Workshop on Bispectrum @ YITP 3/29 – 30 2018

Theory of primordial non-Gaussianity and its constraints by bispectrum

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with Shuichiro Yokoyama (Rikkyo) Phys. Rev. D 91, 123521

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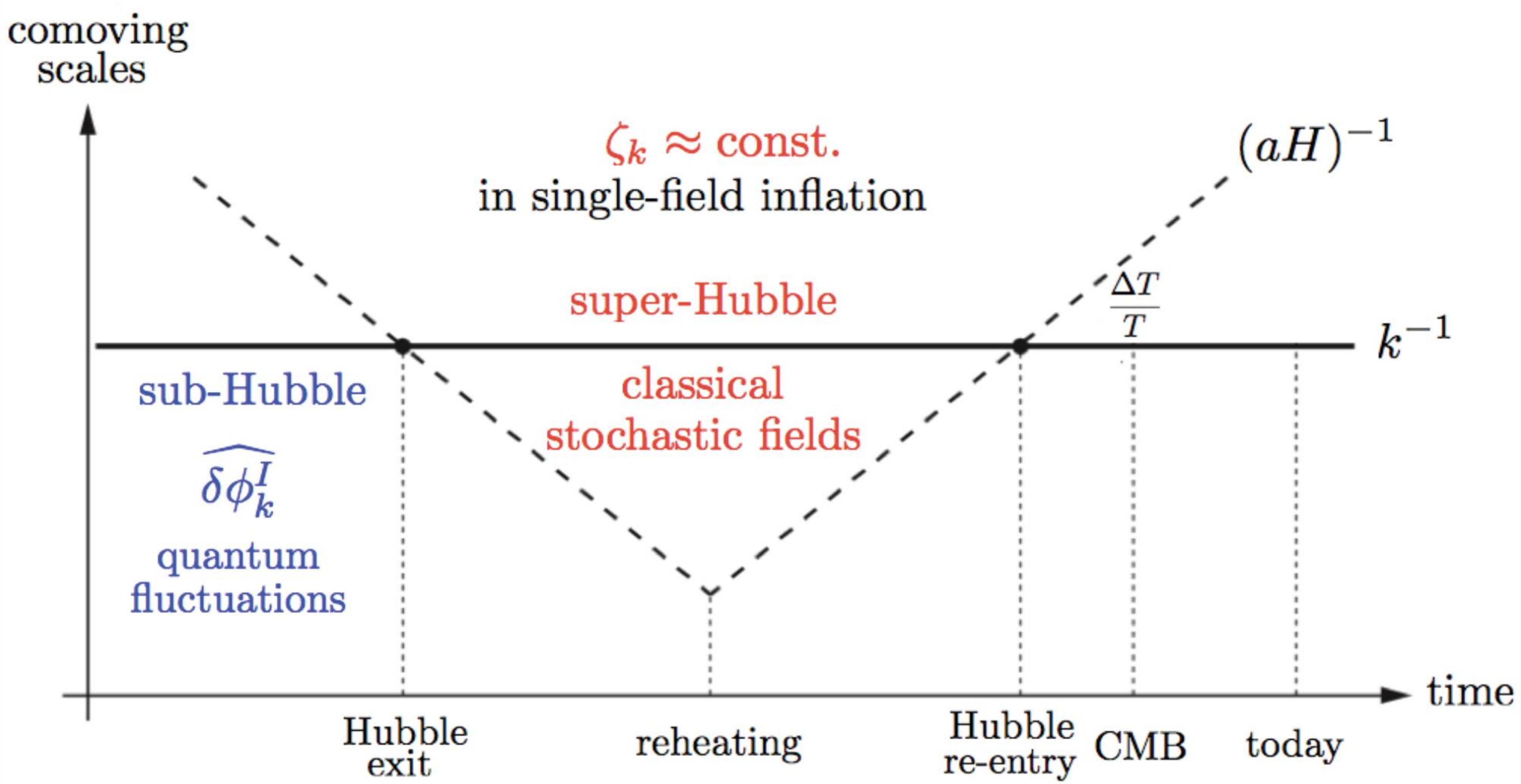
Phys. Rev. D 94, 043532

