Analytic approaches to nonlinear structure formation

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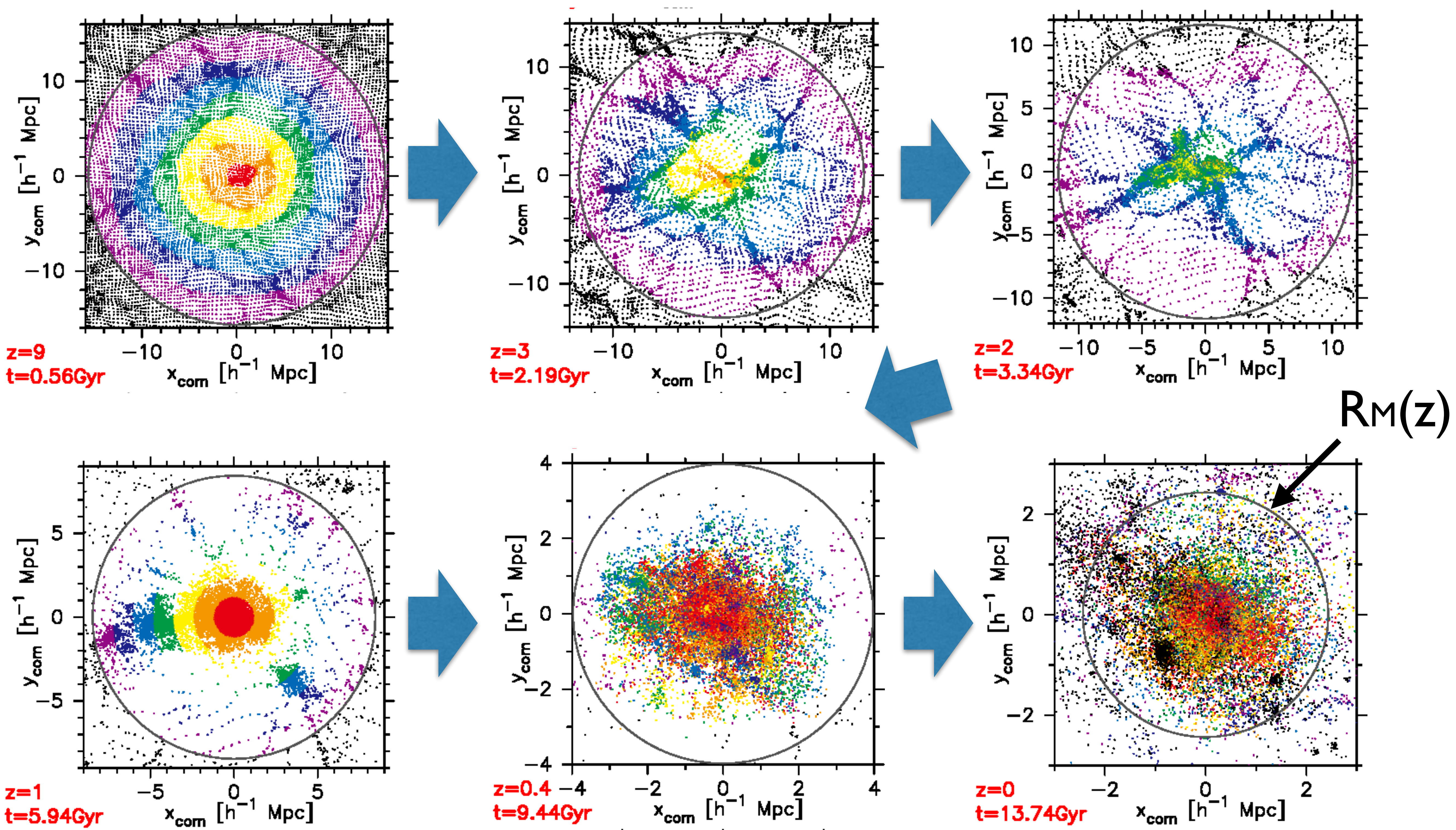
- Spherical collapse model
- Zel’dovich approximation
- Perturbation theory
- Halo model
- Galaxy/halo bias

Spherical collapse model

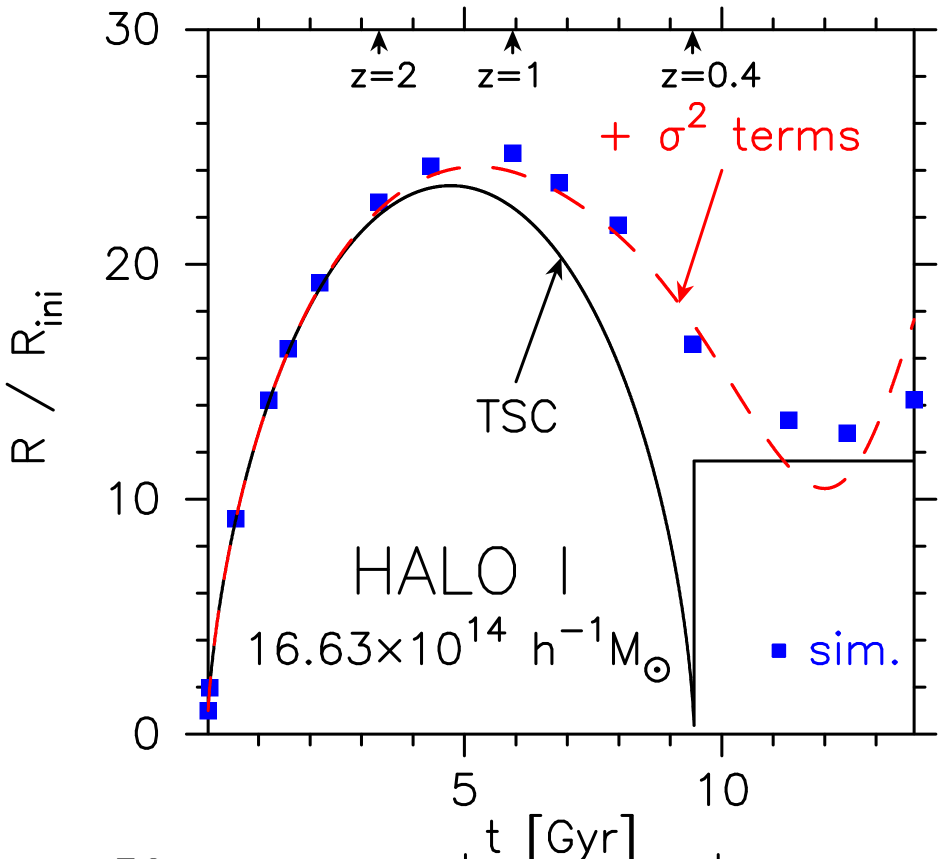
Halo formation

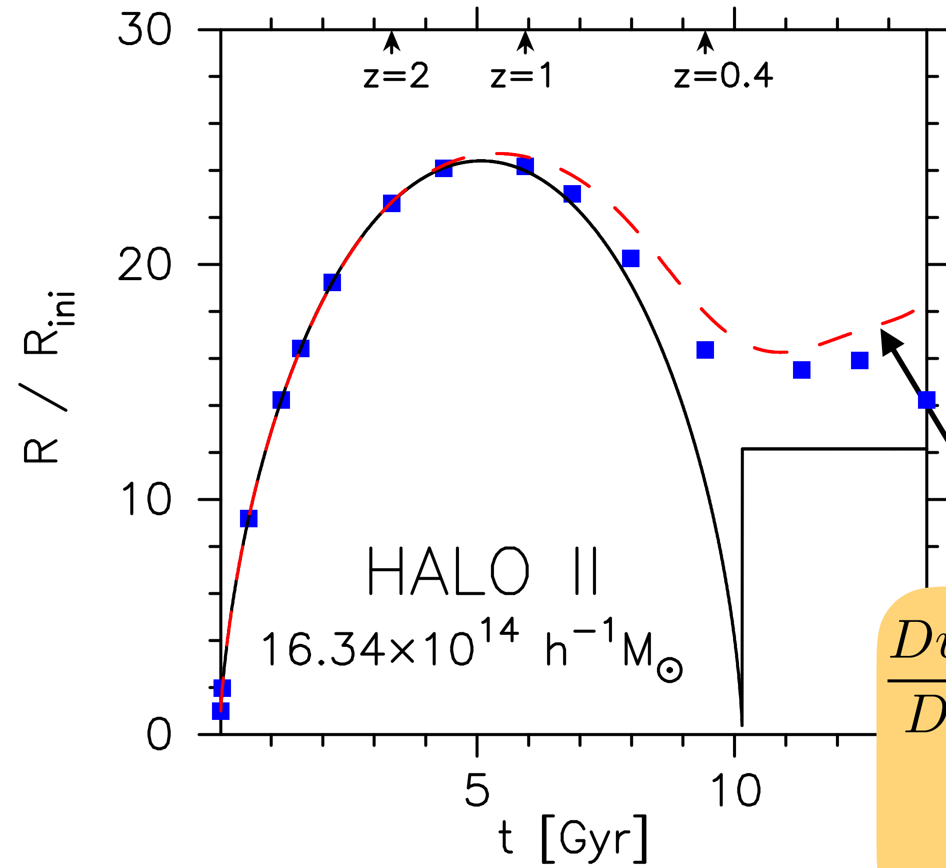
Halo 1

Suto, Kitayama, Osato, Sasaki & Suto (16)



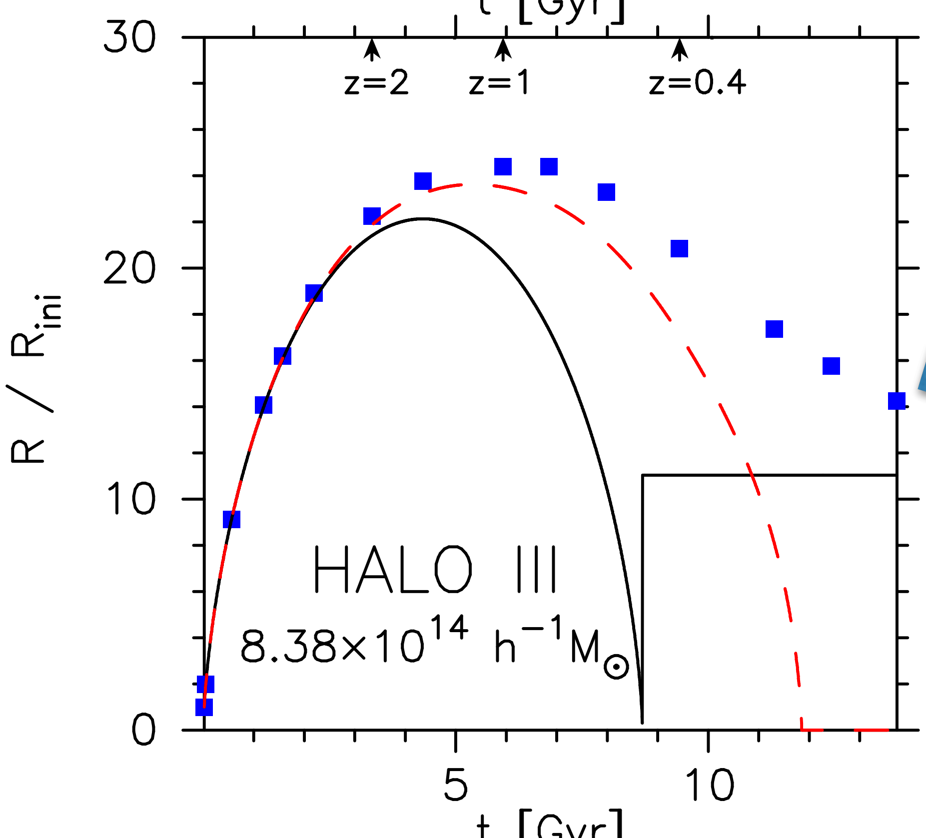
Comparison with SCM

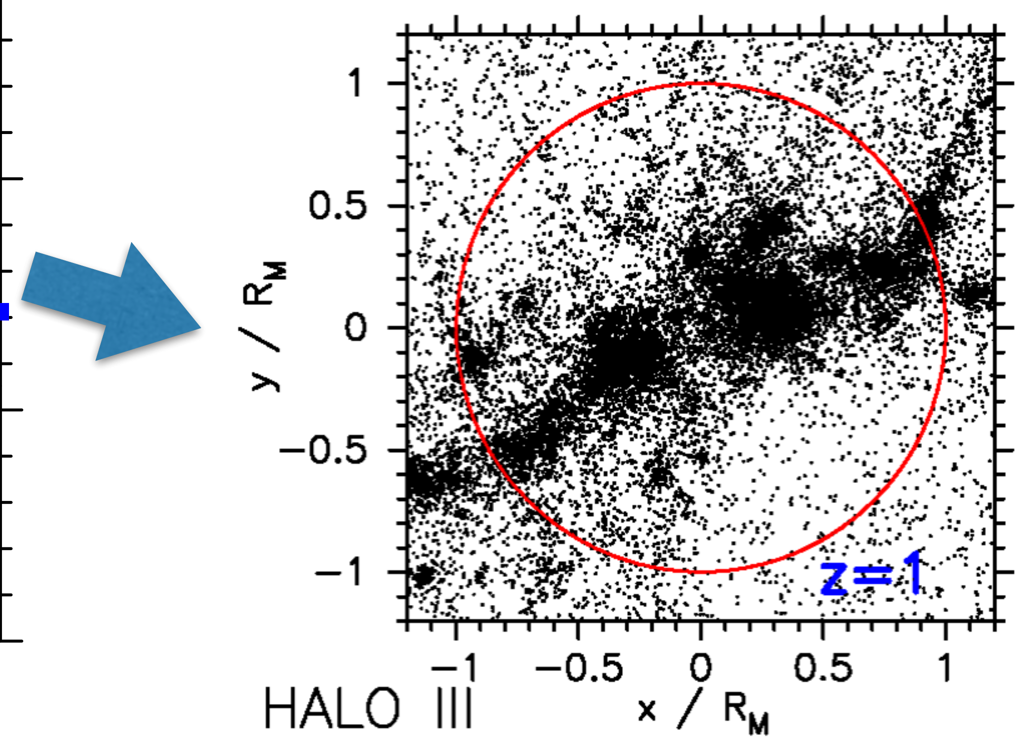




Evolution of $R_M(z)$

$$\frac{Dv_r}{Dt} = -\frac{1}{\rho}\frac{\partial(\rho\sigma_r^2)}{\partial r} - \frac{2\sigma_r^2 - \sigma_{\tan}^2}{r} - \frac{GM}{r^2} \quad \text{(Jeans eq.)}$$



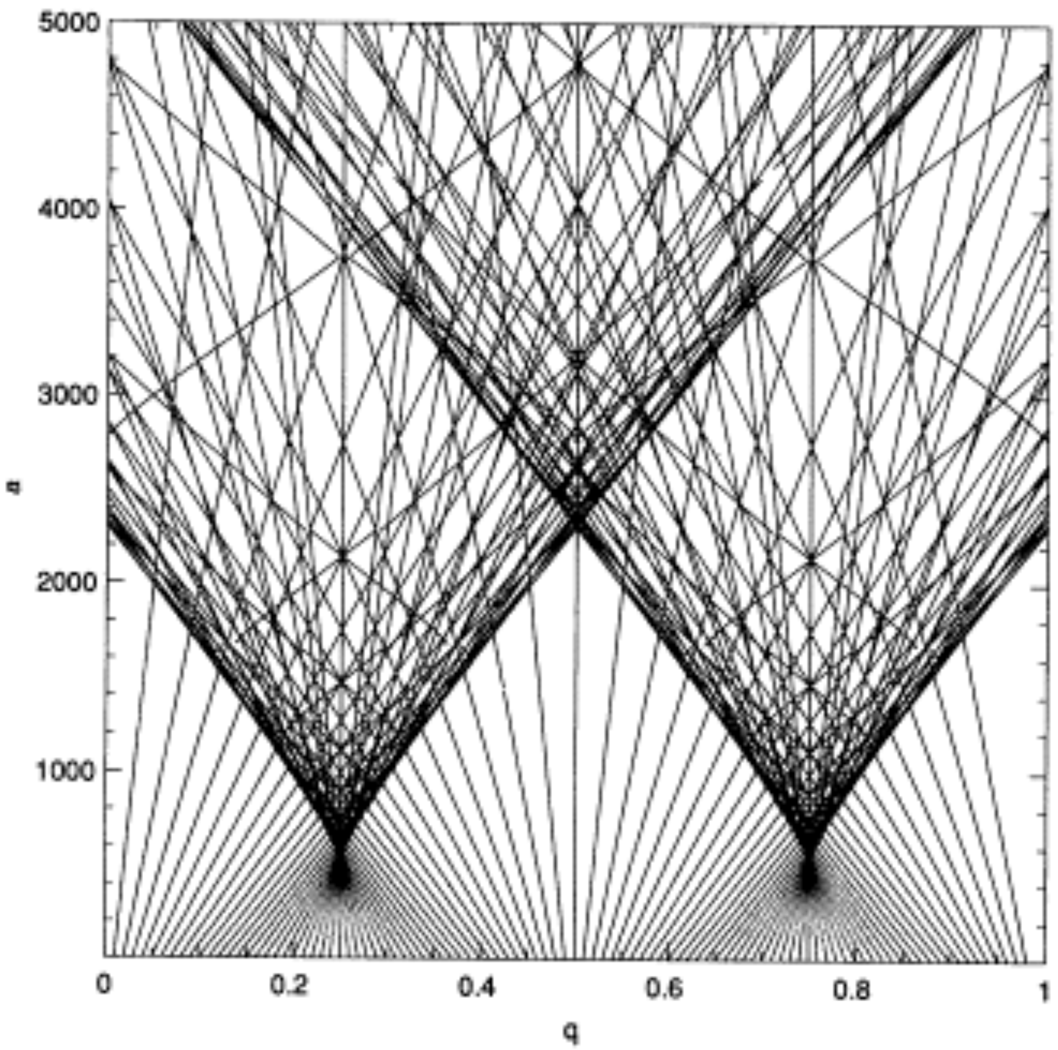


Suto, Kitayama, Osato, Sasaki & Suto (16)

