BISPECTRUM WS
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赤方偏移バイスペクトルの分解方法

SLEPIAN & EISENSTEIN (2017)と応用

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Outline

- §１ イントロダクション、２点相関関数、パワースペクトルの変数
- §２　３点相関関数、バイスペクトルの変数と多重極展開
- §３ ハローアプローチに基づいたバイスペクトルの理論模型
- §４　結論

1.イントロダクション

バイスペクトルの定義(Scoccimarro et al)は数学的にはわかるが、
直感的な振る舞いの理解はよく分からない。
特に、赤方偏移空間を記述する量
Multipole bispectrumの振る舞い。

バイスペクトル $\rightleftarrows$ ３点相関関数の対応

Slepian & Eisenstein (2015～2017)

３点相関関数(3PCF)の展開
～　赤方偏移空間の3PCFの変数と展開法
解説と応用

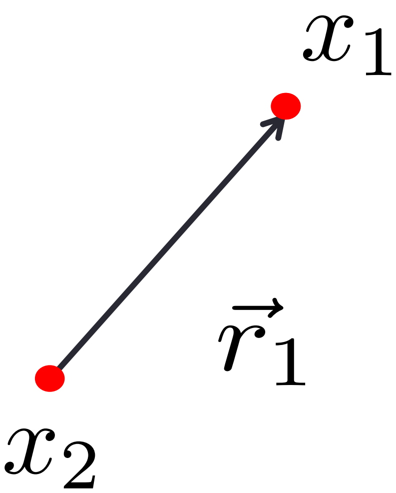
§ 2. 2点関数関数

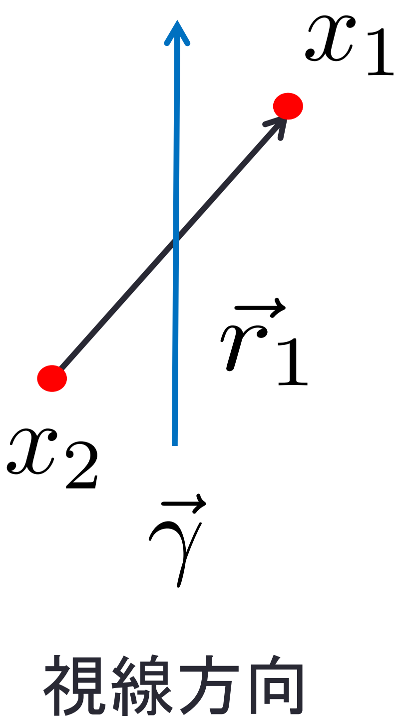
real space

$$\xi(r_1) = \langle \delta(x_1)\delta(x_2) \rangle$$

redshift space

$$\xi(\vec{r}_1, \vec{\gamma})$$



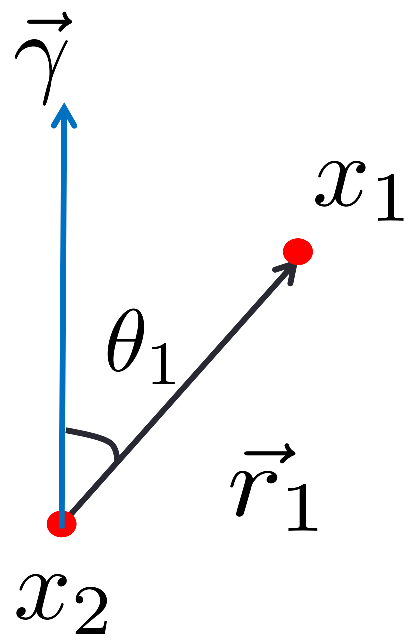


赤方偏移空間での2点相関

視線方向を2つの天体のうち一つの天体の方向に選ぶと
多重極スペクトルが速く計算できる。(Yamamoto, et al 2006)

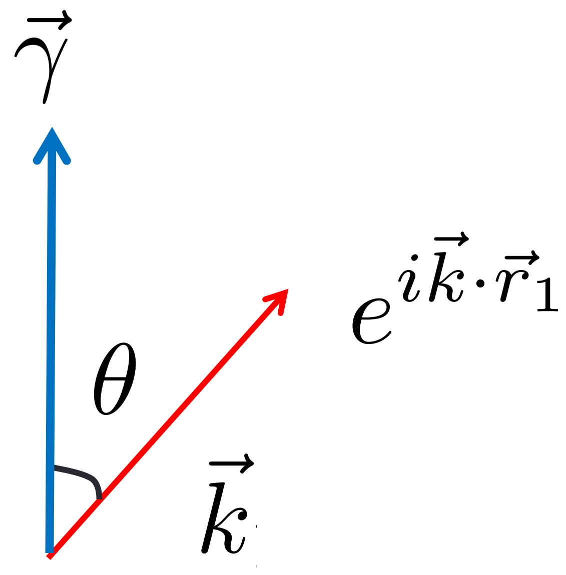
相関関数

$$\xi(r_1, \theta_1)$$



Power spectrum

$$P(k, \theta)$$



$$\xi(r_1,\theta_1) = \sum_\ell \xi_\ell(r_1)\mathcal{L}_\ell(\cos\theta_1) \qquad P(k,\theta) = \sum_\ell P_\ell(k)\mathcal{L}_\ell(\cos\theta)$$

$$\xi_\ell(r_1) = \frac{1}{2\pi^2}\int_0^\infty dk k^2 (-i)^\ell j_\ell(kr_1) P_\ell(k)$$

c.f. Matsubara Suto

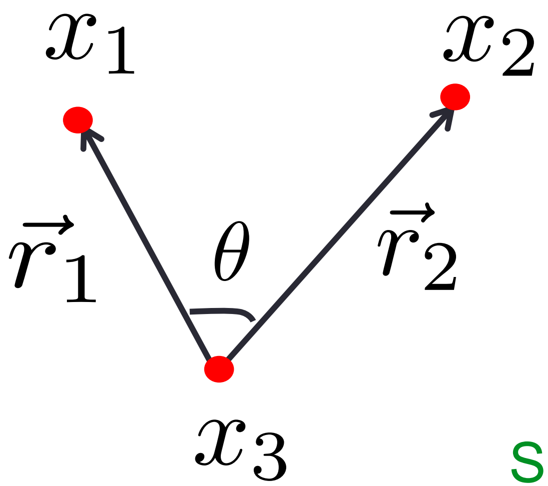
この手続きを、３点相関関数に拡張する。

3. 3点相関関数、バイスペクトルの変数と多重極展開

３点相関関数

Real space（3角形）は3つパラメーターで記述される。

$$\langle\delta(x_1)\delta(x_2)\delta(x_3)\rangle = \zeta(\vec{r}_1, \vec{r}_2)$$



Slepian and Eisenstein (2015)

多重極展開による定式化

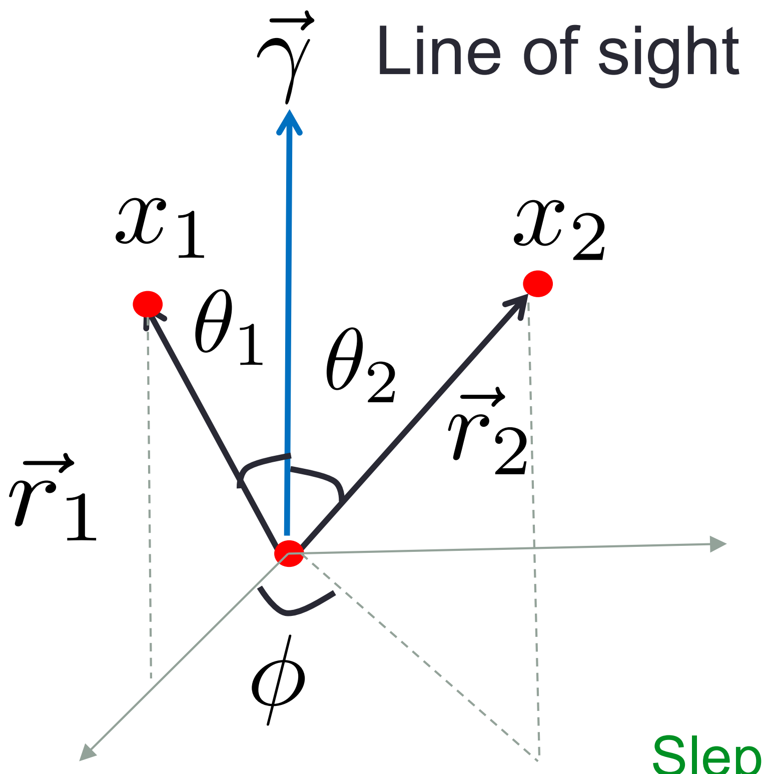
$$\zeta(r_1, r_2, \theta) = \sum_{\ell} \zeta_\ell(r_1, r_2)\mathcal{L}_\ell(\cos\theta)$$

