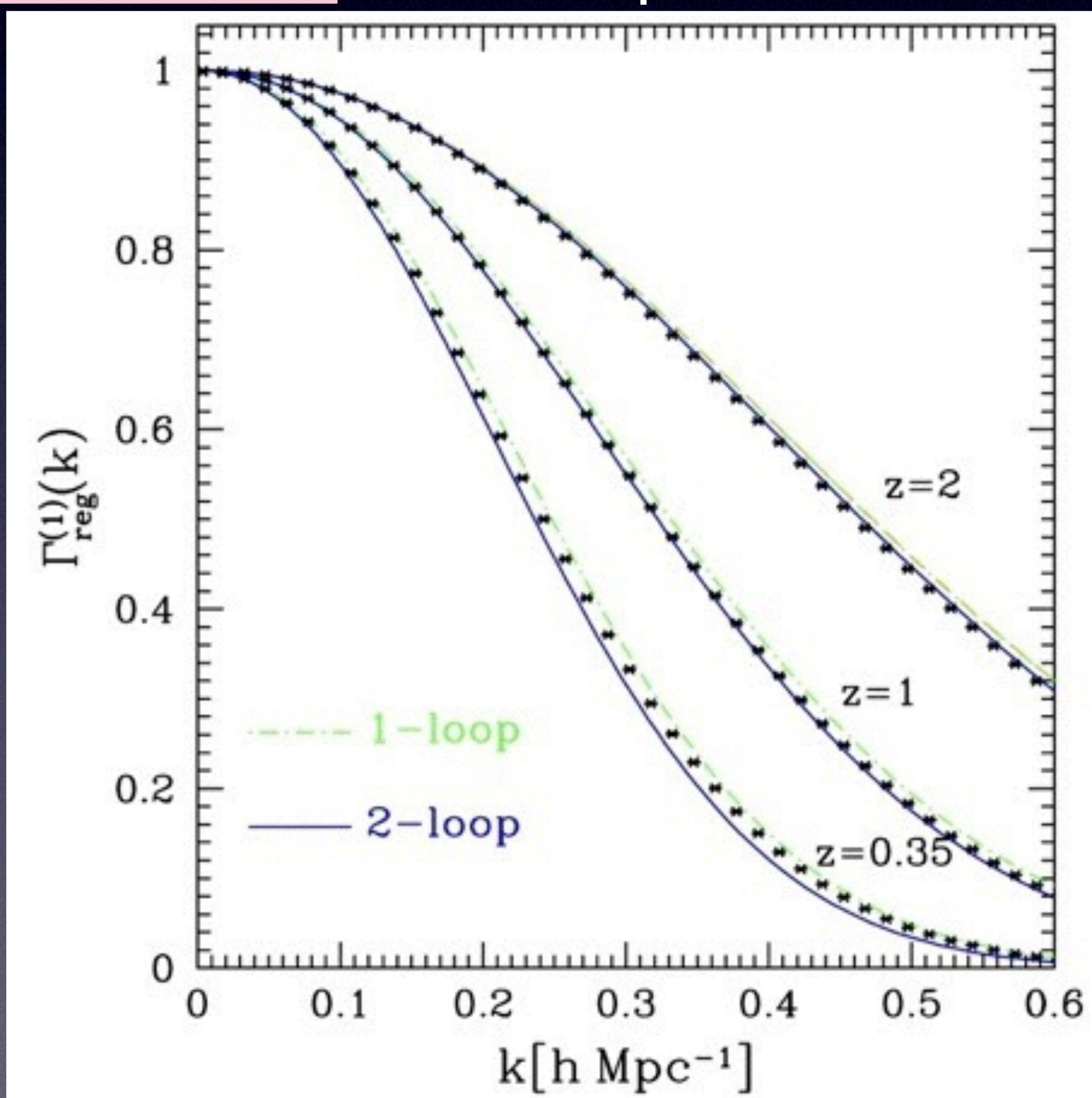
e.g., For IR behavior valid at 2-loop level,

$$\Gamma^{(n)}_{\rm reg} = \left[\Gamma^{(n)}_{\rm tree}\left\{1+\underbrace{\frac{k^2\sigma_{\rm v}^2}{2}+\frac{1}{2}\left(\frac{k^2\sigma_{\rm v}^2}{2}\right)^2}_{\text{counter term}}\right\}+\Gamma^{(n)}_{\text{1-loop}}\left\{1+\underbrace{\frac{k^2\sigma_{\rm v}^2}{2}}_{\text{counter term}}\right\}+\Gamma^{(n)}_{\text{2-loop}}\right]\exp\left\{-\frac{k^2\sigma_{\rm v}^2}{2}\right\}$$