Zel’dovich approximation

Particle trajectories in ZA

ZA



2LPT

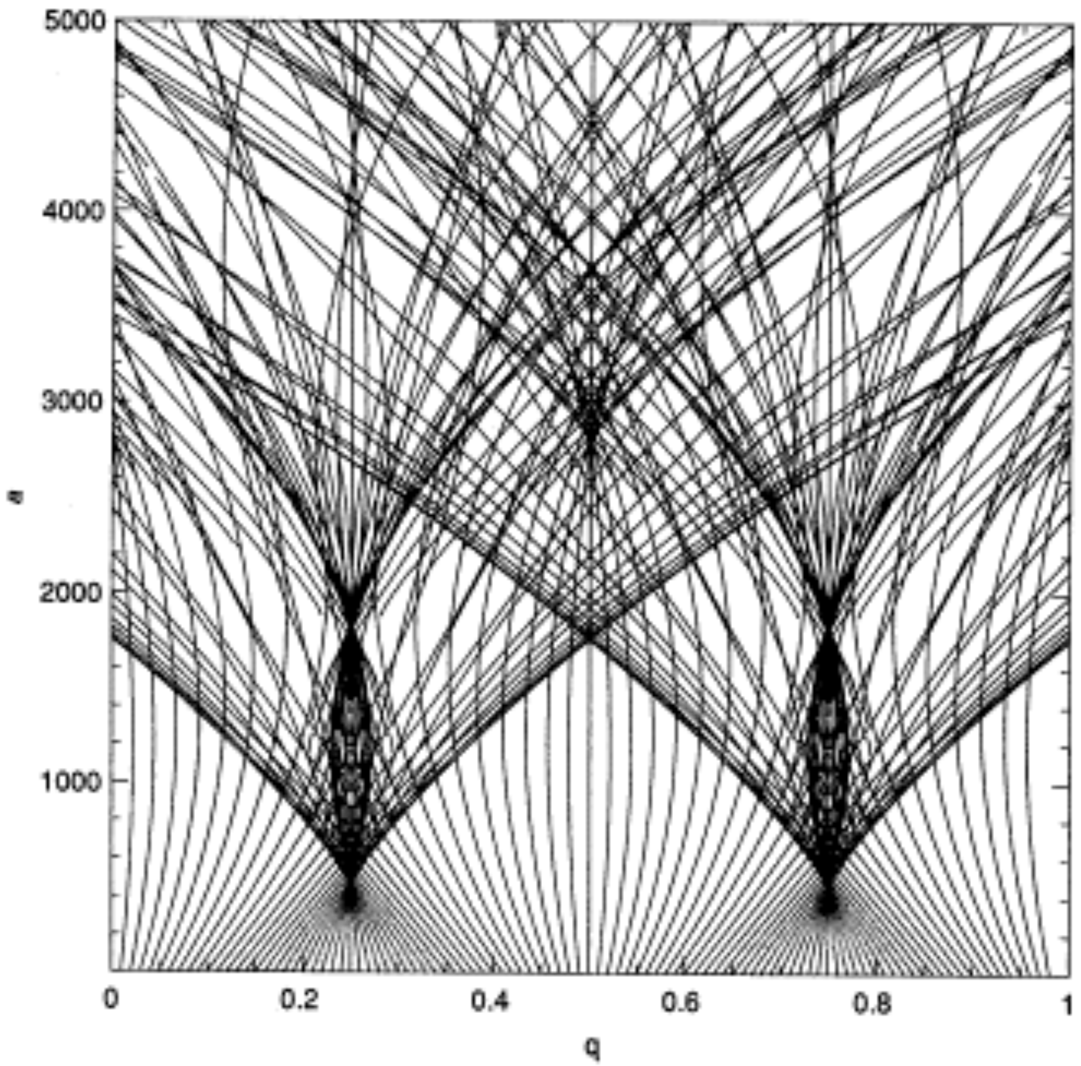
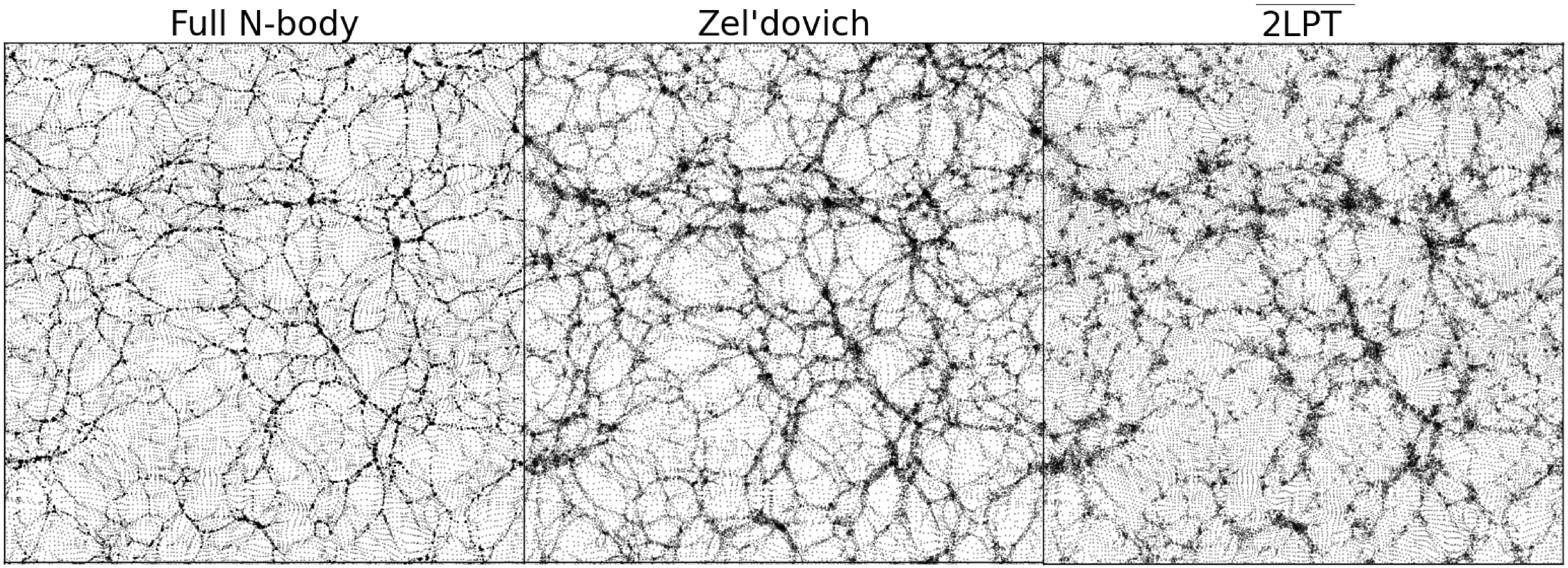


Figure 3. A family of trajectories corresponding to the model presented in Fig. 1 is shown for the first-order (upper panel) and second-order (lower panel) approximations. The trajectories end in the Eulerian space–time section ($y=0.5$, t) centred at a cluster. These plots illustrate that the three-stream system that develops after the first shell-crossing performs a self-oscillation due to the action of self-gravity.

Buchert & Ehlers ('93)



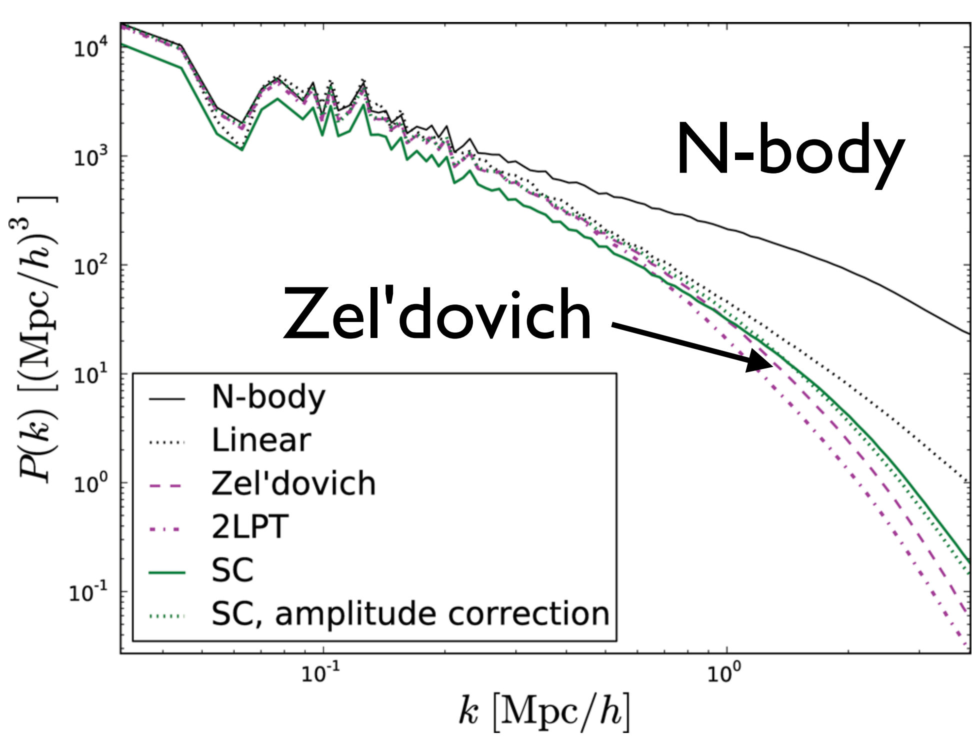
N_particle = 256^3

L=200Mpc/h

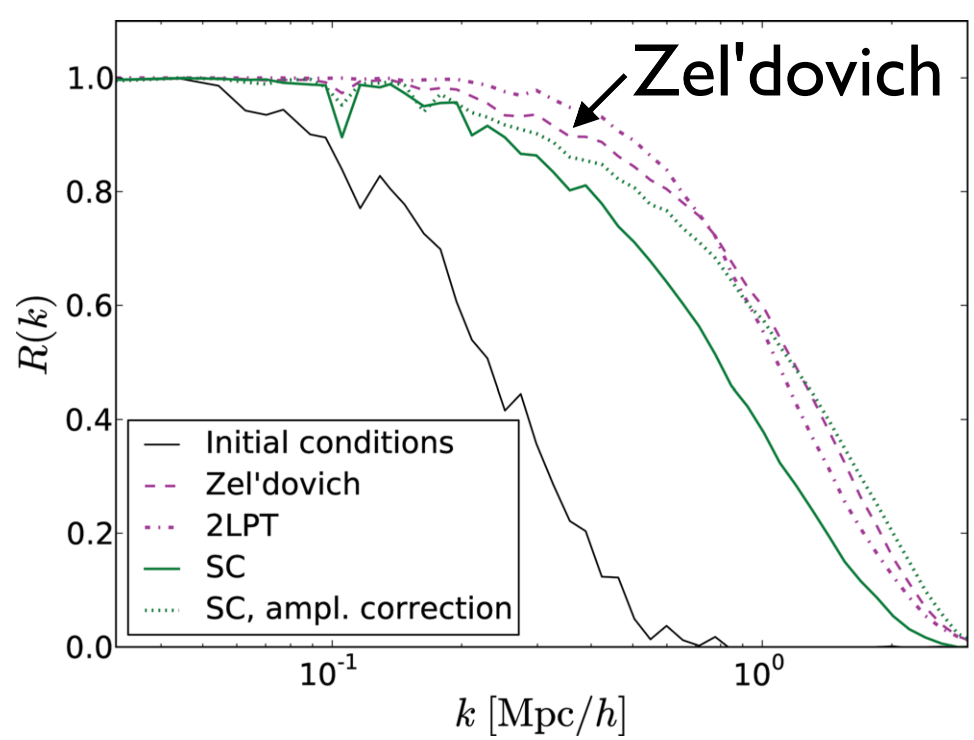
ΛCDM

Neyrink ('13)

power spectrum



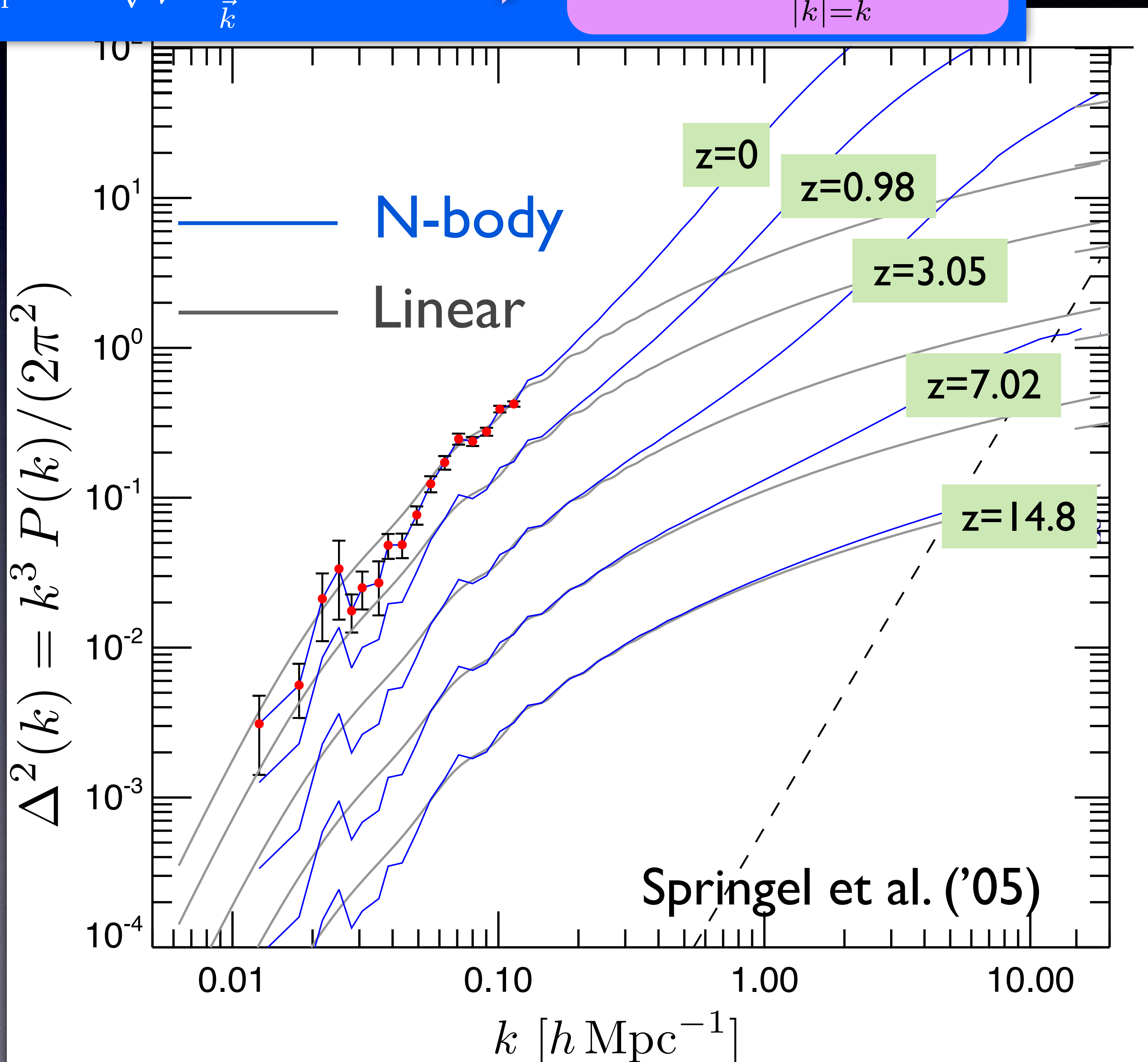
cross correlation coeff.

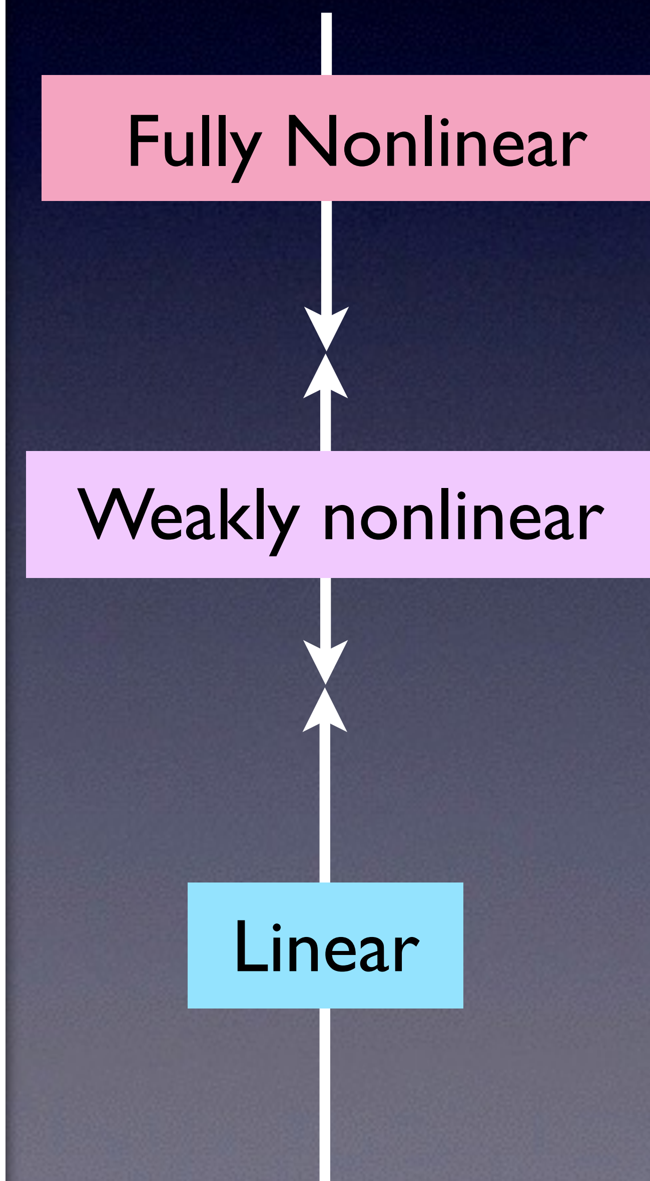


Perturbation theory

Nonlinear gravitational evolution

$$\delta(\vec{x}) \equiv \frac{\delta\rho_{\rm m}(\vec{x})}{\bar{\rho}_{\rm m}} = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \delta(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} \Rightarrow P(k) = \frac{1}{N_k} \sum_{|\vec{k}|=k} |\delta(\vec{k})|^2$$