ルジャンドル多項式

３点相関関数 Redshift-space

Redshift spaceでは(３＋２＝)５つのパラメーターで記述される。

$$\zeta(\vec{r}_1, \vec{r}_2, \vec{\gamma})$$



Configuration space

Slepian and Eisenstein (2017)

$$\zeta(r_1, r_2, \theta_1, \theta_2, \phi) = \sum_{\ell, \ell', m} \zeta_{\ell\ell'}^{m}(r_1, r_2) Y_{\ell}^{m}(\hat{r}_1) Y_{\ell'}^{m*}(\hat{r}_2)$$

$$\zeta(r_1, r_2, \theta_1, \theta_2, \phi) = \sum_{\ell, \ell', m} \zeta^m_{\ell\ell'}(r_1, r_2) \underline{Y^m_\ell(\hat{r}_1) Y^{m*}_{\ell'}(\hat{r}_2)}$$

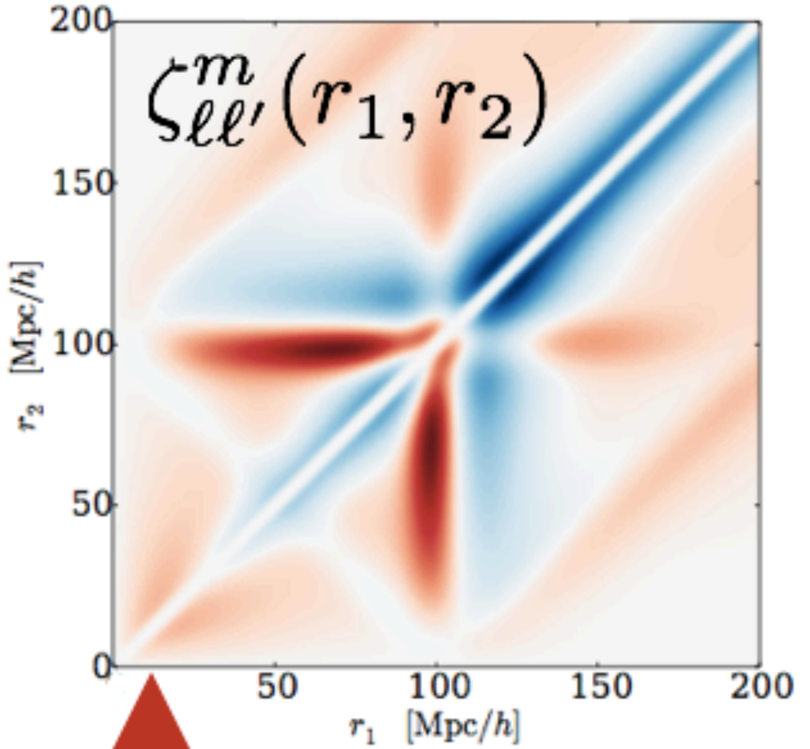
5つのパラメーター

$$\underline{N_{\ell,\ell'} P^{|m|}_\ell(\cos\theta_1) P^{|m|}_{\ell'}(\cos\theta_2) e^{im\phi}}$$

Slepian and Eisenstein (2017)

$$\ell = \ell' = m = 0$$

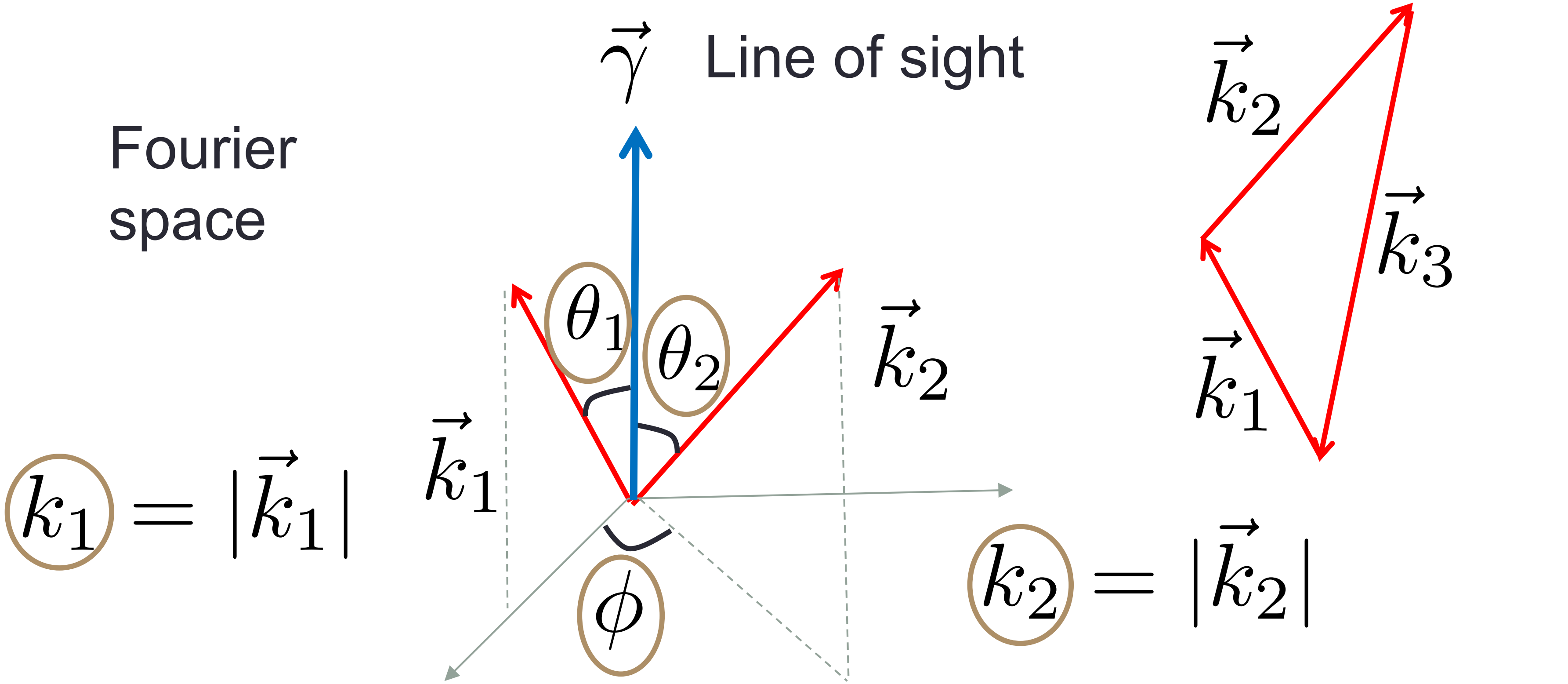
r_2



r_1

バイスペクトルの変数と展開

$$\langle \widetilde{\delta}(\vec{k}_1)\widetilde{\delta}(k_2)\widetilde{\delta}(\vec{k}_3)\rangle = (2\pi)^3 B(\vec{k}_1, \vec{k}_2, \vec{k}_3)\delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$



$$B(\vec{k}_1, \vec{k}_2, \gamma) = \sum_{\ell, \ell', m} B^m_{\ell\ell'}(k_1, k_2) Y^m_\ell(\hat{k}_1) Y^{m*}_{\ell'}(\hat{k}_2)$$

Sugiyama et al. (2018)

$$\zeta(r_1, r_2, \theta_1, \theta_2, \phi) = \sum_{\ell,\ell',m} \zeta^m_{\ell\ell'}(r_1, r_2) Y^m_\ell(\hat{r}_1) Y^{m*}_{\ell'}(\hat{r}_2)$$

$$\Updownarrow$$

$$B(\vec{k}_1, \vec{k}_2, \gamma) = \sum_{\ell,\ell',m} B^m_{\ell\ell'}(k_1, k_2) Y^m_\ell(\hat{k}_1) Y^{m*}_{\ell'}(\hat{k}_2)$$

$$\zeta^m_{\ell\ell'}(r_1, r_2, \gamma) = \frac{i^{\ell-\ell'}}{(2\pi^2)^2} \int_0^\infty dk_1 k_1^2 \int_0^\infty dk_2 k_2^2 j_\ell(k_1 r_1) j_{\ell'}(k_2 r_2) B^m_{\ell\ell'}(k_1, k_2)$$

cf. Sugiyama et al. (2018)

$$B_{\ell\ell L}(k_1, k_2) \propto \sum_m \begin{pmatrix} \ell & \ell' & L \\ m & -m & 0 \end{pmatrix} (-1)^m B^m_{\ell,\ell'}(k_1, k_2)$$