Contents

1. Primordial Non-Gaussianity (PNG)

Generation of primordial perturbations



$$\langle\delta\phi^2\rangle = \int d\ln k\, \mathcal{P}_{\delta\phi}(k)$$

$$\mathcal{P}_{\delta\phi} = \left(\frac{H}{2\pi}\right)^2$$

$$\delta g_{ij} = a^2 e^{2\zeta}\delta_{ij}dx^i dx^j$$

$$\zeta = -\frac{H}{\dot{\phi}}\delta\phi$$

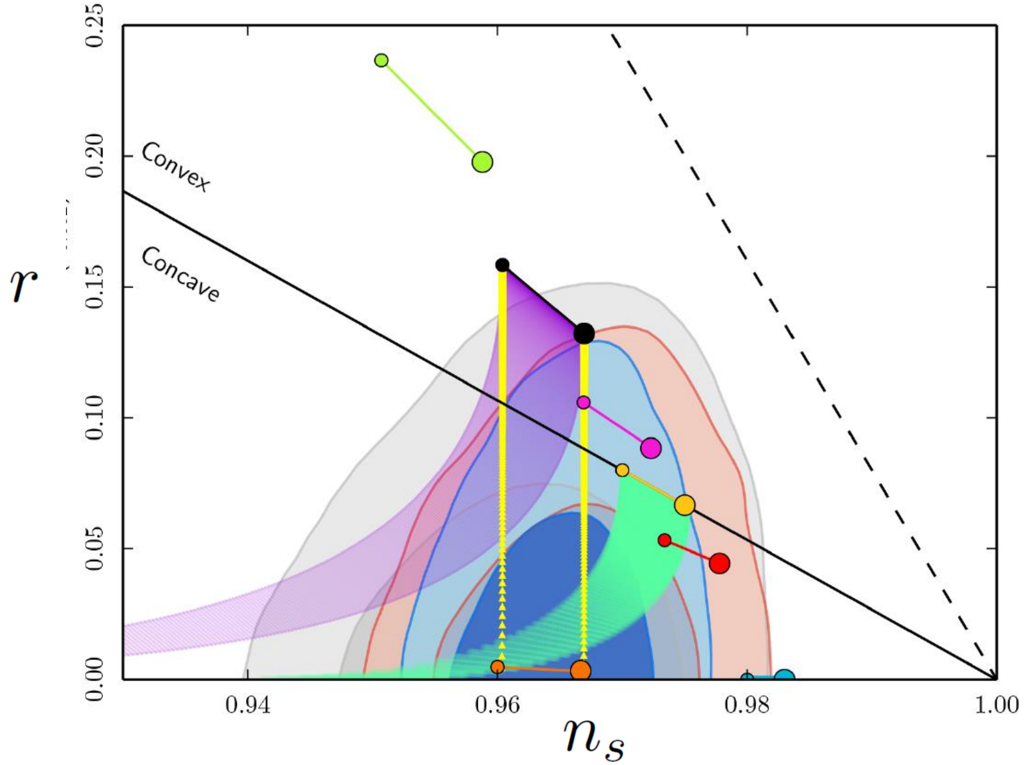
Constraints from primordial power spectrum

- Primordial power spectrum

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_\zeta(k)$$

well approximated by $\frac{k^3}{2\pi^2} P_\zeta(k) = A_s(k_\star) \left(\frac{k}{k_\star} \right)^{n_s - 1}$ $\quad k_\star = 0.05 \mathrm{Mpc}^{-1}$

- Constraints from Planck Ade et al `16



For standard single-field slow-roll inflation models

$$n_s - 1 \simeq -2\epsilon - \eta$$

$$r \equiv \frac{P_h}{P_\zeta} = 16\epsilon$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \qquad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon}$$

Primordial bispectrum

- Primordial bispectrum

dimensionless function

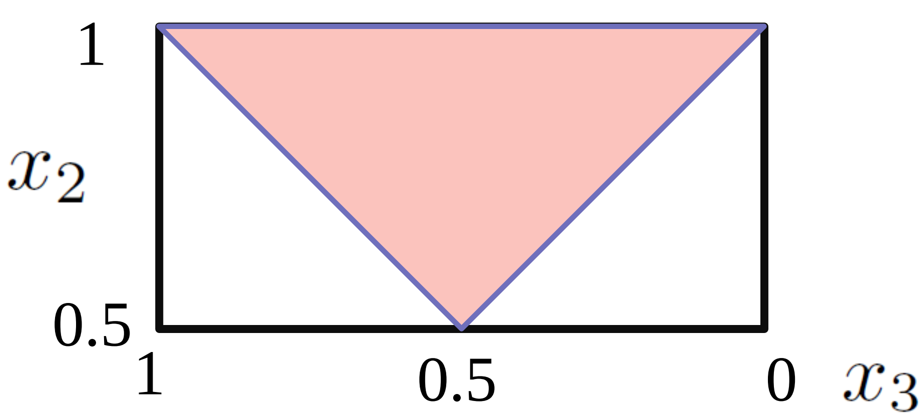
$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle \equiv (2\pi)^3 \delta(\sum_{\mathbf{i}} \mathbf{k_i}) \left[(2\pi)^4 \frac{S(k_1, k_2, k_3)}{(k_1 k_2 k_3)^2} A_s^2 \right]$$