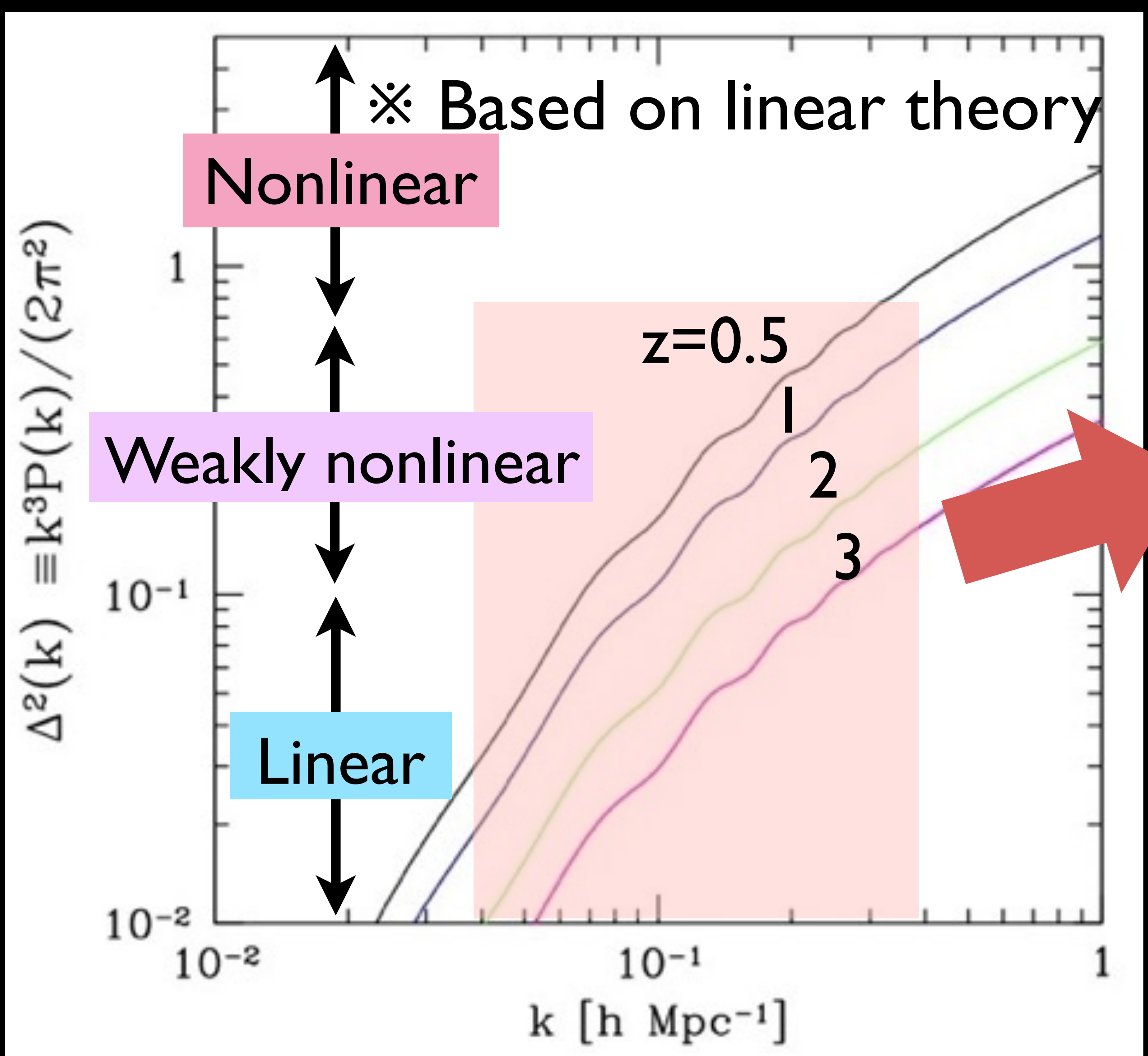


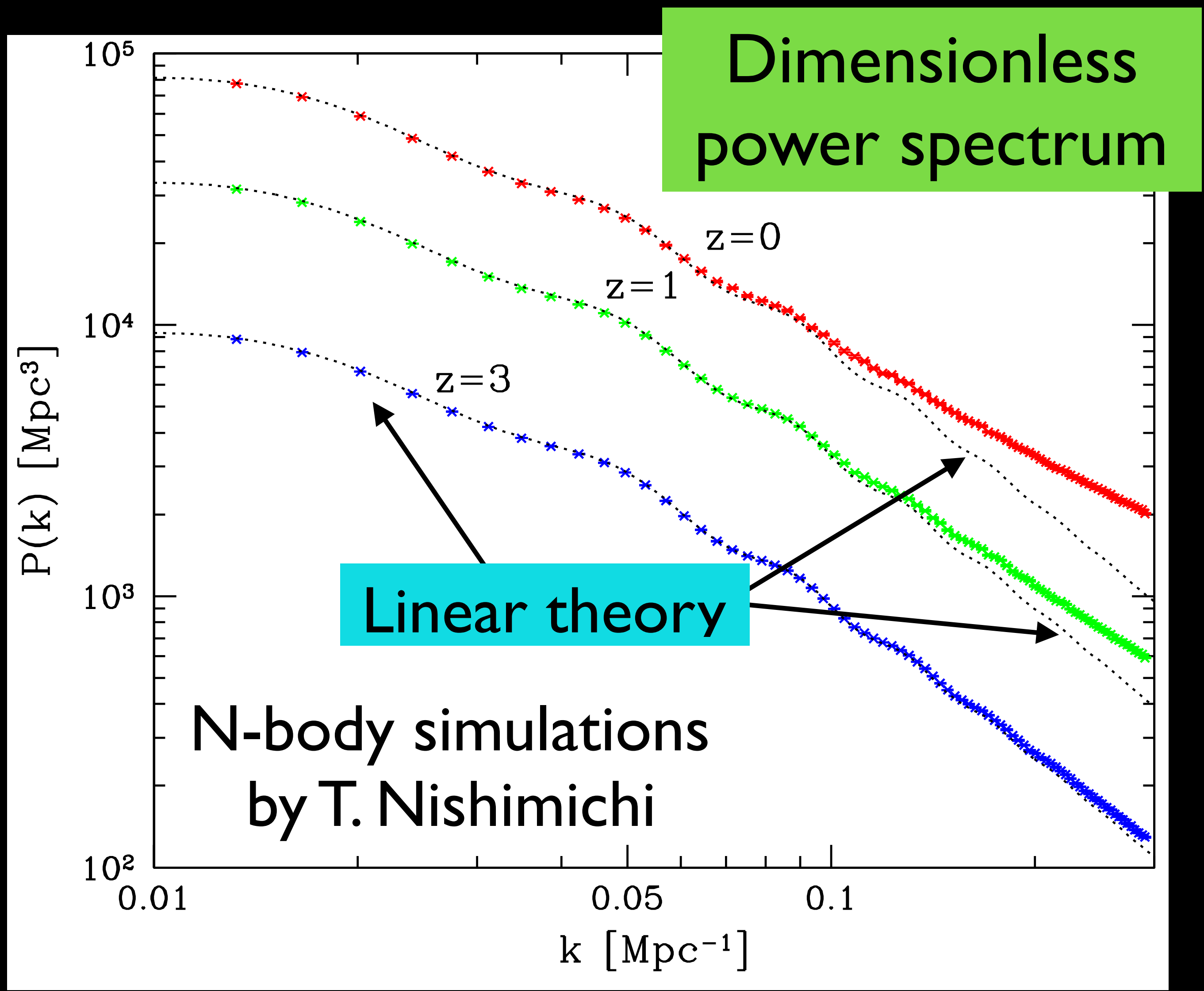


Regime of our interest

Most of interesting cosmological information (BAO, RSD, signature of massive neutrinos, …) lies at k<0.2-0.3 h/Mpc

→ Weakly nonlinear regime





Range of applicability

	Methods (Gravitational evolution)	Other systematics
Fully nonlinear ($\Delta^2 > 1$)	**N-body simulation** most powerful, but extensive & time-consuming **(c.f. fitting formula)**	*very difficult* **Baryon physics** (weak lensing)
weakly nonlinear ($\Delta^2 \lesssim 1$)	**Perturbation theory** limited range of application, but analytical & very fast	• **Galaxy bias** • **Redshift-space distortion** (galaxy surveys)
linear ($\Delta^2 \ll 1$)	**Linear theory** (CMB Boltzmann code)	*relatively easy*

Perturbation theory (PT)

Theory of large-scale structure based on gravitational instability

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Jain & Bertschinger ('94), ...

Cold dark matter + baryons = pressureless & irrotational fluid

Basic eqs.

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\,\bar{\rho}_m\,\delta$$

Single-stream approx. of collisionless Boltzmann eq.

standard PT
$|\delta| \ll 1$

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots \qquad \langle\delta(\boldsymbol{k};t)\delta(\boldsymbol{k}';t)\rangle = (2\pi)^3\,\delta_D(\boldsymbol{k}+\boldsymbol{k}')\,P(|\boldsymbol{k}|;t)$$

Equations of motion

$$\partial_\tau \delta + \partial_i \left[(1+\delta) v^i\right] \; = \; 0 \, ,$$

$$\partial_\tau v^i + \mathcal{H}\, v_l^i + \partial^i \phi + v_l^j \, \partial_j v^i \; = 0$$

$$\triangle \, \phi \; = \; \frac{3}{2} \mathcal{H}^2 \Omega_m \, \delta \; .$$

τ: conformal time

(adτ = dt)

$$\int_{\boldsymbol{q}} \; \equiv \; \int \frac{d^3 \boldsymbol{q}}{(2\pi)^3}$$

Fourier expansion

$$\partial_\tau \delta(\boldsymbol{k},\tau) + \theta(\boldsymbol{k},\tau) \; = \; - \int_{\boldsymbol{q}} \alpha(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q})\, \theta(\boldsymbol{q},\tau) \delta(\boldsymbol{k}-\boldsymbol{q},\tau) \; ,$$

$$\partial_\tau \theta(\boldsymbol{k},\tau) + \mathcal{H}\, \theta(\boldsymbol{k},\tau) + \frac{3}{2} \Omega_m \mathcal{H}^2 \delta(\boldsymbol{k},\tau) \; = \; - \int_{\boldsymbol{q}} \beta(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q})\, \theta(\boldsymbol{q},\tau) \theta(\boldsymbol{k}-\boldsymbol{q},\tau)$$

$$\theta \equiv \nabla \cdot \boldsymbol{v}$$

$$\alpha(\boldsymbol{q}_1, \boldsymbol{q}_2) \; \equiv \; \frac{\boldsymbol{q}_1 \cdot (\boldsymbol{q}_1 + \boldsymbol{q}_2)}{q_1^2} \, , \qquad \beta(\boldsymbol{q}_1, \boldsymbol{q}_2) \; \equiv \; \frac{1}{2} (\boldsymbol{q}_1 + \boldsymbol{q}_2)^2 \, \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1^2 \, q_2^2} \; .$$

Standard perturbation theory

$$\delta(\boldsymbol{k},a) = \sum_{i=1}^{\infty} \delta_{(i)}(\boldsymbol{k},a) , \qquad \theta(\boldsymbol{k},a) = -\mathcal{H}\, f(a) \sum_{i=1}^{\infty} \theta_{(i)}(\boldsymbol{k},a)$$

$$f(a) \equiv d\ln D_1/d\ln a$$

$D_1(a)$: Linear growth factor

Adopting the E-dS approximation,

$$\delta_{(n)}(\boldsymbol{k},a) = \underline{D_1^n(a)}\, \delta_n(\boldsymbol{k}) , \qquad \theta_{(n)}(\boldsymbol{k},a) = \underline{D_1^n(a)}\, \theta_n(\boldsymbol{k}) .$$

$$\delta_n(\boldsymbol{k}) = \int_{\boldsymbol{q}_1} \ldots \int_{\boldsymbol{q}_n} (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}-\boldsymbol{q}_1 \ldots - \boldsymbol{q}_n)\, F_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)\, \delta_0(\boldsymbol{q}_1)\ldots\delta_0(\boldsymbol{q}_n) ,$$

$$\theta_n(\boldsymbol{k}) = \int_{\boldsymbol{q}_1} \ldots \int_{\boldsymbol{q}_n} (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}-\boldsymbol{q}_1 \ldots - \boldsymbol{q}_n)\, G_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)\, \delta_0(\boldsymbol{q}_1)\ldots\delta_0(\boldsymbol{q}_n) ,$$

Linear density field (Gaussian)

standard PT kernel $(F_1 = G_1 = 1)$

Recursion relation for PT kernels

$$\mathcal{F}_a^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) \equiv \left[\begin{array}{c} F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) \\ G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) \end{array}\right]$$

$$\mathcal{F}_a^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) = \sum_{m=1}^{n-1} \sigma_{ab}^{(n)}\, \gamma_{bcd}(\boldsymbol{q}_1,\boldsymbol{q}_2)\, \mathcal{F}_c^{(m)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_m)\, \mathcal{F}_d^{(n-m)}(\boldsymbol{k}_{m+1},\cdots,\boldsymbol{k}_n)$$

$$\boldsymbol{q}_1 = \boldsymbol{k}_1 + \cdots + \boldsymbol{k}_m$$
$$\boldsymbol{q}_2 = \boldsymbol{k}_{m+1} + \cdots + \boldsymbol{k}_n$$

$$\sigma_{ab}^{(n)} = \frac{1}{(2n+3)(n-1)} \left(\begin{array}{cc} 2n+1 & 2 \\ 3 & 2n \end{array}\right)$$

$$\gamma_{abc}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \begin{cases} \frac{1}{2}\left\{1+\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_1}{|\boldsymbol{k}_2|^2}\right\}; & (a,b,c)=(1,1,2) \\ \frac{1}{2}\left\{1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{|\boldsymbol{k}_1|^2}\right\}; & (a,b,c)=(1,2,1) \\ \frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)|\boldsymbol{k}_1+\boldsymbol{k}_2|^2}{2|\boldsymbol{k}_1|^2|\boldsymbol{k}_2|^2}; & (a,b,c)=(2,2,2) \\ 0; & \text{otherwise.} \end{cases}$$

Note—. repetition of the same subscripts (a,b,c) indicates the sum over all multiplet components

PT kernels constructed from recursion relation should be symmetrized

Power spectrum

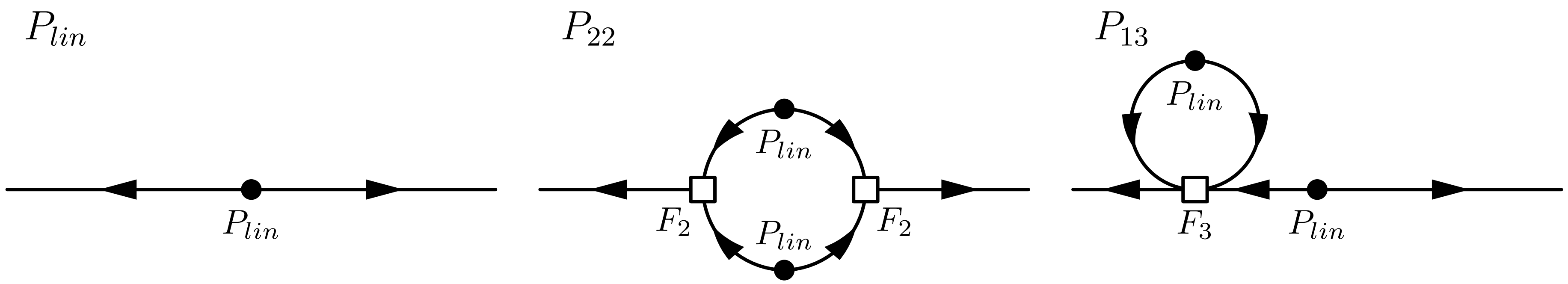
$$\langle \delta(\boldsymbol{k}_1, a)\delta(\boldsymbol{k}_2, a)\rangle \equiv (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)\, P(k_1, a)$$

linear 1-loop

$$P_{SPT}(k) = \underline{P_{lin}(k)} + \underline{P_{22}(k) + P_{13}(k)} + \text{ higher order loops} .$$

$$P_{22}(k) = 2\int_{\boldsymbol{q}} P_{lin}(q) P_{lin}(|\boldsymbol{k}-\boldsymbol{q}|)\, F_2^2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) ,$$

$$P_{13}(k) = 6 P_{lin}(k) \int_{\boldsymbol{q}} P_{lin}(q)\, F_3(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}) ,$$



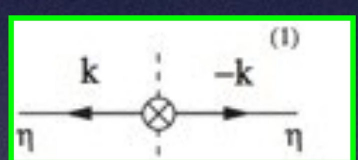
Next-to-next-to leading order

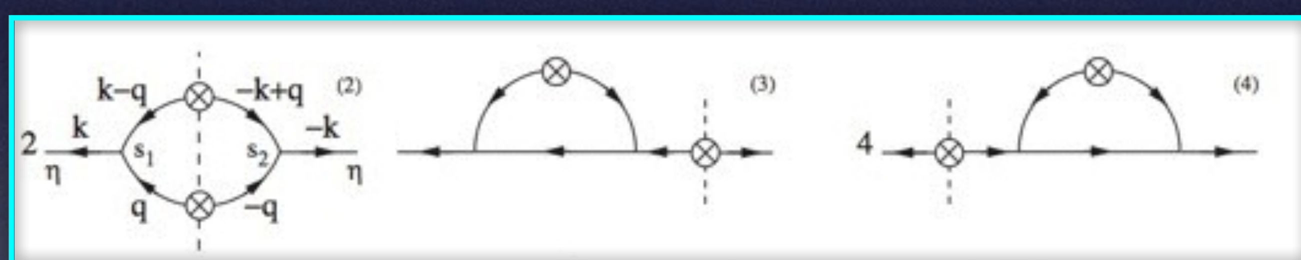
up to 2-loop order

$$P^{(mn)} \simeq \langle \delta^{(m)} \delta^{(n)} \rangle$$

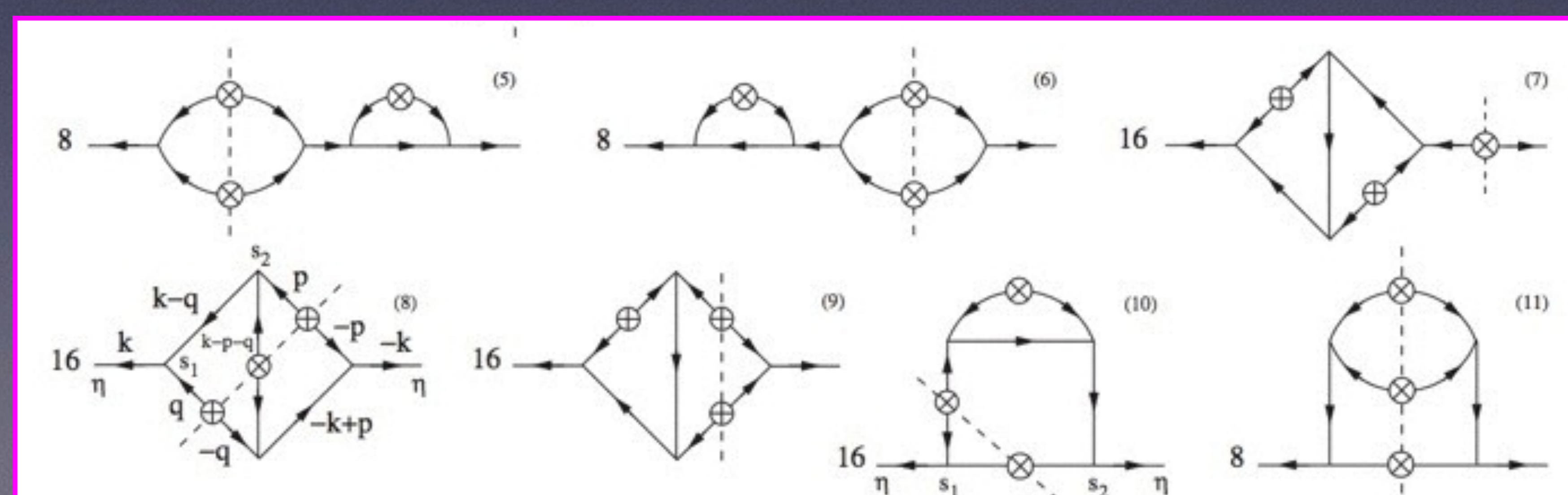
$$P(k) = P^{(11)}(k) + \left(P^{(22)}(k) + P^{(13)}(k)\right) + \left(P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k)\right) + \cdots$$