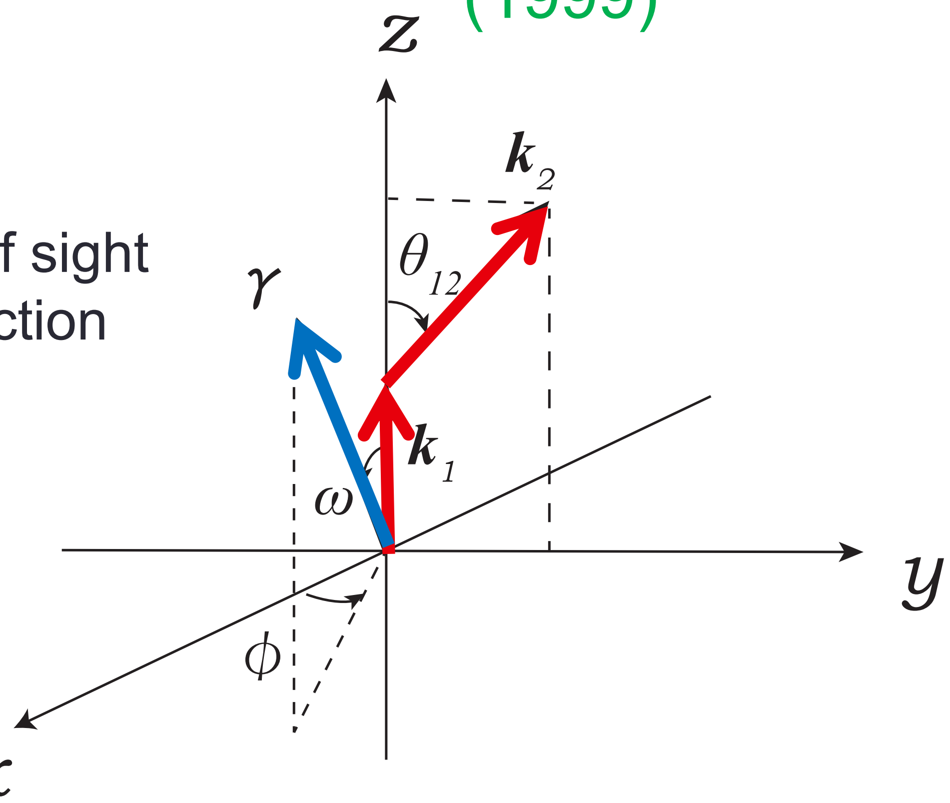
Sugiyama formulaと基本的に同等

Bispectrum in redshift space

Scoccimarro et al (1999)

$$B(\underbrace{k_1, k_2, \theta_{12},}\underbrace{\omega, \phi}_{\gamma})$$

line of sight direction

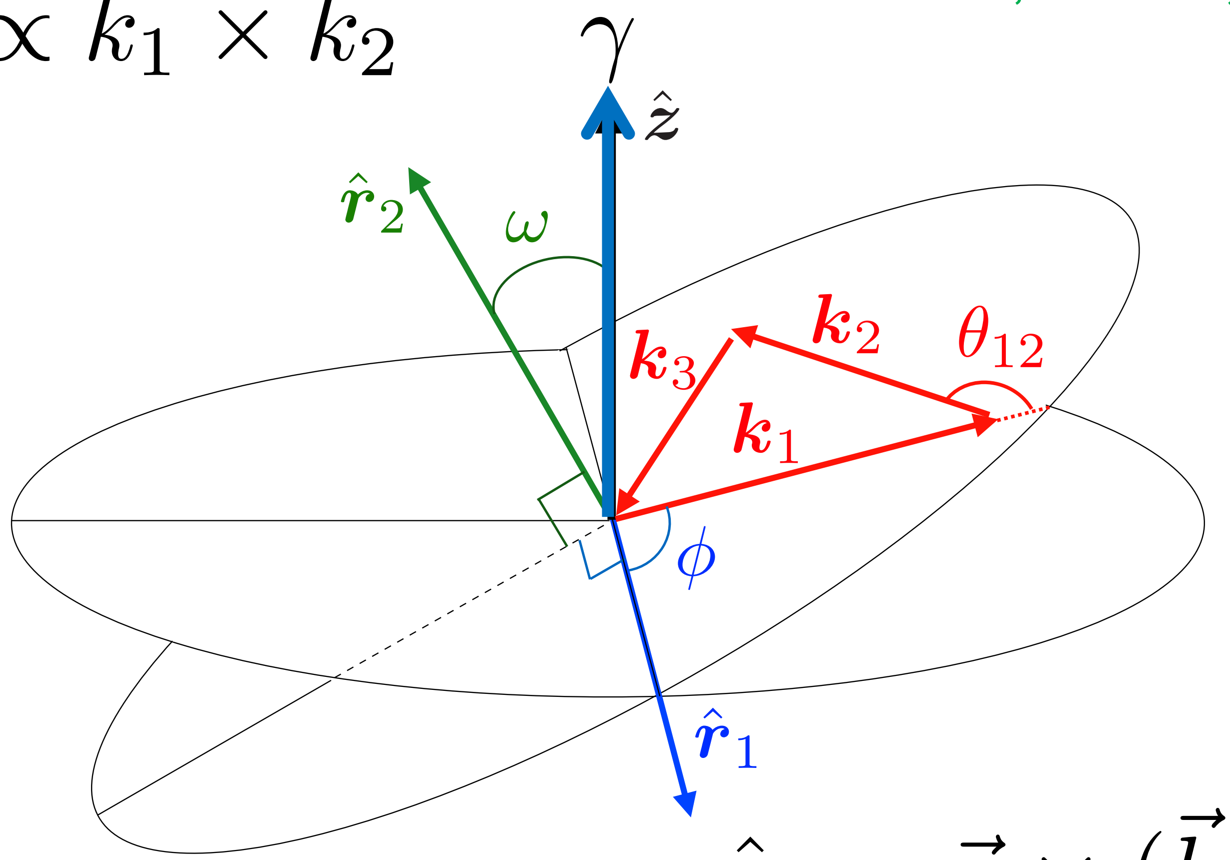


multipole expansion

$$B(k_1, k_2, \theta_{12}, \omega, \phi) = \sum_{\ell, m} B^{\ell, m}(k_1, k_2, \theta_{12}) Y_\ell^m(\omega, \phi)$$

Hashimoto, Rasera, Taruya (2017)

$$\hat{r}_2 \propto \vec{k}_1 \times \vec{k}_2$$

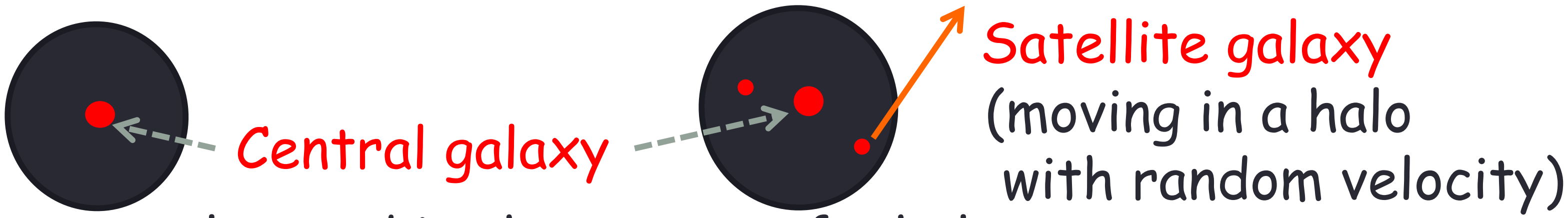


$$\hat{r}_1 \propto \vec{\gamma} \times (\vec{k}_1 \times \vec{k}_2)$$

$$B(k_1, k_2, \theta_{12}, \omega, \phi)$$

§３．ハローアプローチとバイスペクトル

Every DM and galaxy reside in dark matter halos, their distribution is described on the basis of the halo density profile with mass M and halo's correlation.