wavevectors $\mathbf{k_i}$ form a tetrahedron in Fourier space

S has information of amplitude as well as shape-dependence

- Shape-dependence of bispectrum

In most models, S depends only on $x_2 \equiv k_2/k_1$ and $x_3 \equiv k_3/k_1$



For $k_3 \leq k_2 \leq k_1$

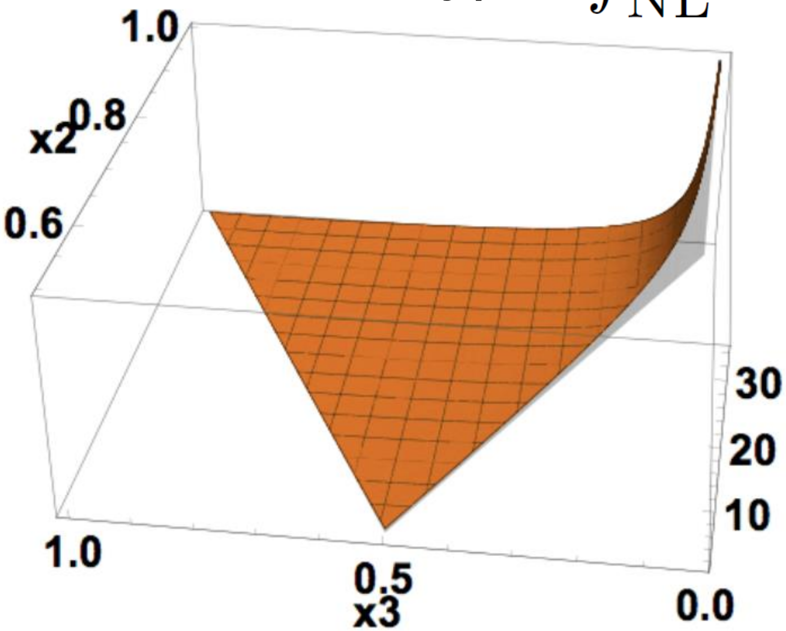
Allowed region is

$$1 \geq x_2 \geq x_3$$

$$1 \leq x_2 + x_3$$

Representative types of primordial bispectra

▪local-type $f_{\rm NL}^{\rm local}$



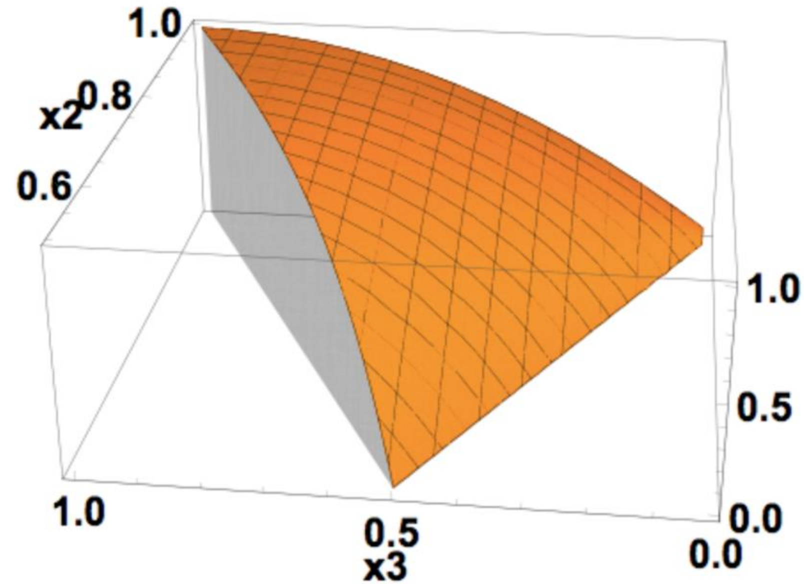
Maximum in the squeezed limit

$$k_3 \ll k_2 \simeq k_1$$

Generated by non-linear dynamics

Ex.) Multi-field model

▪equilateral-type $f_{\rm NL}^{\rm equil}$



Maximum in the equilateral limit

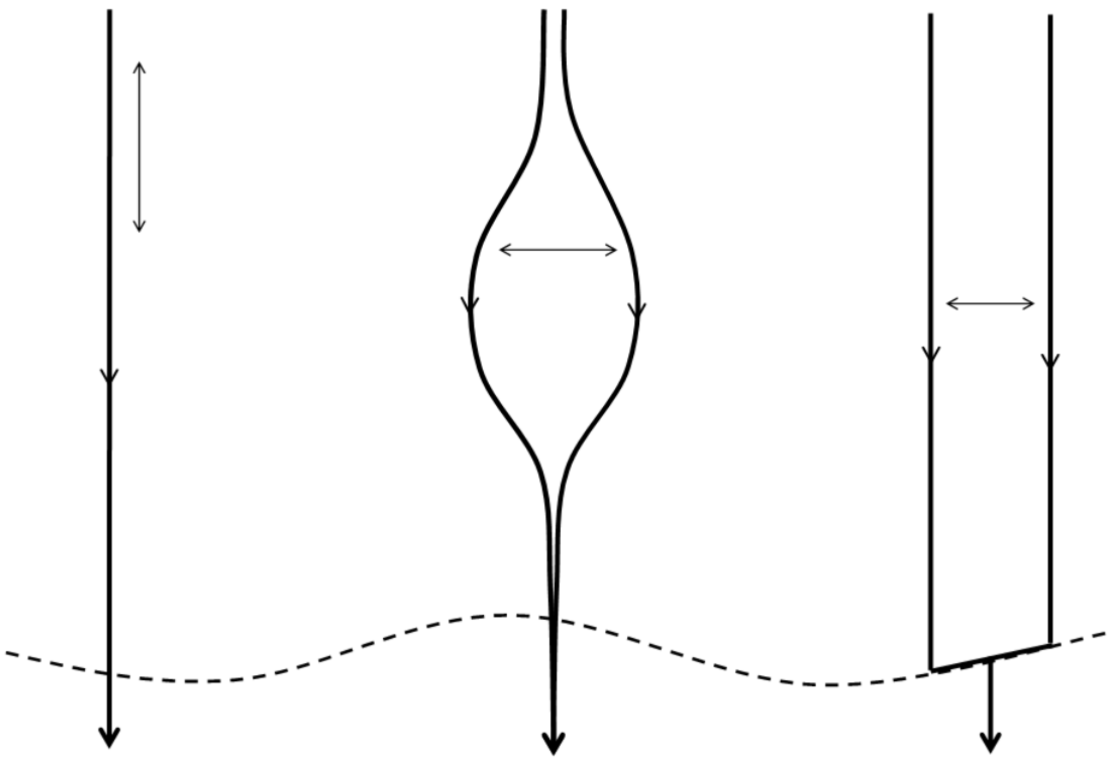
$$k_3 \simeq k_2 \simeq k_1$$

Generated by intrinsic nonlinearity of inflaton perturbation

Local-type PNG from multi-field model

- Schematic graphs for curvature perturbation (δN)

(From, Sasaki, `08)



Single-field

$$\zeta = -\frac{H}{\dot{\phi}}\delta\phi$$

Multi-field inflation

Curvaton, modulated reheating

Curvature perturbation is affected by isocurvature (entropy) perturbation !!

$$f_{\rm NL}^{\rm local} = \frac{5}{6}\frac{N_A N_B N^{AB}}{(N_C N^C)^2}$$

$$g_{\rm NL}^{\rm local} = \frac{25}{54}\frac{N_{ABC} N^A N^B N^C}{(N_D N^D)^3}$$

(Trispectrum)

Consistency relation

- Squeezed-limt bispectrum from single field inflation

Maldacena `03, Creminelli, Zaldarriaga `04

$$\lim_{k_3 \to 0} B_\zeta(k_1, k_2, k_3) = (1 - n_s(k_1)) P_\zeta(k_1) P_\zeta(k_3)$$

Conservation of ζ_k on super-Hubble scales

$\longrightarrow$ k_3 mode locally acts as a background field

$\Rightarrow$ Detection of $f_{\rm NL}^{\rm local}$ rule out most single field inflation models !!