Propagators in N-body simulations

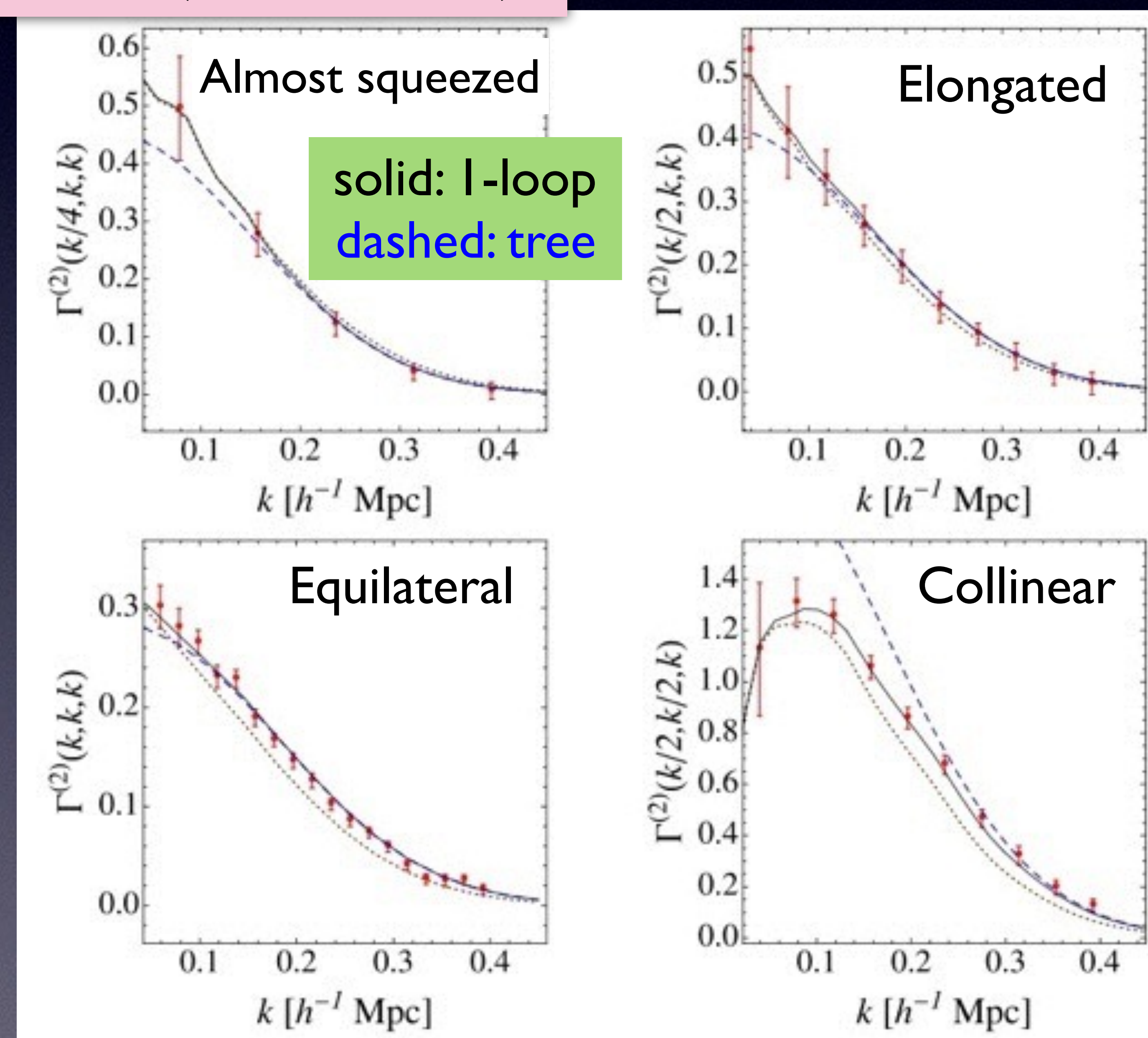
compared with '*Regularized*' propagators constructed analytically