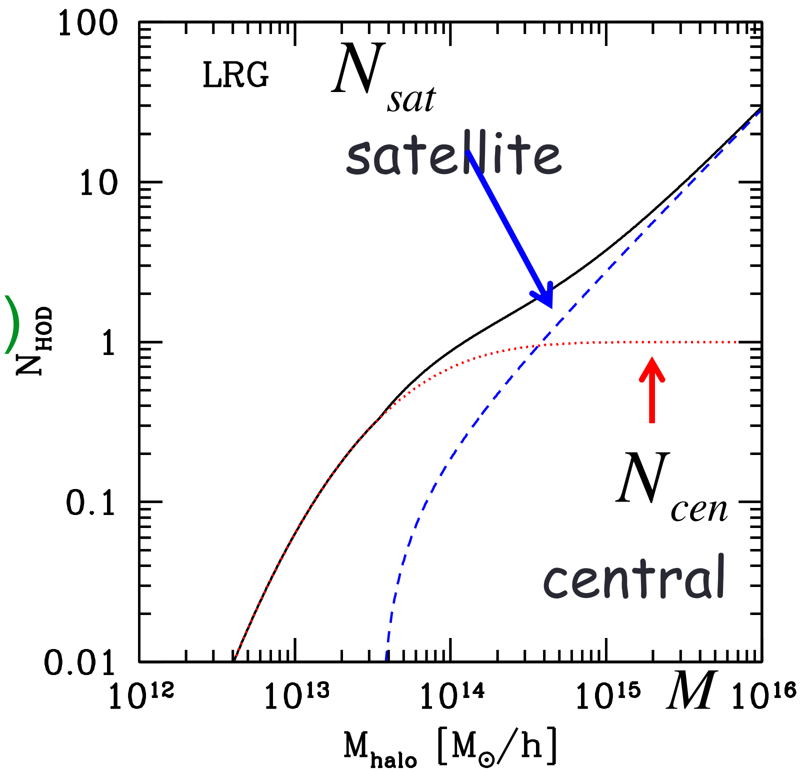


located in the center of a halo,
and the random velocity is negligible.

Halo occupation distribution (HOD)

galaxy number in a halo with mass M

(Hikage et al. 2012)



Satellite galaxy ramdom velocity

Virial random velocity dispersion

$$\sigma^2_{v,off} = \frac{GM}{2r_{\rm vir}}$$

Finger of God effect

Galaxy power spectrum with the halo model in redshift-space

1-halo term

$$P^{1h}(k,\mu) = \frac{1}{\bar{n}^2}\int dM \frac{dn(M)}{dM}\left(\overbrace{2N_{cen}N_{sat}\,p(k,\sigma_V,M)}^{\text{Central-Satellite}} + \overbrace{N_{sat}^2\,p(k,\sigma_V,M)^2}^{\text{Satellite-Satellite}}\right)$$

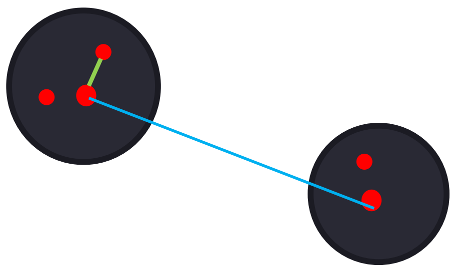
Finger of God effect

2-halo term

$$P^{2h}(k,\mu) \cong \frac{1}{\bar{n}^2}\left[\int dM \frac{dn(M)}{dM}\langle N_{cen}\rangle\left(1+\langle N_{sat}\rangle\, p(k,\sigma_V,M)\right)\left(b(M)+f\mu^2\right)\right]^2 P_m(k)$$

Linear distortion

satellite galaxies have random velocity → Finger of God



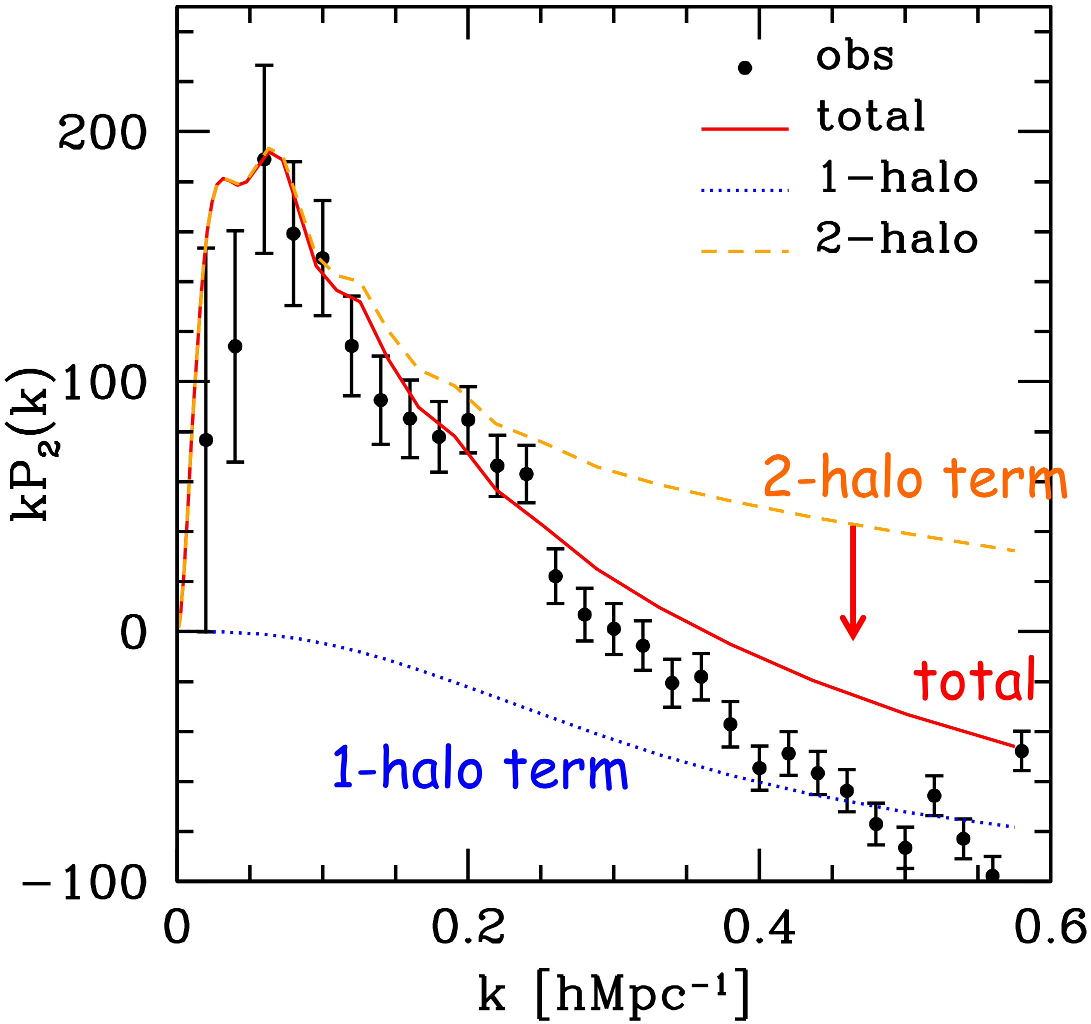
$$p(k,\sigma_V,M) \cong u(k,M)\exp\left[-\frac{\sigma_V^2(M)k^2\mu^2}{2a^2H^2}\right]$$

RSD

- ✓ Halo mass function, halo correlation
- ✓ Halo density profile = satellite galaxy distribution
- ✓ Halo occupation distribution (HOD)

$kP_2(k)$ Quadrupole spectrum

Hikage, Yamamoto (2013)



Significant contribution of 1-halo term to quadrupole spectrum at large k ($k>0.2hMpc^{-1}$)