Linear (tree) 1-loop 2-loop





Crocce & Scoccimarro ('06)



Calculation involves multi-dimensional numerical integration

(time-consuming)

Comparison with simulations

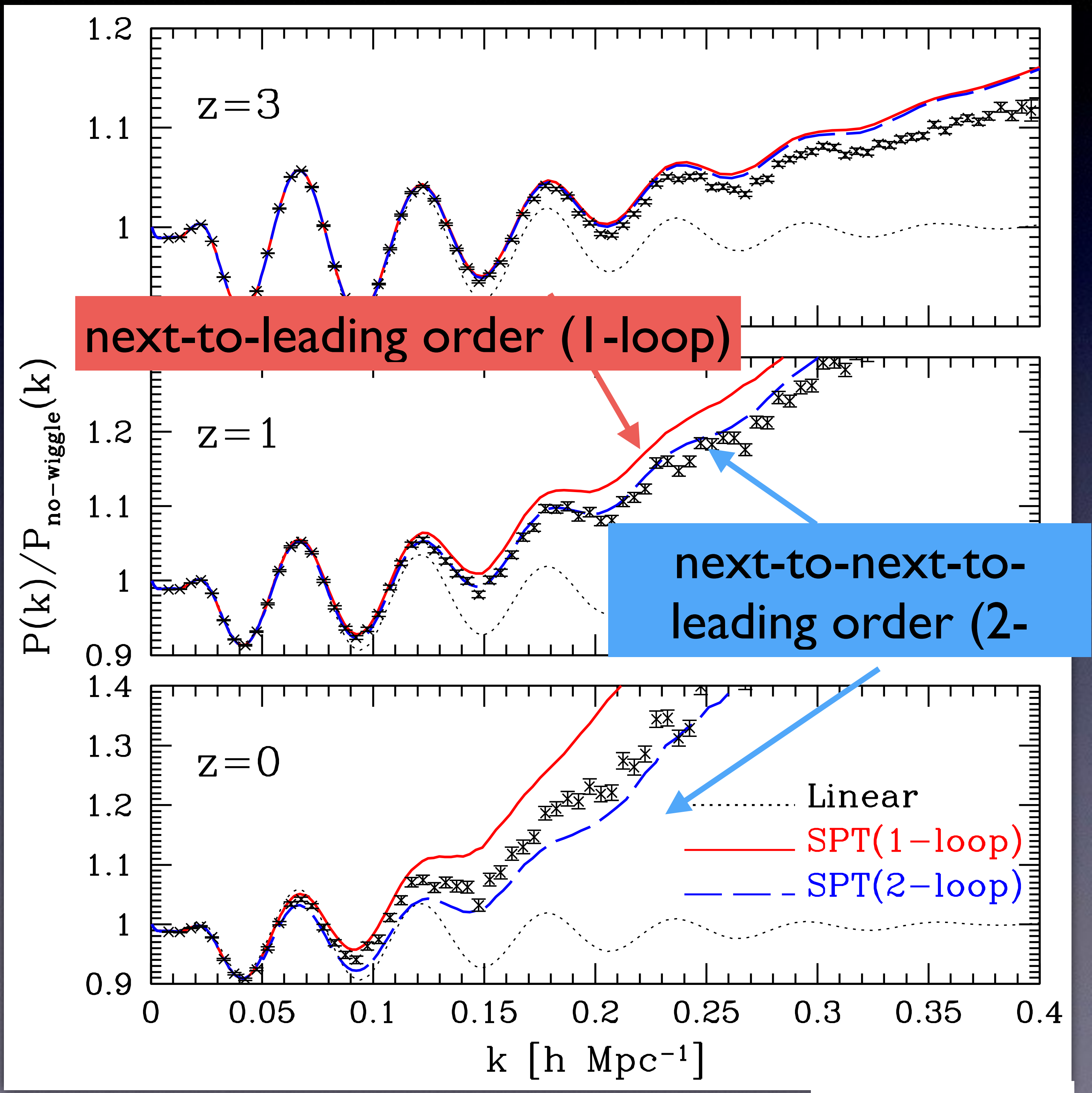
Standard PT qualitatively explains scale-dependent nonlinear growth, however,

1-loop :
overestimates simulations

2-loop :
overestimates at high-z, while it turn to underestimate at low-z

Standard PT produces ill-behaved PT expansion !!

… need to be improved



AT et al. ('09)

Improving PT predictions

Basic idea Reorganizing standard PT expansion by introducing non-perturbative statistical quantities

$\delta_0(\boldsymbol{k})$

initial density field (Gaussian)

$\delta(\boldsymbol{k};z)$

Evolved density field (non-Gaussian)

Initial power spectrum

$P_0(k)$

from linear theory

(CMB Boltzmann code)

Nonlinear mapping

Observables

$P(k;z)$

$B(k_1,k_2,k_3;z)$

$T(k_1,k_2,k_3,k_4;z)$

$\vdots$

of dark matter/galaxies/halos

Concept of '*propagator*' in physics/mathematics may be useful

Propagator in physics

- ✦ Green's function in linear differential equations
- ✦ Probability amplitude in quantum mechanics

Schrödinger Eq.

$$\left(-i\hbar\frac{\partial}{\partial t}+H_x\right)\psi(x,t)=0$$

$$G(x,t;x',t')\equiv\frac{\delta\psi(x,t)}{\delta\psi(x',t')}$$

$$\left(-i\hbar\frac{\partial}{\partial t}+H_x\right)G(x,t;x',t')=-i\hbar\,\delta_D(x-x')\delta_D(t-t')$$

$$\Rightarrow\quad \psi(x,t)=\int_{-\infty}^{+\infty}dx'G(x,t;x',t')\,\psi(x',t')\ ;\quad t>t'$$

Cosmic propagators

Propagator should carry information on
non-linear evolution & statistical properties

Crocce & Scoccimarro ('06)

Evolved (non-linear) density field

$$\left\langle \frac{\delta\,\delta_{\rm m}(\boldsymbol{k};t)}{\delta\,\delta_0(\boldsymbol{k}')} \right\rangle \equiv \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}')\,\Gamma^{(1)}(k;t)$$

Propagator

Initial density field

Ensemble w.r.t randomness of initial condition

Contain statistical information on *full-nonlinear* evolution

(Non-linear extension of Green's function)

Multi-point propagators

Bernardeau, Crocce & Scoccimarro ('08)
Matsubara ('11) ⟶ *integrated PT*

As a natural generalization,

Multi-point propagator

$$\left\langle \frac{\delta^n\,\delta_{\rm m}(\boldsymbol{k};t)}{\delta\,\delta_0(\boldsymbol{k}_1)\cdots\delta\,\delta_0(\boldsymbol{k}_n)}\right\rangle = (2\pi)^{3(1-n)}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}')\,\Gamma^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)$$

With this multi-point prop.

- Building blocks of a new perturbative theory (PT) expansion

  ---------- Γ-expansion or Wiener-Hermite expansion

- A good convergence of PT expansion is expected

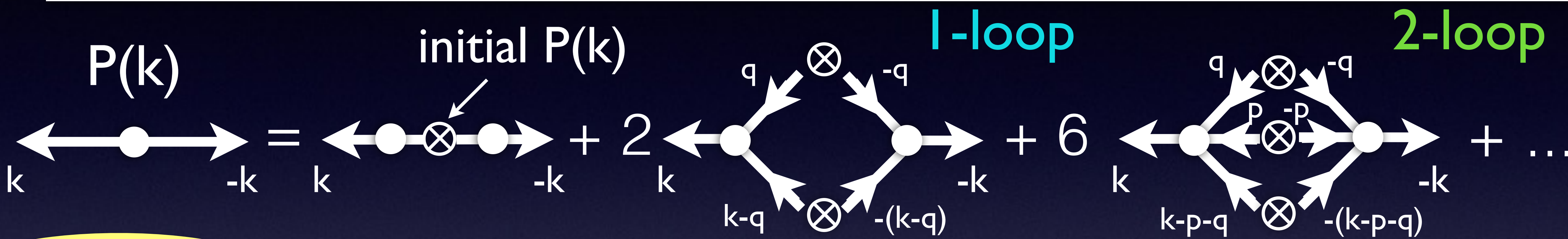
  (c.f. standard PT)

Power spectrum

Initial power spectrum

$$P(k;t) = \left[\Gamma^{(1)}(k;t)\right]^2 P_0(k) + 2\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\Gamma^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q};t)\right]^2 P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$+6\int \frac{d^6\boldsymbol{p}d^3\boldsymbol{q}}{(2\pi)^6}\left[\Gamma^{(3)}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q};t)\right]^2 P_0(p)P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|) + \cdots$$

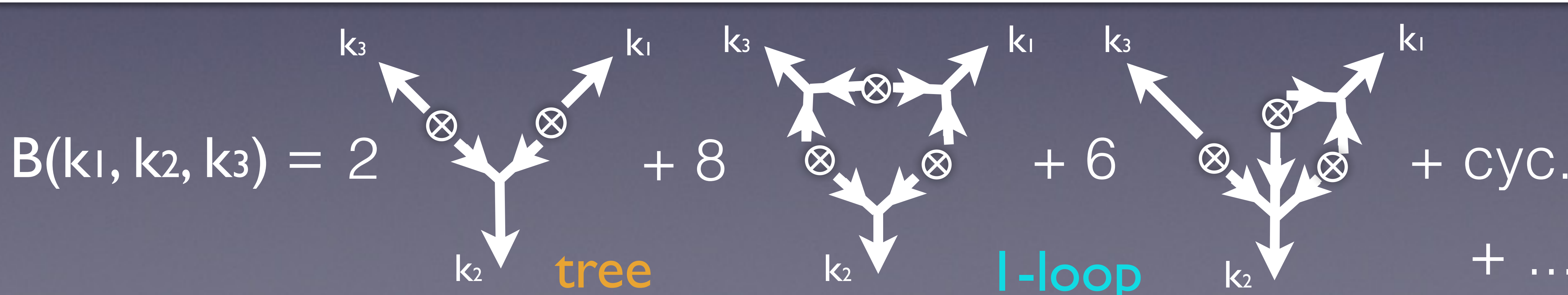


Bispectrum

$$B(k_1,k_2,k_3) = 2\,\Gamma^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\Gamma^{(1)}(k_1)\,\Gamma^{(1)}(k_2)\,P_0(k_1)P_0(k_2) + \text{cyc.}$$

$$+ \Big[8\int d^3q\,\Gamma^{(2)}(\mathbf{k}_1-\mathbf{q},\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{q}-\mathbf{k}_1,-\mathbf{k}_2-\mathbf{q})P_0(|\mathbf{k}_1-\mathbf{q}|)P_0(|\mathbf{k}_2+\mathbf{q}|)P_0(q)$$

$$+ 6\int d^3q\,\Gamma^{(3)}(-\mathbf{k}_3,-\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2-\mathbf{q},\mathbf{q})\Gamma^{(1)}(\mathbf{k}_3)P_0(|\mathbf{k}_2-\mathbf{q}|)P_0(q)P_0(k_3) + \text{cyc.}\Big].$$

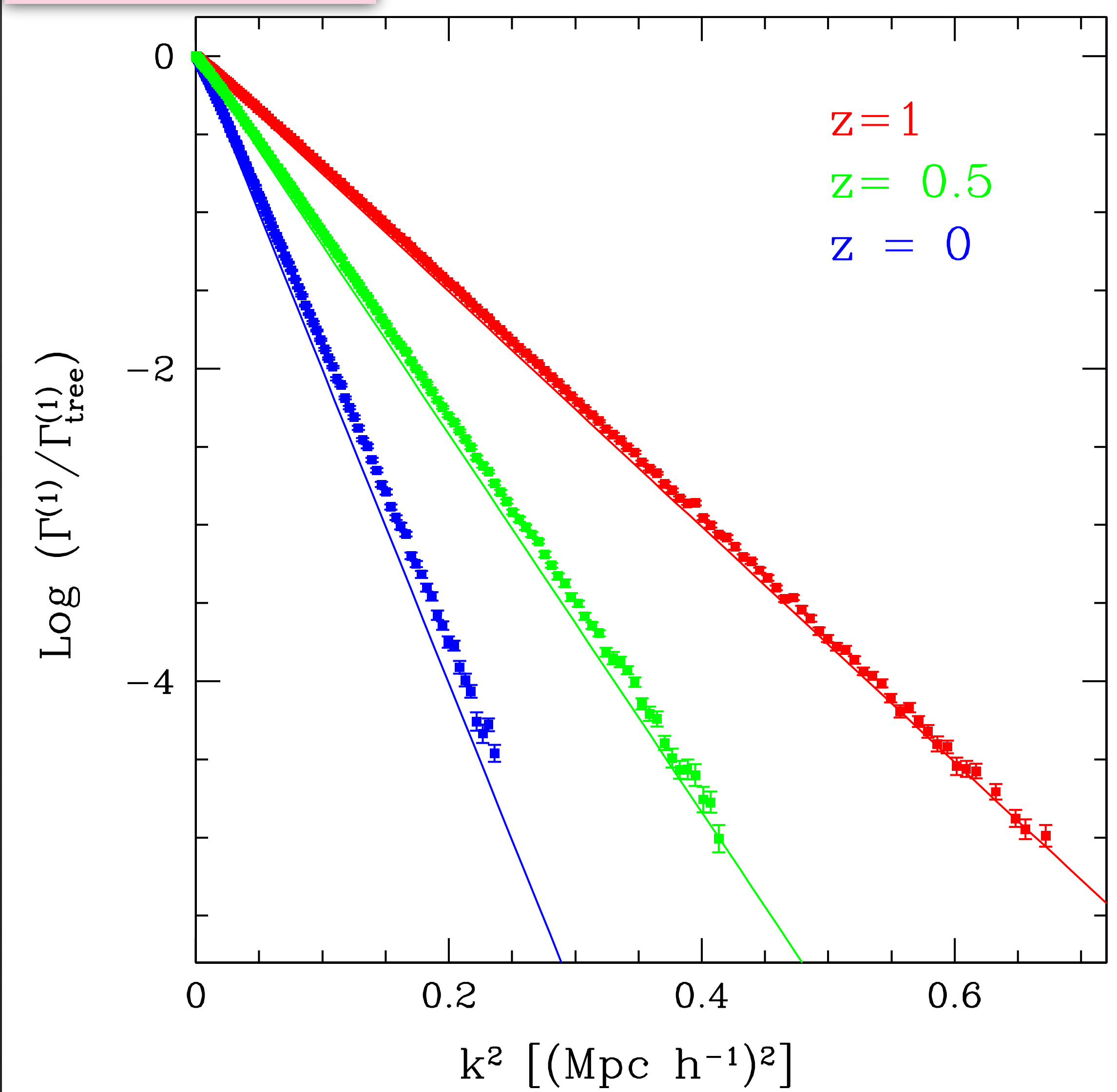


Generic property of propagators

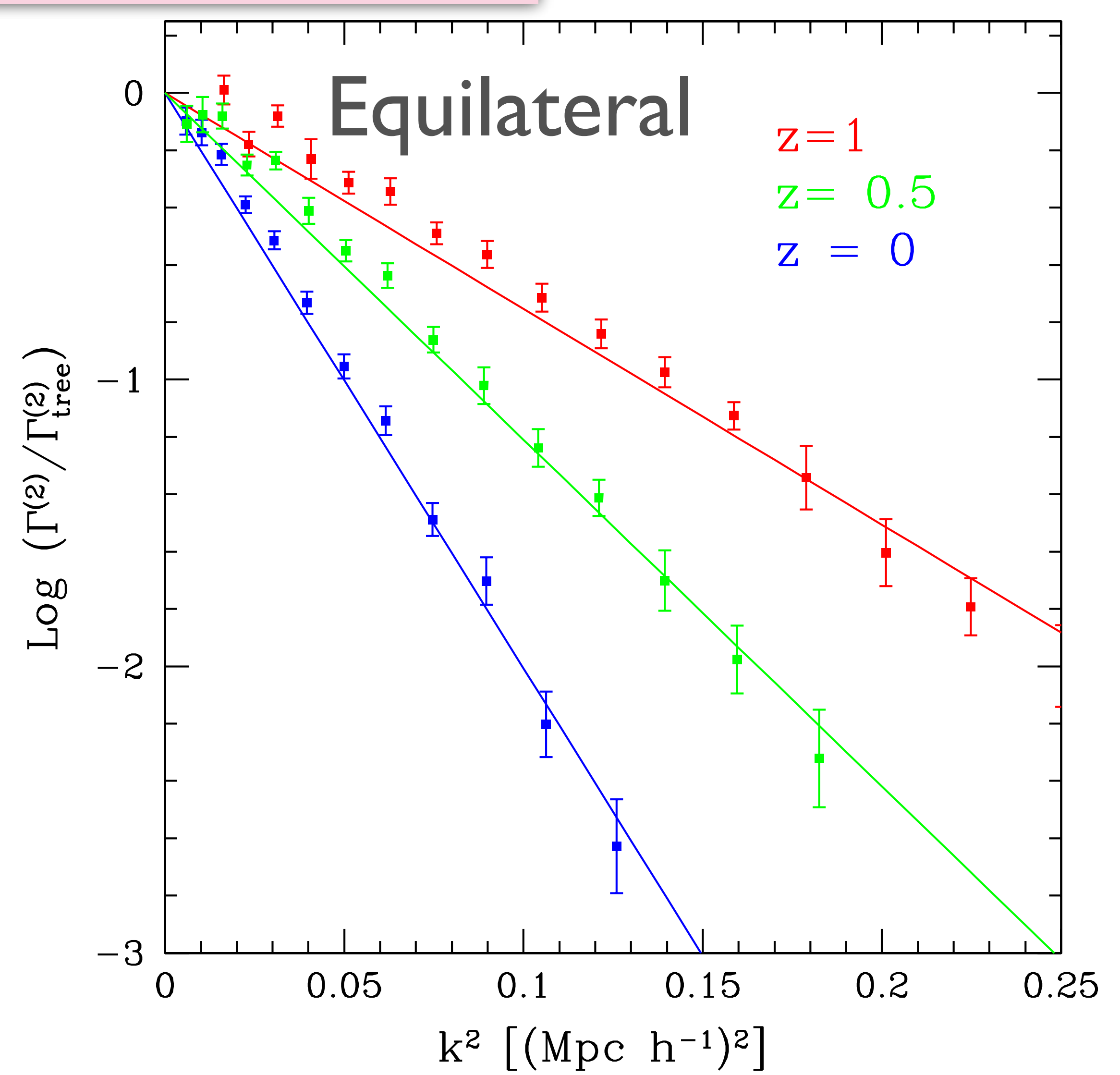
Crocce & Scoccimarro '06, Bernardeau et al. '08

$$\Gamma^{(n)} \xrightarrow{k \to +\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm v}^2/2} \quad ; \quad \sigma_{\rm v}^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)$$