$\Gamma^{(1)}(k)$ predictions up to 2-loop order



Bernardeau, AT & Nishimichi ('12)

$\Gamma^{(2)}(k_1, k_2, k_3)$ predictions up to 1-loop order



Bernardeau et al. ('12)

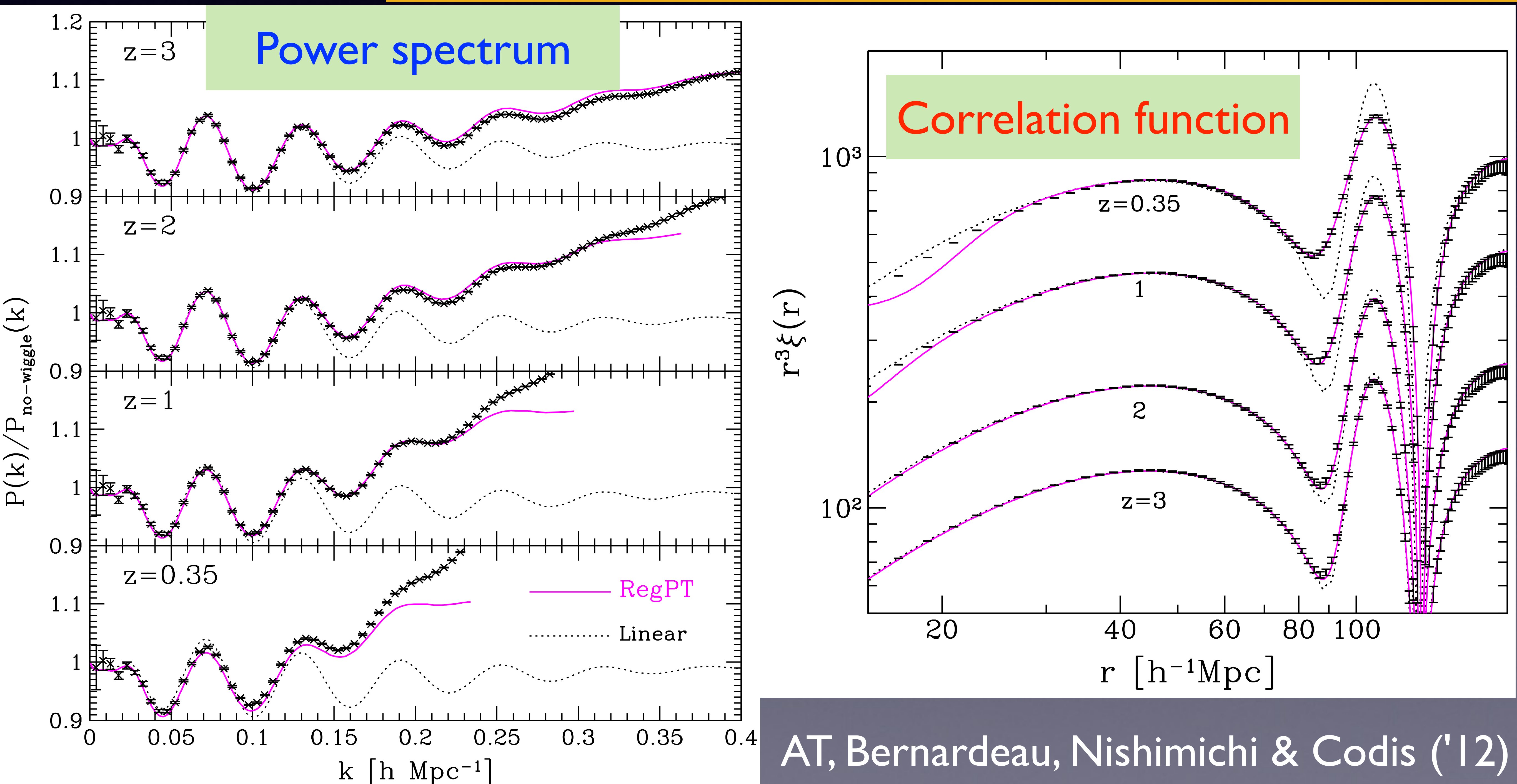
RegPT: fast PT code for P(k) & ξ(r)

few sec.