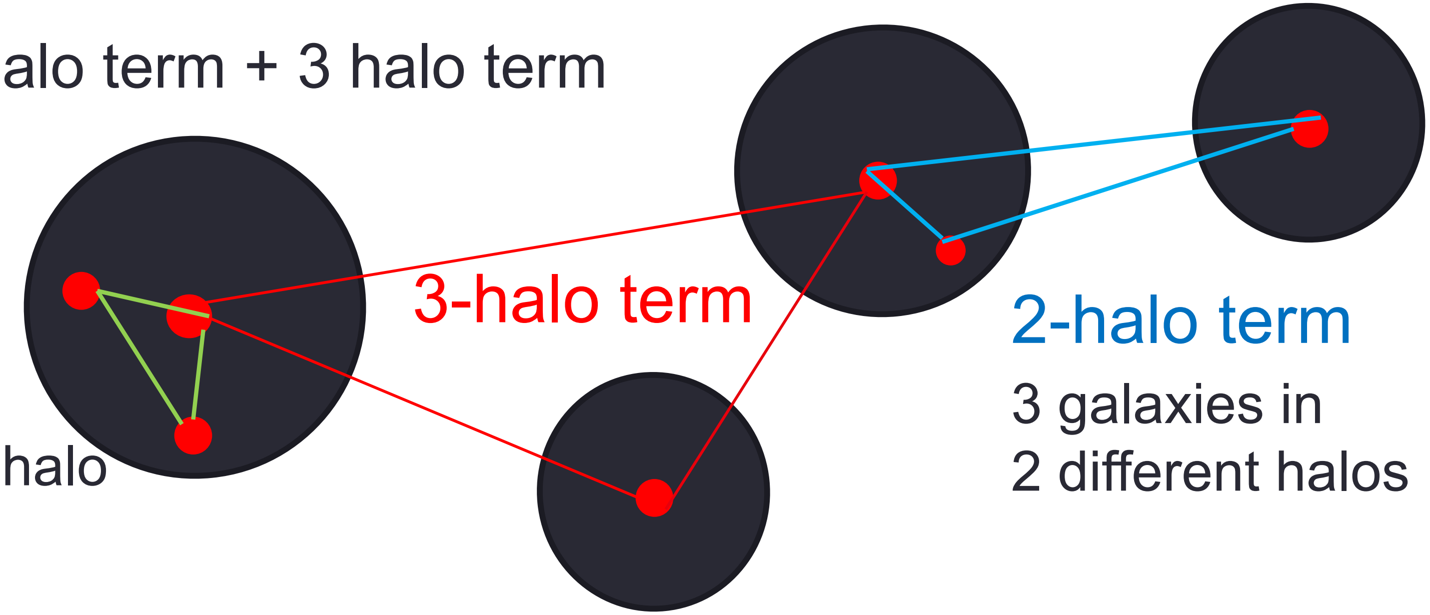
Possible contribution at quasi-linear regime ($k<0.2hMpc^{-1}$)

Quadrupole spectrum 非線形領域の振る舞いを説明する

Bispectrum of galaxies in halo approach

KY, Nan, Hikage (2017)

= 1 halo term + 2 halo term + 3 halo term



3 - halo term, dominant contribution on the large scales

2 - halo term, dominant contribution on the small scales

1 - halo term, Finger of God effect from satellite galaxies

→ redshift-space distortion on the small scale

Theoretical formula of the bispectrum in halo approach

$$B_g(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = B_{g,1h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) + B_{g,2h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) + B_{g,3h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$\begin{aligned} B_{g,1h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \; & \frac{1}{\bar{n}^3}\int dM \frac{dn(M)}{dM}\Big[\langle N_c\rangle \langle N_s(N_s-1)\rangle (\widetilde{u}(\mathbf{k}_1,M)\widetilde{u}(\mathbf{k}_2,M) + 2\text{ cyclic terms}) \\ & + \langle N_s(N_s-1)(N_s-2)\rangle \widetilde{u}(\mathbf{k}_1,M)\widetilde{u}(\mathbf{k}_2,M)\widetilde{u}(\mathbf{k}_3,M)\Big] \end{aligned}$$

$$\begin{aligned} B_{g,2h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = & \frac{1}{\bar{n}^3}\int dM_1 \frac{dn(M_1)}{dM_1}\Big[\langle N_c\rangle \langle N_s\rangle (\widetilde{u}(\mathbf{k}_1,M_1) + \widetilde{u}(\mathbf{k}_2,M_1)) + \langle N_s(N_s-1)\rangle \widetilde{u}(\mathbf{k}_1,M_1)\widetilde{u}(\mathbf{k}_2,M_1)\Big] \\ & \times \int dM_2 \frac{dn(M_2)}{dM_2} (\langle N_c\rangle + \langle N_c\rangle \langle N_s\rangle \widetilde{u}(\mathbf{k}_3,M_2)) P_{2h}(t,\mathbf{k}_3,M_1,M_2) + 2\text{ cyclic terms} \end{aligned}$$

$$B_{g,3h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{1}{\bar{n}^3}\int \prod_{i=1}^{3}\Big[dM_i \frac{dn(M_i)}{dM_i}\langle N_c\rangle (1 + \langle N_s\rangle \widetilde{u}(\mathbf{k}_i,M_i))\Big] P_{3h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,M_1,M_2,M_3)$$

$$P_{2h}(t,\mathbf{k}_3,M_1,M_2) = (b(M_1)+\mu_3^2 f)(b(M_2)+\mu_3^2 f)P_m(t,k_3)$$

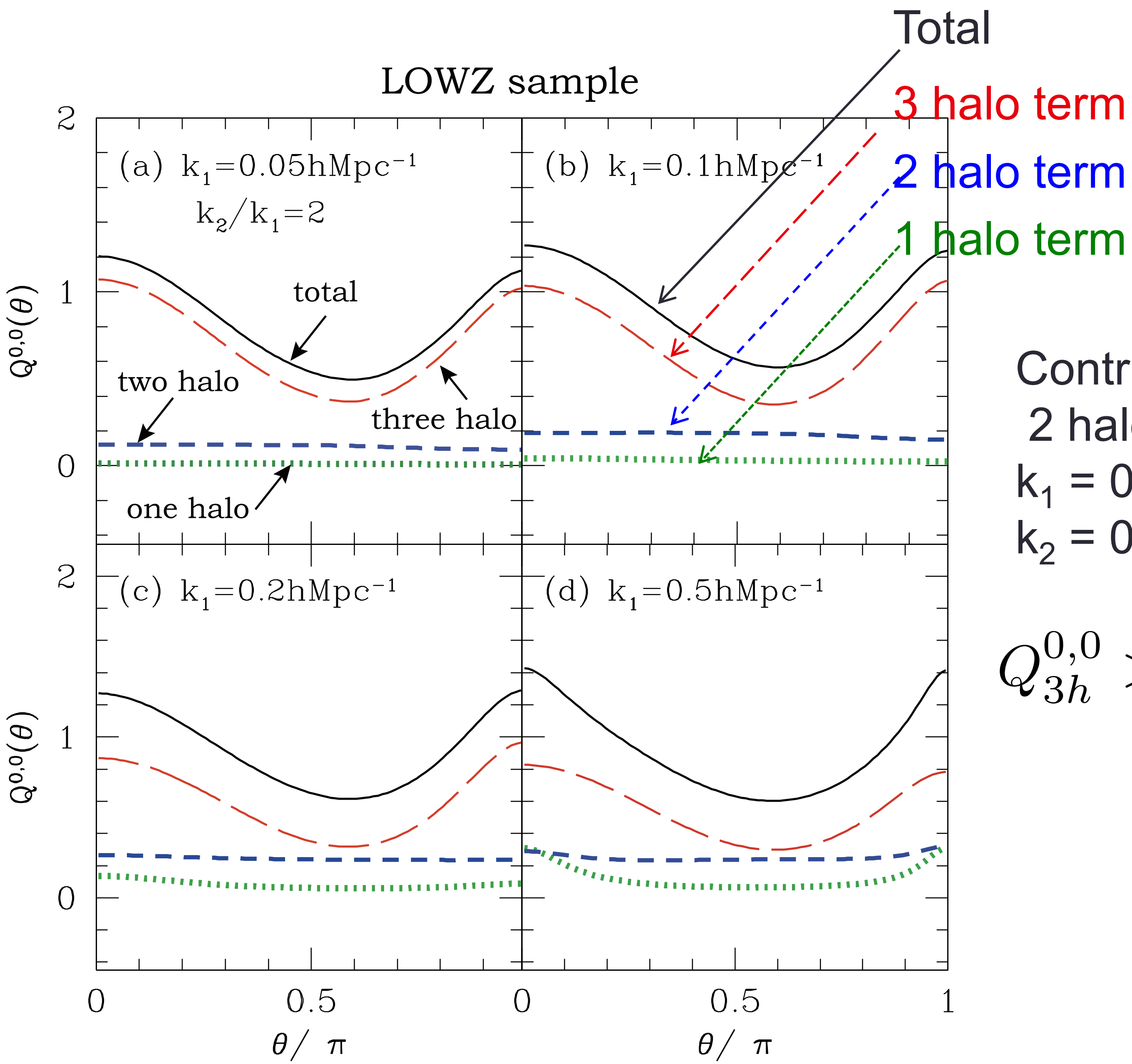
$$P_{3h}(t,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,M_1,M_2,M_3) = 2P_m(t,k_1)P_m(t,k_2)(b(M_1)+f\mu_1^2)(b(M_2)+f\mu_2^2)Z_2(\mathbf{k}_1,\mathbf{k}_2,M_3)$$

$$\begin{aligned} Z_2(\mathbf{k}_1,\mathbf{k}_2,M_3) = b(M_3)F_2(\mathbf{k}_1,\mathbf{k}_2) + \frac{b_2(M_3)}{2} + f\mu_{12}^2 G_2(\mathbf{k}_1,\mathbf{k}_2) \\ + \frac{1}{2} f\mu_{12}k_{12}\left\{\frac{\mu_1}{k_1}(b(M_3)+f\mu_2^2) + \frac{\mu_2}{k_2}(b(M_3)+f\mu_1^2)\right\} \end{aligned}$$

$$\mu_{12} = (\mathbf{k}_1+\mathbf{k}_2)\cdot\gamma/k_{12} \qquad k_{12} = |\mathbf{k}_1+\mathbf{k}_2| \qquad \mu_j = \mathbf{k}_j\cdot\gamma/k_j$$