$\Gamma^{(1)}(k)$



$\Gamma^{(2)}(k_1, k_2, k_3)$



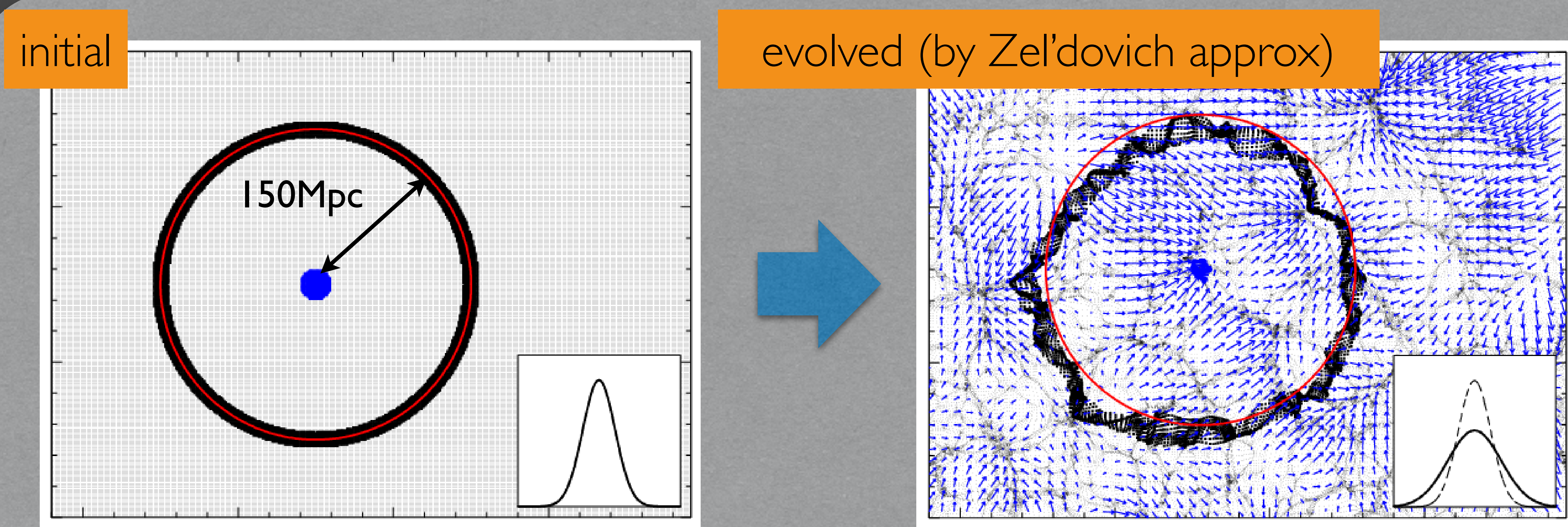
Origin of Exp. damping

For Gaussian initial condition,

$$\langle \delta_{\rm m}(\boldsymbol{k};t)\,\delta_0(\boldsymbol{k}')\rangle = \Gamma^{(1)}(k;t)\,\underline{\underline{\langle \delta_0(\boldsymbol{k})\delta_0(\boldsymbol{k}')\rangle}} \;\; /\!/ \;\; P_0(k)$$

initial power spectrum

➡ Cross correlation between initial & evolved density fields



Padmanabhan et al. ('12)

Initial structure becomes blurred by the *local* cosmic flow

----- origin of Gaussian damping in propagator

Constructing regularized propagators

- UV property (k >>1) :

$$\Gamma^{(n)} \xrightarrow{k \to +\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm v}^2/2} \quad ; \quad \sigma_{\rm v}^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)$$

Bernardeau, Crocce & Scoccimarro ('08), Bernardeau, Van de Rijt, Vernizzi ('11)

- IR behavior (k<<1) can be described by standard PT calculations :

$$\Gamma^{(n)} = \Gamma^{(n)}_{\rm tree} + \Gamma^{(n)}_{\text{1-loop}} + \Gamma^{(n)}_{\text{2-loop}} + \cdots$$

Importantly, each term behaves like $\Gamma^{(n)}_{p\text{-loop}} \xrightarrow{k \to +\infty} \frac{1}{p!}\left(-\frac{k^2\,\sigma_{\rm v}^2}{2}\right)^p \Gamma^{(n)}_{\rm tree}$

A regularization scheme that reproduces both UV & IR behaviors

Bernardeau, Crocce & Scoccimarro ('12)

Regularized propagator

Bernardeau, Crocce & Scoccimarro ('12)

A global solution that satisfies both UV (k>>1) & IR (k<<1) properties:

$$\Gamma^{(n)}_{\rm reg} = \left[\Gamma^{(n)}_{\rm tree}\left\{1+\frac{k^2\sigma_{\rm v}^2}{2}\right\}+\Gamma^{(n)}_{\rm 1\text{-}loop}\right]\exp\left\{-\frac{k^2\sigma_{\rm v}^2}{2}\right\}; \quad \sigma_{\rm v}^2=\int\frac{dq}{6\pi^2}P_{\theta\theta}(q)$$

counter term

.... *IR behavior is valid at 1-loop level*

Precision of IR behavior can be systematically improved by
including higher-loop corrections and adding counter terms

e.g., For IR behavior valid at 2-loop level,

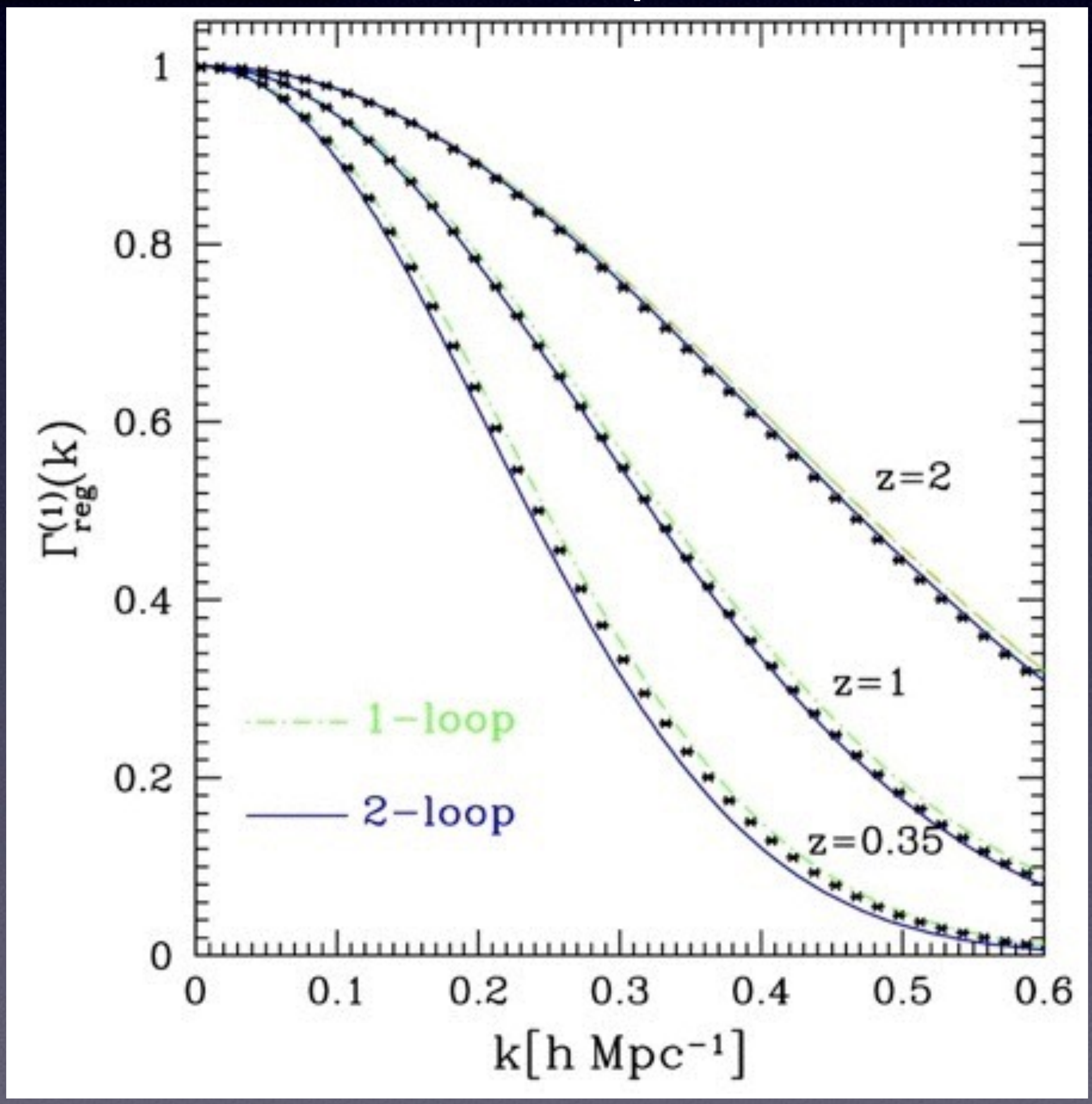
$$\Gamma^{(n)}_{\rm reg} = \left[\Gamma^{(n)}_{\rm tree}\left\{1+\frac{k^2\sigma_{\rm v}^2}{2}+\frac{1}{2}\left(\frac{k^2\sigma_{\rm v}^2}{2}\right)^2\right\}+\Gamma^{(n)}_{\rm 1\text{-}loop}\left\{1+\frac{k^2\sigma_{\rm v}^2}{2}\right\}+\Gamma^{(n)}_{\rm 2\text{-}loop}\right]\exp\left\{-\frac{k^2\sigma_{\rm v}^2}{2}\right\}$$

counter term counter term

Propagators in N-body simulations

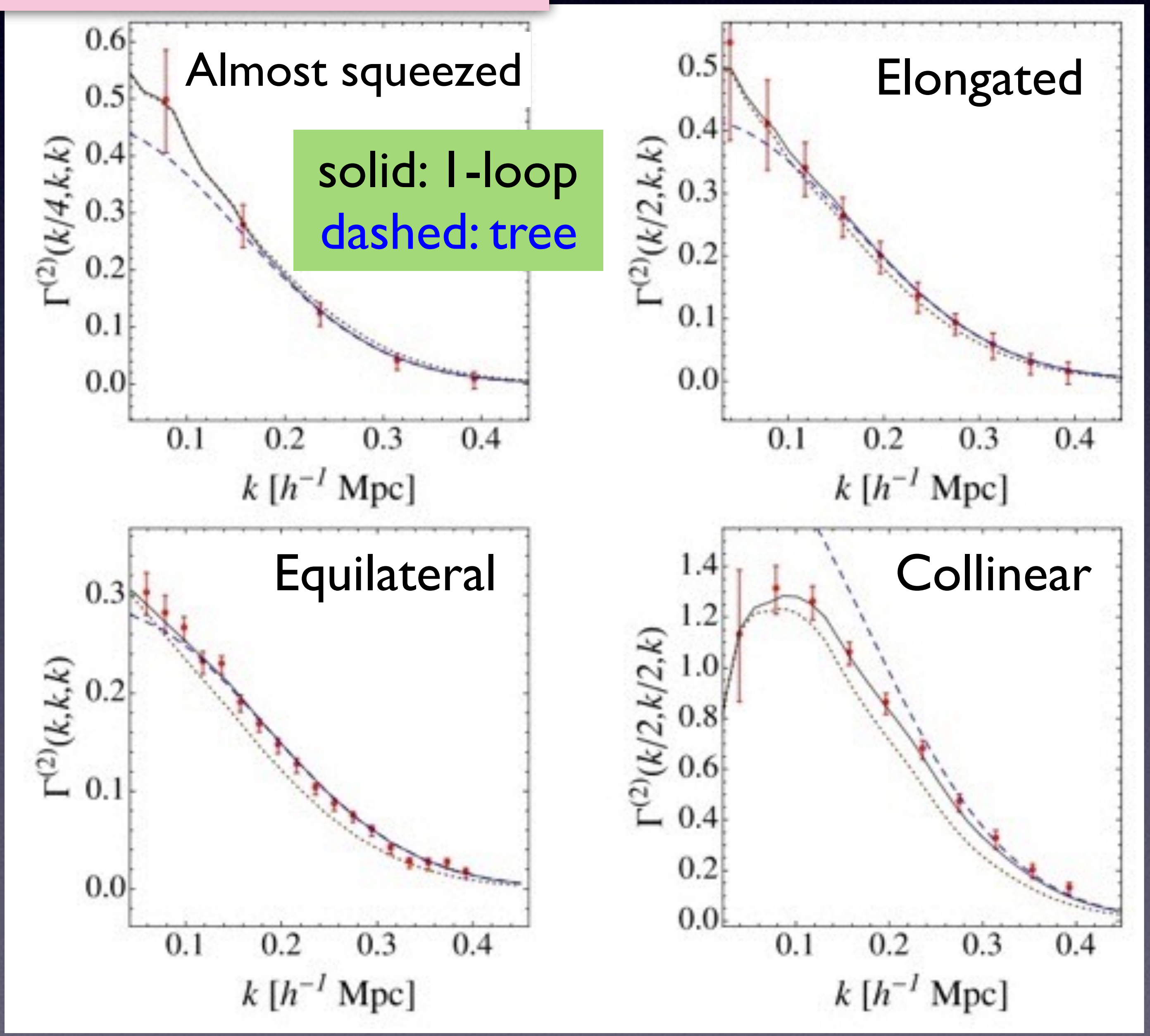
compared with '*Regularized*' propagators constructed analytically

$\Gamma^{(1)}(k)$ predictions up to 2-loop order



Bernardeau, AT & Nishimichi ('12)

$\Gamma^{(2)}(k_1, k_2, k_3)$ predictions up to 1-loop order



Bernardeau et al. ('12)

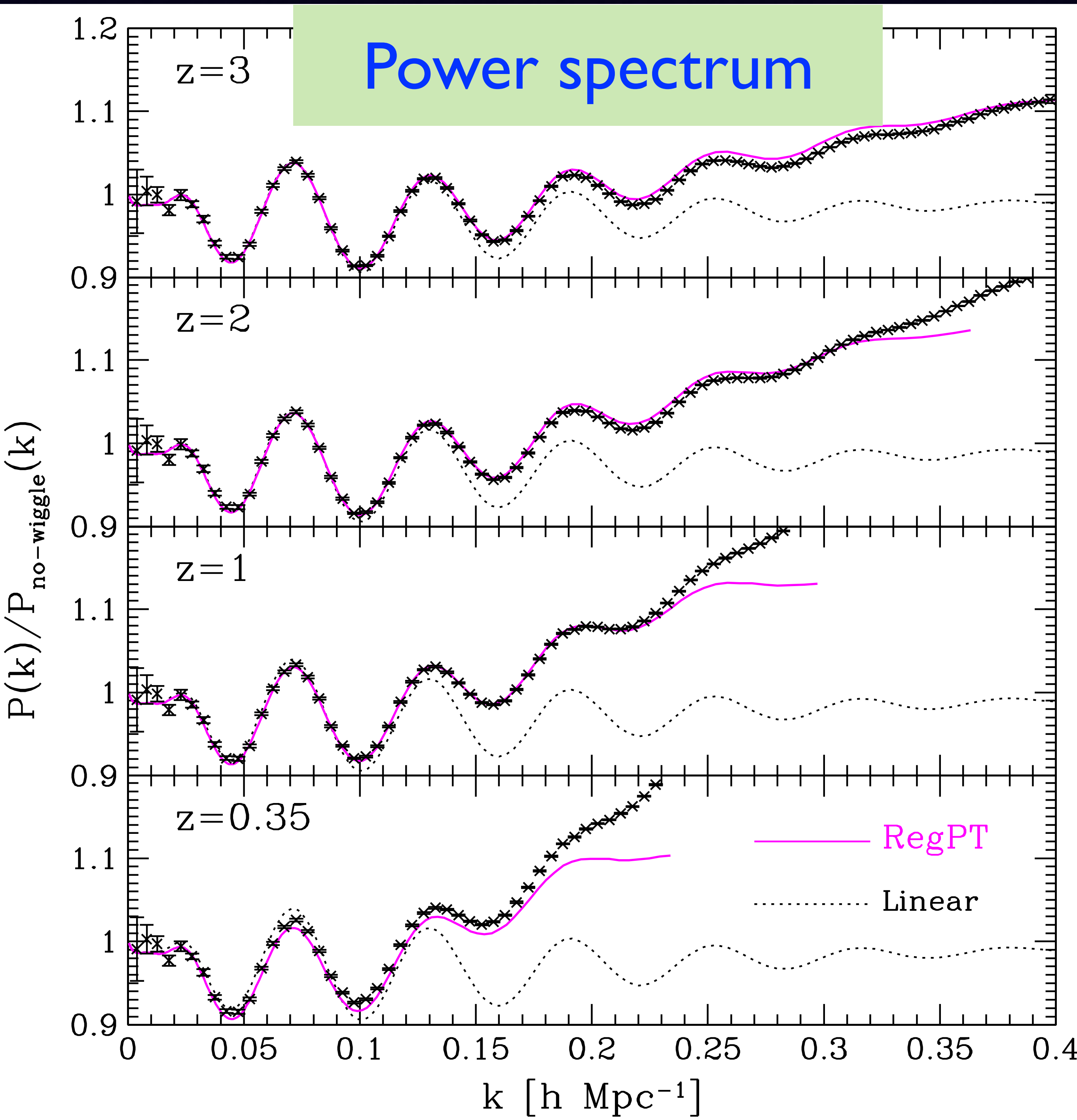
RegPT : fast PT code for P(k) & ξ(r)