(regularized)

A public code based on multi-point propagators at *2-loop order*

http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/regpt_code.html



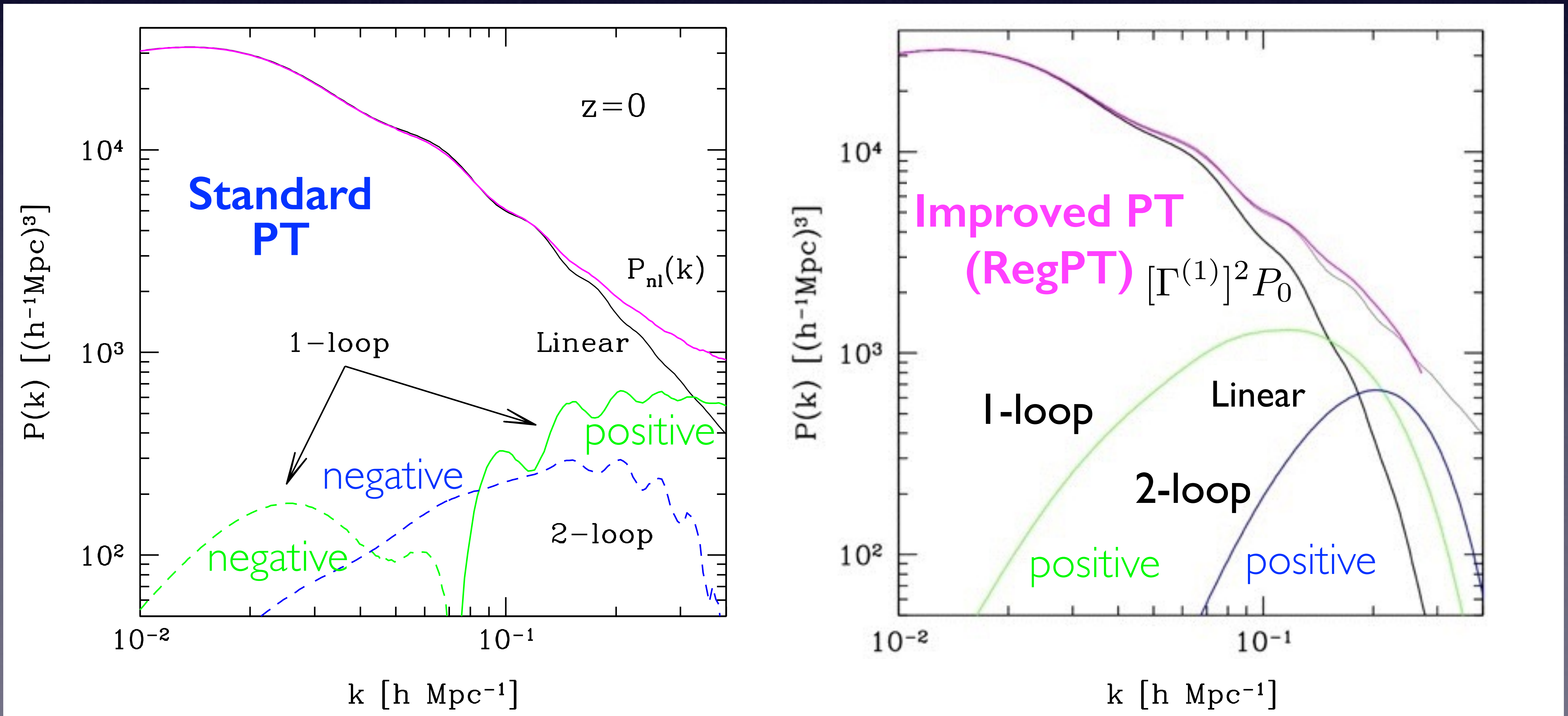
AT, Bernardeau, Nishimichi & Codis ('12)

Why improved PT works well?