Monopole (reduced) bispectrum

KY, Nan, Hikage (2017)

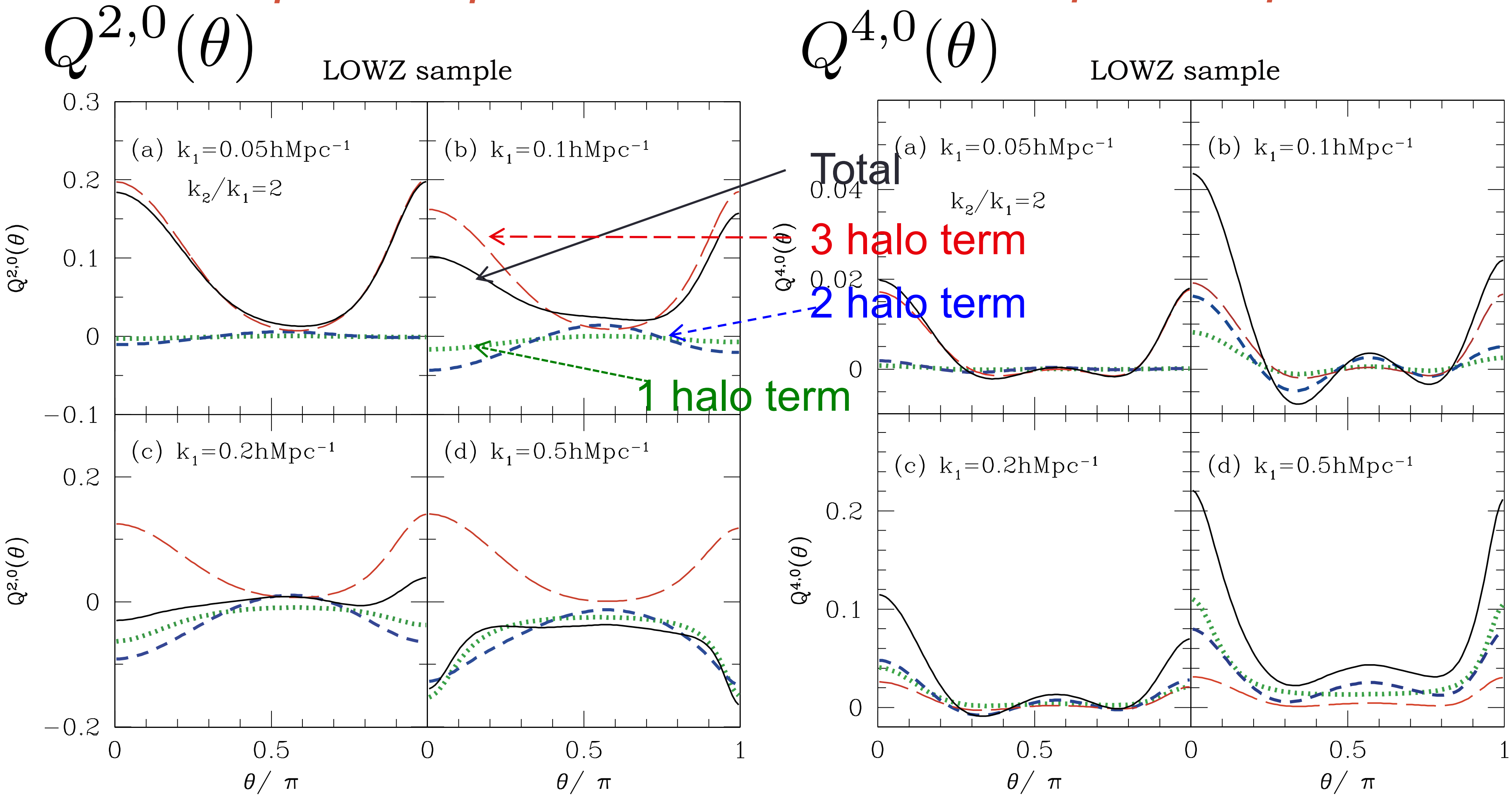


Contribution from
2 halo term at
k_1 = 0.1 hMpc^{-1}
k_2 = 0.2 hMpc^{-1}

$$Q^{0,0}_{3h} > Q^{0,0}_{2h} > Q^{0,0}_{1h}$$

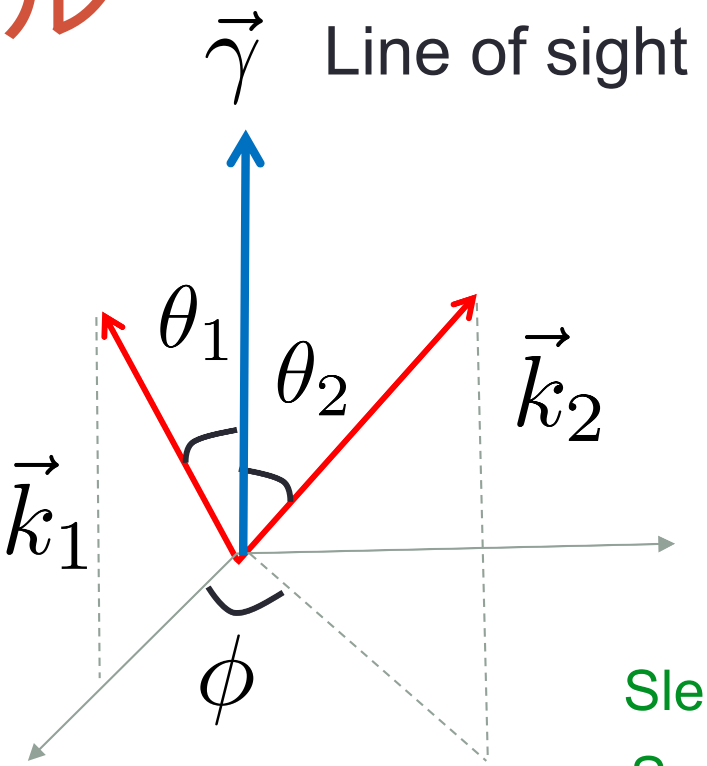
Quadrupole bispectrum

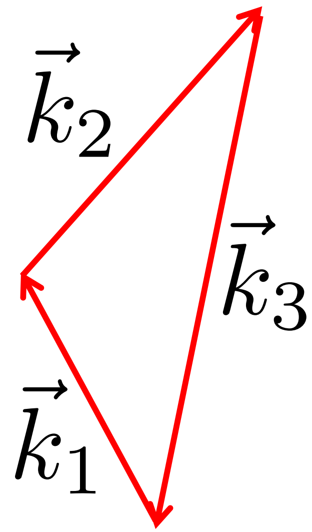
Hexadecapole bispectrum



significant contribution from the 2 halo term (and 1 halo term) on the scale k >~0.1 hMpc^{-1} to the higher multipoles bispectrum

バイスペクトル





Slepian & Eisenstein (2017)