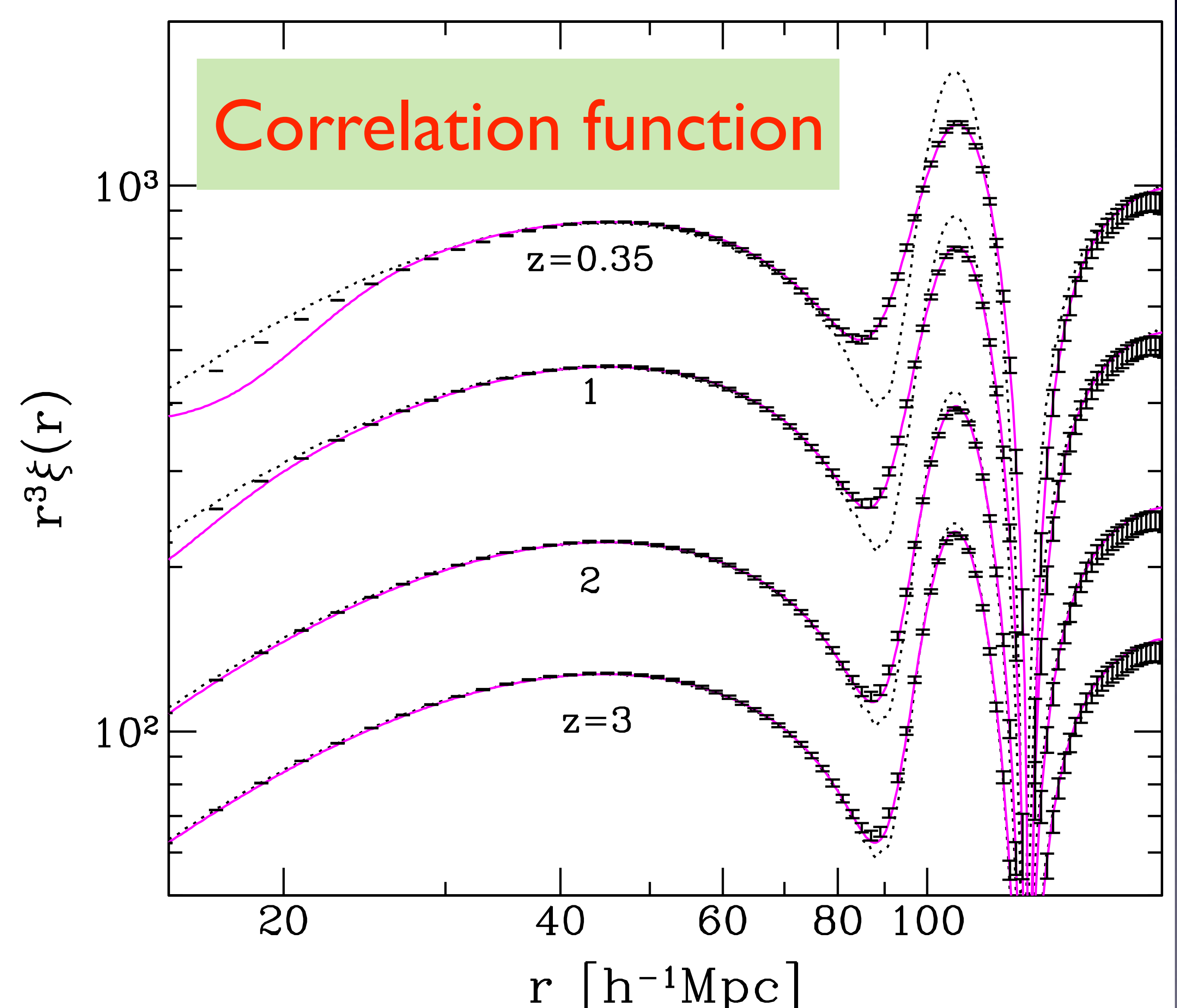
few sec.

(regularized)

A public code based on multi-point propagators at *2-loop order*

http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/regpt_code.html





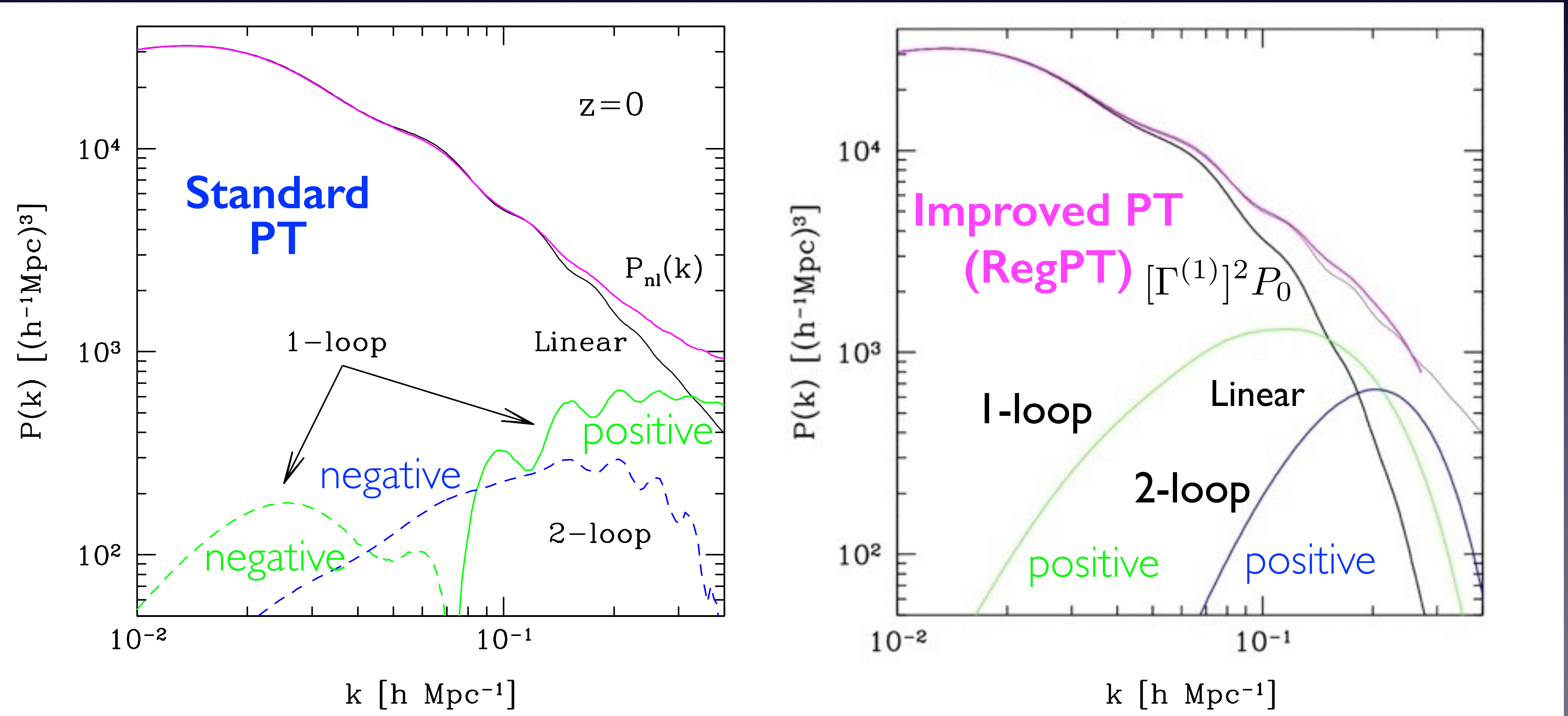
AT, Bernardeau, Nishimichi & Codis ('12)

Why improved PT works well?

AT, Bernardeau, Nishimichi, Codis ('12)
AT et al. ('09)

- All corrections become comparable at low-z.
- Positivity is not guaranteed.

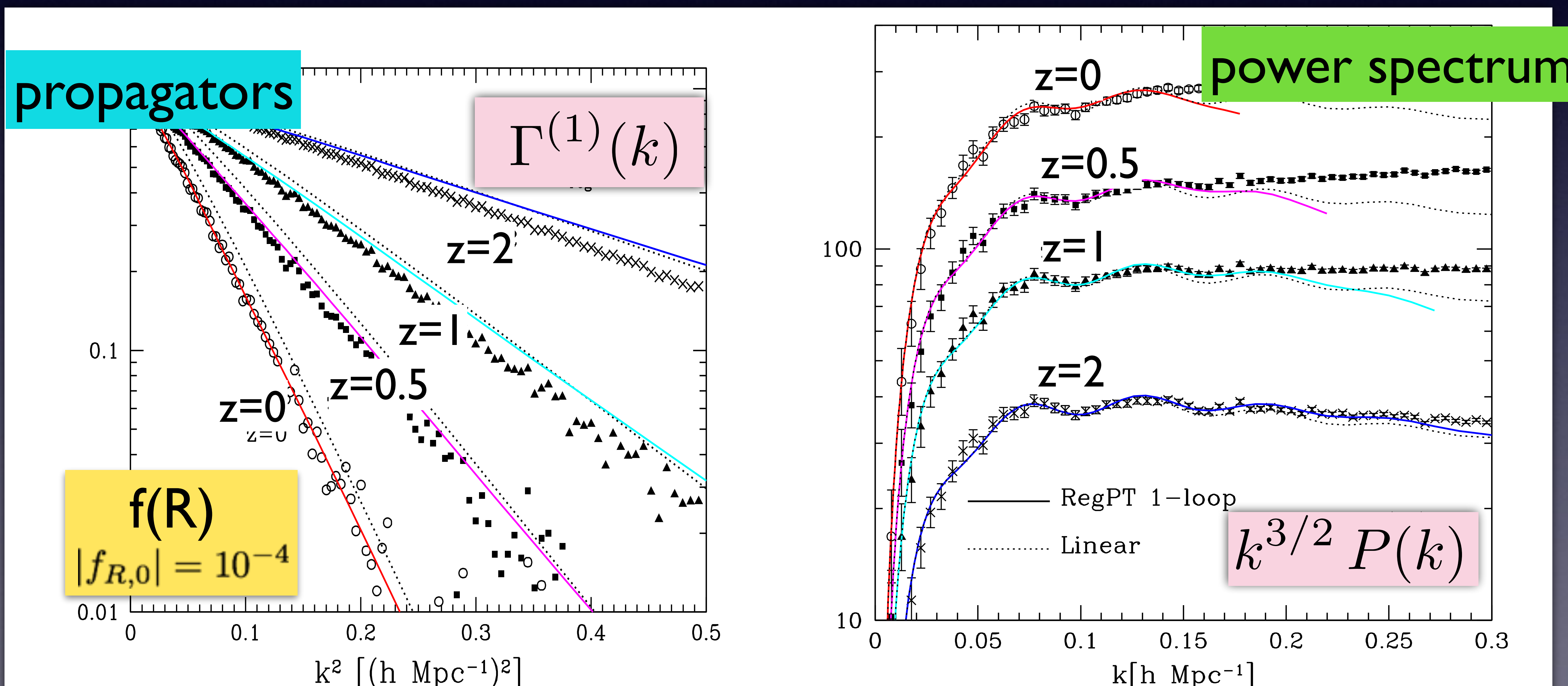
Corrections are positive & localized, shifted to higher-k for higher-loop



RegPT in modified gravity

Good convergence is ensured by
a *generic* damping behavior in propagators $\Gamma^{(n)} \xrightarrow{k\to\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm d}^2/2}$

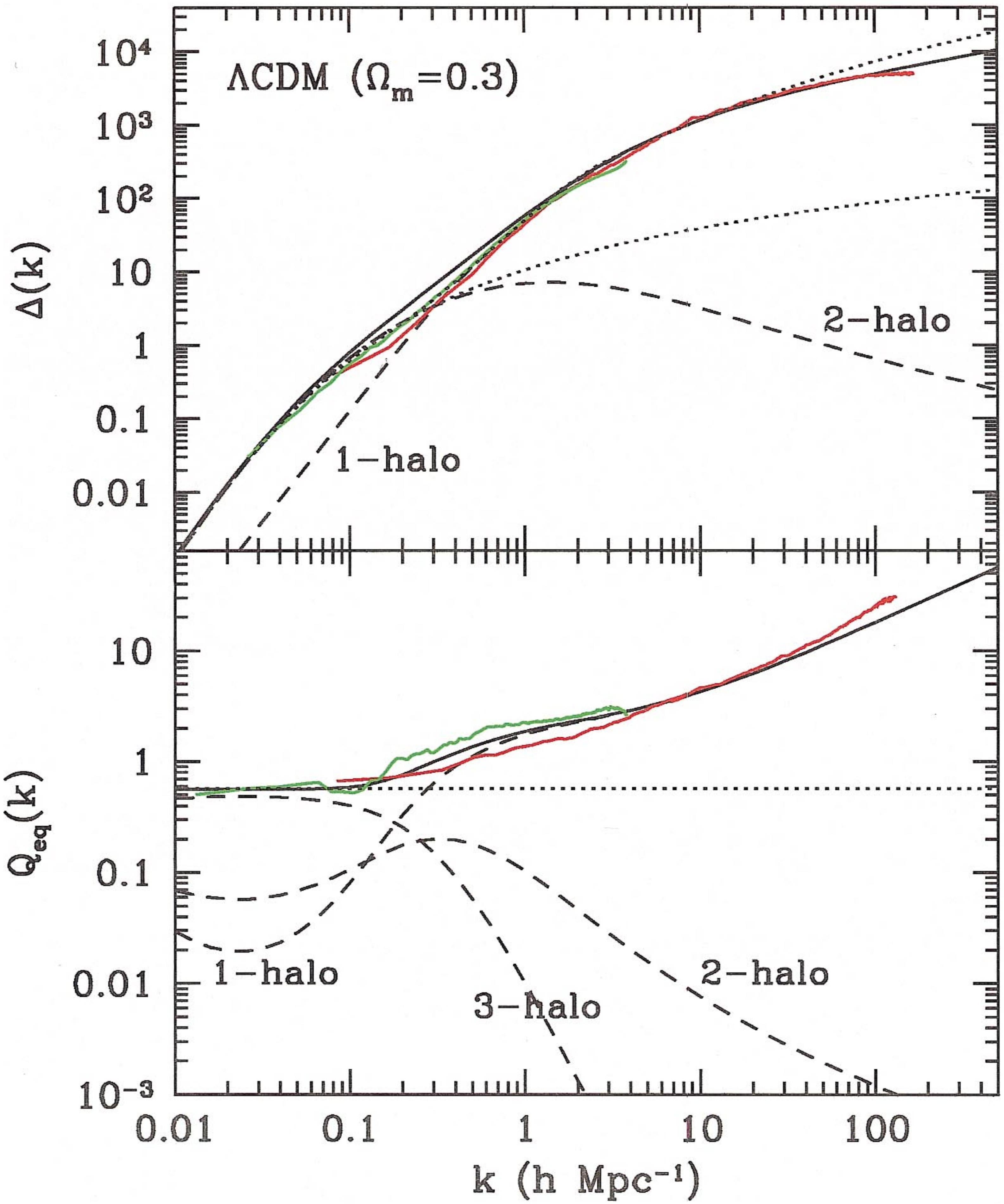
Even in modified gravity, well-controlled expansion with RegPT



N-body data: Baojiu Li

AT, Nishimichi, Bernardeau, et al.('14)

Halo model


ΛCDM (Ω_m=0.3)
10^4
10^3
10^2
10
1
0.1
0.01
$\Delta(k)$
2−halo
1−halo
10
1
0.1
0.01
10^{-3}
$Q_{eq}(k)$
1−halo
3−halo
2−halo
0.01
0.1
1
10
100
k (h Mpc^{-1})

Ma & Fry (2000)

From dark matter to galaxies

Cooray & Sheth ('02)

Assuming that galaxies in each halo follow a Poisson distribution

$$P_{\rm gal}(k) = P^{1h}_{\rm gal}(k) + P^{2h}_{\rm gal}(k),$$

$$P^{1h}_{\rm gal}(k) = \int {\rm d}m\, n(m)\, \frac{\langle N_{\rm gal}(N_{\rm gal}-1)|m\rangle}{\bar{n}^2_{\rm gal}} |u_{\rm gal}(k|m)|^2$$

$$P^{2h}_{\rm gal}(k) \approx P^{\rm lin}(k)\left[\int {\rm d}m\, n(m)\, b_1(m)\, \frac{\langle N_{\rm gal}|m\rangle}{\bar{n}_{\rm gal}} u_{\rm gal}(k|m)\right]^2$$

m: halo mass

$$\bar{n}_{\rm gal} = \int {\rm d}m\, n(m) \langle N_{\rm gal}|m\rangle$$

Needs determine observationally assuming their functional forms

For SDSS LRG or CMASS, the contributions further need to be divided into central ands satellites, i.e., $N_{\rm gal} = N_{\rm cen} + N_{\rm sat}$

(e.g., Zheng et al.'05)

Galaxy power spectrum

Hikage & Yamamoto ('13)

$$P^R(k) = P^{R1h}(k) + P^{R2h}(k)$$

$$P^{R1h}(k) = \frac{1}{\bar{n}^2}\int dM \frac{dn(M)}{dM}\langle N_{cen}\rangle\left[2\langle N_{sat}\rangle\tilde{u}_{\rm NFW}(k;M) + \langle N_{sat}(N_{sat}-1)\rangle\tilde{u}_{\rm NFW}(k;M)^2\right],$$

$$P^{R2h}(k) = \frac{1}{\bar{n}^2}\left[\int dM \frac{dn(M)}{dM}\langle N_{cen}\rangle\left(1+\langle N_{sat}\rangle\tilde{u}_{\rm NFW}(k;M)\right)b(M)\right]^2 P_m(k),$$