AT, Bernardeau, Nishimichi, Codis ('12)
AT et al. ('09)

- All corrections become comparable at low-z.
- Positivity is not guaranteed.

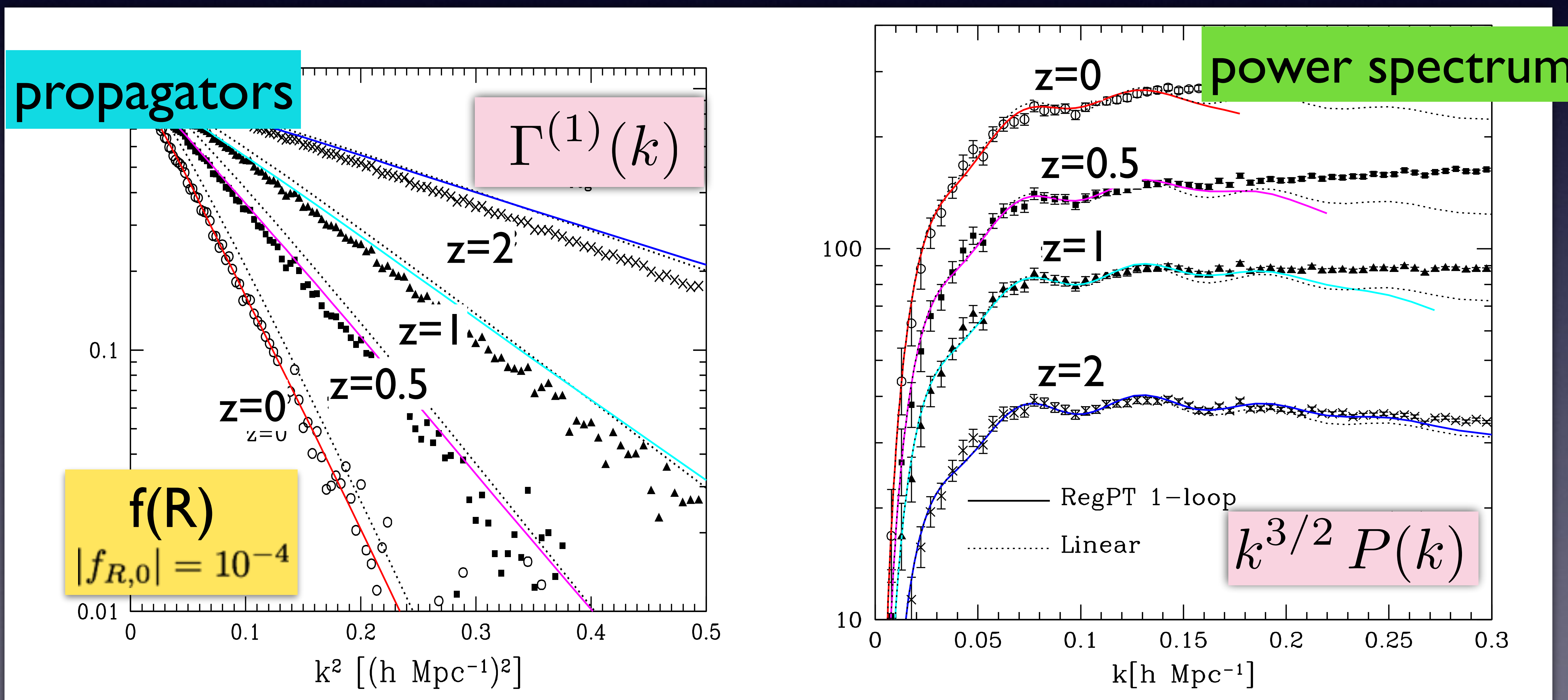
Corrections are positive & localized, shifted to higher-k for higher-loop



RegPT in modified gravity

Good convergence is ensured by
a *generic* damping behavior in propagators $\Gamma^{(n)} \xrightarrow{k\to\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm d}^2/2}$

Even in modified gravity, well-controlled expansion with RegPT



N-body data: Baojiu Li

AT, Nishimichi, Bernardeau,et al.('14)