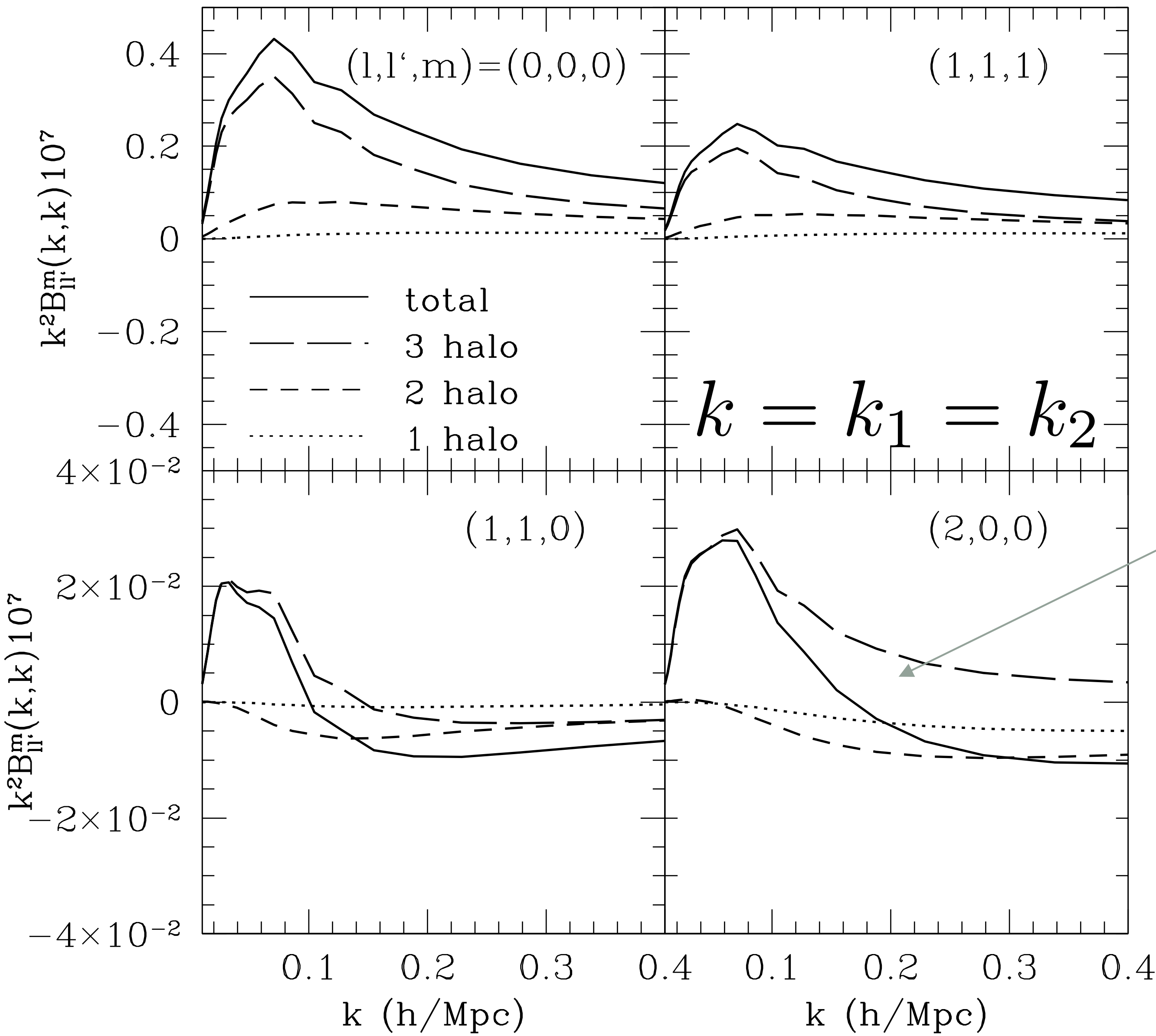
Sugiyama et al. (2018)

$$B(k_1, k_2, \theta_1, \theta_2, \phi) = \sum_{\ell, \ell', m} B_{\ell\ell'}^{m}(k_1, k_2, \gamma) Y_{\ell}^{m}(\hat{k}_1) Y_{\ell'}^{m*}(\hat{k}_2)$$

Slepian & Eisensteinの定義をバイスペクトルに応用した定式化を採用し、ハローモデルの理論予言

$k^2 B^m_{\ell,\ell'}(k,k)$ SDSS III LOWZ sample

HOD (Parejko et al 2013)



c.f. Sugiyama et al.