- Loop hole

Chen, Firouzjahi, Namjoo, Sasaki `13

Based on non-attractor background solution, we can violate this, as the 'decaying' mode can become important on large scales

Intrinsic non-linearity of inflaton perturbation

- Expansion of the action

$$g_{\mu\nu}(t,x) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,x) \qquad \phi(t,x) = \bar{\phi}(t) + \delta\phi(t,x)$$

$$\Rightarrow \quad S = \bar{S} + S^{(2)}(\delta g_{\mu\nu}, \delta\phi) + \underline{S^{(3)}(\delta g_{\mu\nu}, \delta\phi) + \cdots}$$

interactions of inflaton fluctuation

- In-in formalism Calzetta and Hu `87, Weinberg `05

The expectation value of an observable $O(t)$

$$\langle in|O(t)|in\rangle = \langle 0|\left[\bar{T}\exp\left(i\int_{-\infty}^{t} \underline{H_I(t')}dt'\right)\right] O^I(t)\left[T\exp\left(-i\int_{-\infty}^{t} \underline{H_I(t'')}dt''\right)\right]|0\rangle$$

interaction Hamiltonian

At leading order $\Rightarrow$ $$\langle O(t)\rangle = 2\,\mathrm{Re}\left[-i\int_{-\infty}^{t} dt' \langle 0|O^I(t)H_I(t')|0\rangle\right]$$

Equilateral-type PNG from k-inflation

Chen, et al, `07 Arroja, SM, Koyama, Tanaka, `09

- Model

$$\mathcal{L} = P(\phi, X) \quad \text{with} \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Ex1) Canonical $P(\phi, X) = X - V(\phi)$

Ex2) DBI $P(\phi, X) = -f(\phi)^{-1}\sqrt{1 - 2Xf(\phi)} + f(\phi)^{-1} + V(\phi)$

- Sound speed

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

is related with intrinsic non-linearity

$\Rightarrow$ $f_{\rm NL}^{\rm equil} \sim 1/c_s^2$ $g_{\rm NL}^{\rm equil} \sim 1/c_s^4$ (trispectrum)

Also generated in more general higher-derivative scenarios as well as effective field theory of inflation

Constraints from CMB and perspectives

- Constraints from Planck (CMB) Ade et al `16

$f_{\rm NL}^{\rm local} = 0.8 \pm 5.0$ $f_{\rm NL}^{\rm equil} = -4 \pm 43$ (68% CL)

- $f_{\rm NL}^{\rm local}$ from multifield scenarios is very model-dependent, but large class of spectator models predicts $|f_{\rm NL}^{\rm local}| \geq \mathcal{O}(1)$

Suyama, Takahashi, Yamaguchi, Yokoyama `13

- The bounds on $f_{\rm NL}^{\rm equil}$ translate into a limit on c_s

$$c_s \geq 0.020 \quad (95\% \text{ CL})$$

Strong coupling scale in EFT $\Lambda_\star \sim \frac{1}{\sqrt{\zeta f_{\rm NL}^{\rm equil}}} H_{inf}$

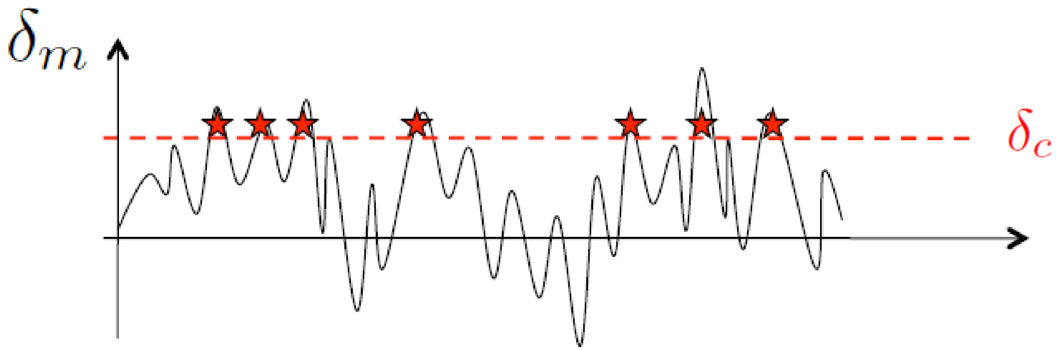
Unless $|f_{\rm NL}^{\rm equil}| \leq 1$, new physics appears much below $M_{\rm Pl}$

Baumann, Green `11

2. Scale-dependent bias from PNG

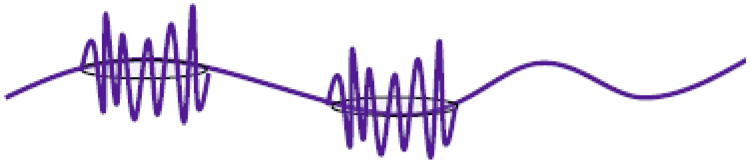
Simple picture of bias

Small scale density peaks exceeding threshold collapse under their own gravity and form virialized objects