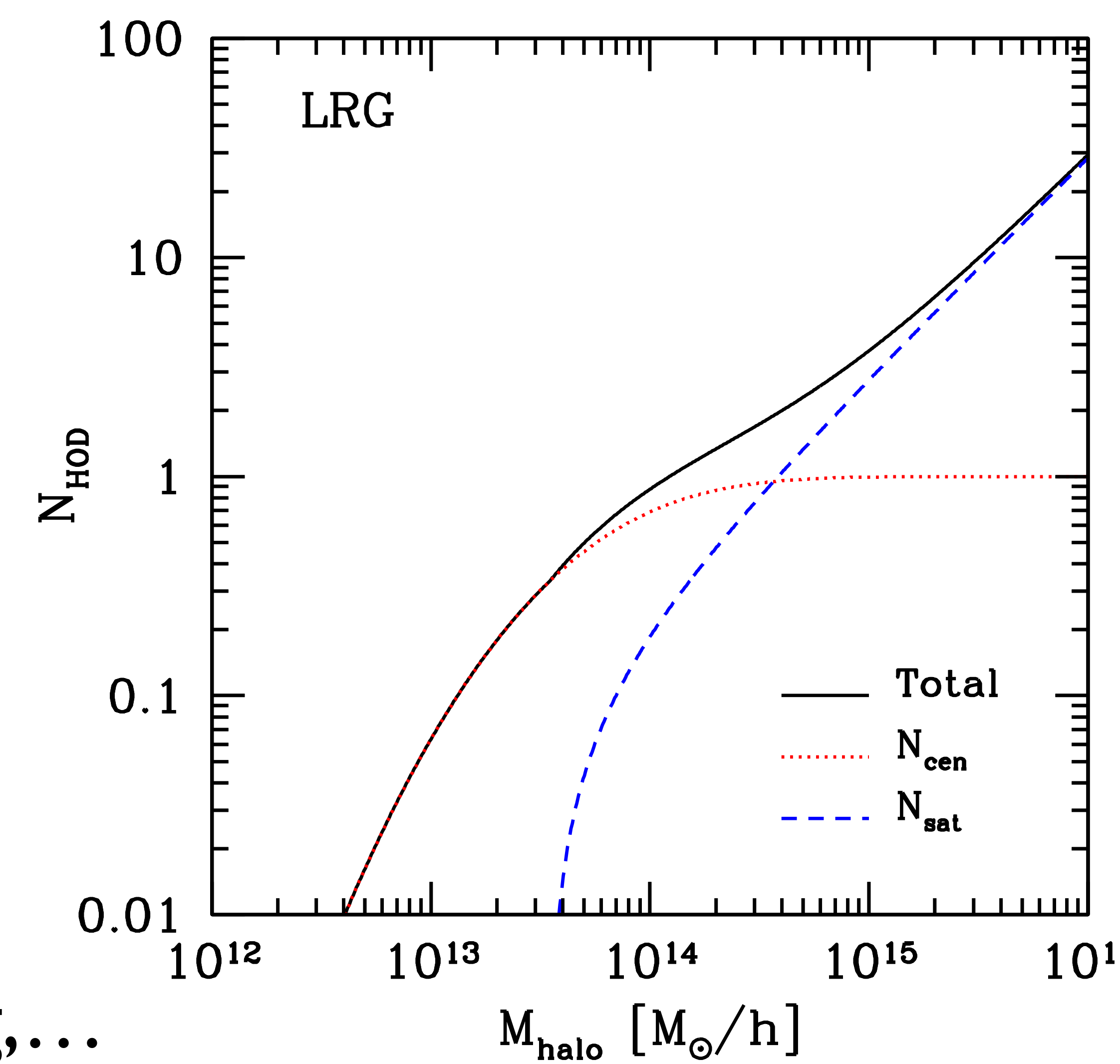
$$\bar{n} = \int dM (dn/dM) N_{\rm HOD}(M)$$

$$N_{\rm HOD}(M) = \langle N_{\rm cen}\rangle(1+\langle N_{\rm sat}\rangle),$$

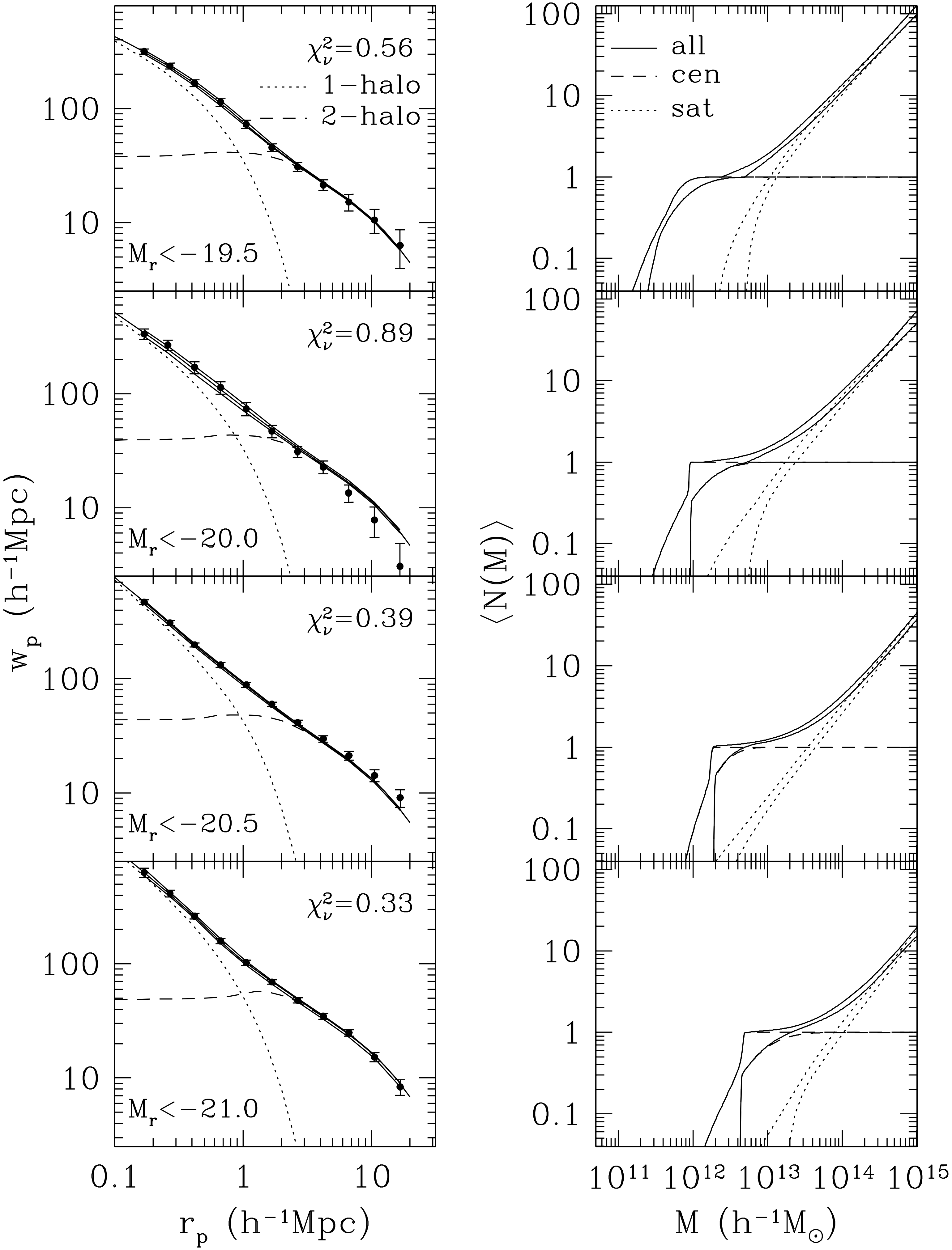
$$\langle N_{\rm cen}\rangle = \frac{1}{2}\left[1+{\rm erf}\left(\frac{\log_{10}(M)-\log_{10}(M_{\rm min})}{\sigma_{\log M}}\right)\right],$$

$$\langle N_{\rm sat}\rangle = f_{\rm col}(M)\left(\frac{M-M_{\rm cut}}{M_1}\right)^{\alpha},$$

Zheng et al. ('07)



Application: RSD, Weak lensing, mock catalog,…

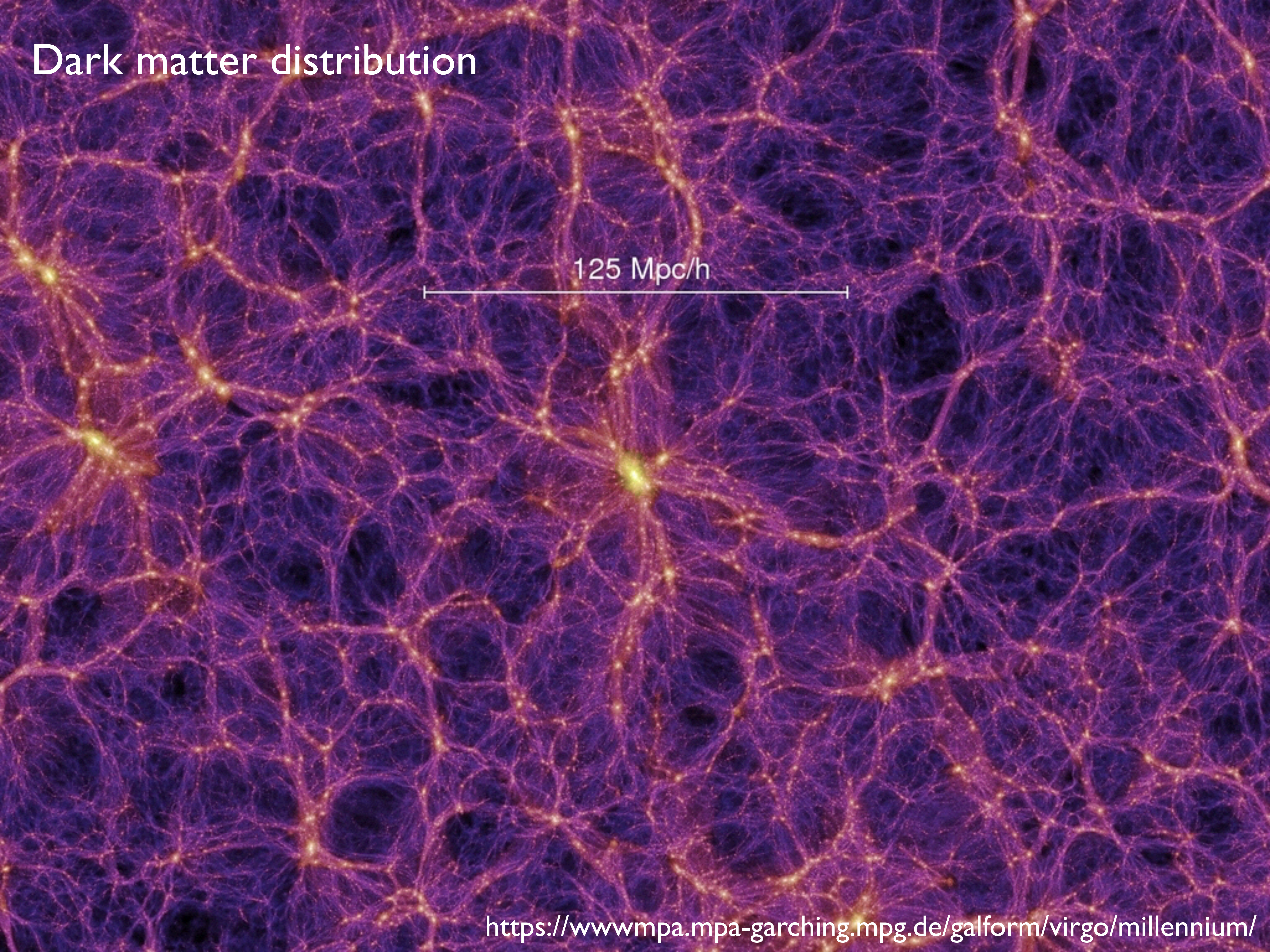


SDSS galaxies (z~0)

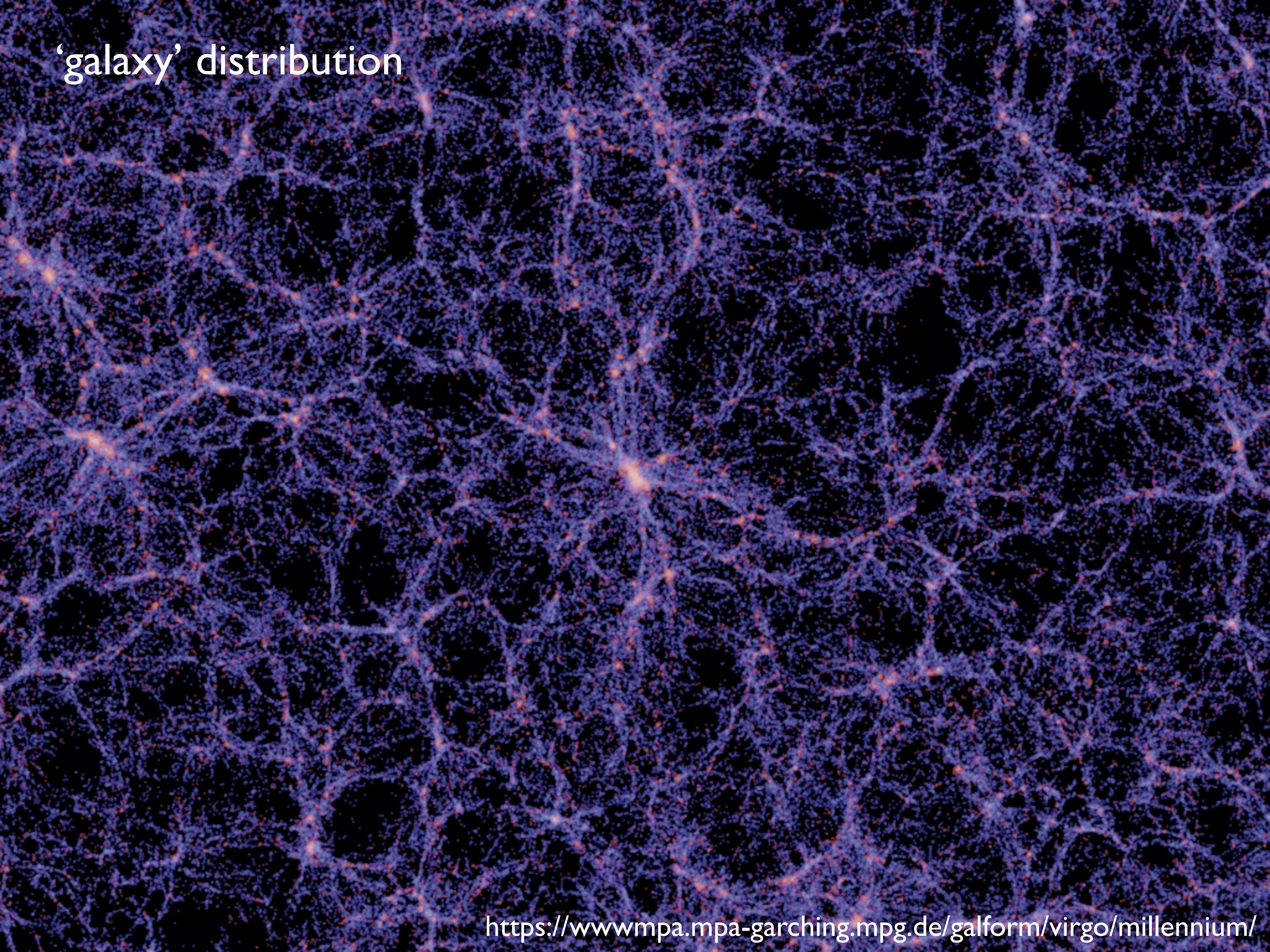
Zheng, Coil & Zehavi ('07)

Galaxy/halo bias

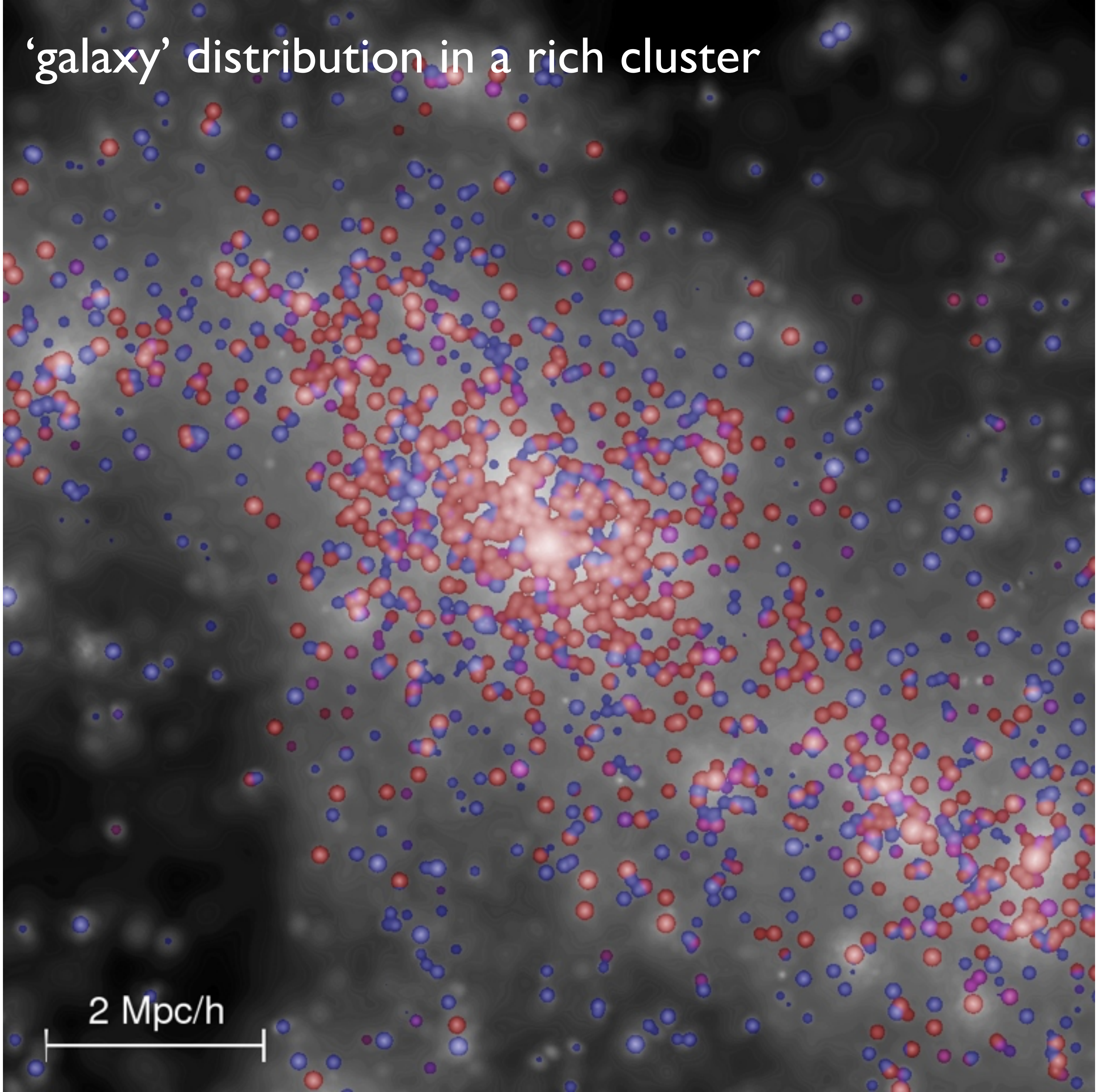
Dark matter distribution



https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/


‘galaxy’ distribution
https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/