2 halo termの寄与が大きい

$P_2(k)$の振る舞いに似ている

Total

3 halo term

2 halo term

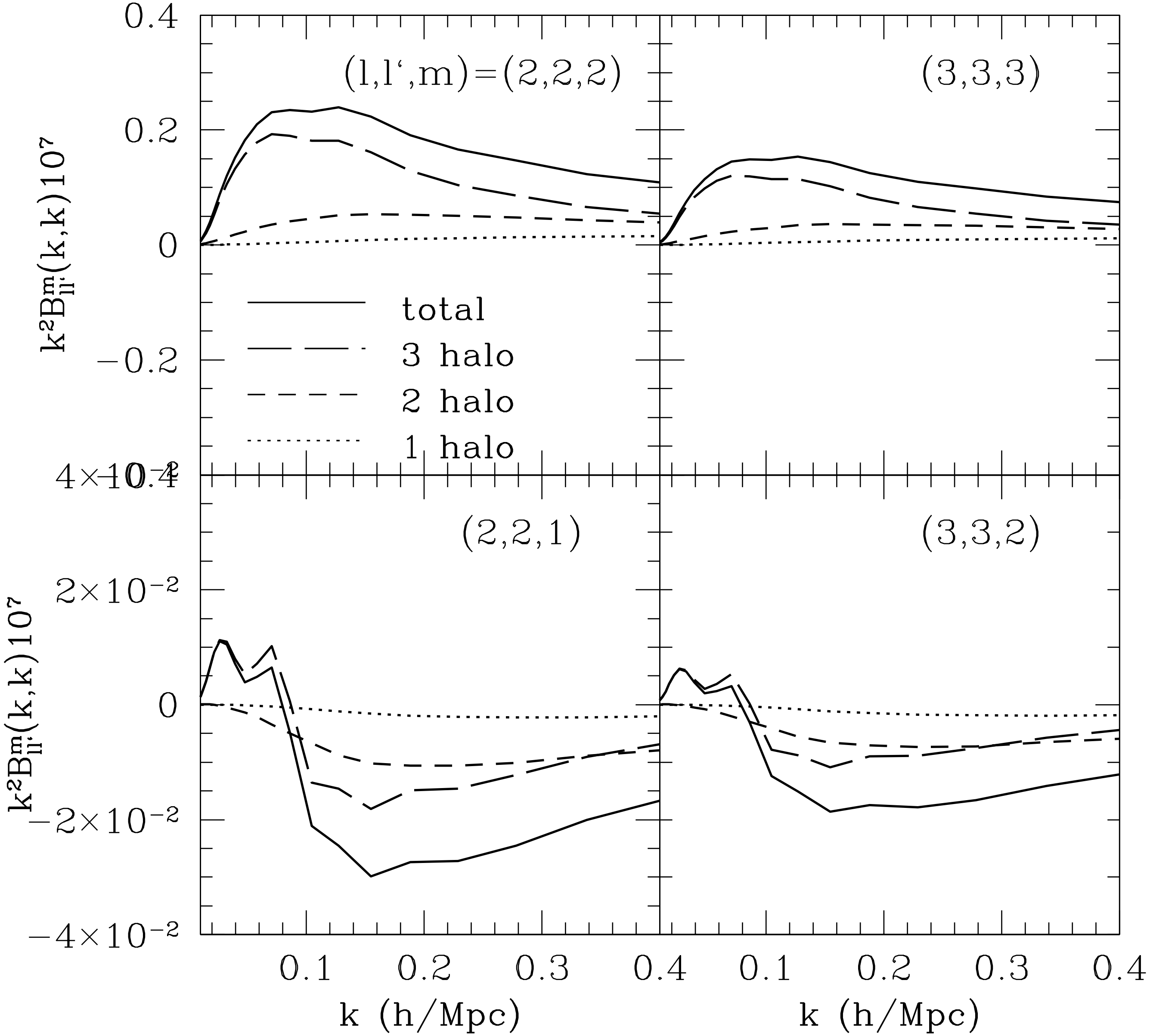
1 halo term

寄与の大きな成分 $B^m_{\ell,\ell'}(k_1, k_2)$

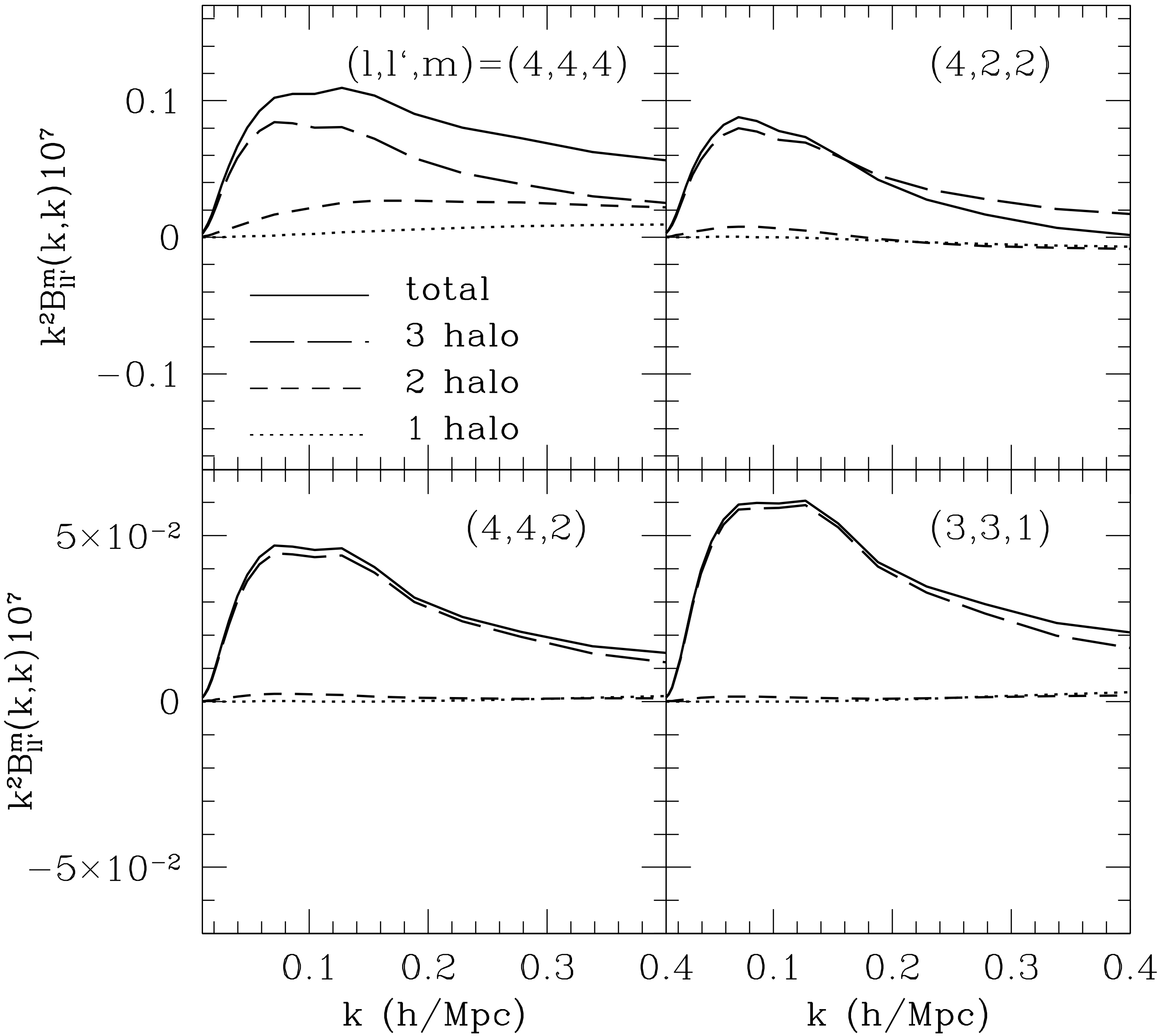
(ℓ, ℓ', m) $\quad \ell = 4$ までの成分

$(0,0,0)$	$(2,0,0)$	$(3,1,1)$	$(4,2,2)$
$(1,1,0)$	$(2,2,0)$	$(3,3,1)$	$(4,4,2)$
$(1,1,1)$	$(2,2,1)$	$(3,3,2)$	$(4,4,3)$
	$(2,2,2)$	$(3,3,3)$	$(4,4,4)$

SDSS III LOWZ sample

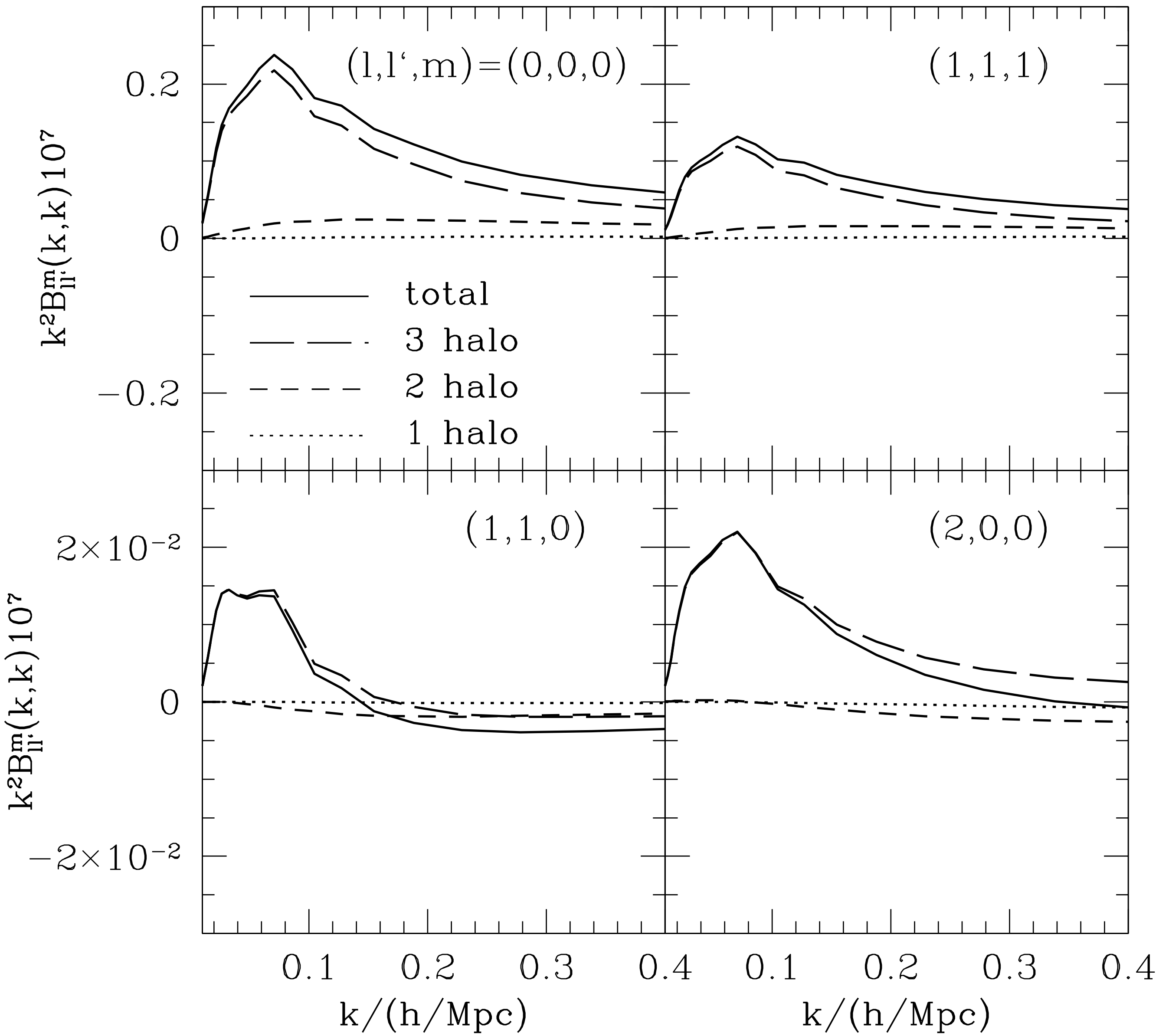


SDSS III LOWZ sample



SDSS III CMASS sample

HOD (Manera et al 2013)



HOD依存性

CMASSでは
2 halo term
の寄与が
LOWZサンプル
に比べて小さい

理論模型模型
の制限に際して
2 halo termの寄与
HOD依存性は、
系統誤差を生む
可能性がある。

4. 結論

✓バイスペクトル、３点相関関数の赤方偏移空間の変数の選び方

Slepian & Eisensteinの3PCFをバイスペクトルに応用した多重極展開

cf. Sugiyama et al (2018)

✓ハローモデルに基づいたバイスペクトル理論模型

２ハロー項の寄与が大きい場合がある HOD依存性

✓今後の課題

赤方偏移空間の多重極バイスペクトルの振る舞いの直感的理解

バイスペクトル測定への応用