- Peak-background split

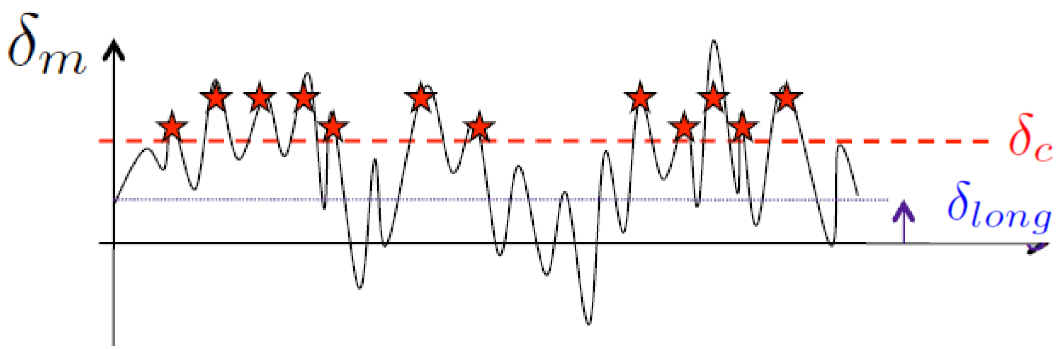
$$\delta_m = \delta_{short} + \delta_{long}$$



Large-scale fluctuations δ_{long} raise local background density,

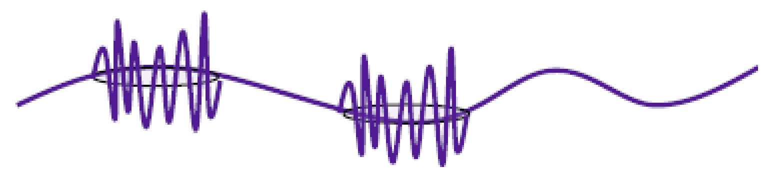
Simple picture of bias

Small scale density peaks exceeding threshold collapse under their own gravity and form virialized objects



- Peak-background split

$$\delta_m = \delta_{short} + \delta_{long}$$



Large-scale fluctuations δ_{long} raise local background density, which lowers effective threshold for collapse and enhances number of peaks above threshold !!

Influence of local-type PNG on bias

Dalal et al, `08, (See also Sloser et al, `08)

- local model of PNG

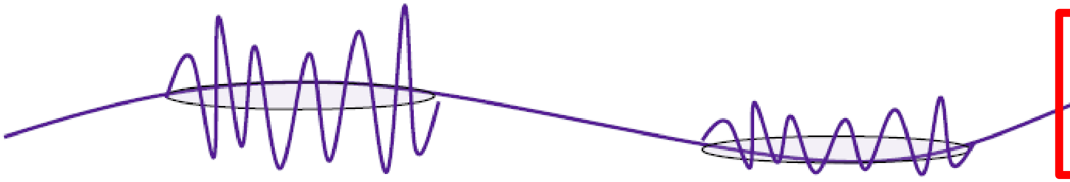
$$\Phi(x) = \phi_G(x) + f_{\rm NL}^{\rm local}(\phi_G^2(x) - \langle\phi_G^2\rangle)$$

peak-background split of Gaussian potential fluctuations:

$$\phi_G(x) = \phi_s(x) + \phi_l(x)$$

$$\Rightarrow \quad \Phi(x) = (1 + \underline{f_{\rm NL}^{\rm local}\phi_l(x)})\phi_s(x) + \phi_l(x) + f_{\rm NL}^{\rm local}(\phi_s^2(x) + \phi_l^2(x) - \langle\phi_s^2(x)\rangle - \langle\phi_l^2(x)\rangle)$$

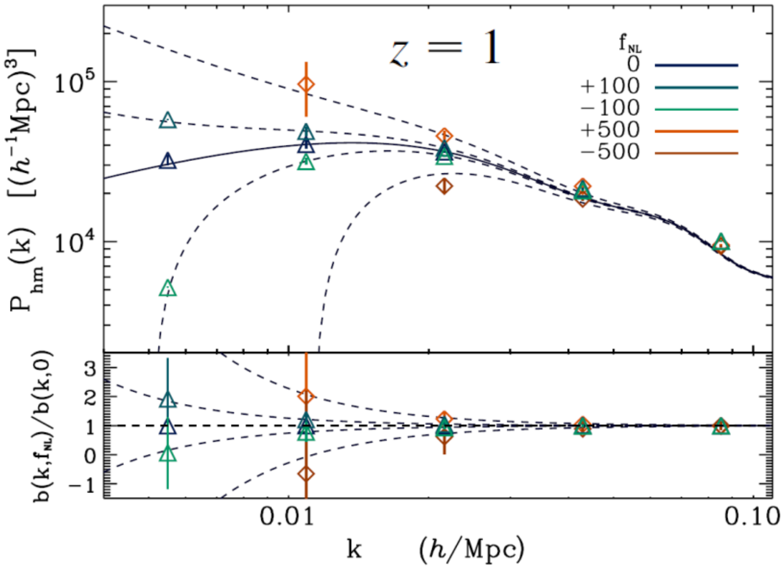
long wavelength modes add to local background density $+\delta_{long}$
and modulate amplitude on small scales $\times f_{\rm NL}\phi_l$



$$\Delta b = f_{\rm NL}^{\rm local}(b_{\rm G} - 1)\frac{3\delta_c\Omega_m H_0^2}{k^2 T(k) D(a)}$$

Constraints on local-type PNG

Dalal et al `08



$1.6 \times 10^{13} M_\odot < M < 3.2 \times 10^{13} M_\odot$

- Current constraints: (95 % CL)

Leistedt, Peiris, Roth `14

$$-49 < f_{\rm NL}^{\rm local} < 31$$

$$-2.7 \times 10^5 < g_{\rm NL}^{\rm local} < 1.9 \times 10^5$$

- Forecast constraints:

Yamauchi et al `14

$$\Delta f_{\rm NL}^{\rm local} \simeq 0.1$$ (SKA)

We expect that the constraint on local-type PNG will be much tighter

How about equilateral-type PNG ?

