Assignment on Special topics in Physics BXVI (35603-1361)

Intensive course on "Cosmology with Large-scale Structure" (15–17 Nov. 2016)

Due date: 22nd Dec.
Submitted to to #208 (教務) @ Faculty of Science Bldg. 1

[1] Derive Eq. (2.52). For a quantitative estimate of $k_{\rm eq}$, you may use $\Omega_{\rm r,0}h^2 = 4.155 \times 10^{-5}$:

$$k_{\rm eq} \equiv a_{\rm eq}\,H_{\rm eq} = \sqrt{\frac{2}{\Omega_{\rm r,0}H_0^2}}\,\frac{\Omega_{\rm m,0}H_0^2}{c} = 0.0095\left(\frac{\Omega_{\rm m,0}h^2}{0.13}\right)\ {\rm Mpc}^{-1}. \tag{2.52}$$

[2] Derive Eq. (2.57):

$$r_s(\eta) = \frac{2}{3\,k_{\rm eq}}\sqrt{\frac{6}{R_{\rm eq}}}\,\ln\left(\frac{\sqrt{1+R(\eta)}+\sqrt{R(\eta)+R_{\rm eq}}}{1+\sqrt{R_{\rm eq}}}\right). \tag{2.57}$$

[3] Using Eqs. (4.2) and (4.4), derive the critical value $\delta_{\rm crit}$ in Eq. (4.5) (see below):

$$\delta_{\rm crit} \equiv \delta_{\rm lin}(t_{\rm coll}) = \frac{3}{20}(12\pi)^{2/3} \simeq 1.68647, \tag{4.5}$$

where $t_{\rm coll}$ is the collapse time, $t_{\rm coll} = t(\theta = \pi/2)$. The $\delta_{\rm lin}$ is the linearized density contrast whose expression is derived by taking the limit, $\theta \ll 1$ [**Hint**: you must take the limit in both $\delta(\theta)$ and $t(\theta)$ to obtain $\delta_{\rm lin}(t)$].

[4] Using Eq. (4.47) and Eq. (4.54), derive Eq. (4.55):

$$B(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \simeq \{D_1(a)\}^4\Big\{2F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)P_0(k_1)P_0(k_2) + (\text{cyclic perm.})\Big\}. \tag{4.55}$$

[5] Consider the gravity-induced non-Gaussianity. A simple non-Gaussian indicator is the skewness defined by:

$$S_3 \equiv \frac{\langle\{\delta(\vec{x})\}^3\rangle}{\langle\{\delta(\vec{x})\}^2\rangle^2},$$

where the quanties in the numerator and denominator are related to the Fourier-space correlators through

$$\langle\{\delta(\vec{x})\}^n\rangle \;=\; \int\frac{d^3\boldsymbol{k}_1\cdots d^3\boldsymbol{k}_n}{(2\pi)^{3n}}\,\langle\delta(\boldsymbol{k}_1)\cdots\delta(\boldsymbol{k}_n)\rangle\,e^{i(\boldsymbol{k}_1+\cdots+\boldsymbol{k}_n)\cdot\vec{x}}.$$

Show that the leading-order calculation based on the perturbation theory up to the second order gives

$$S_3 \simeq \frac{34}{7}.$$

Hint : To compute the denominator of S_3, you may use the linear theory result, $\delta \simeq \delta_1 = D_1(a)\,\delta_0(\boldsymbol{k})$ to obtain

$$\langle\{\delta(\vec{x})\}^2\rangle \simeq D_1(a)^2\int\frac{dk\,k^2}{2\pi^2}\,P_0(k).$$

For the numerator, the leading-order expression of the bispectrum has to be used [see Eq. (4.55)]. With the explicit functional form of the kernel F_2 [Eq. (4.37)], the numerator is then reduced to

$$\langle\{\delta(\vec{x})\}^3\rangle \propto \left[D_1(a)^2\int\frac{dk\,k^2}{2\pi^2}\,P_0(k)\right]^2.$$

Note–. All the equation numbers indicated above are those presented in the lecture note. The lecture note is downloaded from `http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/lecture.html`.