dark matter distribution in a rich cluster


‘galaxy’ distribution in a rich cluster
2 Mpc/h

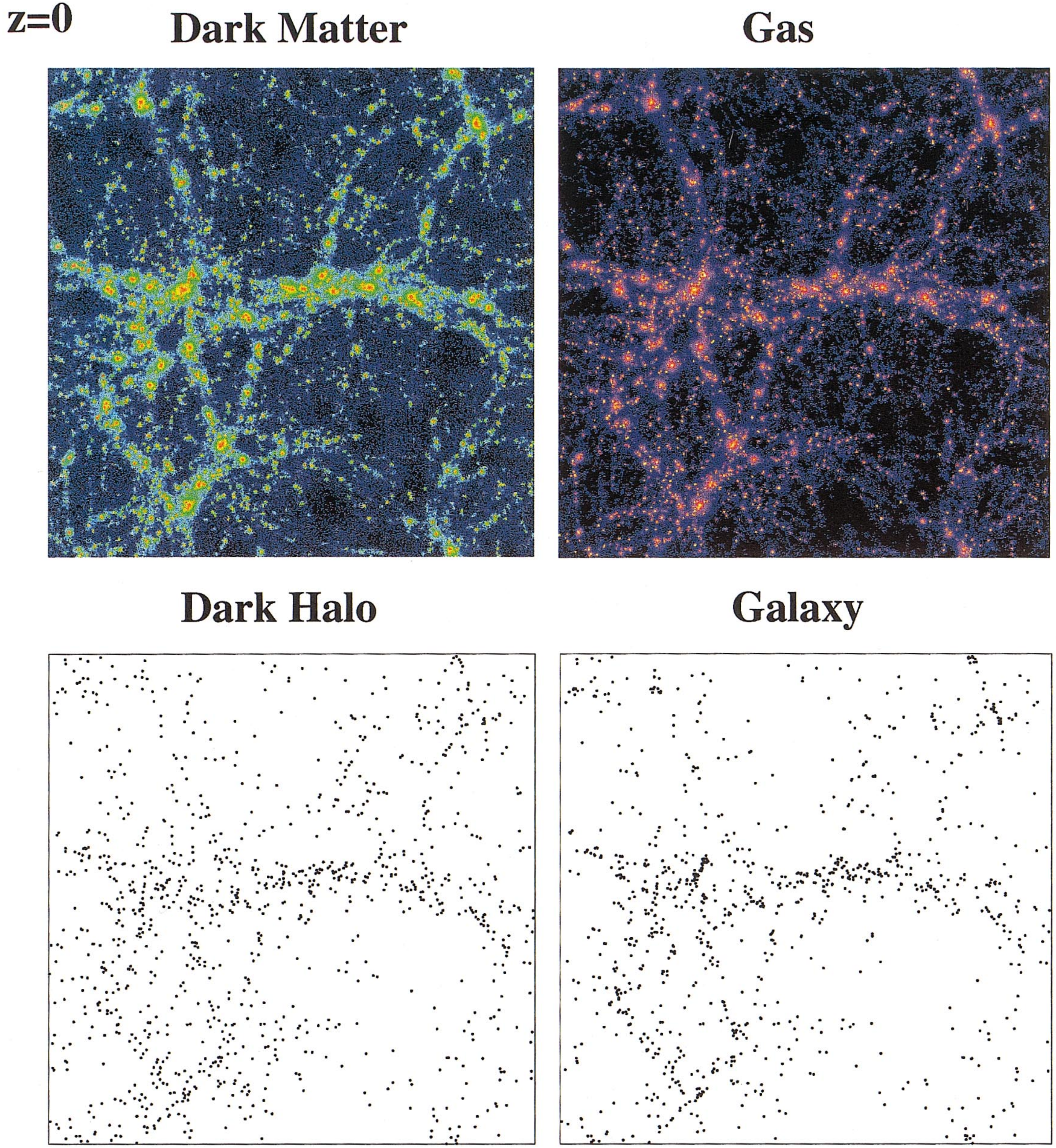


FIG. 2.—Distribution of gas particles, dark matter particles, galaxies, and dark halos in the volume of 75 × 75 × 30 (h^{-1} Mpc)^3 model at $z = 0$. *Upper right*, gas particles; *upper left*, dark matter particles; *lower right*, galaxies; *lower left*, DM cores.

Yoshikawa, et al. ('01)

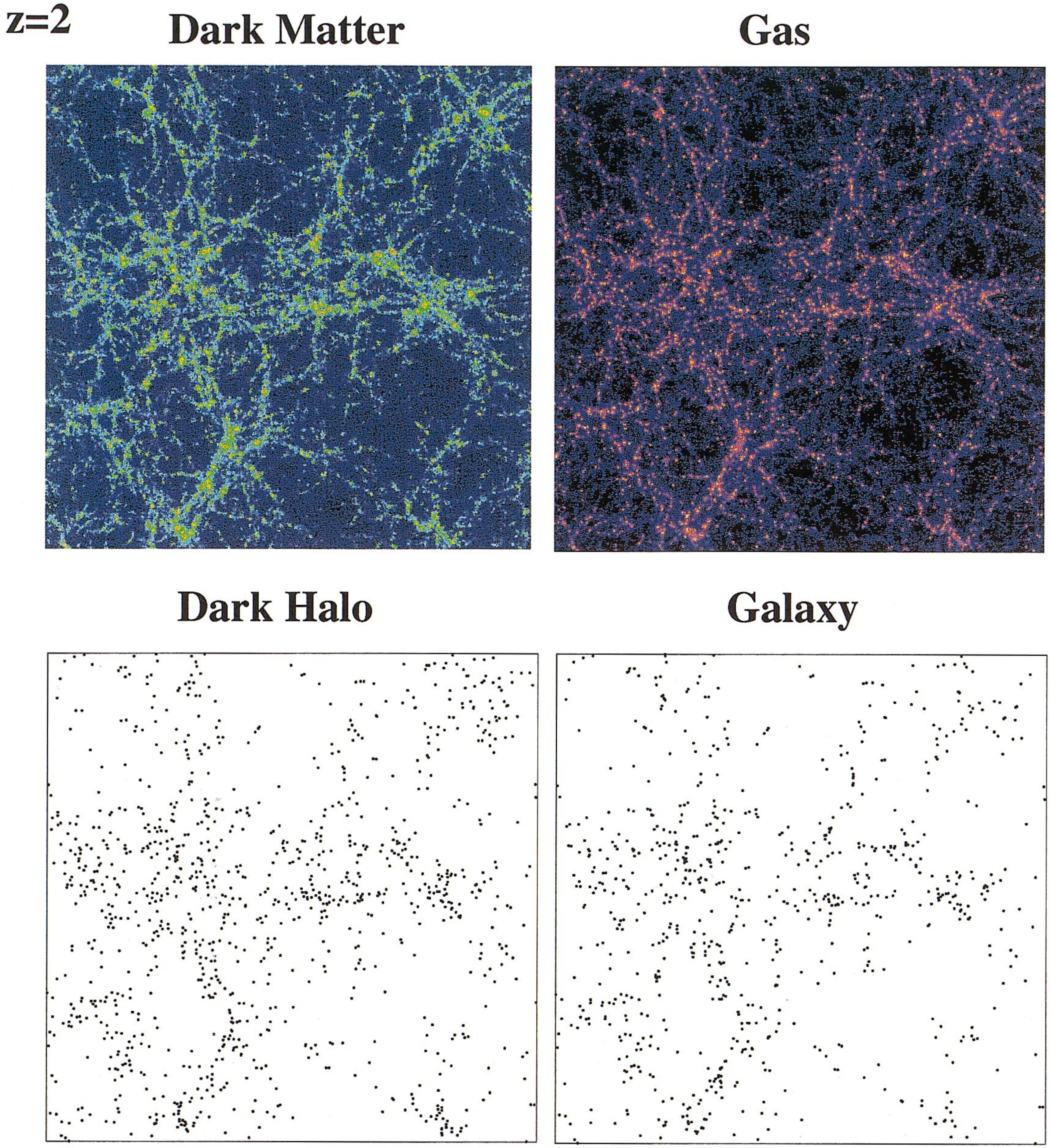
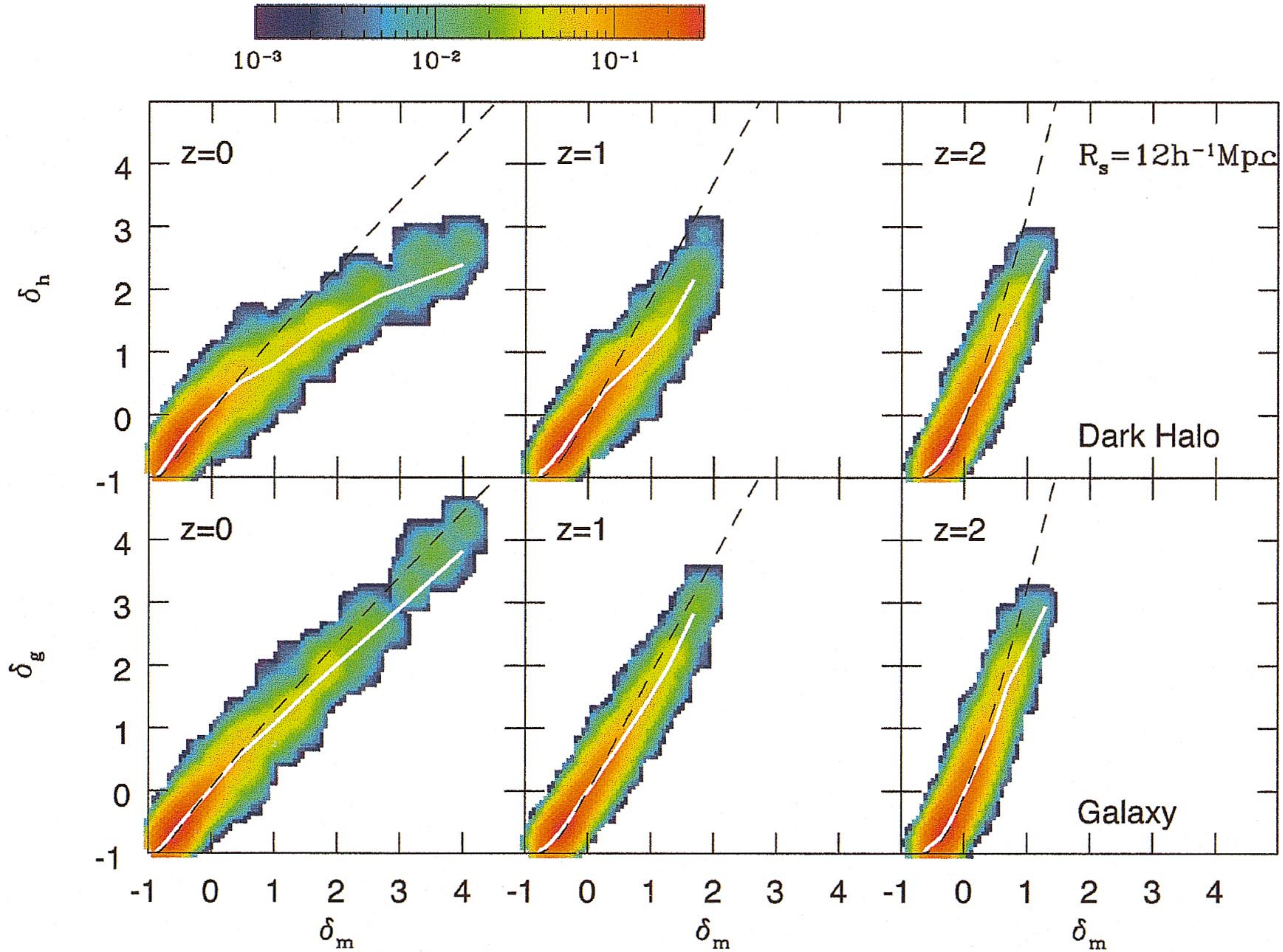
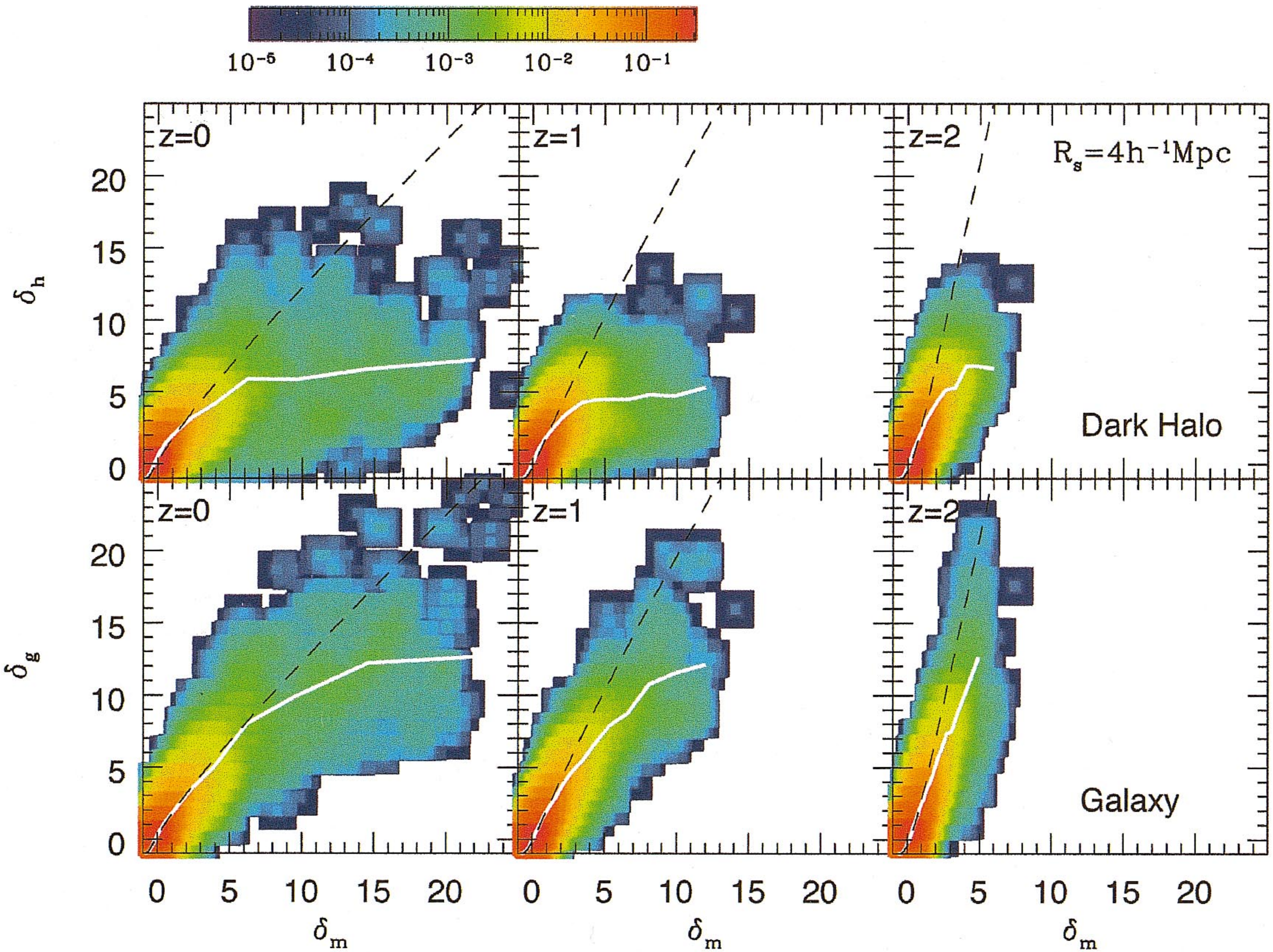


FIG. 3.—Same as Fig. 2, but for $z = 2$

Yoshikawa, et al. ('01)



Yoshikawa, et al. ('01)



10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1}
z=0
z=1
z=2
$R_s=4h^{-1}Mpc$
Dark Halo
Galaxy
δ_h
δ_g
δ_m

Kaiser (1984)

THE ASTROPHYSICAL JOURNAL, **284**:L9–L12, 1984 September 1

ON THE SPATIAL CORRELATIONS OF ABELL CLUSTERS

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ABSTRACT

If rich clusters formed where the primordial density enhancement, when averaged over an appropriate volume, was unusually large, then they give a biased measure of the large-scale density correlation function: $\xi_{\text{clusters}}(r) \approx A\xi_{\text{density}}(r)$. The factor A is determined by the probability distribution of the density fluctuations on a rich cluster mass scale, and if this distribution was Gaussian the correlation function is amplified. The amplification for rich $R \geq 1$ clusters is estimated to be $A \approx 10$, and the predicted trend of A with richness agrees qualitatively with that observed. Some implications of these results for the large-scale density correlations are discussed.

Subject headings: cosmology — galaxies: clustering

then

$$1 + \xi_{>\nu}(r) = \frac{P_2}{P_1^2} = (2/\pi)^{1/2}\left[\operatorname{erfc}(\nu/2^{1/2})\right]^{-2} \times \int_\nu^\infty e^{-1/2y^2} \operatorname{erfc}\left[\left[\frac{\nu - y\xi(r)/\xi(0)}{\{2[1-\xi^2(r)/\xi^2(0)]\}^{1/2}}\right]\right] dy.$$

This result may also be obtained by application of Price's theorem (Price 1958). For $\xi_c \ll 1$ this expression simplifies to

$$\xi_{>\nu}(r) = \left(e^{\nu^2/2}\int_\nu^\infty e^{-1/2y^2}\,dy\right)^{-2} \xi(r)/\sigma^2, \qquad (2)$$

and for $\nu \gg 1$

$$\xi_{>\nu}(r) \approx (\nu^2/\sigma^2)\xi(r). \qquad (3)$$