Integrated Perturbation Theory (iPT)

Matsubara `12, `13, Bernardeau et al `08

- Multi-point propagator of biased objects

$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta\delta_{\rm L}(\mathbf{k}_1)\delta\delta_{\rm L}(\mathbf{k}_2)\cdots\delta\delta_{\rm L}(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta(\mathbf{k}_1+\mathbf{k}_2+\cdots+\mathbf{k}_n)\Gamma_X^{(n)}(\mathbf{k}_1,\mathbf{k}_2,\cdots,\mathbf{k}_n)$$

Gravitational evolution, bias, …

δ_X : number density field of the biased objects (Halo/ Galaxy)

$\delta_{\rm L}$: linear density field which is related with primordial curvature perturbation ζ through

$$\delta_{\rm L}(k) = \mathcal{M}(k)\zeta(k);\ \mathcal{M}(k) = \frac{2}{3}\frac{D(z)}{D(z_*)(1+z_*)}\frac{k^2 T(k)}{H_0^2\Omega_{\rm m0}}$$

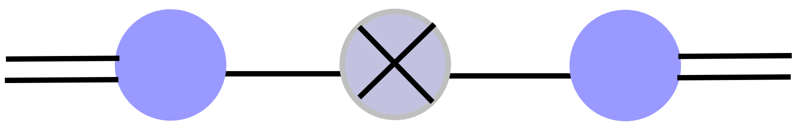
$D(a)$: growth factor $T(k)$: transfer function

⇨ large scale behavior of the spectra of biased objects

Halo/Galaxy power spectrum with PNG

- Diagrams for the power spectrum of the biased objects

P_0



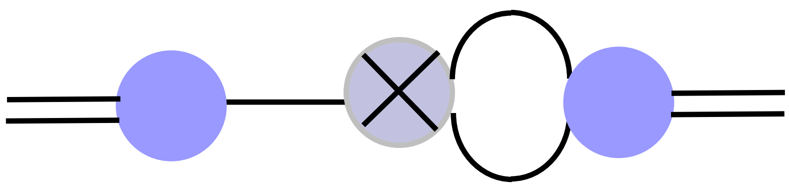
$\Rightarrow \propto \mathcal{M}(k)^2 P_\Phi(k) \propto k$

large scale limit

$k \ll p$

typical scale of the biased objects

$P_{\rm bis}$



large scale limit $\Rightarrow$

$$\begin{cases} \propto \mathcal{M}(k) k^{-3} \propto k^{-1} & \text{with } f_{\rm NL}^{\rm local} \\ \propto \mathcal{M}(k) k^{-1} \propto k & \text{with } f_{\rm NL}^{\rm equil} \end{cases}$$

enhancement (for $\propto k^{-1}$)

no enhancement (for $\propto k$)

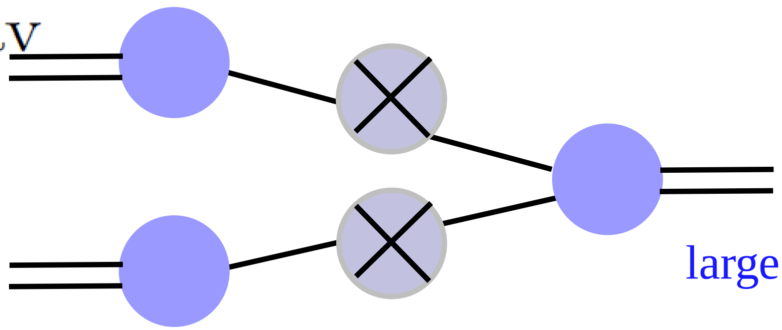
3. Constraints on PNG by Halo/galaxy bispectrum

Halo/galaxy bispectrum with PNG

- Diagrams for the bispectrum of the biased objects

SM, Yokoyama `15

$B_{\rm grav}$

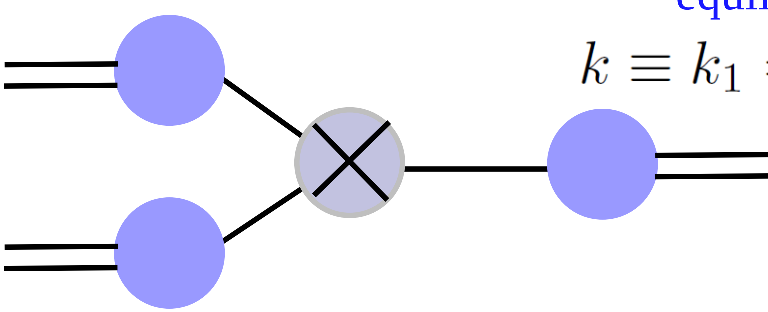


$\propto \mathcal{M}(k)^4 P_\Phi(k)^2 \propto k^2$

large scale limit
&
equilateral config.

$k \equiv k_1 = k_2 = k_3 \ll p$

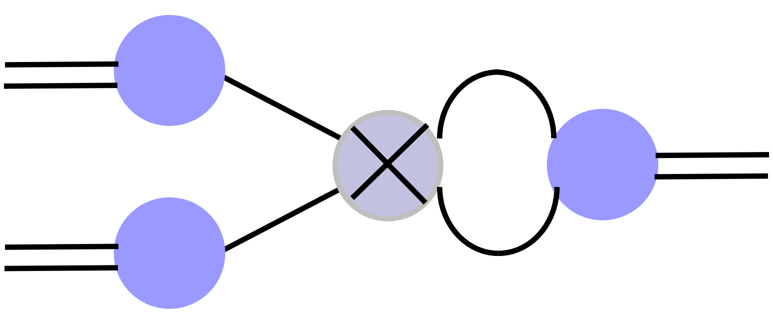
$B_{\rm bis}$



$\propto \mathcal{M}(k)^3 P_\Phi(k)^2 \propto k^0$

with $f_{\rm NL}^{\rm equil}$

$B_{\rm tri}$

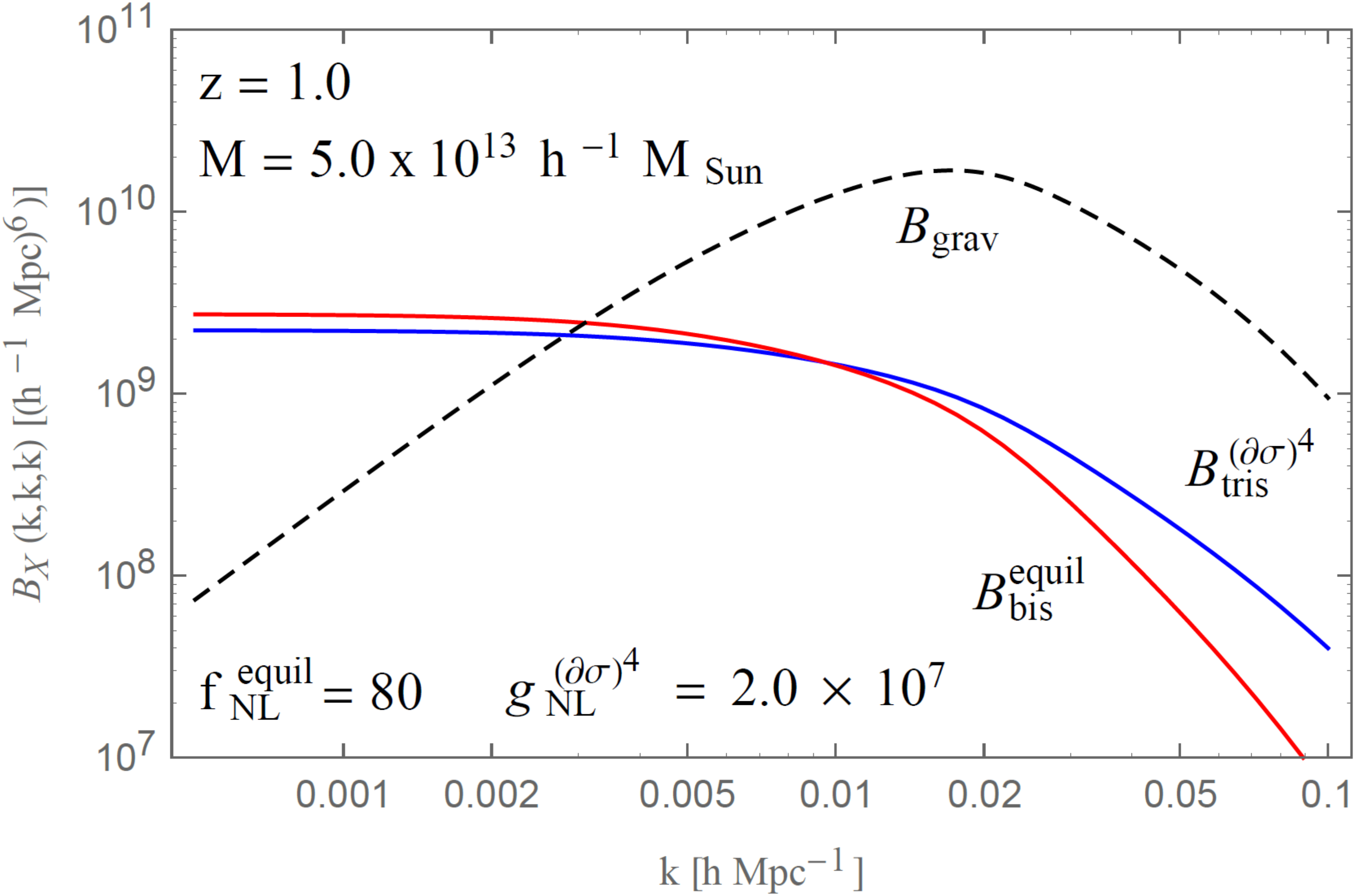


$\propto \mathcal{M}(k)^2 k^{-4} \propto k^0$

with $g_{\rm NL}^{(\partial\sigma)^4}$

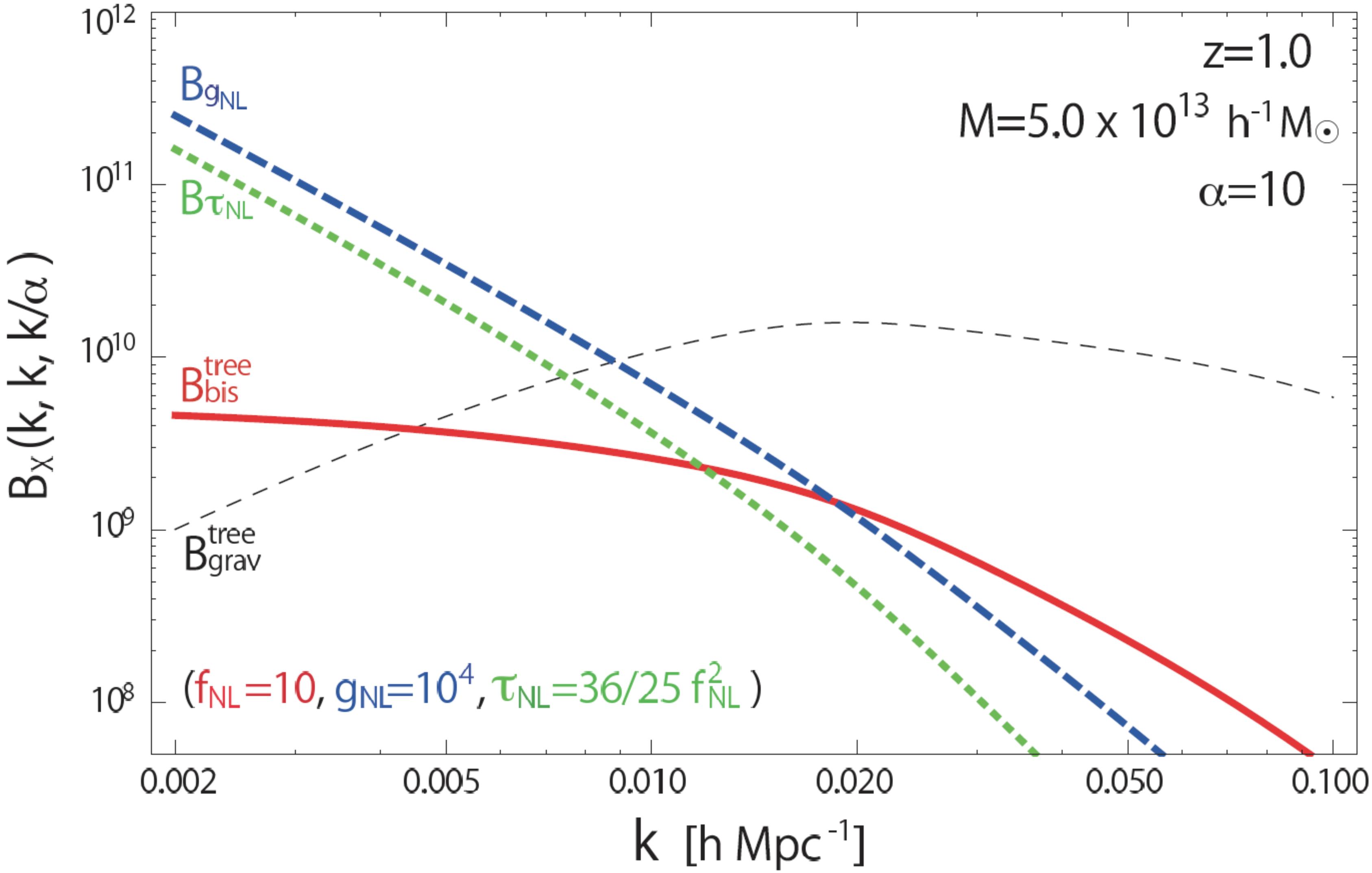
Scale-dependence of halo/galaxy bispectrum (Equilateral-type PNG)

SM, Yokoyama `15



Scale-dependence of halo/galaxy bispectrum (Local-type PNG)

Yokoyama, Matsubara, Taruya `14



Forecast constraints

Hashimoto, Taruya, Matsubara, Namikawa, Yokoyama `16

Hashimoto, SM, Yokoyama `16

Can LSS obtain more severer constraints than CMB ?

- Ongoing/future surveys on LSS

	$f_{\rm sky}$	$z_{\rm m}$	$\bar{n}_{\rm s}$ [arcmin^{-2}]
HSC [21]	0.0375 (1,500 deg^2)	1.0	35
DES [22]	0.125 (5,000 deg^2)	0.5	12
LSST [23]	0.5 (20,000 deg^2)	1.5	100

sky coverage mean source redshift mean number density of source

Expected constraints on PNG

- Local-type from P + B (halo + lens)

	HSC marginalized	DES marginalized	LSST marginalized	CMB un-marginalized
$\sigma(f_{NL})$	15	11	1.0	5.1
$\sigma(g_{NL})$	1.4×10^5	2.0×10^5	4.7×10^3	1.4×10^5
$\sigma(\tau_{NL})$	6.0×10^3	8.9×10^3	10	1.4×10^3

- Equilateral-type from B (halo + lens)

	HSC marginalized	DES marginalized	LSST marginalized	CMB un-marginalized
$\sigma(f_{NL})$	2.1×10^3	1.1×10^3	6.4×10^2	43
$\sigma(g_{NL})$	2.7×10^8	7.7×10^8	4.4×10^7	1.3×10^6

4. Summary and discussions

Summary

• PNG has information on various types of nonlinearity of inflation models and is helpful to distinguish between them

• Currently, from CMB , no significant PNG is observed and the simplest single-field slow-roll inflation models are consistent

• From scale-dependent bias, the future/ongoing projects on LSS can constrain $f_{\rm NL}^{\rm local}$ more and we can expect $\Delta f_{\rm NL}^{\rm local} \simeq 0.1$

• From halo/galaxy bispectrum, we can constrain $f_{\rm NL}^{\rm equil}$, but constraints from LSS are looser than that from CBM

Discussions

• Multitracer technique (Yamauchi et al `16) is helpful and gets $\Delta f_{\rm NL}^{\rm equil} \sim 20$ for SKA, slightly better than Planck

•We can constrain $f_{\rm NL}^{\rm equil}$ more from information of small scales if we specify nonlinear and nonlocal bias (Gleyzes et al `16)

• PNG is also generated by models with Exited initial states, resonance and feature models, models with gauge fields,…